CA FOUNDATION

QUANTITATIVE APTITUDE

Mr. Anurag Chauhan

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PREFACE

At **CA Wallah by PW,** our mission is simple—to help CA Foundation aspirants build strong conceptual clarity through content that is accurate, structured, and fully aligned with ICAI's standards.

We are delighted to introduce the **Second Edition of the CA Foundation Quantitative Aptitude - Concept book by Anurag Chauhan Sir,** a book designed to prioritize clarity, relevance, help students develop concepts systematically, and approach the subject with confidence.

This book has been developed through close collaboration between experienced **faculty, subject matter experts, academic reviewers, and designers.** Their combined expertise ensures that every chapter is presented in a student-friendly manner, enhancing the overall quality while maintaining conceptual depth and alignment with ICAI standards.

The guiding philosophy of this book is **"Strong Concepts with Smart Practice"**. Designed as a detailed concept companion. It works best when paired with the **Revizer** in your regular study plan, especially for last-stage exam preparation.

KEY FEATURES

- **100% Chapterwise ICAI syllabus coverage** ensuring complete syllabus alignment.
- **Dedicated "Basic Maths" chapter included** to strengthen fundamental concepts.
- **Shortcut Tricks and Smart Tips** to enhance speed and accuracy in exams.
- **Chapter-wise weightage and PYQ analysis** for strategic preparation.
- **Step-by-step solved illustrations** for better conceptual clarity.
- **MCQ practice questions with answer keys** for effective self-assessment.

We hope this Concept Book becomes a reliable companion in your CA Foundation preparation and helps you master Quantitative Aptitude with clarity and confidence.

Wishing you focused preparation and great success!

ABOUT THE AUTHOR

Anurag Chauhan Sir is a distinguished educator and mathematician, renowned for his innovative and practical approach to teaching Mathematics and Statistics. He holds a **B.Com (Hons.)**, **M.Com**, and **B.Ed**.

With over **14 years of rich teaching experience**, he has evolved from offline classroom mentoring to becoming a trusted guide for **10+ lakh students globally**.

He specializes in teaching **Class 11 & 12, CUET, NDA, and CA Foundation**, where he simplifies complex concepts and makes them engaging through real-life applications.

Known for his unique style of integrating real-world problems into his lessons, he inspires students to build strong conceptual clarity in Mathematics and Statistics, empowering them to excel in academics as well as competitive examinations

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		Jan	Dec	Jan	Dec	Dec	Jan	Jul	Dec	Jan	Dec	Jan	Dec	June	Sept	Jan	May	Sept	Jan
1.	Ratio and Proportion, Indices and Logarithms	4	4	5	5	6	5	5	9	6	4	5	4	3	4	6	5	4	4
2.	Equations	5	2	2	4	4	3	4	3	3	3	4	6	5	4	2	3	3	3
3.	Linear Inequalities	1	1	3	1	1	1	1	2	1	2	2	–	2	2	2	2	3	3
4.	Mathematics of Finance	6	14	10	13	11	14	14	6	14	14	14	14	14	14	15	14	14	14
7.	Permutations and Combinations	2	4	4	4	4	6	4	4	7	4	4	3	4	4	5	4	4	4
8.	Sequence and Series	4	4	4	4	3	3	3	3	4	2	3	4	4	4	4	4	4	4
9.	Sets, Relations, and Functions	3	4	5	2	4	3	4	3	3	3	5	4	1	4	2	3	4	3
10.	Limits and Continuity	–	–	—		–	–	–	–	–	–	–	–	1	–	1	1	–	1
11.	Basic applications of Differential and Integral Calculus	4	4	5	6	5	2	4	4	3	6	4	4	6	4	3	4	4	4
13.	Number Series, Coding Decoding and Odd Man Out	3	5	4	5	4	5	5	5	5	6	5	5	5	5	5	5	5	5
14.	Direction Test	6	5	4	6	4	3	4	8	5	5	4	4	5	5	5	5	5	5
15.	Seating Arrangement	5	3	4	2	5	4	4	5	4	2	4	5	5	5	5	5	5	5
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24.	Index Numbers	8	3	4	3	3	3	4	5	5	5	4	5	5	5	5	5	5	5

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TOP RANKS EVERY ATTEMPT

KUSHAGRA ROY

1 AIR

CA INTERMEDIATE MAY 2024

ADARSH MAURYA — 13 AIR

ABHIMANYU GOYAL — 14 AIR

SAKSHAM AGARWAL — 15 AIR

ADITI SAPRA — 23 AIR

CHURAMANI REGMI — 25 AIR

PIYUSH GUPTA — 27 AIR

ROHAN GUPTA — 30 AIR

KUSHAGRA HANDA — 31 AIR

SIYA — 31 AIR

AMAN VERMA — 31 AIR

ANKIT GOYAL — 33 AIR

VANSH GUPTA — 34 AIR

MOHIT — 34 AIR

KESHAV LADDHA — 34 AIR

GARVIT — 35 AIR

SHIVAM SINGLA — 36 AIR

SHRISHTI — 37 AIR

ANGELICA — 38 AIR

FATIMA ANSARI — 39 AIR

PRINCE — 39 AIR

ANSH — 41 AIR

KRUNAL RAJKUMAR S. — 41 AIR

ANANT KUMAR GOYAL — 42 AIR

KALMARUDAPPA H N — 43 AIR

KUSH AGARWAL — 45 AIR

BHUMIKA — 46 AIR

VANSH MODI — 46 AIR

ROUNAK RATWANI — 47 AIR

ARMAAN KHAN — 48 AIR

JAY SANJEEV KAROSHI — 49 AIR

YASH MISHRA — 50 AIR

24 AIRs IN LAST 2 ATTEMPTS

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Introductory Overview

Basic Math

In this chapter, we revise essential topics such as Number System, HCF & LCM, Divisibility Rules, Squares, Cubes, Roots, Modulus, and Basic Equations.

A strong grasp of these basics ensures confidence and clarity while solving complex numerical or analytical questions in higher sections.

NUMBER SYSTEM

Numbers are the basic building blocks of mathematics. They help us count, compare, and calculate.

The different types of numbers are classified as follows:

- **Natural Numbers (N):**

  Counting numbers starting from 1.

  Example: 1, 2, 3, 4, 5, …

- **Whole Numbers (W):**

  Natural numbers including 0.

  Example: 0, 1, 2, 3, 4, …

- **Integers (Z):**

  Whole numbers and their negatives.

  Example: …, –3, –2, –1, 0, 1, 2, 3, …

- **Rational Numbers (Q):**

  Numbers that can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

  Example: 1/2, –2/3, 4, 0.25, 1.333…

- **Irrational Numbers:**

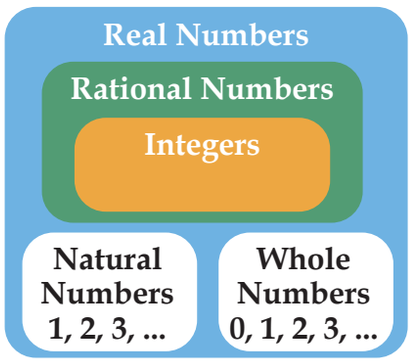
  Numbers that cannot be written as $\frac{p}{q}$.

  They are non-terminating and non-repeating decimals.

  Example: $\sqrt{2}, \pi, \sqrt{3}$

- **Real Numbers (R):**

  All **rational and irrational numbers** together form real numbers.



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EVEN AND ODD NUMBERS

- **Even Numbers:**

  Numbers divisible by 2.

  Examples: 0, 2, 4, 6, 8, 10, …

- **Odd Numbers:**

  Numbers not divisible by 2.

  Examples: 1, 3, 5, 7, 9, 11, …

PRIME AND COMPOSITE NUMBERS

- **Prime Numbers:**

  A number that has exactly two factors — *1 and itself.*

  Examples: 2, 3, 5, 7, 11, 13, …

- **Composite Numbers:**

  Numbers having more than two factors.

  Examples: 4, 6, 8, 9, 10, 12, 14, …

MODULUS OF A NUMBER (ABSOLUTE VALUE)

The modulus (or absolute value) of a number gives its positive value, regardless of sign.

Notation: |x|

- $|x| = x$, if $x \geq 0$
- $|x| = -x$, if $x < 0$

Ex: $|2| = 2$, $|-5| = 5$, $|-6| = 6$, $|7| = 7$ & $|0| = 0$

Hence, modulus always gives a non-negative result.

GREATEST INTEGER FUNCTION (GIF)

The greatest integer function, denoted by [x], gives the largest integer less than or equal to x.

Notation: [x]

Examples:

$[6.2] = 6$, $[7.3] = 7$, $[8.1] = 8$, $[1.9] = 1$, $[-1.5] = -2$

Note: For negative numbers, the GIF moves towards the left on the number line (toward smaller integers).

SQUARES AND SQUARE ROOTS

Square: The square of a number is obtained by multiplying the number by itself.

Formula: $x^2 = x \times x$

Examples:

$5^2 = 5 \times 5 = 25$

$6^2 = 6 \times 6 = 36$

Tip: If $x^2 = n \rightarrow$ then $x = \sqrt{n}$

Square Root: The square root of a number is the number which, when multiplied by itself, gives the original number.

Formula: $\sqrt{x} = p$ such that $p^2 = x$

Examples:

$\sqrt{25} = 5$ (since $5 \times 5 = 25$)

$\sqrt{36} = 6$ (since $6 \times 6 = 36$)

CUBES AND CUBE ROOTS

Cube:

The **cube of a number** is obtained by multiplying the number by itself three times.

Formula: $x^3 = x \times x \times x$

Examples:

$2^3 = 2 \times 2 \times 2 = 8$

$5^3 = 5 \times 5 \times 5 = 125$

Cube Root:

The **cube root** of a number is the number which, when multiplied three times by itself, gives the original number.

Formula: $\sqrt[3]{x} = p$ such that $p^3 = x$

Examples:

$\sqrt[3]{8} = 2$ (since $2 \times 2 \times 2 = 8$)

$\sqrt[3]{27} = 3$ (since $3 \times 3 \times 3 = 27$)

LEAST COMMON MULTIPLE (LCM)

The LCM of two or more numbers is the smallest number that is exactly divisible by each of them.

METHODS TO FIND LCM

1. **Prime Factorization Method:**
   - Express each number as a product of prime factors.
   - Take all the prime factors with the **highest powers.**
   - Multiply them to get the LCM.

Example: Lowest common multiple of 8, 48 and 80

$8 = 2 \times 2 \times 2$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$80 = 2 \times 2 \times 2 \times 2 \times 5$

$\Rightarrow$ LCM (8, 48, 80) $= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 480$

2. **Division Method:**

   Divide the numbers by common prime numbers until all quotients become 1.

   Product of all divisors = LCM

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Example:

LCM of 18, 54, 108:

2	18 – 54 – 108
2	9 – 27 – 54
3	9 – 27 – 27
3	3 – 9 – 9
3	1 – 3 – 3
	1 – 1 – 1

Therefore, LCM (18, 54, 108) = 2 × 2 × 3 × 3 × 3 = 108.

FACTORS

If a number p divides q exactly, then p is a factor of q.

i.e., $q \div p$ leaves no remainder.

Examples:

- Factors of 6 = 1, 2, 3, 6
- Factors of 10 = 1, 2, 5, 10

Example:

60 = 2 × 2 × 3 × 5

→ Factors can be obtained by combining prime factors in all possible ways.

HIGHEST COMMON FACTOR (HCF) OR GCD (GREATEST COMMON DIVISOR)

The HCF/GCD of two or more numbers is the greatest number that divides each of them exactly.

Methods to find HCF:

1. Prime Factorization Method:

- Find the common prime factors with their **lowest powers.**
- Multiply them to get HCF.

Example: HCF of 9 & 12:

9 = 3 × 3

12 = 2 × 2 × 3

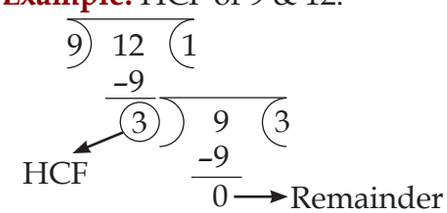
⇒ HCF = 3^1 = 3

2. Division Method:

Divide the larger number by the smaller; then divide the divisor by the remainder. Continue until remainder = 0.

The last divisor = HCF.

- **Example:** HCF of 9 & 12:



❑ Decimals Fraction

$0.1 = \frac{1}{10}$

$0.23 = \frac{23}{100}$

$43.012 = \frac{43012}{1000}$

$\frac{1}{10} = 0.1$

$\frac{145}{1000} = 0.145$

$\frac{21}{10000} = 0.0021$

❑ Addition and Subtraction of like fractions

(i) $\frac{2}{5}+\frac{13}{5}=\frac{2+13}{5}=\frac{15}{5}=3$

(ii) $\frac{7}{5}+\frac{3}{5}-\frac{6}{5}=\frac{7+3-6}{5}=\frac{4}{5}$

❑ Addition of Unlike Fractions

(i) $\frac{2}{5}+\frac{4}{7}$

LCM of 5 and 7 = 35

$=\frac{14+20}{35}=\frac{34}{35}$

(ii) $\frac{2}{8}+\frac{5}{12}$

LCM of 8 and 12 = 96

$=\frac{24+70}{96}=\frac{94}{96}=\frac{47}{48}$

❑ Addition and Subtraction of Fractions

(i) $\frac{4}{6}+\frac{3}{8}-\frac{5}{12}$

LCM of 6, 8 and 12 = 24

$=\frac{16+9-10}{24}$

$=\frac{15}{24}=\frac{5}{8}$

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(ii) $\frac{8}{5}+\frac{6}{12}-\frac{7}{30}$

LCM of 5, 12 and 30 = 60

$=\frac{96+30-14}{60}$

$=\frac{112}{60}=\frac{28}{15}=1\frac{13}{15}$

BODMAS RULE

BODMAS

→ Brackets → Orders → Division → Multiplication → Addition → Subtraction

Example 1:

2 + 56 ÷ (22 + 6) × 2

= 2 + 56 ÷ 28 × 2

= 2 + 2 × 2

= 2 + 4 = 6

Example 2:

{15 × 32 ÷ 2 × 5} ÷ 75

$=\frac{15\times16\times5}{75}$

$=\frac{1200}{75}$

= 16

❑ Some Basic Formula

- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- $(a + b)(a - b) = a^2 - b^2$

Examples

1. $(2x + 3y)^2$

   $= (2x)^2 + (3y)^2 + 2(2x)(3y)$

   $= 4x^2 + 9y^2 + 12xy$

2. $(x + 3)(x - 3)$

   $= x^2 - 3^2$

   $= x^2 - 9$

Q1. Solve x + 5 = 12.

