

QUANTITATIVE APTITUDE

PAPER-3



Chartered Accountant

REVISION BOOK

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CA FOUNDATION

Terms of the Ratio

The quantities a and b are called the terms of the ratio.

- a is called the first term or antecedent.
- b is called the second term or consequent.

The inverse ratio of $a : b$ is:

$$\frac{1}{a} : \frac{1}{b} = b : a$$

Similarly, the inverse ratio of $a : b : c$ is:

$$\frac{1}{a} : \frac{1}{b} : \frac{1}{c} = bc : ac : ab$$

3. Sub-duplicate Ratio

A sub-duplicate ratio is obtained by taking the square root of the terms of the original ratio. If the original ratio is $a : b$, the sub-duplicate ratio is $\sqrt{a} : \sqrt{b}$.

4. Sub-triplicate Ratio

A sub-triplicate ratio is obtained by taking the cube root of the terms of the original ratio. If the original ratio is $a : b$, the sub-triplicate ratio is $\sqrt[3]{a} : \sqrt[3]{b}$.

5. Compound Ratio

A compound ratio is formed by multiplying the corresponding terms of two or more ratios. For example, the compound ratio of $a : b$ and $c : d$ is $(a \times c) : (b \times d)$.

Continued Ratio

1. Duplicate Ratio

A duplicate ratio is obtained by squaring the terms of the original ratio. If the original ratio is $a : b$, the duplicate ratio is $a^2 : b^2$.

2. Triplicate Ratio

A triplicate ratio is obtained by cubing the terms of the original ratio. If the original ratio is $a : b$, the triplicate ratio is $a^3 : b^3$.

Proportion

A proportion is defined as the equality of two ratios.

- If $a : b = c : d$, it is usually expressed as:

$$\frac{a}{b} = \frac{c}{d} \quad \text{or} \quad ad = bc \quad \text{or} \quad a : b :: c : d$$

- In this proportion, d is called the **fourth proportional** (or the highest proportional).

1. Continued Proportion: - If $a : b :: b : c$, then: $\frac{a}{b} = \frac{b}{c} \quad \text{or} \quad b^2 = ac \quad \text{or} \quad b = \sqrt{ac}$ 2. Product of Extremes and Means: - For a proportion $a : b :: c : d$, Product of extremes = Product of means	2. Cross Product Rule: - For three quantities a, b, and c in continued proportion: $a : b = b : c \quad \text{or} \quad \frac{a}{b} = \frac{b}{c} \quad \text{or} \quad b^2 = ac$ - Here, b is the mean proportional, a is the first proportional, and c is the third proportional.
Properties of Proportion 1. Basic Properties 1. Basic Proportion Property - If $a : b = c : d$, then $ad = bc$. 2. Invertendo - If $a : b = c : d$, then $b : a = d : c$. 3. Alternendo - If $a : b = c : d$, then $a : c = b : d$. 4. Componendo - If $a : b = c : d$, then $a + b : b = c + d : d$.	5. Dividendo - If $a : b = c : d$, then $a - b : b = c - d : d.$ 6. Componendo and Dividendo - If $a : b = c : d$, then $a + b : a - b = c + d : c - d.$ 7. Addendo - If $a : b = c : d = e : f = \dots$, then each of these ratios is equal to $\frac{a + c + e + \dots}{b + d + f + \dots}$ 8. Subtrahendo - If $a : b = c : d = e : f = \dots$, then each of these ratios is equal to $\frac{a - c - e - \dots}{b - d - f - \dots}$

Indices (Exponents)

Definition:

- An index (plural: indices) or exponent is the power to which a number (the base) is raised.
- If a number a is multiplied by itself n times, it is written as a^n , where a is the base and n is the index or exponent.

Laws of Indices:

1. Product of Powers (Same Base):

$$a^m \times a^n = a^{m+n}$$

- The bases must be the same.

2. Quotient of Powers (Same Base):

$$\frac{a^m}{a^n} = a^{m-n}$$

3. Power of a Power:

$$(a^m)^n = a^{m \cdot n}$$

4. Power of a Product:

$$(ab)^n = a^n \cdot b^n$$

5. Power of a Quotient:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

6. Zero Exponent:

$$a^0 = 1 \quad (\text{for } a \neq 0)$$

7. Negative Exponent:

$$a^{-m} = \frac{1}{a^m}$$

8. Fractional Exponent:

$$\sqrt[m]{a} = a^{1/m}$$

9. Equality of Exponents:

- If $a^x = a^y$, then $x = y$.

Algebraic Identities

Key Identities:

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a + b)(a - b)$
4. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$5. (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$6. a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$7. a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$8. (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Special Note:

- If $a^{1/3} + b^{1/3} + c^{1/3} = 0$, then $(a + b + c)^3 = 27abc$.

Types of Logarithms

Logarithms can be categorized into two main types:

1. Natural Logarithm

- A logarithm with base "e" (Euler's number) is called a **natural logarithm**.
- It is represented as $\text{Log}_e x$ or $\ln(x)$.
- The constant $e \approx 2.7183$.
- Used frequently in calculus, exponential growth, and scientific applications.

2. Common Logarithm

- A logarithm with base 10 is called a **common logarithm**.
- It is represented as $\text{Log}_{10}x$ or simply $\text{Log}(x)$ when no base is mentioned.
- If the base is not explicitly given, it is assumed to be 10 in most practical and commercial calculations.

Basic Concept:

- If there are three quantities indicated by a , x , and n , they are related as follows:

$$a^x = n$$

- Where $n > 0$, $a > 0$, and $a \neq 1$.
- Then x is said to be the logarithm of the number n to the base a .
- Symbolically, it can be expressed as:

$$x = \log_a n$$

Characteristic & Mantissa:

- The integral part of a common logarithm is called the **characteristic**.
- The non-negative decimal part is called the **mantissa**.

Antilogarithm:

- If x is said to be the logarithm of N to a given base, then N is said to be the antilogarithm of x to that base.

$$\text{If } \log_a N = x \Rightarrow N = \text{antilog } x$$

Additional Notes:

- $\log_a n = x \Rightarrow a^x = n$

Laws of Logarithms 1. Product Rule: $\log_a(xy) = \log_a x + \log_a y$ Where x and y are positive numbers. 2. Quotient Rule: $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ 3. Power Rule: $\log_a(x^k) = k \log_a x$ 4. Change of Base Rule: $\log_b m = \frac{\log_a m}{\log_a b}$	5. Logarithm of 1: $\log_a 1 = 0$ 6. Logarithm of a Number to the Same Base: - The logarithm of a number to the same base is equal to 1. $\log_a a = 1$ 7. Inverse Property: $a^{\log_a x} = x$ 8. Equality Property: - If $\log x = \log y$, then $x = y$. 9. Number of Digits in a Given Numeral: Number of digits in a given numeral = $\log(\text{numeral})$
Simple Equation A simple equation in one unknown x is in the form $ax + b = 0$. Where a, b are known constants and $a \neq 0$. Note: A simple equation has only one root.	Simultaneous Linear Equations in Two Unknowns The general form of a linear equation in two unknowns x and y is $ax + by + c = 0$, where a, b are non-zero coefficients and c is a constant. Two such equations, $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, form a pair of simultaneous equations in x and y. A value for each unknown which satisfies both equations simultaneously will give the roots of the equations.

Quadratic Equation

An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called a quadratic equation or equation of the second degree.

- **Pure Quadratic Equation:** When $b = 0$, the equation is called a pure quadratic equation.
- **Affected Quadratic Equation:** When $b \neq 0$, the equation is called an affected quadratic.

General Solution:

The roots of the quadratic equation $ax^2 + bx + c = 0$ can be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and Product of the Roots:

Let one root be α and the other root be β .

- **Sum of the Roots:**

$$\alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$$

the sum of the roots is:

$$\alpha + \beta = -\frac{b}{a}$$

It can be interpreted as:

$$\text{Sum of roots} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

For the equation $ax^2 + bx + c = 0$, we have:

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

or

$$x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$$

or

$$x^2 - (\text{Sum of the roots})x + (\text{Product of the roots}) = 0$$

Nature of the Roots for the Equation $ax^2 + bx + c = 0$

1. If $b^2 - 4ac = 0$:

- The roots are real and equal.

2. If $b^2 - 4ac > 0$ and a perfect square:

- The roots are real, distinct, and rational.

3. If $b^2 - 4ac > 0$ and not a perfect square:

- The roots are real, distinct, and irrational.