Sol: Given; x + 5 = 12

⇒ x = 12 – 5 = 7

Q2. Solve $x - 9 = 3$.

Sol: Given; $x - 9 = 3$

$\Rightarrow x = 3 + 9 = 12$

Q3. Solve $3x = 27$.

Sol: Given; $3x = 27$

$\Rightarrow x = \frac{27}{3} = 9$

Q4. Solve $\frac{x}{4} = 6$.

Sol: Given; $\frac{x}{4} = 6$

$\Rightarrow x = 4 \times 6 = 24$

Q5. Solve $12 - 3x = 6$.

Sol: Given; $12 - 3x = 6$

$\Rightarrow -3x = 6 - 12 = -6$

$\Rightarrow -3x = -6 \Rightarrow x = \frac{-6}{-3} \Rightarrow x = 2$

Q6. Solve $5x + 3 = 3x + 11$.

Sol: Given; $5x + 3 = 3x + 11$

$\Rightarrow 5x - 3x = 11 - 3 \Rightarrow 2x = 8 \Rightarrow x = 4$

Q7. Solve $2(x + 3) = 4x - 6$.

Sol: Given; $2x + 6 = 4x - 6$

$\Rightarrow 6 + 6 = 4x - 2x \Rightarrow 12 = 12x \Rightarrow x = 6$

Q8. Solve $6(x + 4) = 3(x + 10)$.

Sol: Given; $6(x + 4) = 3(x + 10)$

$\Rightarrow 6x + 24 = 3x + 30$

$\Rightarrow 3x = 6 \Rightarrow x = 2$

PRACTICE QUESTIONS

1. Which of the following is a natural number?

(a) 0 (b) –2 (c) 3 (d) $\sqrt{2}$

2. Which of the following is a whole number but not a natural number?

(a) 1 (b) 0 (c) 5 (d) –1

3. Which of the following is a rational number?

(a) $\sqrt{5}$ (b) π (c) $\frac{3}{4}$ (d) $\sqrt{2}$

4. Which of these is an irrational number?

(a) 7 (b) $\frac{2}{5}$ (c) 1.414213… (d) –6

5. $\sqrt{-9}$ represents which type of number?

(a) Real (b) Imaginary (c) Natural (d) Whole

6. Which of the following is an even number?

(a) 9 (b) 12 (c) 15 (d) 17

7. Which number is both even and prime?

(a) 2 (b) 3 (c) 5 (d) 7

8. Identify the composite numbers:

(a) 4, 6, 8 (b) 2, 3, 5 (c) 7, 11 (d) 13, 17

9. The value of $|-25|$ is:

(a) –25 (b) 0 (c) 25 (d) None of these

10. The value of [7.6] + [–1.2] is:

(a) 7 (b) 6 (c) 5 (d) –5

11. The square of 18 is:

(a) 256 (b) 324 (c) 289 (d) 336

12. The cube of 4 is:

(a) 16 (b) 32 (c) 64 (d) 128

13. $\sqrt{49}$ is equal to:

(a) 6 (b) 7 (c) 8 (d) 9

14. $\sqrt[3]{125}$ is equal to:

(a) 4 (b) 3 (c) 5 (d) 25

15. If $x^2 = 169$, then x = ?

(a) 13 only (b) –13 only (c) ±13 (d) None

16. Which of the following are all factors of 12?

(a) 1, 3, 6, 12 (b) 1, 2, 3, 4, 6, 12

(c) 2, 4, 6, 12 (d) 3, 6, 12

17. Find HCF of 20 and 28.

(a) 2 (b) 4 (c) 6 (d) 8

18. Find LCM of 15 and 20.

(a) 30 (b) 45 (c) 60 (d) 75

19. Solve: x + 5 = 12

(a) 5 (b) 7 (c) 12 (d) 17

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20. Solve: $x - 9 = 3$

(a) 3 (b) 6 (c) 9 (d) 12

21. Solve: $3x = 27$

(a) 6 (b) 8 (c) 9 (d) 12

22. Solve: $\frac{x}{4} = 6$

(a) 12 (b) 18 (c) 20 (d) 24

23. Solve: $12 - 3x = 6$

(a) 1 (b) 2 (c) 3 (d) 4

24. Solve: $5x + 3 = 3x + 11$

(a) 2 (b) 3 (c) 4 (d) 5

25. Solve: $2(x + 3) = 4x - 6$

(a) 4 (b) 5 (c) 6 (d) 8

26. Solve: $6(x + 4) = 3(x + 10)$

(a) 1 (b) 2 (c) 3 (d) 4

Answer Key

1. (c)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (c)
11. (c)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (b)
21. (d)	**22.** (c)	**23.** (d)	**24.** (b)	**25.** (c)	**26.** (c)				

❑❑❑

PART

A

Business Mathematics

CHAPTER

01

Ratio and Proportion, Indices, Logarithms

RATIO

DEFINITION OF RATIO

A ratio is a comparison between two or more quantities of the same kind, where by "same kind" we means that they are similar in nature (like gold is comparable to silver but not to apple).

If a and b are two quantities of the same kind (in same units, E.g. - Kg and gram are two different units of weights).

It is represented by a : b

Then, fraction of a and b is called the ratio of a and b which is written as $\frac{a}{b}$

- The quantities a and b are called **terms** of the ratio.
  E.g.: In 3 : 4, 3 and 4 are terms of ratio.
- a is called the **first term** or **antecedent**.
  E.g.: In 3 : 4, 3 is first term or antecedent.
- b is called the **second term** or **consequent.**
  E.g.: In 3 : 4, 4 is second term or consequent.

Example 1. The ratio of two quantities is 3 : 4. What is antecedent and what is consequent?

(a) 3, 4 (b) 4, 3 (c) 3, 3 (d) None of these

Sol. (a) As per the definition,

The term '3' is the antecedent and the term '4' is the consequent.

Hence, the correct option is (a).

Example 2. The ratio of two quantities is 3 : 4. If the antecedent is 15, the consequent is

(a) 16 (b) 60 (c) 22 (d) 20

Sol. (d) Given: Ratio of two quantities = 3 : 4

Let 'x' be the consequent of the ratio whose antecedent is 15.

According to the problem,

$$\frac{3}{4} = \frac{15}{x}$$

On cross-multiplication, we get

$3x = 4 \times 15$

$3x = 60$

Dividing both sides by 3, we get

$x = \frac{60}{3} = 20$

Alternate solution: Go by choices

Since, $\frac{3}{4} = \frac{15}{x}$ …(i)

Now, for option (a): 16

On substituting x = 16 in eq (i), we have

$\frac{3}{4} = \frac{15}{16}$, which is not true

For option (b): 60

On substituting x = 60 in eq (i), we have

$\frac{3}{4} = \frac{15}{60}$

$\frac{3}{4} = \frac{1}{4}$, which is not true

For option (c): 22

On substituting x = 22 in eq (i), we have

$\frac{3}{4} = \frac{15}{22}$, which is not true

For option (d): 20

On substituting x = 20 in eq (i), we have

$\frac{3}{4} = \frac{15}{20}$

$\frac{3}{4} = \frac{3}{4}$, which is true

Therefore, when the antecedent is 15 in a ratio of 3 : 4, the consequent is 20.

Hence, the correct option is (d).

Example 3. The ratio of height 160 cm and weight 70 kg of a person is

(a) 17 : 6 (b) 17 cm : 60 kg (c) Not a ratio (d) None

Sol. (c) Since, the height and weight are not of same kind, thus it is not a ratio.

Hence, the correct option is (c).

Example 4. The ratio of 25 hours and 45 minutes is

(a) 5 : 9 (b) 100 : 3 (c) Not a ratio (d) None

Sol. (b) The ratio of 25 hours and 45 minutes is given by;

$\frac{25 \text{ hours}}{45 \text{ minutes}} = \frac{25 \times 60 \text{ min}}{45 \text{ min}} = \frac{100}{3} = 100 : 3$

Hence, the correct option is (b).

KEY CONCEPTS REGARDING RATIOS

- Both terms of a ratio can be multiplied or divided by the same (non-zero) number.
  E.g.: If we have a ratio of 2 : 3, we can multiply both terms by 2 to get 4 : 6 and do the similar with division.
- Usually, a ratio is expressed in lowest terms (or simplest form).
  E.g.: If we have a ratio of 6 : 8, we can simplify it to 3 : 4 by dividing both terms by their greatest common divisor, which is 2.
- The order of the terms in a ratio is important.
  E.g.: The ratio 2 : 3 is different from the ratio 3 : 2. They both represent different quantities.
- Ratio exists only between quantities of the same kind.
  E.g.: We can compare the ratio of apples to oranges, but not apples to minutes.

- Quantities to be compared (by division) must be in the same units.

  E.g.: If we want to compare the ratio of the lengths of two objects, one measured in inches and the other in centimeters, we need to convert them to the same unit (e.g., both in inches or both in centimeters) before dividing them.
- To compare two ratios, convert them into equivalent like fractions.

  E.g.: If we have the ratio 2 : 3 and the ratio 4 : 5, we can convert them into equivalent fractions by converting to a common denominator. The ratios become $\frac{10}{15}$ and $\frac{12}{15}$, respectively, and we can then compare them easily.

Example 5. Which ratio is greater?

I. $3\frac{1}{2} : 4\frac{1}{2}$ or $2.5 : 4.5$

(a) $3\frac{1}{2} : 4\frac{1}{2}$ (b) $2.5 : 4.5$

(c) Both are equal (d) Cannot be determined

II. $2\frac{1}{5} : 3\frac{1}{5}$ or $3.6 : 4.0$

(a) $2\frac{1}{5} : 3\frac{1}{5}$ (b) 3.6 : 4.0 (c) Both are equal (d) Cannot be determined

Sol. I. (a) We know,

$3\frac{1}{2} : 4\frac{1}{2}$ can be simplified to 3.5 : 4.5

Now, on comparing 3.5 : 4.5 and 2.5 : 4.5, i.e., $\frac{3.5}{4.5}$ and $\frac{2.5}{4.5}$ we see that the denominators are equal thus we can compare the numerators.

Clearly, 3.5 is greater than 2.5

Thus, 3.5 : 4.5 is greater than 2.5 : 4.5.

Therefore, the ratio $3\frac{1}{2} : 4\frac{1}{2}$ is greater than 2.5 : 4.5.

Another method:

As, $\frac{3.5}{4.5} = 0.778$ & $\frac{2.5}{4.5} = 0.556$

Clearly, the ratio $3\frac{1}{2} : 4\frac{1}{2}$ is greater than 2.5 : 4.5

Hence, the correct option is (a).

II. (b) We know,

$2\frac{1}{5} : 3\frac{1}{5}$ can be simplified to $\frac{11}{5} : \frac{16}{5} = 11 : 16 = \frac{11}{16}$

Also, $3.6 : 4.0 = \frac{36}{40} = \frac{9}{10}$

Making the denominators equal to 80 i.e.,

$\frac{11}{16} \times \frac{5}{5} = \frac{55}{80}$ and $\frac{9}{10} \times \frac{8}{8} = \frac{72}{80}$

On comparing both the ratios, we see

$55 < 72$

$\frac{55}{80} < \frac{72}{80}$

i.e., $2\frac{1}{5} : 3\frac{1}{5} < 3.6 : 4.0$

Therefore, the ratio 3.6 : 4.0 is greater than $2\frac{1}{5} : 3\frac{1}{5}$

Hence, the correct option is (b).

- If a quantity increases or decreases in the ratio a: b. The fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio.

  E.g.: Let's say you have a quantity of 100 apples and it increases in the ratio of 2 : 3
- Then, New quantity = Original quantity × Factor multiplying ratio
- New quantity $= 100 \times \frac{3}{2} = 150$ apples

Example 6. Earlier shoe company produced 2000 shoes in a day. They increased their production 5 : 6. What will be the new production?

(a) 2500 (b) 2400 (c) 2000 (d) 3000

Sol. (b) Given: Original production of the shoe company in a day = 2000

Since, the increased production ratio is given as 5 : 6, thus the new production is 6/5 times the original production.

Thus, New production $= 2000 \times \left(\frac{6}{5}\right) = 2400$

Alternate Solution

Given;

$\frac{\text{Old production}}{\text{New production}} = \frac{5}{6}$

$\Rightarrow \frac{2000}{x} = \frac{5}{6}$

$\Rightarrow x = 2400$

Therefore, the new production of the shoe company is 2400 shoes in a day.

Hence, the correct option is (b).

Example 7. If Rajni eats 12 chapati in a day. If she reduces her chapati by 6 : 5. How many chapati does she eat now?

(a) 12 (b) 8 (c) 10 (d) 15

Sol. (c) Given: Original number of chapatis Rajni eats = 12

Reduced chapati intake ratio = 6 : 5

Thus, New quantity $= 12 \times \left(\frac{5}{6}\right) = \frac{60}{6} = 10$

Therefore, Rajni now eats 10 chapatis.

Hence, the correct option is (c).

Example 8. Simplify the ratio $\frac{1}{2} : \frac{2}{3} : \frac{4}{5}$

(a) 15 : 20 : 24 (b) 10 : 25 : 24 (c) 2 : 3 : 5 (d) 1 : 2 : 4

Sol. (a) We know that,

L.C.M. (2, 3, 5)

2	2	–3	–5
3	1	–3	–5
5	1	–1	–5
	1	–1	–1

L.C.M. = 2 × 3 × 5 = 30

Thus, making the denominator of each ratio equal to 30, we get

$\frac{1}{2} \times \frac{15}{15} = \frac{15}{30}$

$\frac{2}{3} \times \frac{10}{10} = \frac{20}{30}$

$\frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$

Therefore, the simplified form of given ratios is 15 : 20 : 24.

Hence, the correct option is (a).

Example 9. Simplify the ratio $\frac{1}{3} : \frac{1}{8} : \frac{1}{6}$ **(ICAI)**

(a) 8 : 3 : 4 (b) 8 : 5 : 1 (c) 7 : 2 : 6 (d) 1 : 3 : 5

Sol. (a) We know that,

L.C.M (3, 8, 6)

2	3	–8	–6
2	3	–4	–3
2	3	–2	–3
3	3	–1	–3
	1	–1	–1

L.C.M (3,8,6) = 2 × 2 × 2 × 3 = 24

Thus, making the denominator of each ratio equal to 24, we get

$\frac{1}{3}\times\frac{8}{8}=\frac{8}{24}$

$\frac{1}{8}\times\frac{3}{3}=\frac{3}{24}$

$\frac{1}{6}\times\frac{4}{4}=\frac{4}{24}$

Therefore, the simplified form of given ratios is 8 : 3 : 4

Hence, the correct option is (a).

Example 10. Anand earns ₹80 in 7 hours and Promod ₹90 in 12 hours. The ratio of their earnings is **(ICAI)**

(a) 32 : 21 (b) 23 : 12 (c) 8 : 9 (d) None of these

Sol. (a) Given that,

Earning of Anand in 7 hours = ₹80

Earning of Promod in 12 hours = ₹90

Thus, earning of Anand in 1 hour $=\frac{80}{7}$

Earning of Promod in 1 hour $=\frac{90}{12}$

Therefore, Ratio of their earnings $=\frac{80}{7}\div\frac{90}{12}=\frac{80\times12}{7\times90}=\frac{32}{21}$

Hence, the correct answer is option (a) i.e., 32 : 21

Example 11. Mr X left 10,000 rupees for his two sons A and B who divides this money in 2:3. What is the share of A and B?

(a) ₹4000, ₹6000 (b) ₹2000, ₹8000

(c) ₹6000, ₹4000 (d) None of the above

Sol. (a) Given; Ratio of shares of A and B = 2 : 3

Total shares = ₹10,000

Let the share of A be 2x and B be 3x, then

2x + 3x = 10000

5x = 10000

x = 2000

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Therefore, the share of A = 2(2000) = ₹4000

Share of B = 3(2000) = ₹6000

Example 12. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 km in 5 hours, the speed of the first train is **(ICAI)**

(a) 10 km/hr (b) 50 km/hr (c) 70 km/hr (d) None of these

Sol. (c) Let the speed of two trains be 7x km/hr and 8x km/hr.

It is given that the second train runs 400 km in 5 hours, thus we have

Speed = Distance/Time

$$8x = \frac{400}{5}$$

$8x = 80 \Rightarrow x = 10$

Therefore, the speed of the first train is $7 \times 10 = 70$km/hr

Hence, the correct answer is option (c) i.e., 70 km/hr.

Example 13. The ratio of two numbers is 7 : 10 and their difference is 105. The numbers are **(ICAI)**

(a) (200, 305) (b) (185, 290) (c) (245, 350) (d) None of these

Sol. (c) Given that, Ratio of numbers = 7 : 10

Difference between the numbers = 105

Then, the numbers are 7x and 10x respectively.

As per question,

Difference between the numbers = 105

10x - 7x = 105

$\Rightarrow 3x = 105 \Rightarrow x = 35$

Therefore, the numbers are:

$7x = 7 \times 35 = 245$ and $10x = 10 \times 35 = 350$

Hence, the correct option is (c) i.e., (245, 350).

Example 14. Division of ₹324 between X and Y is in the ratio 11 : 7. X & Y would get rupees

(a) (204, 120) (b) (200, 124) (c) (180, 144) (d) None of these **(ICAI)**

Sol. (d) We need to divide ₹324 between X and Y in the ratio 11 : 7.

Let X's shares be 11a

Let Y's shares be 7a

Since, sum of their shared amount = ₹324

11a + 7a = 324

$\Rightarrow 18a = 324$

$$\Rightarrow a = \frac{324}{18}$$

$\Rightarrow a = 18$

Therefore, X's shares = 11 × 18 = 198

And Y's shares = 7 × 18 = 126

Hence, the correct option is (d).

Example 15. If the salary of P is 25% lower than that of Q and the salary of R is 20% higher than that of Q, the ratio of the salary of R and P will be

(a) 5 : 8 (b) 8 : 5 (c) 5 : 3 (d) 3 : 5

Sol. (b) Given, Salary of P is 25% lower than that of Q i.e.,

$$P = Q - 25\% \text{ of } Q = Q - \frac{25}{100}Q = \frac{75}{100}Q = 0.75Q$$

$$\Rightarrow \frac{P}{Q} = 0.75 \quad ...(i)$$

Also, salary of R is 20% higher than that of Q i.e,

$$R = Q + 20\% \text{ of } Q = Q + \frac{20}{100}Q = \frac{120}{100}Q = 1.2Q$$

$$\Rightarrow \frac{R}{Q} = 1.2 \quad ...(ii)$$

Thus, $\frac{R}{P} = \frac{R}{Q} \times \frac{Q}{P} = \frac{1.2}{0.75}$

$$\Rightarrow \frac{R}{P} = 1.6 = \frac{16}{10} = \frac{8}{5}$$

Therefore, the ratio of the salary of R and P is 8 : 5.

Hence, the correct option is (b).

Example 16. The ratio of the number of boys to the number of girls in a school of 720 students is 3 : 5. If 18 new girls are admitted in the school, then find how many new boys may be admitted so that the ratio of the number of boys to the number of girls may change to 2 : 3.

(a) 40 (b) 42 (c) 45 (d) None of these

Sol. (b) Given: Total students = 720

Ratio of number of boys to girls = 3 : 5

Let the number of boys and number of girls be 3a and 5a.

Thus, 3a + 5a = 720

$\Rightarrow$ 8a = 720

$\Rightarrow$ a = 90

So, number of boys = 3(90) = 270 and number of girls = 5(90) = 450

Now, 18 new girls are admitted in the school

Thus, new girls = 450 + 18 = 468

Let 'x' be the number of boys admitted in the school.

According to the question,

$$\frac{270 + x}{468} = \frac{2}{3}$$

$\Rightarrow$ 810 + 3x = 936

$\Rightarrow 3x = 126$

$\Rightarrow x = 42$

Therefore, the new number of boys admitted in the school is 42.

Hence, the correct option is (b).

INVERSE RATIO

One ratio is the inverse of the other if their product is 1.

E.g.: b : a is the inverse ratio of a: b since $\frac{a}{b} \times \frac{b}{a} = 1$

'or' Inverse ratio of a : b = b : a

Example 17. The inverse ratio of 11 : 15 is

(a) 15 : 11 (b) $\sqrt{11} : \sqrt{15}$ (c) 121 : 225 (d) None of these

Sol. (a) As per the definition,

The inverse ratio of 11 : 15 is 15 : 11

Hence, the correct option is (a).

Example 18. The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 25, the antecedent is

(a) 7 (b) $\sqrt{35}$ (c) 35 (d) None of these

Sol. (c) Given: Ratio of the quantities = 5 : 7

Thus, the inverse ratio of the given ratio = 7 : 5

Also, the consequent of its inverse ratio = 25

Let x be the antecedent of the ratio whose consequent is 25, thus

$\frac{7}{5} = \frac{x}{25}$

$\Rightarrow x = 7 \times 5 \Rightarrow x = 35$

Therefore, the required antecedent is 35.

Hence, the correct option is (c).

Greater Inequality Ratio	Less Inequality Ratio
A ratio a : b will be of greater Inequality if a > b	A ratio a : b will be of less inequality if a < b

The ratio 2 : 1 represents a greater inequality because the first term (2) is greater than the second term (1).

The ratio 3 : 4 represents less inequality because the first term (3) is smaller than the second term (4).

Example 19. The ratio of 8 : 15 is

(a) Greater inequality ratio (b) Less inequality ratio

(c) Equal to 1 (d) None of these

Sol. (b) We know that,

A ratio a: b will be of less inequality if a < b

In ratio 8 : 15,

8 < 15 i.e, a < b

Therefore, the ratio of 8 : 15 is less inequality ratio.

Hence, the correct option is (b).

Example 20. The ratio of 20 : 15 is

(a) Greater inequality ratio (b) Less inequality ratio

(c) Equal to 1 (d) None of these

Sol. (a) We know that,

A ratio a : b will be of greater inequality if a > b.

In ratio 20:15,

20 > 15 i.e, a > b

Therefore, the ratio of 20 : 15 is a greater inequality ratio.

Hence, the correct option is (a).

COMPOUNDED RATIO

Product of two or more ratios is called ratio compounded.

E.g.: If there are ratios 5 : 6, 7 : 8 and 9 : 10 then the compounded ratio will be:

(5 × 7 × 9) : (6 × 8 × 10)

$$= \frac{5\times7\times9}{6\times8\times10} = \frac{7\times3}{2\times8\times2} = \frac{21}{32}$$

Example 21. The ratio compounded of 5 : 6,9 : 2,4 : 3 and 1 : 5 is

(a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) None of these

Sol. (a) We know that,

The compounded ratio of 5 : 6, 9 : 2, 4 : 3 and 1 : 5 is:

$$= \frac{5\times9\times4\times1}{6\times2\times3\times5} = \frac{180}{180} = 1:1$$

Hence, the correct option is (a).

Example 22. The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is

(a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) None of these

Sol. (a) We know that,

The compounded ratio of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is:

$$= \frac{2\times9\times5\times8}{3\times4\times6\times10} = \frac{720}{720} = 1:1$$

Hence, the correct option is (a).

Example 23. If A : B = 2: 3, B : C = 5 : 7, then A : B : C = ?

(a) 10 : 15 : 21

(b) 2 : 3 : 7

(c) 4 : 8 : 9

(d) none of these

Sol. (a) Given; A : B = 2 : 3 and B : C = 5 : 7

i.e., $\frac{A}{B}=\frac{2}{3}$ and $\frac{B}{C}=\frac{5}{7}$

Then,

$\frac{A}{B}=\frac{2}{3}\times\frac{5}{5}=\frac{10}{15}$

$\frac{B}{C}=\frac{5}{7}\times\frac{3}{3}=\frac{15}{21}$

A : B : C = 10 : 15 : 21

Another method:

a : b : c : d

2 : 3 : 5 : 7

10 : 15 : 21

Hence, the correct option is (a).

Example 24. If A : B = 1 : 2, B : C = 1 : 3 and C : D = 3 : 4. Find A : B : C : D.

(a) 4 : 8 : 24 : 27 (b) 1 : 2 : 6 : 8 (c) 3 : 6 : 8 : 10 (d) None

Sol. (b) Given; $\frac{A}{B}=\frac{1}{2}, \frac{B}{C}=\frac{1}{3}$ and $\frac{C}{D}=\frac{3}{4}$

$\frac{A}{B}=\frac{1}{2}$

$\frac{B}{C}=\frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$

$\frac{C}{D}=\frac{3}{4}\times\frac{2}{2}=\frac{6}{8}$

Therefore, A : B : C : D = 1 : 2 : 6 : 8

Trick: Go by options

For option (a): 4 : 8 : 24 : 27

$4:8=\frac{4}{8}=\frac{1}{2}, 8:24=\frac{8}{24}=\frac{1}{3}$ and $\frac{24}{27}=\frac{8}{9}$, which does not satisfies the given conditions

For option (b): 1 : 2 : 6 : 8

$1:2=\frac{1}{2}, 2:6=\frac{1}{3}$ and $\frac{6}{8}=\frac{3}{4}$ which satisfies all the given conditions

For option (c): 3 : 6 : 8 : 10

$3:6=\frac{3}{6}=\frac{1}{2}, 6:8=\frac{3}{4}$ and $\frac{8}{10}=\frac{4}{5}$, which does not satisfies the given conditions

Hence, the correct option is (b).

Example 25. If a : b = 2 : 3, b : c = 4 : 5 and c : d = 6 : 7, then a : d is

(a) 24 : 35 (b) 8 : 15

(c) 16 : 35 (d) 7 : 15

Sol. (c) Given, a : b = 2 : 3, b : c = 4 : 5 and c : d = 6 : 7

i.e., $\frac{a}{b}=\frac{2}{3}, \frac{b}{c}=\frac{4}{5}$ and $\frac{c}{d}=\frac{6}{7}$

Thus, $\frac{a}{d}=\frac{a}{b}\times\frac{b}{c}\times\frac{c}{d}=\frac{2}{3}\times\frac{4}{5}\times\frac{6}{7}=\frac{16}{35}$

Therefore, a : d = 16 : 35

Hence, the correct option is (c).

Example 26. P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 16 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is

(a) 22 : 27 (b) 27 : 22

(c) 32 : 33 (d) None of these

Sol. (b) Given: Ratio of average temperature between P and Q = 11 : 12

Ratio of average temperature between P and R = 9 : 8

i.e., $\frac{P}{Q}=\frac{11}{12}$ and $\frac{P}{R}=\frac{9}{8}$

Thus, $\frac{Q}{R}=\frac{P}{R}\div\frac{P}{Q}$

$=\frac{9}{8}\div\frac{11}{12}=\frac{9}{8}\times\frac{12}{11}=\frac{27}{22}$

Therefore, the ratio between the average temperature of Q and R is 27 : 22.

Hence, the correct option is (b).

- When the ratio of two similar quantities can be expressed as a ratio of two integers, the quantities are considered to be **Commensurable**.
- On the other hand, if the ratio cannot be expressed as a ratio of two integers, the quantities are considered to be **Incommensurable**.