4. If $b^2 - 4ac < 0$:

- The roots are imaginary.

1. If one root of the quadratic equation is $P + \sqrt{q}$: - Then the other root will be $P - \sqrt{q}$. 2. If one root is the reciprocal of the other: - Then $\alpha\beta = 1$ (since $\alpha \cdot \frac{1}{\alpha} = 1$).	If two roots are in the ratio $\alpha : \beta$: $\alpha\beta b^2 = (\alpha + \beta)^2 ac$
Cubic Equation Definition: A polynomial in x having degree 3 General form: $ax^3 + bx^2 + cx + d = 0$	Sum of the roots: Sum of the roots $= -\frac{b}{a}$ Product of the roots: Product of the roots $= -\frac{d}{a}$
Definition of an Inequality An inequality is a statement that compares two real numbers or two algebraic expressions using the symbols $<$, $>$, $\leq$, $\geq$. For example: $3 < 5$ or $7 > 5$ (numerical inequalities). $x < 5, y \geq 2, x \geq 3, y \leq 4$ (inequalities involving variables, sometimes called literal inequalities).	Inequalities in Two Variables An inequality in two variables x and y often appears in the form: $ax + by < c, \quad ax + by > c, \quad ax + by \leq c, \quad ax + by \geq c,$ where a, b, and c are real constants. The solution set (also called the solution region or solution space) is the set of all points (x, y) in the plane that satisfy the inequality.

Steps to Graph a Given Inequality

1. Plot the Boundary Line:

- Convert the given inequality into an equation by replacing the inequality sign ($<$, $>$, $\leq$, $\geq$) with $=$.
- The resulting equation represents a straight line.
- If the inequality is **strict** ($<$ or $>$), use a **dashed** boundary line (since points on the line are not included).
- If the inequality is **non-strict** ($\leq$ or $\geq$), use a **solid** boundary line (since points on the line are included).

2. Choose a Test Point:

- Pick a test point (often the origin $(0, 0)$ if it is not on the line).
- Substitute this point into the original inequality.
- If the inequality holds true, shade the **half-plane** containing the test point.
- If the inequality does not hold true, shade the **other half-plane**.

Common Region for Given Inequalities

Understanding Feasibility of a Solution Region

- If a **common region** exists where all the given inequalities overlap, the solution is said to be **feasible**.
- If no common region exists, the inequalities are considered **infeasible** (i.e., no solution satisfies all inequalities simultaneously).

Using Arrows to Indicate the Solution Region

- **Arrows** are used to indicate which half-plane satisfies the inequality.
- If the **coefficient of y is positive**:
 - For **less than** ($<$ or $\leq$), arrows should be **below the line**.
 - For **greater than** ($>$ or $\geq$), arrows should be **above the line**.
- If the **coefficient of y is negative**:
 - For **less than** ($<$ or $\leq$), arrows should be **above the line**.
 - For **greater than** ($>$ or $\geq$), arrows should be **below the line**.

Solving Problems on Linear Inequalities

Key Rule for Multiplication with a Negative Number

- When an inequality is **multiplied or divided by a negative number**, the inequality sign **reverses**.
- That is:
 - $<$ changes to $>$
 - $>$ changes to $<$
 - $\leq$ changes to $\geq$
 - $\geq$ changes to $\leq$

Bounded and Unbounded Solution Regions - If the common region is enclosed within boundaries, it is called feasible and bounded. - If the common region extends infinitely in any direction, it is called feasible and unbounded.	Key Guidelines for Setting Up Inequalities - For time-related constraints, use "$\leq$" (less than or equal to). - For work-related constraints, use "$\geq$" (greater than or equal to).
Basic Understanding of Accumulated Amount or Balance - Formula: $A = \text{Principal} + \text{Interest}$ - Interest Types: Simple Interest: Interest is calculated on the principal amount only. Compound Interest: Interest is calculated on the principal amount and any previously earned interest. Simple Interest Definition Simple interest is the additional money paid for using somebody else's money (principal) over a period of time at a simple rate.	Simple Interest (SI) = $\frac{P \times R \times T}{100}$ $\text{Amount}(A) = P + SI$ $\text{Amount}(A) = P + \frac{P \times R \times T}{100}$
When two amounts (in terms of Rs) and their respective time durations are given, the objective is to determine the principal (P) and the simple interest (SI) rate per annum. Note: $\text{SI rate per annum} = \frac{\text{SI for 1 year}}{P} \times 100$	When the principal is the same $\text{Total SI} = \frac{P}{100}(T_1R_1 + T_2R_2 + T_3R_3 + \dots)$ When the principal amounts are different $\text{Total SI} = \frac{P_1T_1R_1 + P_2T_2R_2 + \dots}{100}$

To find the time (T) when an amount (in terms of principal, P) and the simple interest (SI) rate per annum are given, use the following formula: $T = \frac{\text{Total SI rate}}{\text{SI rate per annum}}$	Formula for Amount (A) in Compound Interest $A = P(1 + i)^n$ Formula for Compound Interest (CI) $CI = P [(1 + i)^n - 1]$
Types of Compoundings & Corresponding Conversions (i) Annual (Yearly) Compounding • n = Number of years • i = Interest rate per annum (ii) Semi-Annual (Half-Yearly) Compounding • $n = 2 \times$ Number of years • $i = \frac{\text{Interest rate per annum}}{2}$	(iii) Quarterly Compounding • $n = 4 \times$ Number of years • $i = \frac{\text{Interest rate per annum}}{4}$ (iv) Monthly Compounding • $n = 12 \times$ Number of years • $i = \frac{\text{Interest rate per annum}}{12}$
Type 2: Finding A When P and Different Interest Rates for Different Years Are Given $A = P(1 + i_1)(1 + i_2)(1 + i_3) \dots (1 + i_n)$	Difference Between CI and SI $\text{Difference} = P [(1 + i)^n - 1 - TR\%]$

Effective Rate of Interest (ERI) Relationship Between NRI and ERI - Annual Compounding: $ERI = NRI$ (No compounding effect) - More Frequent Compounding: $ERI > NRI$ - Semi-annual, quarterly, and monthly compounding increase the ERI.	Formula for ERI $ERI = (1 + i)^n - 1$ Formula for Percentage ERI $\%ERI = [(1 + i)^n - 1] \times 100$
Formula for Depreciation $A = P \times (1 - i)^n$ Where: A = Future value or scrap value P = Present value or cost price	Types of Annuity 1. Annuity Regular (Ordinary Annuity): - Payments are made at the end of each period (e.g., end of year, half-year, quarter, or month). - Examples: Rent, EMI, etc. 2. Annuity Immediate (Annuity Due): - Payments are made at the beginning of each period. - Examples: Recurring Deposits (RD), insurance premiums, etc.

For Annuity Regular: $\text{Future Value (FV) of AR} = \frac{a((1+i)^n - 1)}{i}$ For Annuity Due (AD): $\text{Future Value (FV) of AD} = \frac{a((1+i)^n - 1)}{i} \times (1+i)$	Relationship Between Future Value and Present Value Future Value (FV) and Present Value (PV) are related by the following formulas: $FV = PV \times (1+i)^n$ $PV = \frac{FV}{(1+i)^n}$
Applications of Annuity 1. Leasing Definition: Leasing is essentially renting. Decision Making: When deciding between purchasing a machine or leasing it, compare the present values of both options. - If the PV of leasing < PV of purchasing, leasing is preferred. - If the PV of leasing > PV of purchasing, purchasing is preferred. - If the PV of leasing = PV of purchasing, both options are equally good.	2. Investment Decision - Rule: If the PV of cash inflows (returns) is greater than or equal to the PV of cash outflows (investment), the investment is considered good; otherwise, it's a loss. Net Present Value (NPV) Formula: $NPV = PV \text{ of Cash Inflows} - PV \text{ of Cash Outflows}$ - An investment is considered good if $NPV \geq 0$.
Bond Valuation Formula Present Value (PV) of the bond: $PV = a \left(\frac{(1+i)^n - 1}{i(1+i)^n} \right) + \frac{\text{Par value}}{(1+i)^n}$ Where: a = Coupon Payment = Par Value $\times$ Nominal Rate	Types of Perpetuity 1. Normal Perpetuity (Multi-Period Perpetuity): Formula: $PV \text{ of Multiperiod Perpetuity} = \frac{R}{i}$