DUPLICATE RATIO

A ratio compound to itself is called a Duplicate ratio. For ratio a : b, its duplicate ratio will be $a^2 : b^2$

E.g.: For ratio 2 : 3, its duplicate ratio will be $2^2 : 3^2 = 4 : 9$

TRIPLICATE RATIO

For ratio a : b, its triplicate ratio will be $a^3 : b^3$

E.g.: For ratio 2 : 3, its triplicate ratio will be $2^3 : 3^3 = 8 : 27$

Example 27. The duplicate ratio of 5 : 9 is

(a) 81 : 25 (b) 9 : 5 (c) 25 : 81 (d) None of these

Sol. (c) We know that, $a^2 : b^2$ is the duplicate ratio of a : b.

Thus, the duplicate ratio of 5 :9 is

$5^2 : 9^2 = 25 : 81$

Hence, the correct option is (c).

Example 28. The triplicate ratio of 5 : 7 is

(a) 25 : 49 (b) 5 : 7 (c) 125 : 343 (d) None of these

Sol. (c) We know that, $a^3 : b^3$ is the triplicate ratio of a : b

Thus, the triplicate ratio of 5 : 7

$= 5^3 : 7^3 = 125 : 343$

Hence, the correct option is (c).

SUB DUPLICATE RATIO

For a : b, the sub duplicate ratio will be $\sqrt{a} : \sqrt{b}$

E.g.: For ratio 4 : 9, its sub duplicate ratio will be $\sqrt{4} : \sqrt{9} = 2 : 3$

SUB TRIPLICATE RATIO

For a : b, the sub triplicate ratio will be $\sqrt[3]{a} : \sqrt[3]{b}$

E.g.: For ratio 8 : 27, its sub triplicate ratio will be $\sqrt[3]{8} : \sqrt[3]{27} = 2 : 3$

Example 29. The sub-duplicate ratio of 121 : 25 is

(a) 6 : 5 (b) 36 : 25 (c) 50 : 72 (d) 11 : 5

Sol. (d) We know that, $\sqrt{a} : \sqrt{b}$ is the sub-duplicate ratio of a : b.

Therefore, the sub – duplicate ratio of 121 : 25 is

$\sqrt{121} : \sqrt{25} = 11 : 5$

Hence, the correct option is (d).

Example 30. The sub - triplicate ratio of 64 : 27 is

(a) 16 : 9 (b) 4 : 3 (c) 27 : 64 (d) None of these

Sol. (b) We know that, $\sqrt[3]{a} : \sqrt[3]{b}$ is the sub triplicate ratio of a : b.

Therefore, the sub – triplicate ratio of 64 : 27 is

$\sqrt[3]{a} : \sqrt[3]{b} = \sqrt[3]{64} : \sqrt[3]{27} = 4 : 3$

Hence, the correct option is (b).

Example 31. The ratio compounded of triplicate ratio of 2 : 3 and the duplicate ratio of 3 : 4 is

(a) 1 : 4 (b) 2 : 3 (c) 1 : 6 (d) None of these

Sol. (c) We know that,

$a^2 : b^2$ is the duplicate ratio of a : b.

$a^3 : b^3$ is the triplicate ratio of a : b.

Thus, the triplicate ratio of 2 : 3 is $2^3 : 3^3 = 8 : 27$

The duplicate ratio of 3 : 4 is $3^2 : 4^2 = 9 : 16$

Therefore, the compounded ratio of 8 : 27 and 9 : 16 is

$$= \frac{8 \times 9}{27 \times 16} = \frac{1}{6}$$

Hence, the correct option is (c).

Example 32. The ratio compounded of duplicate ratio of 4 : 5, triplicate ratio of 3 : 2, sub duplicate ratio 25 : 81 and sub - triplicate ratio of 1000 : 27 is

(a) 1 : 4 (b) 3 : 25 (c) 4 : 1 (d) None of these

Sol. (c) As per the definitions,

The duplicate ratio of 4 : 5 is $4^2 : 5^2 = 16:25$

The triplicate ratio of 3 : 2 is $3^3 : 2^3 = 27:8$

The sub duplicate ratio 25 : 81 is $\sqrt{25} : \sqrt{81} = 5 : 9$

The sub – triplicate ratio of 1000 : 27 is $\sqrt[3]{1000} : \sqrt[3]{27} = 10 : 3$

Therefore, the compounded ratio of 16 : 25, 27 : 8, 5 : 9 and 10 : 3 is

$$= \frac{16 \times 27 \times 5 \times 10}{25 \times 8 \times 9 \times 3} = \frac{2 \times 2}{1} = \frac{4}{1}$$

Therefore, the required compounded ratio is 4 : 1

Hence, the correct option is (c).

Example 33. If A : B = 2 : 5, then (10A + 3B): (5A + 2B) is equal to:

(a) 7 : 4 (b) 7 : 3 (c) 6 : 5 (d) 7 : 9

Sol. (a) Given: A : B = 2 : 5

Let A = 2x and B = 5x

Then, (10A + 3B) : (5A+2B)

$$= \frac{10A + 3B}{5A + 2B} = \frac{10(2x) + 3(5x)}{5(2x) + 2(5x)} = \frac{20x + 15x}{10x + 10x} = \frac{35x}{20x} = \frac{7}{4}$$

Hence, the correct option is (a).

Example 34. If 2s : 3t is the duplicate ratio of 2s - p : 3t - p, then **(ICAI)**

(a) $p^2 = 6st$ (b) $p = 6st$ (c) $2p = 3st$ (d) None of these

Sol. (a) We know that,

$a^2 : b^2$ is the duplicate ratio of a : b.

Since, 2s : 3t is the duplicate ratio of 2s - p : 3t - p

So, the duplicate ratio of $2s - p : 3t - p = (2s - p)^2 : (3t - p)^2$

According to given condition,

$\Rightarrow (2s - p)^2 : (3t - p)^2 = 2s : 3t$

$\Rightarrow 4s^2 + p^2 - 4sp : 9t^2 + p^2 - 6tp = 2s : 3t$

On cross multiplying,

$\Rightarrow 12s^2t + 3p^2t - 12spt = 18st^2 + 2sp^2 - 12stp$

$\Rightarrow 12s^2t + 3p^2t = 18st^2 + 2sp^2$

$\Rightarrow p^2(3t - 2s) = 6st(3t - 2s)$

$\Rightarrow p^2 = 6st$

Hence, the correct option is (a).

Example 35. If p : q is the sub-duplicate ratio of $p - x^2 : q - x^2$, then x^2 is **(ICAI, (2006, 2019))**

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$ (c) $\frac{pq}{p+q}$ (d) None of these

Sol. (c) **Detailed method:**

Given: p : q is the sub-duplicate ratio of $p - x^2 : q - x^2$

Thus, $\frac{p}{q} = \frac{\sqrt{p - x^2}}{\sqrt{q - x^2}}$

On squaring both the sides, we get

$\frac{p^2}{q^2} = \frac{p - x^2}{q - x^2}$

On cross-multiplication, we get

$p^2q - x^2p^2 = pq^2 - q^2x^2$

$\Rightarrow p^2q - pq^2 = x^2(p^2 - q^2)$

$\Rightarrow pq(p - q) = x^2(p - q)(p + q)$

$\Rightarrow pq = x^2(p + q)$

$\Rightarrow x^2 = \frac{pq}{p+q}$

Alternate solution:

Go by choices: For option (c)

When $x^2 = \frac{pq}{p+q}$, then

$$p - x^2 = p - \frac{pq}{p+q} = \frac{p^2}{p+q}$$

And $q - x^2 = q - \frac{pq}{p+q} = \frac{q^2}{p+q}$

Thus, the sub-duplicate ratio of $p - x^2 : q - x^2$, is

$$\sqrt{\frac{p^2}{p+q}} : \sqrt{\frac{q^2}{p+q}} = \frac{\sqrt{\frac{p^2}{p+q}}}{\sqrt{\frac{q^2}{p+q}}} = \frac{\sqrt{p^2}}{\sqrt{q^2}} = \frac{p}{q}$$

= p : q, which is true

Hence, the correct option is (c).

CONTINUED RATIO

It is the comparison between the magnitudes of three or more quantities of the same kind. The continued ratio of three similar quantities a,b,c is written as a : b : c.

E.g.: Consider three similar quantities: Lengths of three sides of a triangle.

Let's say the lengths of the sides are a = 2 units, b = 4 units and c = 6 units.

The continued ratio of these side lengths is written as a : b : c, which in this case is 2 : 4 : 6.

This indicates the side lengths in comparison to each other. The ratio can be simplified to 1 : 2 : 3 by dividing all the terms by their greatest common divisor, which in this case is 2.

Example 36. The angles of a triangle are in ratio 2 : 7 : 11. The angles are **(ICAI)**

(a) (20°, 70°, 90°) (b) (30°, 70°, 80°)

(c) (18°, 63°, 99°) (d) None of these

Sol. (c) Given: Ratio of angles of triangle = 2 : 7 : 11

Let the angles of the triangle be 2x, 7x, 11x.

We know that, the sum of interior angles of a triangle is 180°.

$\Rightarrow$ 2x + 7x + 11x = 180°

$\Rightarrow$ 20x = 180°

$\Rightarrow x = \frac{180°}{20}$

$\Rightarrow$ x = 9

Thus, the angles of the triangle is 2 × 9°, 7 × 9°, 11 × 9° i.e., (18°, 63°, 99°)

Hence, option (c) is correct i.e., (18°, 63°, 99°).

Example 37. Find three numbers which are in the ratio 1 : 2 : 3, such that the sum of their squares is equal to 504. **(Dec 2013)**

(a) 6, 12, 18 (b) 3, 6, 9 (c) 4, 8, 12 (d) 5, 10, 15

Sol. (a) Given: Ratio of three numbers = 1 : 2 : 3

Let the three numbers be x, 2x and 3x

According to the question,

Sum of their squares = 504

$\Rightarrow x^2 + (2x)^2 + (3x)^2 = 504$

$\Rightarrow x^2 + 4x^2 + 9x^2 = 504$

$\Rightarrow 14x^2 = 504$

$\Rightarrow x^2 = 36$

$\Rightarrow x = 6$

Therefore, the three numbers are 6, 6 × 2, 6 × 3 i.e., 6, 12, 18.

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART-A)

1. The antecedent and consequent in the ratio 5 : 16 is ____ and ____ respectively.

(a) 5, 5 (b) 16, 5 (c) 5, 16 (d) 16, 16

2. The ratio of two quantities is 2 : 5. If the antecedent is less than the consequent by Rs. 9000, the consequent will be …….

(a) 3000 (b) 6000 (c) 9000 (d) 15000

3. The ratio of two quantities is 5 : 8. If the consequent is 40, then the antecedent is

(a) 25 (b) 40 (c) 5 (d) 8

4. In a school, where the ratio of boys to girls is 7 : 5, if the total number of students is 2400, how many girls are enrolled in the school?

(a) 500 (b) 1000 (c) 2000 (d) None of these

5. A bag contains 25 paise, 10 paise, 5 paise are in the ratio 3 : 2 : 1. The total value of coins is ₹40, then the number of 5 paise coin is:

(a) 40 (b) 80 (c) 240 (d) 480

6. If the division of ₹252 between A and B is in the ratio 5 : 4, how much will A receive?

(a) ₹140 (b) ₹120 (c) ₹112 (d) ₹100

7. The ratio of boys and girls in a college is 7 : 5. If the number of boys increases by 40% and the number of girls increases by 20%, what will be the new ratio of boys and girls?

(a) 4 : 2 (b) 30 : 49 (c) 49 : 30 (d) 49 : 47

8. The ratio of two numbers is 2 : 9, and their difference is 91. What are the numbers?

(a) (13, 117) (b) (26, 91) (c) (26, 117) (d) None of these

9. Three friends A, B, and C shared a total amount of ₹3,000 in the ratio of 5 : 3 : 2. How much did B receive?

(a) ₹700 (b) ₹750 (c) ₹900 (d) ₹300

10. If x : y = 5 : 2 then the value of (8x + 9y) : (8x + 2y) is

(a) 22 : 29 (b) 26 : 61 (c) 29 : 22 (d) 61 : 26

11. Find the compounded ratio of 3 : 5, 2 : 3, 5 :1 and 4 : 3.

(a) 4 : 1 (b) 4 : 5 (c) 3 : 8 (d) 8 : 3

12. The compounded ratio of inverse ratio of 15 : 28, sub-duplicate ratio of 36 : 49 and the triplicate ratio of 5 : 4 is

(a) 25 : 7 (b) 15 : 4 (c) 25 : 8 (d) None of these

13. Find the compounded ratio of 7 : 4, duplicate ratio of 7 : 8, triplicate ratio of 2 : 7, sub-duplicate ratio of 16 : 25 and sub-triplicate ratio of 125 : 343.

(a) 1 : 40 (b) 4 : 45 (c) 1 : 56 (d) 7 : 58

14. If p : q = 2 : 3 and x : y = 4 : 5, then the value of 5px + 3qy : 10px + 4qy is:

(a) $\frac{2}{9}$ (b) $\frac{19}{28}$ (c) $\frac{4}{19}$ (d) None of above

Answer Key

1. (c) **2.** (d) **3.** (a) **4.** (b) **5.** (a) **6.** (a) **7.** (c) **8.** (c) **9.** (c) **10.** (c)
11. (d) **12.** (c) **13.** (c) **14.** (b)

PROPORTION

WHAT IS PROPORTION?

- An equality of two ratios is called a proportion. If we take four quantities a, b, c, d, they will be said to be in proportion if a/b = c/d i.e., ad = bc
- a, b, c and d are called its **terms** of the proportion.
- First and fourth terms are called **extremes.**
- Second and third terms are called **means (or middle terms).**
- Also, it can be written as **a : b :: c : d.**

E.g.: Consider four quantities: a = 2, b = 4, c = 3, and d = 6.

In this case, the terms of the proportion are 2, 4, 3 and 6.

The first and fourth terms, a = 2 and d = 6, are called the extremes.

The second and third terms, b = 4 and c = 3, are called the means or middle terms.

We can express this proportion as a : b :: c : d, which indicates the equality of the ratios a/b and c/d.

The ratio a/b is 2/4, which can be simplified to 1/2.

The ratio c/d is 3/6, which can also be simplified to 1/2.

Thus, we can say a = 2, b = 4, c = 3, and d = 6 are in proportion.

CROSS PRODUCT RULE

Product of extremes = Product of means

Let's consider the following proportion: a, b, c and d

i.e., $\frac{a}{b}=\frac{c}{d}$

In this proportion, a and d are the extremes, while b and c are the means.

The cross product rule states that the product of the extremes (a × d) is equal to the product of the means (b × c)

Example 38. 8, 12, *, 15 are in proportion. Then * is

(a) 20 (b) 10 (c) 15 (d) None of these

Sol. (b) We know that,

If a, b, c and d are in proportion, then a/b = c/d.

Let * be represented by x.

Since, 8, 12, *, 15 are in proportion then

$\frac{8}{12}=\frac{x}{15}$

$\Rightarrow x=\frac{8}{12}\times 15$

$\Rightarrow x = 2 \times 5$

$\Rightarrow x = 10$

Therefore, * is 10.

Hence, the correct option is (b).

Example 39. The fourth proportional to 2, 4, 8 is

(a) 16 (b) 32 (c) 48 (d) None of these

Sol. (a) Let the fourth proportion be x.

We know that,

If a, b, c and d are in proportion, then $\frac{a}{b}=\frac{c}{d}$.

Since, 2, 4, 8 and x are in proportion then

$\frac{2}{4}=\frac{8}{x}$

$\Rightarrow x=\frac{8\times 4}{2}$

$\Rightarrow x = 16$

Therefore, the fourth proportional to 2, 4, 8 is 16.

Hence, the correct option is (a).

Example 40. The fourth proportional to 2a, a^2, c is **(ICAI)**

(a) $\frac{ac}{2}$ (b) ac (c) $\frac{2}{ac}$ (d) None of these

Sol. (a) Let the fourth proportion be x.

We know that,

If a, b, c and d are in proportion, then $\frac{a}{b}=\frac{c}{d}$.

Since, 2a, a^2, c and x are in proportion, then

$$\frac{2a}{a^2}=\frac{c}{x}$$

$$\Rightarrow x=\frac{c\times a^2}{2a}$$

$$\Rightarrow x=\frac{c\times a}{2}$$

$$\Rightarrow x=\frac{ac}{2}$$

Hence, the correct option is (a).

Example 41. If four numbers $\frac{1}{2},\frac{1}{3},\frac{1}{5},\frac{1}{x}$ are proportional then x is **(ICAI)**

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{15}{2}$ (d) None of these

Sol. (c) We know that,

If a, b, c and d are in proportion, then a/b = c/d.

Since, $\frac{1}{2},\frac{1}{3},\frac{1}{5},\frac{1}{x}$ are proportional then

Product of extremes = Product of means

$$\frac{1}{2}\times\frac{1}{x}=\frac{1}{3}\times\frac{1}{5}$$

$$\Rightarrow \frac{1}{2x}=\frac{1}{15}$$

$$\Rightarrow 2x=15$$

$$\Rightarrow x=\frac{15}{2}$$

Hence, the correct option is (c).

Example 42. The number which has the same ratio to 22 that 5 has to 11 is

(a) 11 (b) 10 (c) $\frac{15}{2}$ (d) None of these

Sol. (b) Let the number which has the same ratio to 22 that 5 has to 11 be x, then

x : 22 :: 5 : 11

$$\Rightarrow \frac{x}{22} = \frac{5}{11}$$

$$\Rightarrow x = \frac{22 \times 5}{11}$$

$$\Rightarrow x = 2 \times 5$$

$$\Rightarrow x = 10$$

Therefore, the required number is 10.

Hence, the correct option is (b).

CONTINUOUS PROPORTION

- If 3 quantities: a, b, c are of same kind (in same units), they will be in continuous proportion if a : b = b : c

  i.e., $\frac{a}{b} = \frac{b}{c}$ or $b^2 = ac$
- The middle term b is called the **mean proportional** between a and c.
- a is **first proportional** and c is the **third proportional.**

E.g.: Consider three quantities: a = 2, b = 4 and c = 8

Here, $\frac{a}{b} = \frac{2}{4} = \frac{1}{2}$

$$\frac{b}{c} = \frac{4}{8} = \frac{1}{2}$$

Since, $\frac{a}{b} = \frac{b}{c}$ thus a, b, c are in continuous proportion.

We can say, 2 and 8 are first and third proportional respectively and 4 is the mean proportional.

Example 43. The mean proportional between 16, 25 is

(a) 18 (b) 19 (c) 20 (d) None of these

Sol. (c) We know that,

If b is the mean proportional of a and c then $b^2 = ac$.

Let b be the mean proportional between 16 and 25 then

$b^2 = 16 \times 25$

$$\Rightarrow b = \sqrt{16 \times 25}$$

$$\Rightarrow b = 4 \times 5$$

$$\Rightarrow b = 20$$

Therefore, the mean proportional between 16, 25 is 20.

Hence, the correct option is (c).

Example 44. The third proportional to 2, 6 is

(a) 12 (b) 18 (c) 24 (d) None of these

Sol. (b) Let the third proportional to 2, 6 is x, then

$2 : 6 :: 6 : x$

$\Rightarrow \frac{2}{6} = \frac{6}{x}$

$\Rightarrow x = \frac{6 \times 6}{2}$

$\Rightarrow x = \frac{36}{2}$

$\Rightarrow x = 18$

Therefore, the third proportional to 2, 6 is 18.

Hence, the correct option is (b).

Example 45. The mean proportional between $12x^2$ and $27y^2$ is **(ICAI)**

(a) 18xy (b) 81xy (c) 8xy (d) None of these

Sol. (a) We know that,

If b is the mean proportional of a and c then $b^2 = ac$.

Let b be the mean proportional between $12x^2$ and $27y^2$ then

$b^2 = 12x^2 \times 27y^2$

$\Rightarrow b = \sqrt{12x^2 \times 27y^2}$

$\Rightarrow b = 3 \times 3 \times 2 \times x \times y$

$\Rightarrow b = 18xy$

Therefore, the mean proportional between $12x^2$ and $27y^2$ is 18xy.

Hence, the correct option is (a).

Example 46. Find 2 numbers such that the mean proportional between them is 18 and 3rd proportional between them is 144. **(Dec 2012)**

(a) 9; 36 (b) 8; 32 (c) 7; 28 (d) 6; 24

Sol. (a) We know that,

If b is the mean proportional of a and c then $b^2 = ac$

Let the two numbers be x and y

Then, $xy = (18)^2 = 324$ …(i)

Also, 3rd proportional between them = 144

Then, x, y, 144 are in continued proportion

i.e., $\frac{x}{y} = \frac{y}{144}$

$\Rightarrow y^2 = 144x$

$$\Rightarrow y^2 = 144\left(\frac{324}{y}\right)$$

$\Rightarrow y^3 = 144 \times 324$

$\Rightarrow y^3 = 12 \times 12 \times 12 \times 3 \times 3 \times 3$

Thus, $y = 12 \times 3 = 36$

So, $x = \frac{324}{36} = 9$

Therefore, the numbers are 9 and 36.

INVERSE PROPORTION

If a ratio is equal to the reciprocal of the other, then either of them is in inverse (or reciprocal) proportion of the other.

Note: In a ratio a : b, both quantities must be of the same kind while in a proportion i.e., a : b = c : d, all the four quantities need not be of the same type.

❑ Properties of Proportion:

1. If a : b = c: d, then ad = bc **(Product of extremes = Product of means)**

   E.g.: If 2 : 3 = 4 : 6, then (2)(6) = (3)(4), which simplifies to 12 = 12.

2. If a: b = c: d, then b : a = d : c **(Invertendo)**

   E.g.: If 5 : 10 = 2 : 4, then 10 : 5 = 4 : 2.

3. If a : b = c : d, then a : c = b : d **(Alternendo)**

   E.g.: If 3 : 9 = 2 : 6, then 3 : 2 = 9 : 6.

4. If a : b = c : d, then (a + b) : b = (c + d) : d **(Componendo)**

   E.g.: If 4 : 6 = 2 : 3, then (4 + 6) : 6 = (2 + 3) :3, which simplifies to 10 :6 = 5 : 3.

5. If a : b = c : d, then (a - b) : b = (c - d) : d **(Dividendo)**

   E.g.: If 9 : 4 = 18 : 8, then (9 - 4) : 4 = (18 - 8) : 8, which simplifies to 5 : 4 = 10 : 8

6. If a : b = c : d, then (a + b) : (a - b) = (c + d) : (c - d) **(Componendo and Dividendo)**

   E.g.: If 5 : 2 = 10 : 4, then (5 + 2) : (5 - 2) = (10 + 4) : (10 - 4), which simplifies to 7 : 3 = 14 : 6.

7. If a : b = c : d = e : f = ..., then each of these ratios is equal to

   (a + c + e +...) : (b + d + f + ----) **(Addendo)**

   E.g.: If 2 : 4 = 3 : 6 = 1 : 2, then each of these ratios is equal to (2 + 3 + 1) : (4 + 6 + 2), which simplifies to 6 : 12.

8. If a : b = c : d = e : f = ….. , then each of these ratios is equal to (a - c - e - …) : (b - d - f - …..) (Subtrahendo)

   E.g.: If 5 : 10 =3 : 6 = 1 : 2, then each of these ratios is equal to (5 - 3 - 1) : (10 - 6 - 2), which simplifies to 1 : 2.

Example 47. If $\frac{x}{y}=\frac{z}{w}$, implies $\frac{y}{x}=\frac{w}{z}$, then the process is called **(ICAI)**

(a) Dividendo (b) Componendo (c) Alternendo (d) None of these

Sol. (d) We know that,

By the process of Invertendo,

If a : b = c : d, then b : a = d : c

According to the question

If $\frac{x}{y}=\frac{z}{w}$, implies $\frac{y}{x}=\frac{w}{z}$, it is the process of Invertendo.

Hence, the correct option is (d).

Example 48. If $\frac{p}{q}=\frac{r}{s}=\frac{p-r}{q-s}$, the process is called **(ICAI)**

(a) Subtrahendo (b) Addendo (c) Invertendo (d) None of these

Sol. (a) We know that,

By the process of Subtrahendo,

If a : b = c : d = a – c : b - d

According to the question, we have

$$\frac{p}{q}=\frac{r}{s}=\frac{p-r}{q-s}$$

It is the process of Subtrahendo.

Hence, the correct option is (a).

Example 49. $\frac{a}{4}=\frac{b}{5}=\frac{c}{9}$, then $\frac{a+b+c}{c}$ is **(ICAI)**

(a) 4 (b) 2 (c) 7 (d) None of these

Sol. (b) Given: $\frac{a}{4}=\frac{b}{5}=\frac{c}{9}$

Let $\frac{a}{4}=\frac{b}{5}=\frac{c}{9}=k$, then

a = 4k, b = 5k and c = 9k

Thus, $\frac{a+b+c}{c}=\frac{4k+5k+9k}{9k}=\frac{18k}{9k}$

$=\frac{18}{9}=\frac{2}{1}=2$

Hence, the correct option is (b).

Example 50. If $\frac{a}{4}=\frac{b}{5}$ then **(ICAI)**

(a) $\frac{a+4}{a-4}=\frac{b-5}{b+5}$ (b) $\frac{a+4}{a-4}=\frac{b+5}{b-5}$

(c) $\frac{a-4}{a+4}=\frac{b+5}{b-5}$ (d) None of these

Sol. (b) We know that,

By the process of Componendo and Dividendo,

If a : b = c : d, then a + b : a – b = c + d : c – d

Since, $\frac{a}{4}=\frac{b}{5}$ then

$\frac{a+4}{a-4}=\frac{b+5}{b-5}$

Hence, the correct option is (b).

Example 51. If $\frac{x}{2}=\frac{y}{3}=\frac{z}{7}$, then the value of x : y is

(a) $\frac{6}{23}$ (b) $\frac{23}{6}$ (c) $\frac{2}{3}$ (d) $\frac{17}{6}$

Sol. (c) Given: $\frac{x}{2}=\frac{y}{3}=\frac{z}{7}$

Since, x/2 = y/3

i.e., $\frac{x}{2}=\frac{y}{3}$

$\Rightarrow \frac{x}{y}=\frac{2}{3}$

Hence, the correct option is (c).

Example 52. If $\frac{5x-3y}{5y-3x}=\frac{3}{4}$, then the value of x : y is

(a) 2 : 9 (b) 7 : 2 (c) 7 : 9 (d) None of these

Sol. (d) Given: $\frac{5x-3y}{5y-3x}=\frac{3}{4}$

$\Rightarrow$ 4(5x – 3y) = 3(5y – 3x)

$\Rightarrow$ 20x – 12y = 15y – 9x

$\Rightarrow$ 20x + 9x = 15y + 12y

$\Rightarrow$ 29x = 27y

$\Rightarrow \frac{x}{y}=\frac{27}{29}$

Thus, the value of x : y is 27 : 29

Hence, the correct option is (d).

Example 53. $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$, then (b - c)x + (c - a)y + (a - b)z is equal to

(a) 1 (b) 0 (c) 5 (d) None of these **(ICAI)**

Sol. (b) Given: $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$

Let $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}=k$

Thus, $\frac{x}{b+c-a}=k, \frac{y}{c+a-b}=k$ and $\frac{z}{a+b-c}=k$

So, x = k(b + c – a), y = k(c + a – b) and z = k(a + b – c)

Now, Putting the values of x, y, z in (b – c)x + (c – a)y + (a – b)z, we get

(b – c)k(b + c – a) + (c – a)k(c + a – b) + (a – b)k(a + b – c)

= k((b – c)(b + c – a) + (c – a)(c + a – b) + (a – b)(a + b – c))

= k((b – c)(b + c) – a(b – c) + (c – a)(c + a) – b(c – a) + (a – b)(a + b) – c(a – b))

= k($b^2 – c^2 – ab + ac + c^2 – a^2 – bc + ab + a^2 – b^2 – ac + bc$) = k(0) = 0

Therefore, the value of (b – c)x + (c – a)y + (a – b)z is 0.