2. Growing Perpetuity: - A stream of cash flows that grows at a constant rate forever. - Formula: $PV \text{ of Growing Perpetuity} = \frac{R}{i - g}$ - Where: g = Growth rate R = Fixed payment (Receipt amount) i = Interest rate	$\%CAGR = \left[\left(\frac{V_n}{V_0} \right)^{\frac{1}{t_n - t_0}} - 1 \right] \times 100$ Where: t_n = End period t_0 = Beginning period V_n = Value at the end period (t_n) V_0 = Value at the beginning period (t_0) $(1 + \text{option})^{t_n - t_0} = \frac{V_n}{V_0}$
1. Permutation: Arrangement of items. 2. Combination: Selection of items.	Factorial of n (n!): Formula: $n! = n \times (n - 1) \times (n - 2) \cdots \times 1$
Permutation Formula: Arrangement of n items, taken r at a time: ${}^n P_r = \frac{n!}{(n - r)!}$	

Properties of Permutations:

1. Full arrangement (all items):

$${}^n P_n = n!$$

2. Zero selection:

$${}^n P_0 = 1$$

3. Recursive relation:

$${}^n P_r = n \times {}^{n-1} P_{r-1}$$

4. Special cases:

- ${}^n P_n = n!$
- ${}^n P_{n-1} = 2! \cdot {}^{n-2} P_{n-2}$
- ${}^n P_{n-2} = 3! \cdot {}^{n-3} P_{n-3}$
- ${}^n P_{n-3} = 4! \cdot {}^{n-4} P_{n-4}$

5. Last term of the expansion:

$${}^n P_r = (n - r + 1)$$

Formula for Permutation with Repeated Letters

If a word contains repeated letters, the total number of unique arrangements is calculated using the formula:

$$\frac{n!}{n_1! \times n_2! \times n_3! \times \dots}$$

Special Case: No Repeated Letters

- If no letter is repeated, the number of unique arrangements simplifies to:

$$n!$$

Permutation with Conditions

- The condition must be satisfied first before applying permutation rules.
- The word "or" in a problem translates to **addition (+)** in calculations.
- The word "and" in a problem translates to **multiplication (×)** in calculations.

1. When two specific items must always stay together:

- Treat the two items as a single unit (block).
- The total number of arrangements is:

$$(n - 1)! \times 2!$$

- $(n - 1)!$: Arrange the block and the remaining items.
- $2!$: Arrange the two items within the block.
- The number of arrangements where two elements are **never together** is found using:

$$(n - 1)! \times (n - 2)$$

• Shortcut for boys and girls occupying alternate positions (unequal in number): Total arrangement = (smaller number)! $\times$ (larger number)! • Shortcut for boys and girls occupying alternate positions (equal in number): Total arrangement = $2 \times \text{boys!} \times \text{girls!}$ • Shortcut for no two boys being together: Total arrangement = $\text{girls!} \times {}^{(\text{girls}+1)}P_{\text{boys}}$	• Shortcut for no two boys being together: Total arrangement = $\text{girls!} \times {}^{(\text{girls}+1)}P_{\text{boys}}$ • Shortcut for no two girls being together: Total arrangement = $\text{boys!} \times {}^{(\text{boys}+1)}P_{\text{girls}}$
Total number of ways to answer 'n' questions: • Each question has two choices: answer it or leave it unanswered. • Formula:	Ways to answer at least one question out of 'n' questions: • This is the total ways to answer all questions minus the case where no question is answered. • Formula:
Circular Permutation (Without Condition) $(n - 1)!$ Key Idea: • Linear Arrangement counts all possible orders. • Circular Arrangement avoids overcounting identical rotations by fixing one item as the reference point.	1. Shortcut for no two girls being together in a circular manner Total arrangement = $(\text{boys} - 1)! \times {}^{\text{boys}}P_{\text{girls}}$ 2. Shortcut for no two boys being together Total arrangement = $(\text{girls} - 1)! \times {}^{\text{girls}}P_{\text{boys}}$

1. Number of Ways of Arranging 'r' Things Taken from 'n' Things, Such That One Particular Thing Is Always Included Formula: $r \cdot (n-1)P_{(r-1)}$ • r: The position occupied by the specific item. • $(n-1)P_{(r-1)}$: Permutations of $r - 1$ items from $n - 1$ items.	2. Number of Ways of Arranging 'r' Things Taken from 'n' Things, Such That One Particular Thing Is Always Excluded Formula: $(n-1)P_r$
Combination (Selection of 'r' Things from 'n' Things) A combination is used when the order of selection does not matter. It gives the number of ways to choose r items from n items. Formula: ${}^nC_r = \frac{{}^nP_r}{r!}$ Where: • ${}^nP_r = \frac{n!}{(n-r)!}$: The number of permutations of r items from n items. • $r!$: Accounts for the fact that the order of selection doesn't matter.	Basic Properties of Combination: 1. ${}^nC_0 = 1$: Selecting 0 items from n items is always 1. 2. ${}^nC_1 = n$: Selecting 1 item from n items can be done in n ways. 3. ${}^nC_n = 1$: Selecting all n items from n is always 1. 4. ${}^nC_r = {}^nC_{n-r}$: Symmetry property—selecting r items is the same as excluding r items. 5. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$: Pascal's identity.

Applications in Geometry and Other Scenarios: 1. Number of straight lines formed using n points, where m are collinear: ${}^nC_2 - {}^mC_2 + 1$ 2. Number of triangles formed using n points, where m are collinear: ${}^nC_3 - {}^mC_3$ 3. Number of matches played by n teams (one match per pair): ${}^nC_2$ If each team plays two matches against every other team: $2 \cdot {}^nC_2$	4. Number of handshakes among n people: ${}^nC_2$ 5. Number of quadrilaterals (or parallelograms) formed by n parallel lines intersecting m parallel lines: ${}^nC_2 \times {}^mC_2$ 6. Number of diagonals in a polygon with n sides: ${}^nC_2 - n$ 7. Number of chords in a circle formed by n points on its circumference: ${}^nC_2$
Arithmetic Progression (AP): Formula for the n-th Term: $a_n = a_1 + (n - 1)d$	Sum of n Terms of an Arithmetic Progression (AP): $S_n = \frac{n}{2}(a + a_n)$
1. Relation Between Sum and Middle Term $S_n = \text{Middle term} \times n$	2. Finding the Middle Term $\text{Middle term} = \frac{\text{Sum of } n \text{ terms of an AP}}{n}$

General Form of GP: $a, ar, ar^2, ar^3, \dots, ar^{n-1}$ • Where: • a: First term • r: Common ratio • n: Number of terms	• Common Ratio: $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \dots$ Formula for the n-th Term: $a_n = a \cdot r^{n-1}$
Sum of n Terms in a Geometric Progression (GP): 1. For $r > 1$: $S_n = a \cdot \frac{r^n - 1}{r - 1}$ 2. For $r < 1$: $S_n = a \cdot \frac{1 - r^n}{1 - r}$	The product of the first n terms of a geometric progression is given by: Product of n terms of GP = $(G_m)^n$ where G_m is the middle term of the GP.
Sum of Infinite Terms in Geometric Progression (GP) 1. Formula: $S_\infty = \frac{a}{1-r}, \text{ where } r < 1.$	Sum of Infinite Terms with Combination of A.P. and G.P. 1. Formula: $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Notes on Combination of A.P., G.P., and H.P.

1. Arithmetic Progression (A.P.):

For three terms a, b, c to be in A.P.:

$$b = \frac{a + c}{2}$$

Example: $a, b, c = 1, 2, 3$.

2. Geometric Progression (G.P.):

For three terms a, b, c to be in G.P.:

$$b = \sqrt{ac}$$

Example: $a, b, c = 2, 4, 8$.