Hence, the correct option is (b) i.e., 0.

Cyclic order Tricks:

TYPE I (a - b) + (b - c) + (c - a) = 0

TYPE II a(b - c) + b(c - a) + c(a - b) = 0

TYPE III ($a^2 - b^2$) + ($b^2 - c^2$) + ($c^2 - a^2$) = 0

TYPE IV ($a^3 - b^3$) + ($b^3 - c^3$) + ($c^3 - a^3$) = 0

TYPE V (b - c)(b + c - a) + (c - a)(c + a - b) + (a - b)

TYPE VI $\frac{1}{(a-b)(b-c)}+\frac{1}{(b-c)(c-a)}+\frac{1}{(c-a)(a-b)}=0$

Clearly, (b - c)x + (c - a)y + (a – b)z is following a cyclic order.

Therefore, the required value is 0.

Hence, the correct option is (b) i.e., 0.

Example 54. The sum of the ages of 3 persons is 150 years. 10 years ago, their ages were in the ratio 7 : 8 : 9. Their present ages are

(a) (45, 50, 55) (b) (40, 60, 50)

(c) (35, 45, 70) (d) None of these

Sol. (a) Given: Ratio of ages of 3 persons 10 years ago = 7 : 8 : 9

Let the ages of three persons 10 years ago were 7x, 8x and 9x.

Thus, their present ages i.e, their ages after 10 years are

7x + 10, 8x + 10 and 9x + 10

According to the question,

Sum of the ages of 3 persons = 150 years

$\Rightarrow 7x + 10 + 8x + 10 + 9x + 10 = 150$

$\Rightarrow 24x + 30 = 150$

$\Rightarrow 24x = 120$

$\Rightarrow x = 5$

Therefore, the present ages of three persons are:

7(5) + 10, 8(5) + 10 and 9(5) + 10 = 45, 50, 55

Hence, the correct option is (a).

Example 55. If a : b = 4 : 1, then $\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}$ is : **(ICAI)**

(a) $\frac{5}{2}$ (b) 4 (c) 5 (d) None of these

Sol. (a) Given, a : b = 4 : 1

i.e., $\frac{a}{b}=\frac{4}{1}$

Thus, $\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}$

$=\sqrt{\frac{4}{1}}+\sqrt{\frac{1}{4}}=2+\frac{1}{2}=\frac{5}{2}$

Hence, the correct option is (a).

Example 56. A person has assets worth ₹1,48,200. He wishes to divide it amongst his wife, son and daughter in the ratio 3 : 2 : 1 respectively. From this asset, the share of his son will be:

(a) ₹24,700 (b) ₹49,400 (c) ₹74,100 (d) ₹37,050

Sol. (b) Given: Shares of wife, son and daughter = 3 : 2 : 1

Value of assets = ₹1,48,200

Let the shares of wife, son and daughter be 3x, 2x and x respectively.

According to the question,

3x + 2x + x = 1, 48,200

$\Rightarrow$ 6x = 1,48,200

$\Rightarrow$ x = 24,700

Therefore, the share of the son is 2x.

= 2(24,700) = 49,400

Hence, the correct option is (b).

Example 57. The students of two classes are in the ratio 5 : 7, if 10 students left from each class, the remaining students are in the ratio of 4 : 6 then the number of students in each class is:

(a) 30, 40 (b) 25, 24 (c) 40, 60 (d) 50, 70

Sol. (d) Given: Ratio of students of two classes = 5 : 7

Let the students of two classes be 5x and 7x respectively.

According to the question,

$$\frac{5x-10}{7x-10}=\frac{4}{6}$$

On cross-multiplication, we get

$6(5x - 10) = 4(7x - 10)$

$\Rightarrow 30x - 60 = 28x - 40$

$\Rightarrow 30x - 28x = 60 - 40$

$\Rightarrow 2x = 20$

$\Rightarrow x = 10$

Therefore, the number of students in each class are 5x and 7x i.e., 50 and 70.

Hence, the correct option is (d).

Example 58. A dealer mixes rice costing ₹13.85 per kg with rice costing ₹15.54 and sell the mixture at ₹17.60 per kg so, he earned a profit of 14.6% on his sale price. the proportion in which he mixes the two qualities of rice is

(a) 3 : 7 (b) 5 : 7 (c) 7 : 9 (d) 9 : 11

Sol. (a) Given,

Cost of type 1 rice = ₹13.85

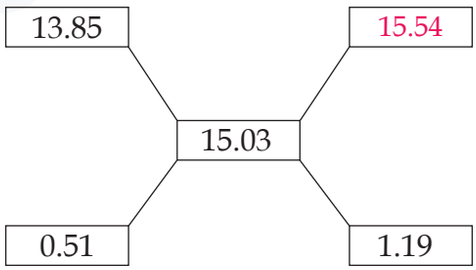
Cost of type 2 rice = ₹15.54

Selling price of mixture = ₹17.60

Profit percent = 14.6%

Thus, profit $=17.60\times\frac{14.6}{100}=2.57$

Cost price = 17.6 – 2.57 = 15.03



Therefore, the required proportion $=\frac{0.51}{1.19}=\frac{51}{199}=\frac{3}{7}$

Hence, the correct option is (a).

Example 59. Two numbers are in the ratio 7 : 8. If 3 is added to each of them, their ratio becomes 8 : 9. The number are: **(Feb. 2007)**

(a) 14, 16 (b) 24, 27 (c) 21, 24 (d) 16, 18

Sol. (c) Given: Ratio of numbers = 7 : 8

Let the numbers be 7x and 8x.

According to the question,

If 3 is added to each of them, their ratio becomes 8 : 9 i.e.,

$$\frac{7x+3}{8x+3}=\frac{8}{9}$$

$\Rightarrow 9(7x + 3) = 8(8x + 3)$

$\Rightarrow 63x + 27 = 64x + 24$

$\Rightarrow 63x - 64x = 24 - 27$

$\Rightarrow -x = -3 \Rightarrow x = 3$

Therefore, the numbers are 7(3) and 8(3) i.e., 21 and 24.

Hence, the correct option is (c).

Example 60. Two numbers are in the ratio 2 : 3 and the difference of their squares is 320. The numbers are

(a) 12, 18 (b) 16, 24 (c) 14, 21 (d) None of these

Sol. (b) Given: Ratio of two numbers = 2 : 3

Let the two numbers are 2x and 3x.

According to the question,

The difference between their squares is 320.

i.e., $(3x)^2 - (2x)^2 = 320$

$\Rightarrow 9x^2 - 4x^2 = 320 \Rightarrow 5x^2 = 320 \Rightarrow x^2 = 64 \Rightarrow x = 8$

Therefore, the numbers are 2(8) and 3(8) i.e., 16 and 24.

Hence, the correct option is (b).

Example 61. The monthly income of A and B are in the ratio 4 : 5 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹150 per month, find their monthly incomes.

(a) (₹40, ₹50) (b) (₹50, ₹40) (c) (₹400, ₹500) (d) None of these

Sol. (c) Given: Ratio of monthly incomes of A and B = 4 : 5

Ratio of their expenditures = 5 : 7

Let the monthly income of A be 4x and that of B be 5x

Let the monthly expenses of A be 5y and that of B be 7y

According to the question,

$4x - 5y = 150$ ...(i)

$5x - 7y = 150$ ...(ii)

Multiply eq (i) with 5 and (ii) by 4, thus we get

$20x - 25y = 750$

$20x - 28y = 600$

On solving both equations, we get

$3y = 150$

$y = 50$

Thus, $4x - 250 = 150$

$4x = 400$

$x = 100$

Therefore, the monthly income of A = $4x = 4(100) = ₹400$

Monthly income of B = $5x = 5(100) = ₹500$

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART-B)

1. If $A = \frac{B}{2} = \frac{C}{5}$, then A : B : C is

(a) 1 : 1 : 1 (b) 5 : 2 : 1 (c) 1 : 2 : 5 (d) None of these

2. If A : B = 5 : 6 and B : C = 7 : 8, what is the value of A : B : C?

(a) 48 : 42 : 35 (b) 5 : 42 : 8 (c) 60 : 42 : 58 (d) 35 : 42 : 48

3. The third proportional to 7, 14 is

(a) 21 (b) 28 (c) 35 (d) None of these

4. The mean proportional of 16 and 9 is

(a) 12 (b) 14 (c) 15 (d) None of these

5. Find the fourth proportional to 4, 8 and 5

(a) 8 (b) 10 (c) 15 (d) None of these

6. If a : b : c = 2 : 1 : 3 and 2a – 3b + c = 8 then a + b + c is

(a) 8 (b) 6 (c) 12 (d) 15

7. Find 2 numbers such that the mean proportional between them is 14 and 3rd proportional between them is 112.

(a) 7, 28 (b) 6, 36 (c) 8, 54 (d) None of these

8. A bag contains ₹187 in the form of 1 rupee, 50 paise and 10 paise coins in the ratio 3 : 4 : 5. Find the number of each type of coins: **(May, 2007)**

(a) 102, 136, 170 (b) 136, 102, 170

(c) 170, 102, 136 (d) None of these

9. Ratio of earnings of A and B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8 : 7. What is A's earning? **(August, 2007)**

(a) ₹21,000 (b) ₹26,000 (c) ₹28,000 (d) Data inadequate

10. The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves ₹1,500, then B's income is: **(Nov, 2007)**

(a) ₹6,000 (b) ₹4,500 (c) ₹3,000 (d) ₹7,500

11. In a 40 litres mixture of glycerine and water, the ratio of glycerine and water is 3 : 1. The quantity of water added in the mixture in order to make this ratio 2 : 1 is:

(a) 15 litres (b) 10 litres (c) 8 litres (d) 5 litres **(February, 2008)**

12. In what ratio should tea worth ₹10 per kg be mixed with tea worth ₹14 per kg, so that the average price of the mixture may be ₹11 per kg? **(June, 2008)**

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 4 : 3

13. If A, B and C started a business by investing ₹1,26,000, ₹84,000 and ₹2,10,000. If at the end of the year profit is ₹2,42,000 then the share of each is: **(2008, December)**

(a) 72,600; 48,400; 1,21,000 (b) 48,400; 1,21,000; 72,600

(c) 72,000; 49,000; 1,21,000 (d) 48,000; 1,21,400; 72,600

14. In a film shooting, A and B received money in a certain ratio and B and C also received the money in the same ratio. If A gets ₹1,60,000 and C gets ₹2,50,000. Find the amount received by B? **(June, 2011)**

(a) ₹2,00,000 (b) ₹2,50,00 (c) ₹1,00,000 (d) ₹1,50,000

15. If $15(2p^2 - q^2) = 7pq$, where p and q are positive, then p : q will be:

(a) 5 : 6 (b) 5 : 7 (c) 3 : 5 (d) 8 : 3

16. The ratio of third proportion of 12, 30 to the mean proportion of 9, 25 is:

(a) 2 : 1 (b) 5 : 1 (c) 7 : 15 (d) 3 : 5 **(Dec. 2015)**

17. What number must be added to each of the numbers 10, 18, 22, 38 to make the numbers in proportion? **(Dec. 2015)**

(a) 2 (b) 4 (c) 8 (d) None of these

18. If x : y = 2 : 3, y : z = 4 : 3 then x : y : z is

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 2 : 4 (d) None of these

19. The numbers 14, 16, 35 are not in proportion. The fourth term for which they will be in proportion is:

(a) 45 (b) 40 (c) 32 (d) None of these

20. The expenditures and savings of a person are in the ratio 4:1. If his saving are increased by 25% of his income, then what is the new ratio of his expenditure and savings?

(a) 11:9 (b) 8:5 (c) 7:5 (d) 7:4

Answer Key

1. (c)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (c)
11. (d)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (a)

INDICES

The power, also known as the **index**, tells you how many times you have to multiply the number by itself and number will be called as **base.**

In other words: If n is a positive integer, and 'a' is a real number, i.e. n ε N and a ε R (where N is the set of positive integers and R is the set of real numbers).

Then, $a^n = a \times a \times a \times a$.... to n factors

Example 62. The values of

(I) 2^4 (II) 3^3 (III) 18^2

(a) 8, 9, 36 (b) 16, 27, 36 (c) 16, 27, 324 (d) None of these

Sol. (c) $2^4 = 2 \times 2 \times 2 \times 2 = 16$

Now, $3^3 = 3 \times 3 \times 3 = 27$

Also, $18^2 = 18 = 324$

Therefore, $2^4 = 16, 3^3 = 27$ and $18^2 = 324$

Hence, the correct option is (c).

- Any base to the power (or index) 0 is 1 i.e., $a^0 = 1$ for $a \neq 0$.

Example 63. The values of

(I) 12° (II) 32° (III) $\left(\frac{1}{18}\right)^\circ$

(a) 12, 32, 18 (b) 12, 32, $\frac{1}{18}$ (c) 1, 1, 1 (d) None of these

Sol. (c) We know that,

$a^\circ = 1$ for $a \neq 0$

Here, $12^\circ = 1$

Also, $32^\circ = 1$

Now, $\frac{1}{18^\circ} = \frac{1}{1} = 1$

Therefore, $12^\circ = 1$, $32^\circ = 1$ and $\frac{1}{18^\circ} = 1$

Hence, the correct option is (c).

Finding the roots : If you have to find n^{th} root of base a i.e. $\sqrt[n]{a}$ it will give value equal to $a^{\frac{1}{n}}$.

Negative Powers: If you have power as negative integer, -n (let) for base a, then $a^{-n} = \frac{1}{a^n}$.

Calculator trick to find a^n

1. Type 'a' on the screen of calculator
2. Press ''×'' for (n - 1) times

Calculator trick to find $a^{-n} = 1/a^n$

1. Type 'a' on the screen of caluclator
2. Press ''÷''
3. Press ''='' for n times

Calculator trick to find n^{th} root: $a^{1/n}$

1. Type the number
2. Press '√' for 12 times
3. Subtract 1 from it and then divide by n
4. Add '1'
5. Press '× =' for 12 times

E.g.: To find $(16)^{\frac{1}{4}}$

1. Type '16'
2. Press '√' for 12 times
3. Subtract 1 from it and then divide by 4
4. Add '1'
5. Press '× =' for 12 times

We will get $2.00035 \approx 2$

Therefore, $(16)^{\frac{1}{4}} = 2$

Example 64. Find out the values of each of the indices:

(I) $\sqrt{4}$ (II) $16^{-1/4}$ (III) $(3)^{-3}$ (IV) 9^{-5}

(a) $2, \frac{1}{2}, \frac{1}{9}, \frac{1}{3^{10}}$ (b) $2, \frac{1}{2}, \frac{1}{27}, \frac{1}{3^5}$

(c) $2, \frac{1}{2}, \frac{1}{27}, \frac{1}{3^{10}}$ (d) None of these

Sol. (c) (I) $\sqrt{4} = \sqrt{2 \times 2} = 2$

(II) We have, $16^{-1/4}$

$= (2^4)^{-1/4} = 2^{-1} = \frac{1}{2}$

Thus, $16^{-1/4} = \frac{1}{2}$

(III) $(3)^{-3} = \frac{1}{3^3} = \frac{1}{27}$

Thus, $(3)^{-3} = \frac{1}{27}$

(IV) $9^{-5} = \frac{1}{9^5} = \frac{1}{(3^2)^5} = \frac{1}{3^{10}}$

Thus, $9^{-5} = \frac{1}{3^{10}}$

Hence, the correct option is (c).

LAW 1

$a^m \times a^n = a^{m+n}$

- **Case-I:** When m and n are positive integers; then $a^m \times a^n = a^{m+n}$
  E.g.: $6^7 6^2 = 6^{7+2} = 6^9$
- **Case-II:** When n = –m, m is positive integer then $a^m \times a^{-m} = a^\circ = 1$
  E.g.: $2^3 2^{-3} = 2^{3-3} = 2^\circ = 1$
- **Case-III:** When m and n are any integers
  E.g.: $7^{\frac{4}{3}} 7^{-2} = 7^{\frac{4}{3}-2} = 7^{\frac{4-6}{3}} = 7^{-\frac{2}{3}}$

Example 65. Find the value of $5^{\frac{3}{5}}.5^{-\frac{7}{2}}$.

(a) $5^{\frac{-4}{7}}$ (b) $5^{-\frac{41}{10}}$ (c) $5^{\frac{29}{10}}$ (d) $5^{-\frac{29}{10}}$

Sol. (d) We have, $5^{\frac{3}{5}}.5^{-\frac{7}{2}}$

Thus, $5^{\frac{3}{5}+\left(-\frac{7}{2}\right)} = 5^{\frac{6-35}{10}} = 5^{-\frac{29}{10}}$

Hence, the correct option is (d).

Example 66. The value of $8^{\frac{1}{3}}$ is **(ICAI)**

(a) $3\sqrt{2}$ (b) 4 (c) 2 (d) None of these

Sol. (c) To find: $8^{\frac{1}{3}}$

$= \sqrt[3]{2\times 2\times 2} = 2$

Hence, the correct option is (c).

Example 67. The value of $2\times(32)^{\frac{1}{5}}$ is **(ICAI)**

(a) 2 (b) 10 (c) 4 (d) None of these

Sol. (c) To find: $2\times(32)^{\frac{1}{5}}$

$= 2\times\sqrt[5]{2\times 2\times 2\times 2\times 2} = 2\times 2 = 4$

Therefore, the value of $2\times(32)^{\frac{1}{5}}$ is 4.

Hence, the correct option is (c).

Example 68. The value of $2(256)^{-\frac{1}{8}}$ is **(ICAI)**

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) None of these

Sol. (a) To find: $2(256)^{-\frac{1}{8}}$

$$= \frac{2}{(256)^{\frac{1}{8}}} = \frac{2}{\sqrt[8]{2\times2\times2\times2\times2\times2\times2\times2}} = \frac{2}{2} = 1$$

Therefore, the value of $2(256)^{-\frac{1}{8}}$ is 1.

Hence, the correct option is (a).

Example 69. $4x^{-1/4}$ is expressed as

(a) $-4x^{1/4}$ (b) x^{-1} (c) $4/x^{1/4}$ (d) None of these

Sol. (c) We have, $4x^{-1/4} = \frac{4}{x^{\frac{1}{4}}}$

Therefore, $4x^{-1/4} = \frac{4}{x^{\frac{1}{4}}}$

Hence, the correct option is (c).

Example 70. Simplify $4x^{1/3} \times 2x^{-1}$ if x = 4.

(a) $2^{\frac{1}{3}}$ (b) 2 (c) $2^{\frac{5}{3}}$ (d) None of these

Sol. (c) We have, $4x^{1/3} \times 2x^{-1}$

At x = 4

$$4x^{1/3}\times2x^{-1} = 4(4)^{\frac{1}{3}}\times2(4)^{-1}$$

$$= 4^{1+\frac{1}{3}-1}\times2 = 4^{\frac{1}{3}}\times2 = (2^2)^{\frac{1}{3}}\times2 = 2^{\frac{2}{3}+1} = 2^{\frac{5}{3}}$$

Hence, the correct option is (c).

LAW 2

$\frac{a^m}{a^n} = a^{m-n}$, when m and n are positive integers and m > n.

E.g.: $\frac{2^3}{2^4} = 2^{3-4} = 2^{-1} = \frac{1}{2}$

Example 71. Which of the following option is correct when you simplify the following:

(I) $4/(16)^{1/4}$ (II) $(27)^{2/3}/(9)^{3/2}$

(a) $2, \frac{1}{3}$ (b) $\frac{1}{2}, \frac{1}{3}$ (c) 2, -3 (d) None of these

Sol. (a) (I) $4/(16)^{1/4}$

$$= \frac{4}{16^{\frac{1}{4}}} = \frac{4}{(2\times2\times2\times2)^{\frac{1}{4}}} = \frac{4}{2} = 2$$

(II) $(27)^{2/3}/\ (9)^{3/2}$

$$= \frac{27^{\frac{2}{3}}}{9^{\frac{3}{2}}} = \frac{(3^3)^{\frac{2}{3}}}{(3^2)^{\frac{3}{2}}} = \frac{3^2}{3^3} = 3^{2-3} = 3^{-1} = \frac{1}{3}$$

Hence, the correct option is (a).

Example 72. The value of $\frac{4}{(32)^{\frac{1}{5}}}$ is **(ICAI)**

(a) 8 (b) 2 (c) 4 (d) None of these

Sol. (b) We have, $\frac{4}{(32)^{\frac{1}{5}}}$

$$= \frac{4}{(32)^{\frac{1}{5}}} = \frac{4}{(2\times2\times2\times2\times2)^{\frac{1}{5}}} = \frac{4}{2} = 2$$

Therefore, $\frac{4}{(32)^{\frac{1}{5}}} = 2$

Hence, the correct option is (b).

LAW 3

$(a^m)^n = a^{mn}$ when m and n are positive integers

E.g.: $((16)^{1/4})^5 = (16)^{\frac{1}{4}\times5} = 16^{\frac{5}{4}} = (2^4)^{\frac{5}{4}} = 2^{4\times\frac{5}{4}} = 2^5$

Example 73. Find the value of $\left[(27)^{\frac{2}{3}}\cdot(9)^{\frac{3}{2}}\right]^{\frac{1}{5}}$

(a) 27 (b) 3 (c) 3^5 (d) None of these

Sol. (b) We have, $\left[(27)^{\frac{2}{3}}\cdot(9)^{\frac{3}{2}}\right]^{\frac{1}{5}}$

$$= \left[(3^3)^{\frac{2}{3}} \cdot (3^2)^{\frac{3}{2}} \right]^{\frac{1}{5}} = \left[3^{3\times\frac{2}{3}} \cdot 3^{2\times\frac{3}{2}} \right]^{\frac{1}{5}} = \left[3^2 \cdot 3^3 \right]^{\frac{1}{5}} = \left[3^5 \right]^{\frac{1}{5}} = 3$$

Hence, the correct option is (b).

Example 74. The value of $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ is **(ICAI)**

(a) 2/3 (b) 3/2 (c) 2/9 (d) None of these

Sol. (a) We have, $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

$$= \left(\frac{2^3}{3^3}\right)^{\frac{1}{3}} = \left(\frac{2}{3}\right)^{3\times\frac{1}{3}} = \left(\frac{2}{3}\right)^1 = \frac{2}{3}$$

Hence, the correct option is (a).

Example 75. $\left[(x^n)^{n-\frac{1}{n}} \right]^{\frac{1}{n+1}} =$ **(ICAI)**

(a) x^n (b) x^{n+1} (c) x^{n-1} (d) None of these

Sol. (c) We have, $\left[(x^n)^{n-\frac{1}{n}} \right]^{\frac{1}{n+1}} =$

$$= \left[(x^n)^{\frac{n^2-1}{n}} \right]^{\frac{1}{n+1}} = \left[(x^n)^{\frac{(n-1)(n+1)}{n}} \right]^{\frac{1}{n+1}} = (x^n)^{\frac{(n-1)(n+1)}{n(n+1)}}$$

$$= (x^n)^{\frac{(n-1)}{n}} = x^{\frac{n(n-1)}{n}} = x^{n-1}$$

Hence, the correct option is (c).

Example 76. The value of $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ is **(ICAI)**

(a) 1 (b) 0 (c) 2 (d) None of these

Sol. (a) We have,

$$\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$$

$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)} = x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} = x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$

Hence, the correct option is (a).

Example 77. If $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, then the simplified form of **(ICAI)**

$$\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l}\right]^{l^2+ln+n^2}$$

(a) 0 (b) 1 (c) x (d) None of these

Sol. (b) We have, $\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l}\right]^{l^2+ln+n^2}$

$= \left[x^{l-m}\right]^{l^2+lm+m^2} \times \left[x^{m-n}\right]^{m^2+mn+n^2} \times \left[x^{n-l}\right]^{l^2+ln+n^2}$

We know that,

$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$= \left[x^{(l-m)\left(l^2+lm+m^2\right)}\right] \times \left[x^{(m-n)\left(m^2+mn+n^2\right)}\right] \times \left[x^{(n-l)\left(l^2+ln+n^2\right)}\right]$

$= x^{l^3-m^3} \times x^{m^3-n^3} \times x^{n^3-l^3} = x^{l^3-m^3+m^3-n^3+n^3-l^3} = x^0 = 1$

Hence, the correct option is (b).

Example 78. If $(25)^{150} = (25x)^{50}$, then the value of x will be

(a) 5^3 (b) 5^4 (c) 5^2 (d) 5

Sol. (b) Given: $(25)^{150} = (25x)^{50}$

$\Rightarrow (25)^{150} = (25^{50})(x^{50})$

$\Rightarrow x^{50} = \frac{(25)^{150}}{(25)^{50}}$

$\Rightarrow x^{50} = (25)^{150-50}$

$\Rightarrow x^{50} = (25)^{100}$

$\Rightarrow x^{50} = (5^2)^{2\times50}$

$\Rightarrow x^{50} = (5^4)^{50}$

On comparing, we get

$x = 5^4$

Hence, the correct option is (b).

Example 79. The value of $\frac{64(b^4a^3)^6}{\left[4(a^3b)^2 \times (ab)^2\right]}$ is

(a) $16a^{10}b^{20}$ (b) $4a^{20}b^{10}$ (c) $8a^{10}b^{20}$ (d) $4a^{10}b^{20}$

Sol. (a) $\dfrac{64(b^4a^3)^6}{\left[4(a^3b)^2\times(ab)^2\right]}$

$=\dfrac{64b^{24}a^{18}}{\left[4a^6b^2\times a^2b^2\right]}=\dfrac{64b^{24}a^{18}}{\left[4a^8b^4\right]}=16b^{24-4}a^{18-8}=16b^{20}a^{10}$

Hence, the correct option is (a).

LAW 4

$(ab)^n = a^n b^n$ when n can take all of the values.

E.g.: 6^3 can be written as $(2 \times 3)^3 = 2^3 \times 3^3$

Example 80. The value of $(8)^3(27)^3$ is

(a) 6 (b) 6^{18} (c) 6^9 (d) None of these

Sol. (c) We have, $(8)^3(27)^3$

$= (23)^3(3^3)^3 = (2^9).(3^9) = (2 \times 3)^9 = 6^9$

Hence, the correct option is (c).

Example 81. Simplify $6a^2b^3c^2 \times 4b^{-3}c^{-2}d$. **(ICAI)**

(a) 0 (b) $24a^2d$ (c) $24a^2bcd$ (d) None of these

Sol. (b) We have, $6a^2b^3c^2 \times 4b^{-3}c^{-2}d$

$= 6 \times 4 \times a^2 \times b^3 \times b^{-3} \times c^2 \times c^2 \times c^{-2} \times d$

$= 24 \times a^2 \times b^{3-3} \times c^{2-2} \times d = 24a^2b^0c^0d = 24a^2d$

Hence, the correct option is (b).

Example 82. The simplified value of $16x^{-3}y^2 \times 8^{-1}x^3y^{-2}$ is **(ICAI)**

(a) 2xy (b) $\dfrac{xy}{2}$ (c) 2 (d) None of these

Sol. (c) We have, $16x^{-3}\,y^2 \times 8^{-1}x^3\,y^{-2}$

$=\dfrac{16}{8}\times x^{-3}\times x^3\times y^2\times y^{-2}=2\times x^{-3+3}\times y^{2-2}=2\times x^0\times y^0=2$

Therefore, $16x^{-3}y^2 \times 8^{-1}x^3y^{-2} = 2$

Hence, the correct option is (c).