3. Harmonic Progression (H.P.):

For three terms a, b, c to be in H.P.:

$$b = \frac{2ac}{a + c}$$

Key Formulas for Sums of Natural Numbers

1. Sum of First n Natural Numbers:

$$\sum_{i=1}^n i = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

2. Sum of Squares of First n Natural Numbers:

$$\sum_{i=1}^n i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Sum of Cubes of First n Natural Numbers:

$$\sum_{i=1}^n i^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2} \right)^2$$

4. Sum of First n Even Natural Numbers:

$$\sum_{i=1}^n 2i = 2 + 4 + 6 + \dots = n(n+1)$$

5. Sum of First n Odd Natural Numbers:

$$\sum_{i=1}^n (2i-1) = 1 + 3 + 5 + \dots = n^2$$

Sets Definition: A set is a collection of well-defined elements. Forms of Sets Roster Form: - Elements are listed explicitly within curly brackets $\{\}$. - Example: $A = \{a, e, i, o, u\}$ Set Builder Form: - Describes the set using a property or rule that its elements satisfy. - Example: $A = \{x : x \text{ is a vowel}\}$	Null Set Definition: An empty set is called a null set. It contains no elements. Notation: $A = \{\}$ or $A = \phi$ Cardinal Number: The cardinal number of a null set is zero. Complement of a Set Definition: If A is a set, then A^1 represents the complement of A. Formula: $A^1 = U - A$ (Elements in the universal set U but not in A)
Formulas for Union of Sets For Any Two Sets A and B: The number of elements in the union of A and B is given by: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ For Any Three Sets A, B, and C: The number of elements in the union of A, B, and C is given by: $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$	For Any Two Sets A and B: - At least one of A or B $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ - Only A $n(A - B) = n(A) - n(A \cap B)$ - Only B $n(B - A) = n(B) - n(A \cap B)$ - Exactly one of A or B $n(A - B) + n(B - A) = n(A) + n(B) - 2n(A \cap B)$ - Neither A nor B $= n(U) - n(A \cup B)$

For Any Three Sets A, B, C: - At least one of A, B, C $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$ - Only A $n(A - (B \cup C)) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$ - Only B $n(B - (A \cup C)) = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$ - Only C $n(C - (A \cup B)) = n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$ - Exactly one of A, B, C $n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$ - Neither A, B, nor C $= n(U) - n(A \cup B \cup C)$	De Morgan's Laws - Complement of Union: $(A \cup B)^c = A^c \cap B^c$ Additional Notes - Complement of Union: $(A \cup B)^c = U - (A \cup B)$ - Formula for Complement of Union Count: $n((A \cup B)^c) = n(U) - n(A \cup B)$ - Usage: To calculate elements that belong to neither A nor B.
Number of Subsets: - For a set with n elements, the number of subsets is: 2^n Number of Proper Subsets: - For a set with n elements, the number of proper subsets is: $2^n - 1$	Note: The cardinal number of $A \times B$ is given by: $n(A \times B) = n(A) \times n(B)$ The number of relations from A to B is given by: $2^{n(A) \times n(B)}$

Additional Types of Relations 1. Equivalence Relation Reflexive: $(a, a) \in R$ for all $a \in A$. Symmetric: If $(a, b) \in R$, then $(b, a) \in R$. Transitive: If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.	2. Partial Order Relation Reflexive: $(a, a) \in R$ for all $a \in A$. Antisymmetric: If $(a, b) \in R$ and $(b, a) \in R$, then $a = b$. Transitive: If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.
Types of Functions 1. One-to-One Function (Injective): Definition: A function is one-to-one if every element of the domain maps to a distinct element in the codomain. 2. Onto Function (Surjective): Definition: A function is onto if every element of the codomain has at least one pre-image in the domain. Note: If a function is not onto, it is called an "into function."	3. One-to-One and Onto (Bijective Function): Definition: A function is bijective if it is both one-to-one and onto. 4. Many-to-One Function: Definition: A function is many-to-one if two or more elements in the domain map to the same element in the codomain.

Finding $f^{-1}(x)$ When $f(x)$ is Given as an Expression:

To find $f^{-1}(x)$ when $f(x)$ is given as a mathematical expression:

1. **Step 1:** Assume $y = f(x)$.
2. **Step 2:** Solve for x in terms of y .
3. **Step 3:** Replace y with x to get $f^{-1}(x)$.

Properties of Limits:

1. **Sum Rule:**

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

2. **Product Rule:**

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

3. **Quotient Rule:**

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \text{ if } \lim_{x \rightarrow a} g(x) \neq 0$$

4. **Constant Rule:**

$$\lim_{x \rightarrow a} c = c$$

Constant Multiplication Rule:

If C is a constant, then:

$$\lim_{x \rightarrow a} [C \cdot f(x)] = C \cdot \lim_{x \rightarrow a} f(x)$$

Important Limits

1. Polynomial Limit:

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$$

2. Exponential Limit:

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a \quad (\text{where } a > 0)$$

3. Natural Exponential Limit:

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

4. Logarithmic Limit:

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

5. Limit Definition of e (Infinity Form):

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

7. Expansion Limit:

$$\lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x} = n$$

Basic Rules of Differentiation

1. Sum and Difference Rule:

$$\frac{d}{dx} [f(x) \pm g(x)] = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

- The derivative of a sum (or difference) of two functions is the sum (or difference) of their derivatives.

Power Rule:

$$\frac{d}{dx} (x^n) = nx^{n-1}$$

Exponential Function with a Constant Base: $\frac{d}{dx}(a^x) = a^x \cdot \ln(a)$ Where a is a constant. Exponential Function with Base e: $\frac{d}{dx}(e^x) = e^x$ Variable as Exponent: $\frac{d}{dx}(x^x) = x^x(1 + \ln(x))$	Logarithmic Function: $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$ Square Root Function: $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$ Logarithmic Function with Base a: $\frac{d}{dx}(\log_a(x)) = \frac{1}{x \cdot \ln(a)}$
Sum Rule Formula: $\frac{d}{dx}(u \pm v) = \frac{d(u)}{dx} \pm \frac{d(v)}{dx}$	Product Rule $\frac{d}{dx}(I \times II) = I \frac{d(II)}{dx} + II \frac{d(I)}{dx}$
Quotient Rule Formula: $\frac{dy}{dx} = \frac{v \frac{d(u)}{dx} - u \frac{d(v)}{dx}}{v^2}$	Shortcut Rule for Rational Functions $y = \frac{af(x) + b}{cf(x) + d}$ $\frac{dy}{dx} = \frac{(ad - bc) \cdot f'(x)}{[cf(x) + d]^2}$

For a composite function $Y = [f(g(x))]$, the derivative is: $\frac{dY}{dx} = f'(g(x)) \cdot g'(x)$	If: $x = f(t) \quad \text{and} \quad y = g(t),$ then the derivative $\frac{dy}{dx}$ is calculated as: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad (\text{provided } \frac{dx}{dt} \neq 0).$
Implicit Functions Implicit functions are those where x and y are not explicitly separated (e.g., x and y are on the same side of the equation). Shortcut Formula: $\frac{dy}{dx} = - \frac{\text{Differentiate with respect to } x, \text{ treating } y \text{ as constant}}{\text{Differentiate with respect to } y, \text{ treating } x \text{ as constant}}$	If a function is given in the form: $x^m \cdot y^n = (x + y)^{m+n},$ then the derivative is simplified as: $\frac{dy}{dx} = \frac{y}{x}.$
Logarithmic Differentiation Shortcut Formula: $\frac{dy}{dx} = y \left[\frac{P}{B} \cdot \frac{d(B)}{dx} + \ln(B) \cdot \frac{d(P)}{dx} \right]$	Derivatives of Infinite Series When dealing with a function that represents an infinite power series, like: $y = f(x)^{f(x)^{f(x) \dots}}$ Shortcut Formula: $\frac{dy}{dx} = \frac{y^2}{1 - y \ln f(x)} \cdot \frac{f'(x)}{f(x)}$

Maximum and Minimum Value of a Function

To determine whether a function $y = f(x)$ has a maximum or minimum value at a given point, we use the first and second derivatives.

Steps:

1. Find the First Derivative:

- Compute $\frac{dy}{dx}$ and set it equal to zero:

$$\frac{dy}{dx} = 0$$

- Solve for x to find the critical points.

2. Find the Second Derivative:

- Compute $\frac{d^2y}{dx^2}$.
- Substitute the critical points (values of x) into $\frac{d^2y}{dx^2}$ to determine the nature of the critical points.

Conditions:

- If $\frac{d^2y}{dx^2} < 0$, the function has a **maximum** value at that point.
- If $\frac{d^2y}{dx^2} > 0$, the function has a **minimum** value at that point.
- If $\frac{d^2y}{dx^2} = 0$, the test is inconclusive, and the point may be a saddle point (neither maximum nor minimum).

Application of Derivatives in Cost and Revenue

1. Average Cost (AC):

$$\text{Avg Cost} = \frac{C(x)}{x}$$

Where $C(x)$ is the cost function, and x is the number of units of production.

2. Marginal Cost (MC):

$$\text{Marginal Cost} = \frac{d}{dx}[C(x)]$$

3. Average Revenue (AR):

$$\text{Avg Revenue} = \frac{R(x)}{x}$$

Where $R(x)$ is the revenue function.

4. Marginal Revenue (MR):

$$\text{Marginal Revenue} = \frac{d}{dx}[R(x)]$$

1. Revenue Function:

$$R(x) = \text{Demand Function} \times x$$

2. Profit Maximization:

- Profit is maximized at the equilibrium point under perfect competition when:

$$\text{Marginal Revenue} = \text{Marginal Cost.}$$

3. Break-Even Point:

- Profit is zero when:

$$\text{Total Revenue} = \text{Total Cost.}$$

Integrals $\int e^x dx = e^x + C$ $\int e^{ax} dx = \frac{e^{ax}}{a} + C$ (where $a \neq 0$) $\int \frac{1}{x} dx = \ln x + C$ $\int a^x dx = \frac{a^x}{\ln a} + C$ (where $a > 0$) $\int 1 dx = x + C$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)	Problems on Sum Rule in Integration The Sum Rule for Integration states: $\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$ Key Properties of Definite Integrals Linearity: $\int_a^b (f(x) + g(x)) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$ $\int_a^b c \cdot f(x) dx = c \cdot \int_a^b f(x) dx$
Steps to Evaluate Definite Integrals - Find the Indefinite Integral: Compute $F(x)$, the antiderivative of $f(x)$ - Apply Limits: Use the Fundamental Theorem of Calculus: $\int_a^b f(x) dx = F(b) - F(a)$	Property 5: Symmetry in Definite Integrals $\int_a^b f(x) dx = \int_a^b f((a+b)-x) dx$
Special Definite Integral Formula $\int_a^b \frac{x^n}{x^n + (a+b-x)^n} dx = \frac{b-a}{2}$	Important Formula $\int e^x (f(x) + f'(x)) dx = e^x f(x) + C$

Integration by Parts

$$\int u v dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx$$

Shortcut:

For repeated integration by parts, the solution can be written as:

$$\int uv dx = uv_1 - u'v_2 + u''v_3 - \dots$$

Standard Integrals for Common Forms

1. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left(x + \sqrt{x^2 - a^2} \right) + C$
2. $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log \left(x + \sqrt{x^2 + a^2} \right) + C$
3. $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left(x + \sqrt{x^2 + a^2} \right) + C$
4. $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left(x + \sqrt{x^2 - a^2} \right) + C$
5. $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left(\frac{x - a}{x + a} \right) + C$
6. $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left(\frac{a + x}{a - x} \right) + C$



Class Width = Upper Class Limit – Lower Class Limit Class Mark = $\frac{\text{Lower Limit} + \text{Upper Limit}}{2}$ Frequency Density = $\frac{\text{Class Frequency}}{\text{Class Width}}$ $RF = \frac{\text{Class Frequency}}{\text{Total Frequency}}$ Percentage Frequency = $RF \times 100$	How to Determine the Number of Classes in a Frequency Distribution 1: Using Range and Class Length Number of Classes = $\frac{\text{Range}}{\text{Class Length}}$ 2: Sturges' Rule Number of Classes = $1 + 3.322 \log N$
Parts of a Table in Tabulation A well-structured table consists of four main parts: 1. Caption: - The topmost part of the table. - Describes columns or sub-columns. - Also called the Column Heading.	2. Box Head: - The entire upper part of the table. - Includes the caption, column names, and units of measurement. 3. Stubs: - The leftmost part of the table. - Lists row headings (e.g., names, categories).

4. Body:

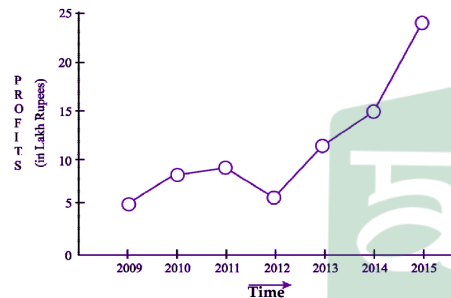
- The main section where the actual data (numbers, values) are arranged in rows and columns.

5. Footnote (Optional):

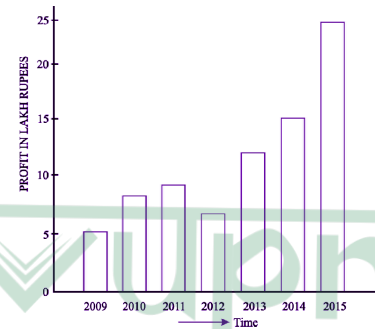
- Located at the bottom of the table.
- Provides source information or any missed details.

Types of Diagrams

1. Line Diagram



2. Bar Diagram (Bar Chart)

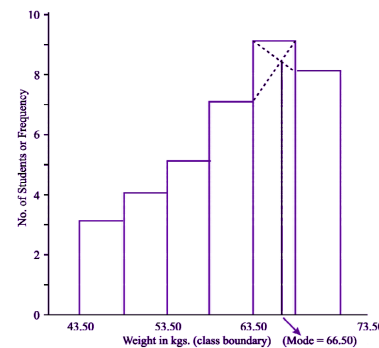


Formula for Central Angle

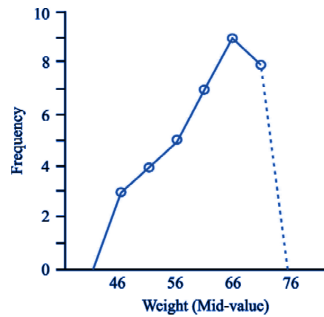
Each segment (category) in a pie chart is represented as a sector with a central angle given by:

$$\text{Central Angle} = \left(\frac{x}{\sum x} \right) \times 360$$

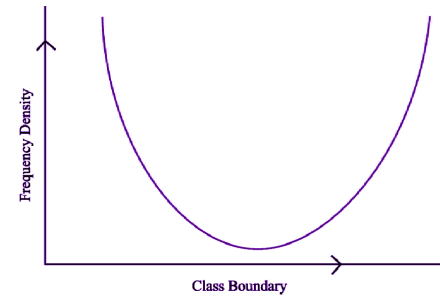
Histogram



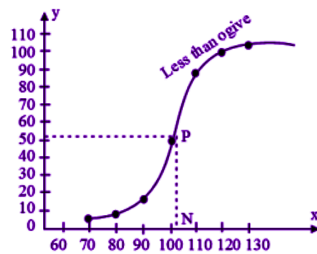
Frequency Polygon



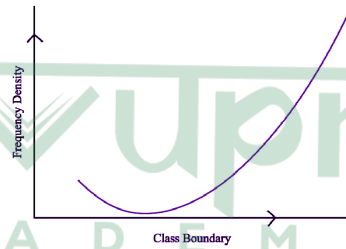
U-Shaped Curve



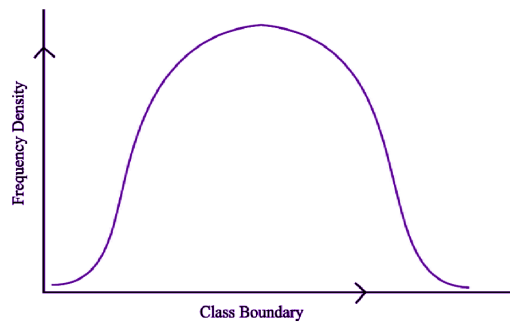
Inverted J-Shaped (S-Shaped) Curve



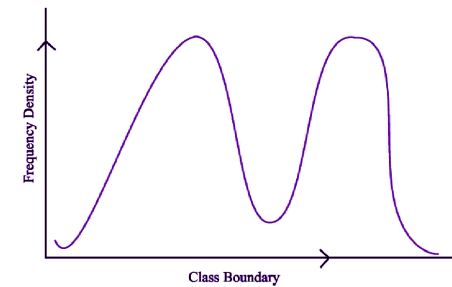
J-Shaped Curve



Bell-Shaped Curve (Normal Distribution)



Mixed Curve



With Replacement (SRSWR): Total possible samples = N^n. Without Replacement (SRSWOR): Total possible samples = N_C_n (combinations of N taken n at a time).	
Standard Error of Mean (SE of $\bar{x}$) For Simple Random Sampling With Replacement (SRSWR): $SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}$	For Simple Random Sampling Without Replacement (SRSWOR): $SE(\bar{x}) = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$ - The term $\sqrt{((N-n)/(N-1))}$ is the finite population correction (fpc). - If n is very large or N is infinite, the fpc ≈ 1 and can be ignored.
Standard Error for Proportion (SE of P) For Simple Random Sampling With Replacement (SRSWR): $SE(P) = \sqrt{\frac{Pq}{n}}$ For Simple Random Sampling Without Replacement (SRSWOR): $SE(P) = \sqrt{\frac{Pq}{n} \times \frac{N-n}{N-1}}$	For Addition/Subtraction: <i>New AM = Original AM $\pm$ k</i> For Multiplication/Division: <i>New AM = Original AM $\times$ k or New AM = Original AM $\div$ k</i> $\bar{x}_c = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$
Arithmetic Mean for Grouped Data <i>Midpoints of each interval = $\frac{UCB+LCB}{2}$</i>	$\sum (x_i - \bar{x}) = \sum [f_i(x_i - \bar{x})] = 0$

$\bar{x} = \frac{\sum f \times x}{N}$	GM For Ungrouped data $G = \left(x_1 \times x_2 \times x_3 \times \dots \times x_n \right)^{\frac{1}{n}}$
$G = \left(x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \times \dots \times x_n^{f_n} \right)^{\frac{1}{N}}$	$GM(xy) = GM(x) \times GM(y)$ $GM\left(\frac{x}{y}\right) = \frac{GM(x)}{GM(y)}$
Harmonic mean(HM) For Ungrouped $HM = \frac{n}{\sum \frac{1}{x}}$	Harmonic mean(HM) For Grouped $HM = \frac{N}{\sum \frac{f}{x}}$

$\text{Combined HM} = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$	Relationship between AM, GM and HM If the observations are positive and equal AM = GM = HM. If the observations are positive and distinct AM > GM > HM
$GM = \sqrt{AM \times HM}$	Median of Ungrouped Data If the number of observations (n) is odd, the median (M) is the $\left(\frac{n+1}{2}\right)^{th}$ observation. If the number of observations (n) is even, the median (M) is the average (AM) of $\frac{n}{2}^{th}$ observation and $\left(\frac{n+2}{2}\right)^{th}$ observation.
Median for Continuous grouped data $M = l_1 + \left(\frac{\frac{N}{2} - cf}{f}\right) \times c$	Mode for continuous grouped Distribution $Z = l_1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times C$

Relationship between mean, Median & mode Perfectly symmetrical distribution. Mean= Median= Mode For Positively Skewed (Skewed Right) Distribution: Positively skewed distribution. mean > median > mode. For Negatively Skewed (Skewed Left) Distribution: mode > median > mean	Arithmetic Mean (AM): Affected by extreme values and sampling fluctuations. The sum of deviations from the mean is always zero: $\sum (x - \bar{x}) = 0$ Median: Not affected by extreme values or sample fluctuations. The sum of absolute deviations is minimum when taken from the median: $\sum x - M$ is minimum Mode: May or may not be affected by extreme values. Mode represents the most frequently occurring value in a dataset.
Mean-mode = 3(mean-Median)	Quartiles Ungrouped data $Q_i = \left[\frac{i}{4}(n + 1) \right]^{th} \text{ Value} \quad i = 1, 2, 3$ $\text{Median } M = Q_2 = \frac{Q_1 + Q_3}{2}$

Deciles Ungrouped data $D_i = \left[\frac{i}{10}(n + 1) \right]^{th} \text{Value}$	Percentiles Ungrouped data $P_i = \left[\frac{i}{100}(n + 1) \right]^{th} \text{Value}$
Partitions For Ungrouped data $Q_i = \left[\frac{i}{4}(n + 1) \right]^{th} \text{Value}$ $D_i = \left[\frac{i}{10}(n + 1) \right]^{th} \text{Value}$ $P_i = \left[\frac{i}{100}(n + 1) \right]^{th} \text{Value}$	For discrete grouped data $Q_i = \left[\frac{i}{4}(N + 1) \right]^{th} \text{Value}$ $D_i = \left[\frac{i}{10}(N + 1) \right]^{th} \text{Value}$ $P_i = \left[\frac{i}{100}(N + 1) \right]^{th} \text{Value}$

Changes in Origin & Scale If x & y are 2 variables related as $ax + by + c = 0$ & a Central tendency of x is given. Then how will you find the central tendency of y. Step 1 Solve y Step 2 Replace x by central tendency of x & Replace y by central tendency of y	Range & Coefficient of Range $R = L - S$ $\text{Coefficient of Range} = \left(\frac{L-S}{L+S} \right) \times 100$
Mean Deviation (MD) & Coefficient of Mean Deviation $\text{MD about mean} = \frac{\sum f \cdot x - \bar{x} }{n}$ $\text{MD about median} = \frac{\sum f \cdot x - M }{n}$	$\text{Coefficient of MD about the Mean} = \frac{\text{MD about mean}}{\text{mean}}$ $\text{Coefficient of MD about Median} = \frac{\text{MD about Median}}{\text{Median}}$
Standard deviation (SD) For Ungrouped data $SD = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$ $SD = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2} \quad \text{where } \bar{x} = \frac{\sum x}{n}$	Standard deviation (SD) For Grouped data $SD = \sqrt{\frac{\sum f \cdot (x - \bar{x})^2}{N}} \quad \text{where } N = \sum f$ Or $SD = \sqrt{\frac{\sum fx^2}{N} - (\bar{x})^2}$

Combined SD = $\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$ $d_1^2 = (\bar{x}_1 - \bar{x}_c)^2$ & $d_2^2 = (\bar{x}_2 - \bar{x}_c)^2$ $\bar{x}_c$ is the combined mean $\bar{x}_c = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$	SD of the first n natural numbers = $\sqrt{\frac{n^2 - 1}{12}}$
Quartile Deviation (QD) $QD = \frac{Q_3 - Q_1}{2}$, QD is also known as the Semi-Interquartile Range, $Q_3 - Q_1$ is called as the inter quartile range Q_3, Q_1 are Third & First Quartile	Coefficient of Quartile Deviation Coefficient of QD = $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{QD}{M}$ M is the median = $\frac{Q_3 + Q_1}{2}$
Relationship between SD, MD, & QD for Normal or symmetrical distribution $2SD = 2.5 MD = 3QD$ Note: $SD > MD > QD$	Dispersion Relationship in Linear Variables: Dispersion of $y = \left \frac{a}{b} \right \times \text{Dispersion of } x$

Probability is the **chance of occurrence of an event**.

If A is an event, the probability of occurrence of A is given by:

$$P(A) = \frac{n(A)}{n(S)}$$

3. **Complementary Events:**

- The sum of the probabilities of an event and its non-occurrence equals 1:

$$P(A) + P(A') = 1$$

Where A' is the complement of A , meaning the event does not occur.

4. **Simple Event:**

- An event that produces only one outcome is called a **simple event**.

Example: Getting a 2 when rolling a die.

Key Properties of Probability

1. **Range of Probability:**

- Probability of an event always lies between 0 and 1 (both inclusive):
 $0 \leq P(A) \leq 1$

2. **Special Cases:**

- If $P(A) = 0$: The event is **impossible** or **improbable**.
- If $P(A) = 1$: The event is a **sure event**.
- If $0 < P(A) < 1$: The event is **possible** or **probable**.

General Rule:

For n tosses of a coin:

$$n(S) = 2^n$$

Where n is the number of tosses.

The probability of getting heads and tails alternatively on tossing a coin n times

$$P = \frac{2}{2^n}$$

For tossing a die n times, the total number of outcomes is $n(S) = 6^n$ Sample Space for a Deck of Cards Total number of cards: $n(S) = 52$	Number of Face Cards: • Face cards: King, Queen, Jack • Total face cards per suit = 3 • Total face cards across all suits:
1. Probability of $A \cup B$ (At least one of A or B occurs): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ • $P(A \cup B)$: Probability of A or B. • $P(A \cap B)$: Probability of A and B. Note: This is also known as the Addition Theorem of Probability.	2. Mutually Exclusive Events: • Two events A and B are mutually exclusive if they have no common outcomes: $P(A \cap B) = 0$ • Probability simplifies to: $P(A \cup B) = P(A) + P(B)$

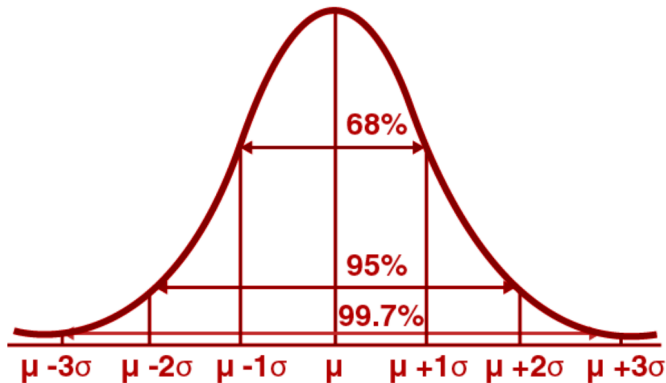
3. Exhaustive Events: Two events A and B are exhaustive if their union includes all possible outcomes: $n(A \cup B) = n(S)$ $P(A \cup B) = 1$	4. Mutually Exclusive and Exhaustive Events: If A and B are both mutually exclusive and exhaustive: $P(A \cup B) = 1$ $P(A \cap B) = 0$ - Formula simplifies to: $1 = P(A) + P(B)$
5. Conditional Probability: If A and B are two events: 1. Probability of A given B (Event B has already occurred): $P(A B) = \frac{P(A \cap B)}{P(B)}$	2. Probability of B given A (Event A has already occurred): $P(B A) = \frac{P(A \cap B)}{P(A)}$ Where: $P(A B)$: Conditional probability of A given B. $P(B A)$: Conditional probability of B given A.
6. Independent Events	Multiplication Theorem for Independent Events: If A and B are independent, then: $P(A \cap B) = P(A) \times P(B)$

Key Properties: 1. $P(A B) = P(A)$ 2. $P(B A) = P(B)$ 3. $P(A B') = P(A)$ 4. $P(B A') = P(B)$	7. Probability of Occurrence of Only A (Out of A & B): $P(A - B) = P(A \cap B') = P(A) - P(A \cap B)$ 8. Probability of Occurrence of Only B: $P(B - A) = P(B \cap A') = P(B) - P(A \cap B)$
9. Probability of Occurrence of Only One Event (Either A or B) $ \begin{aligned} P(\text{Only one event}) &= P(A - B) + P(B - A) \\ &= P(A) - P(A \cap B) + P(B) - P(A \cap B) \\ &= P(A) + P(B) - 2P(A \cap B) \end{aligned} $	10. Equally Likely Events: If A and B are equally likely events: $P(A) = P(B) = K$
The probability of neither A nor B occurring can be calculated using: $P(A \cup B)' = 1 - P(A \cup B)$	

Sample Space for Leap Year and Non-Leap Year Problems Non-Leap Year: • Total Days: 365 • Odd Days: 1 • Sample Space: {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday} • $n(S)$: 7 • Key Notes: • $P(A \cap B) = 0$, since there are no overlapping days for non-leap years.	Leap Year: • Total Days: 366 • Odd Days: 2 • Sample Space: {Sunday-Monday, Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday, Thursday-Friday, Friday-Saturday, Saturday-Sunday} • $n(S)$: 7 • Key Notes: • $P(A \cap B) = 0$ if the given days are non-successive. • $P(A \cap B) = \frac{1}{7}$ if the days are successive.
Mathematical Expectation of x • Definition: The expected value or mean of a random variable is the weighted average of all possible values of x, weighted by their probabilities. • Formulas: • $E(x) = \sum x \cdot P(x)$	Variance of x • Definition: Variance measures the spread or dispersion of the random variable around its mean. It tells us how far the values of x are likely to be from the expected value $E(x)$. • Formula: $V(x) = E(x^2) - [E(x)]^2$
Detailed Explanation of Properties of $E(x)$: 1. $E(K) = K$: 2. $E(x + y) = E(x) + E(y)$: 3. $E(XY) = E(X) \cdot E(Y)$ (For Independent Variables):	Bayes' Theorem Explained: Bayes' Theorem helps us calculate the probability of an event E_i, given that another event A has already occurred. This is particularly useful when dealing with conditional probabilities. The Formula: $P(E_i/A) = \frac{P(A/E_i) \cdot P(E_i)}{\sum_{i=1}^n P(A/E_i) \cdot P(E_i)}$

4. $E(a + bX) = a + bE(X)$:	
Odds in Favor of an Event If the odds in favor of an event are given as a:b: • a represents the number of favorable outcomes. • b represents the number of outcomes against the event. • The total number of outcomes is $a + b = n(S)$, where $n(S)$ is the total sample space.	Odds Against an Event If the odds against an event are given as a:b: • b represents the number of favorable outcomes. • a represents the number of outcomes against the event. • Again, the total number of outcomes is $a + b = n(S)$.
Key Features of Binomial Distribution: 1. Based on Bernoulli Trials: • A Bernoulli trial is a random experiment with exactly two possible outcomes: "success" "failure." 2. Discrete Probability Distribution: • The distribution is for discrete random variables. 3. Biparametric Distribution: • Two parameters define it: • n: Number of trials • p: Probability of success in a single trial	Probability Mass Function (PMF): The probability of exactly x successes in n trials $P(x) = nC_x p^x q^{n-x}$ Where: • nC_x (combination formula) is: $nC_x = \frac{n!}{x!(n-x)!}$ • x: Number of successes • $q = 1 - p$: Probability of failure

1. Mean of a Binomial Distribution: - The mean of a binomial distribution is given by: $\text{Mean} = np$ n: Number of trials. p: Probability of success in each trial. - This represents the expected number of successes in n trials.	2. Standard Deviation of a Binomial Distribution: - The standard deviation measures the spread of the distribution and is given by: $\sigma = \sqrt{npq}$ $q = 1 - p$: Probability of failure. 3. Variance of a Binomial Distribution: - The variance quantifies the dispersion of the distribution and is given by: $\text{Variance} = npq$
4. Mode of a Binomial Distribution: - The mode is the most frequent value in the distribution. - Conditions for the mode: - If $(n + 1)p$ is a non-integer, the mode is the integral part of $(n + 1)p$. - If $(n + 1)p$ is an integer, the binomial distribution is bimodal, with modes: $\text{Modes} = (n + 1)p \text{ and } (n + 1)p - 1$	5. Symmetry and Skewness: - If $p = q = 0.5$: The distribution is symmetric. - If $p < 0.5$: The distribution is positively skewed. - If $p > 0.5$: The distribution is negatively skewed.
Additive Property of Binomial Distribution: If $X \sim B(n_1, p)$ and $Y \sim B(n_2, p)$, then: $(X + Y) \sim B(n_1 + n_2, p)$	Poisson Distribution (PD) Formulas: - Mean = m - Variance = m - Standard Deviation = $\sqrt{m}$

Mode: - If m is a non-integer: The mode is the integral part of m. - If m is an integer: The distribution is bimodal, and the modes are m and $m - 1$. Probability Mass Function (PMF): $P(x) = \frac{e^{-m} m^x}{x!} \quad \text{for } x = 0, 1, 2, \dots, \infty$	Additive Property: If $X \sim P(m_1)$ and $Y \sim P(m_2)$, then $(X + Y) \sim P(m_1 + m_2)$.
Normal Distribution Probability Density Function (PDF): $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	Symmetry: - The curve is perfectly symmetrical around the mean (μ). - The left and right sides of the distribution are mirror images. Skewness: - A normal distribution has zero skewness, meaning it is not tilted to the left or right.
Equality of Measures: - For a normal distribution: Mean = Median = Mode. Shape Independence: - The shape of the distribution remains consistent regardless of the values of μ and σ. The mean shifts the center, while the variance scales the spread. Unimodal: - The distribution is unimodal, meaning it has a single peak at the mean.	 The diagram shows a normal distribution curve with the following labels and percentages: 68% of the data lies between $\mu - 1\sigma$ and $\mu + 1\sigma$. 95% of the data lies between $\mu - 2\sigma$ and $\mu + 2\sigma$. 99.7% of the data lies between $\mu - 3\sigma$ and $\mu + 3\sigma$.

1. Points of Inflection: - The points of inflection occur at $\mu - \sigma$ and $\mu + \sigma$, where: μ: Mean σ: Standard deviation - At these points, the curve transitions from concave to convex or vice versa.	2. Quartile and Mean Deviations: - Quartiles (Q1 and Q3): $Q1 = \mu - 0.6745\sigma$ $Q3 = \mu + 0.6745\sigma$ - The median is exactly in the middle of Q1 and Q3.
3. Area Under the Normal Curve: - The total area under the curve is 1, representing 100% of the probability - The area is symmetric around the mean (μ): - Area from $-\infty$ to 0 = 0.5. - Area from 0 to $+\infty$ = 0.5.	- Specific areas between standard deviations: - Between $\mu - 3\sigma$ and $\mu + 3\sigma$: 0.9973 (99.73%). - Between $\mu - 2\sigma$ and $\mu + 2\sigma$: 0.9546 (95.46%). - Between $\mu - \sigma$ and $\mu + \sigma$: 0.6828 (68.28%).
For Single Value: $P(X < a)$ Conversion to Standard Normal Form: The normal variable X is converted into the standard normal variable Z using: $Z = \frac{X - \mu}{\sigma}$	Probability Expression: $P(a < X < b) = \phi(K_2) - \phi(K_1)$ Here, $\phi(K_2)$ is the area under the curve up to K_2, and $\phi(K_1)$ is the area up to K_1

1. Symmetry of the normal curve: • $\phi(-K) = 1 - \phi(K)$ • This property allows calculation of areas for negative K. 2. Boundary Area Formulae: • Area from $-\infty$ to 0: 0.5, • Area from 0 to K: $\phi(K) - 0.5$ (for $K > 0$).	Central Moments by Distribution 1. First Central Moment (μ_1) • Represents the mean deviation. • For symmetric distributions: • Binomial: $\mu_1 = 0$ (No mean deviation from itself). • Poisson: $\mu_1 = 0$ (Mean deviation is zero). • Normal: $\mu_1 = 0$ (Mean deviation is zero).
2. Second Central Moment (μ_2) • Represents the variance. • For each distribution: • Binomial: $\mu_2 = npq$, where n is the number of trials, p is the probability of success, $q = 1 - p$ is the probability of failure. • Poisson: $\mu_2 = m$, where m is the mean (in Poisson, mean = variance). • Normal: $\mu_2 = \sigma^2$, where σ^2 is the variance.	3. Third Central Moment (μ_3) • Represents skewness, which describes asymmetry in the distribution. • For each distribution: • Binomial: $\mu_3 = npq(q - p)$. Indicates the direction of skewness based on p and q. • Poisson: $\mu_3 = m$. The distribution is positively skewed for $m > 0$. • Normal: $\mu_3 = 0$. Normal distribution is symmetric.
4. Fourth Central Moment (μ_4) • Represents kurtosis, which describes the "tailedness" of the distribution. • For each distribution: • Binomial: $\mu_4 = 3n^2p^2q^2 + npq(1 - 6pq)$. Combines variance and higher-order terms tailedness. • Poisson: $\mu_4 = m(3m + 1)$. Indicates the relative height of the distribution curve. • Normal: $\mu_4 = 3\sigma^4$. The coefficient of 3 indicates "mesokurtic" (normal-tailed).	Correlation Coefficient (r) • The value of r ranges between -1 and +1, where: • $r = 1$: Perfect positive correlation. • $r = -1$: Perfect negative correlation. • $r = 0$: No correlation or zero correlation.

Karl Pearson's Product-Moment Correlation Coefficient $r_{xy} = \frac{\text{Cov}(x,y)}{S_x \cdot S_y}$ $r_{xy} = \frac{n \sum xy - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$ When Deviations Are Taken From the Mean: $r_{uv} = \frac{\sum dx \cdot dy}{\sqrt{\sum dx^2 \cdot \sum dy^2}} \text{ Where:}$	(c) Spearman's Rank Correlation Coefficient Formula for r_r: 1. With Ties: $r_r = 1 - \left[\frac{6 \left(\sum d^2 + \frac{\sum(t^3-t)}{12} \right)}{n^3 - n} \right]$ 2. Without Ties: $r_r = 1 - \frac{6 \sum d^2}{n^3 - n}$
(d) Coefficient of Concurrent Deviations $r_{cd} = \pm \sqrt{\frac{2c - m}{m}}$	Probable Error (PE): • Definition: It is used to determine the limits of the correlation coefficient. • Formula: $PE = 0.6745 \frac{(1 - r^2)}{\sqrt{n}}$

Standard Error (SE): • Definition: It provides an extended range for the probable error. • Formula: $SE = 1.5 PE$	Coefficient of Determination (r^2): • Definition: Represents the proportion of explained variation in the dependent variable. • Value: The square of the correlation coefficient, r^2.
Coefficient of Non-Determination: • Definition: Represents the unexplained variation. • Formula: $1 - r^2$	$r = \pm \sqrt{b_{xy} \cdot b_{yx}}$ • Calculation of b_{xy}: $b_{xy} = r \cdot \frac{s_x}{s_y} = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum y^2 - (\sum y)^2}$ • Calculation of b_{yx}: $b_{yx} = r \cdot \frac{s_y}{s_x} = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$

Sign of r: r is positive (+) if both b_{xy} and b_{yx} are positive. r is negative (−) if both b_{xy} and b_{yx} are negative.	Price Relative Formula: $\text{Price Relative} = \frac{P_n}{P_0}$ To express it as a percentage, it is multiplied by 100: $\text{Price Relative} = \left(\frac{P_n}{P_0} \right) \times 100$
Simple Aggregative Price Index (SAPI) $SAPI = \left(\frac{\sum P_1}{\sum P_0} \right) \times 100$	Simple Relative Price Index (SRPI) $SRPI = \left(\frac{\sum \left(\frac{P_1}{P_0} \right)}{N} \right) \times 100$
Types of Weighted Index Numbers 1. Laspeyres' Index (L) $L = \left(\frac{\sum (P_1 q_0)}{\sum (P_0 q_0)} \right) \times 100$	2. Paasche's Index (P) $P = \left(\frac{\sum (P_1 q_1)}{\sum (P_0 q_1)} \right) \times 100$ where: q_1 = Current year quantity

3. Bowley's Index (B) It is the Arithmetic Mean (AM) of Laspeyres' and Paasche's Index. $B = \frac{L + P}{2}$	4. Fisher's Index (F) It is the Geometric Mean (GM) of Laspeyres' and Paasche's Index. $F = \sqrt{L \times P}$
5. Marshall's Index (M) $M = \left(\frac{\sum P_1(q_0 + q_1)}{\sum P_0(q_0 + q_1)} \right) \times 100$ Marshall's Index is a good approximation to Fisher's Index.	Formula for CPI: $CPI = \frac{\sum(I \times W)}{\sum W}$ where: - I = Group index - W = Weight (importance of the item in the market basket)
Deflated Value: $\text{Deflated Value} = \frac{\text{Current Price}}{\text{Current CPI}}$	Shifted Price Index $\text{Shifted Price Index} = \left(\frac{\text{Original Price Index}}{\text{Price Index of the year to which it has to be shifted}} \right) \times 100$
Chain Index Number (CIN) $CIN = \left(\frac{\text{Link Relative of Current Year} \times \text{CIN of Previous Year}}{100} \right)$	Purchasing Power of Money $\text{Purchasing Power of Money} = \frac{1}{\text{Price Index Number}}$

Real Wages

$$\text{Real Wages} = \left(\frac{\text{Current Year Wages}}{\text{Current Year CPI}} \right) \times \text{Base Year CPI}$$

Finding Current Year Salary & Dearness Allowance (DA)

If the Base Year Salary, Base Year CPI, and Current Year CPI are given, the Current Year Salary is calculated as:

$$\text{Current Year Salary} = \left(\frac{\text{Base Year Salary} \times \text{Current Year CPI}}{\text{Base Year CPI}} \right)$$

Dearness Allowance (DA) Calculation:

$$DA = \text{Current Year Salary} - \text{Base Year Salary}$$

Note: If the Base Year CPI is not given, it is taken as 100.

Tests of Adequacy (Tests of Consistency)

1) Unit Test

2) Time Reversal Test (TRT)

- This test checks whether an index number remains consistent when the time order is reversed.
- Condition for satisfying TRT:

$$P_{01} \times P_{10} = 1$$

3) Factor Reversal Test (FRT)

- This test checks whether the value index can be obtained by multiplying the price index and the quantity index.
- Condition for satisfying FRT:

$$P_{01} \times Q_{01} = V_{01}$$

4) Circular Test

- This test ensures consistency over multiple time periods.
- Condition for satisfying Circular Test:

$$P_{01} \times P_{12} \times P_{20} = 1$$

- Fisher's Index fails to satisfy the Circular Test.