Example 83. If $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{z}}$ and xyz = 1, then the value of p + q + r is **(ICAI)**

(a) 1 (b) 0 (c) $\dfrac{1}{2}$ (d) None of these

Sol. (b) Given: $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}}$ and xyz =1

Let $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}} = k$

Since $x^{\frac{1}{p}} = k$, thus

$$\left(x^{\frac{1}{p}}\right)^p = k^p \Rightarrow x = k^p \quad \left[\because (x^m)^n = x^{mn}\right]$$

$(\therefore (x^m)^n = x^{mn})$

Similarly, $y^{\frac{1}{q}} = k$

$$\left(y^{\frac{1}{q}}\right)^q = k^q \Rightarrow y = k^q$$

Also, $z^{1/r} = k$

$$\left(x^{\frac{1}{r}}\right)^r = k^r \Rightarrow z = k^r$$

Substituting the values of x, y and z in xyz =1, we get

$k^p \times k^q \times k^r = 1$

$\Rightarrow k^{p+q+r} = 1 \ (\therefore a^m \cdot a^n = a^{m+n})$

$\Rightarrow k^{p+q+r} = k^0 \ (\therefore a^0 = 1)$

On comparing, we get p + q + r = 0

Hence, option (b) is correct.

Example 84. On simplification $\frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 6^{y+1}}{6^{x+1} \times 10^{y+3} \times 15^x}$ reduces to

(a) 0 (b) 1 (c) -1 (d) None of above

Sol. (b) $\frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 2^{y+1} \times 3^{y+1}}{2^{x+1} \times 3^{x+1} \times 2^{y+3} \times 5^{y+3} \times 3^x \times 5^x}$

$$\Rightarrow \frac{2^{x+3} \times 2^{y+1}}{2^{x+1} \times 2^{y+3}} \times \frac{3^{2x-y} \times 3^{y+1}}{3^{x+1} \times 3^x} \times \frac{5^{x+y+3}}{5^{y+3} \times 5^x}$$

$\Rightarrow 2^{x+3+y+1-x-1-y-3} \times 3^{2x-y+y+1-x-1-x} \times 5^{x+y+3-y-3-x}$

$\Rightarrow 2^0 \times 3^0 \times 5^0 = 1$

Hence, the correct option is (b).

Example 85. If $2^a = 3^b = (12)^c$, then the value of ab – c(a + 2b) reduces to **(ICAI)**

(a) 0 (b) 1 (c) 2 (d) 3

Sol. (a) Given: $2^a = 3^b = (12)^c$

Let us assume $2^a = 3^b = (12)^c = k$

So, $2^a = k$

$\Rightarrow 2 = k^{\frac{1}{a}}$ ...(i)

Also, $3^b = k$

$\Rightarrow 3 = k^{\frac{1}{b}}$ ...(ii)

Now, $(12)^c = k$

$\Rightarrow 12 = k^{\frac{1}{c}}$

$\Rightarrow 2 \times 2 \times 3 = k^{\frac{1}{c}}$

$\Rightarrow k^{\frac{1}{a}} \times k^{\frac{1}{a}} \times k^{\frac{1}{b}} \times k^{\frac{1}{c}}$ (from eq. (i) and (ii))

$\Rightarrow k^{\frac{2}{a}+\frac{1}{b}} = k^{\frac{1}{c}}$ ($\because a^m \cdot a^n = a^{m+n}$)

On comparing both sides of equation, we get

$$\frac{1}{c} = \frac{2}{a} + \frac{1}{b}$$

$$\Rightarrow \frac{1}{c} = \frac{2b+a}{2a}$$

On cross multiplying, we have

$ab = c(a + 2b)$

$\Rightarrow ab - c(a + 2b) = 0$

Therefore, the value of $ab - c(a + 2b)$ reduces to 0.

Hence, the correct answer is option (a) i.e., 0.

Example 86. If $a^b = b^a$, then the value of $\left(\frac{a}{b}\right)^{\frac{a}{b}} - a^{\frac{a}{b}-1}$ is **(ICAI)**

(a) a (b) b (c) 0 (d) None

Sol. (c) Given: $a^b = b^a$

$$\therefore \left(\frac{a}{b}\right)^{\frac{a}{b}} - a^{\frac{a}{b}-1} = \left[\left(\frac{a}{b}\right)^a\right]^{\frac{1}{b}} - a^{\frac{a}{b}-1} \quad (\because (x^m)^n = x^{mn})$$

$$= \left(\frac{a^a}{b^a}\right)^{\frac{1}{b}} - a^{\frac{a}{b}-1} \quad \left[\because \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}\right]$$

$$= \left(\frac{a^a}{a^b}\right)^{\frac{1}{b}} - a^{\frac{a}{b}-1} \qquad (\because\ b^a = a^b)$$

$$= \left(a^{a-b}\right)^{\frac{1}{b}} - a^{\frac{1}{b}-1} \qquad \left(\because \frac{x^m}{x^n} = x^{m-n}\right)$$

$$= a^{(a-b)\times\frac{1}{b}} - a^{\frac{a}{b}-1}$$

$$= a^{\frac{a-b}{b}} - a^{\frac{a}{b}-1}$$

$$= a^{\frac{a}{b}-\frac{b}{b}} - a^{\frac{a}{b}-1}$$

$$= a^{\frac{a}{b}-1} - a^{\frac{a}{b}-1} = 0$$

Therefore, the value of $\left(\frac{a}{b}\right)^{\frac{a}{b}} - a^{\frac{a}{b}-1}$ is 0.

Hence, the correct answer is option (c) i.e., 0.

PRACTICE QUESTIONS (PART-C)

1. Find out the values of each: (i) $16^{\frac{1}{4}}$ and (ii) $343^{\frac{1}{3}}$

(a) 2, 7 (b) 4, 343 (c) 3, 27 (d) None of these

2. The value of $16^{-\frac{3}{4}}$ is

(a) 8 (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) None of these

3. Simplify $\left(\frac{1}{5}\right)^{-2}$.

(a) 5 (b) 10 (c) 25 (d) $\frac{1}{25}$

4. The value of $\left(\frac{8}{125}\right)^{\frac{1}{3}}$ is

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{2}{25}$ (d) None of these

5. The value of $\left(\frac{2p^2q^3}{3xy}\right)^0$ where $p, q, x, y \neq 0$ is equal to

(a) 0 (b) 1 (c) 2 (d) None of these

6. The value of $4^{-3} \times 4^5$ is

(a) 4 (b) 8 (c) 16 (d) None of these

7. The value of $9^2 \times 9^3 \times 9^5$ is

(a) 90 (b) 9^{10} (c) 9^{30} (d) None of these

8. The value of $5^3 \times 7^2 \times 5^4 \times 7^3$ is

(a) 35^{12} (b) $5^3 \times 7^2$ (c) $5^7 \times 7^5$ (d) None of these

9. The value of $(125)^{\frac{2}{3}} \times (625)^{\frac{3}{4}}$ is

(a) 150 (b) 625 (c) 3125 (d) None of these

10. The value of $(10)^7 \div (10)^5$ is

(a) 1 (b) 10 (c) 20 (d) 100

11. The value of $3^5 \times 3^6 \div 3^2$ is

(a) 3 (b) 3^3 (c) 3^9 (d) 3^{12}

12. The value of $12y^6 \div 6y^{-5}$ is

(a) 2 (b) 2y (c) $2y^{-1}$ (d) None of these

13. Simplify $\dfrac{2a^{\frac{1}{2}} \times a^{\frac{2}{3}} \times 6a^{-\frac{7}{3}}}{9a^{\frac{-5}{3}} \times a^{\frac{3}{2}}}$ if a = 4

(a) 3 (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) 4

14. The value of $(4^2)^5$ is

(a) 4^7 (b) 4^{10} (c) 4^{20} (d) None of these

15. If $(3^5)^6 = 3^m$, then the value of m is

(a) 10 (b) 30 (c) 20 (d) None of these

16. On simplification, $\dfrac{1}{1+a^{m-n}+a^{m-p}} + \dfrac{1}{1+a^{n-m}+a^{n-p}} + \dfrac{1}{1+a^{p-m}+a^{p-n}}$ is equal to **(ICAI)**

(a) 0 (b) a (c) 1 (d) $\frac{1}{a}$

17. $(\{(2)^{1/2}.\ (4)^{3/4}.\ (8)^{5/6}.\ (16)^{7/8}.\ (32)^{9/10}\}^4)^{3/25}$ is **(ICAI)**

(a) A fraction (b) an integer (c) 1 (d) None of these

18. If $3^{x+2} = 27$, what is the value of x?

(a) 1 (b) 0 (c) 2 (d) None of these

19. If $2^x = 3^y = 6^{-z}, \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}$ is **(ICAI)**

(a) 1 (b) 0 (c) 2 (d) None of these

20. The value of $\left[\frac{(x+y)^{\frac{2}{3}} \times (x-y)^{\frac{3}{2}}}{\sqrt{x+y} \times \sqrt{(x-y)^3}}\right]^6$ is **(ICAI)**

(a) $(x + y)^2$ (b) $(x - y)$ (c) $x + y$ (d) None of these

21. $\{(3^3)^2 \times (4^2)^3 \times (5^3)^2\} / \{(3^2)^3 \times (4^3)^2 \times (5^2)^3\}$ is **(ICAI)**

(a) 3/4 (b) 4/5 (c) 4/7 (d) None of these

22. if $2^a = 3^b = (12)^c$, then $\frac{1}{c} - \frac{1}{b} - \frac{2}{a}$ reduces to _____ **(ICAI)**

(a) 1 (b) 0 (c) 2 (d) None of these

Answer Key

1. (a)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (c)	**10.** (d)
11. (c)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (c)
21. (d)	**22.** (b)								

LOGARITHM

WHAT IS LOGARITHM?

Definition:

- The logarithm is the inverse operation of an exponential equation of the form $n = a^x$, where 'a' represents the base and 'x' represents the exponent. The logarithm can be found by interchanging the positions of 'x' (index) and 'n' in the equation.
  E.g.: For $2^3 = 8$, then $\log_2 8 = 3$
- For all real numbers n and all positive numbers a and x where $a \neq 1$, $x = \log_a n$ if and only if $n = a^x$.
  E.g.: For n = 1000 and a = 10.
  Here, $n = 1000 = (10)^3$ which is of the form $n = a^x$
- The expression **$\log_a n$** represents the base or index to which the base "a" must be raised in order to obtain x.

Example 87. $\log_2 8$ is equal to

(a) 2 (b) 8 (c) 3 (d) None of these

Sol. (c) We have, $\log_2 8$

We know that, $\log_a b = \frac{\log b}{\log a}$

$$\log_2 8 = \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2} = \frac{3\log 2}{\log 2} = 3 \times 1 = 3$$

Therefore, $\log_2 8$ is equal to 3.

Hence, the correct option is (c).

Example 88. $\log_{\sqrt{2}} 64$ is equal to

(a) 12 (b) 6 (c) 1 (d) None of these

Sol. (a) We have, $\log_{\sqrt{2}} 64$

We know that, $\log_a b = \frac{\log b}{\log a}$

Thus, $\log_{\sqrt{2}} 64 = \frac{\log 64}{\log \sqrt{2}}$

$= \frac{\log 2^6}{\log 2^{\frac{1}{2}}} = \frac{6 \log 2}{\frac{1}{2} \log 2} = 6 \times 2 = 12$

Therefore, $\log_{\sqrt{2}} 64 = 12$

Hence, the correct option is (a).

Example 89. $\log_{2\sqrt{3}} 1728$ is equal to

(a) $2\sqrt{3}$ (b) 2 (c) 6 (d) None of these

Sol. (c) We have, $\log_{2\sqrt{3}} 1728$

$= \log_{2\sqrt{3}}(2^6 \times 3^3) = \log_{2\sqrt{3}}(2^6 \times (\sqrt{3})^6)$ $(\because 3 = \sqrt{3} \times \sqrt{3} \Rightarrow 3^3 = (\sqrt{3})^6)$

$= \log_{2\sqrt{3}}(2\sqrt{3})^6$ $\left[\because (x \times y)^m = x^m \times y^m\right]$

$= 6 \log_{2\sqrt{3}}(2\sqrt{3})$ $\left[\because \log_a x^n = n \log_a x\right]$

$= 6 \times 1 = 6$ $\left[\because \log_a a = 1\right]$

Therefore, the value of $\log_{2\sqrt{3}} 1728$ is 6.

Hence, option (c) is correct.

Characteristic

The characteristic of the logarithm of any number greater than 1 is positive and is one less than the number of digits to the left of the decimal point in the given number.

Mantissa: The mantissa is the fractional part of the logarithm of a given number.

E.g.: log 27 = 1.431, here 1 is the characteristic and 0.431 is mantissa.

❑ **Calculator Trick to find out Logarithm:**

To find $\log_{10} m$:

1. Type number (m)
2. Press root for 19 times.
3. Subtract 1
4. Multiply by 227695

We can get the required value of $\log_{10} m$.

❑ **Another Calculator Trick to find out Logarithm:**
1. Type number (m) and press root for 13 times.
2. Subtract 1
3. Multiply by 3558

Example 90. Find

(I) log 4.221 (II) log 0.2312 (III) log 0.001294

Sol.: (I) First type 4.221 and press root for 19 times. After that subtract 1 from it. Now multiply it by 227695, thus we get

log 4.221 = 0.6254

(II) Following the above steps, we get

log 0.2312 = -0.636

(III) Following the above steps, we get

log 0.001294 = -2.88758

FUNDAMENTAL LAW OF LOGARITHMS

LAW 1

Logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers to the same base, i.e. $\log_a mn = \log_a m + \log_a n$

E.g.: $\log_a(10) = \log_a(2 \times 5) = \log_a 2 + \log_a 5$

Example 91. log 20 can be expressed as

(a) log 2 + log 5
(b) log 5 + log 8
(c) log 5 + log 4
(d) None of these

Sol. (c) We know that, log ab = log a + log b

We have, log 20

= log(4 × 5) = log 4 + log 5

Hence, the correct option is (c).

Example 92. log 6 + log 5 is expressed as

(a) log 11 (b) log 30 (c) log 5/6 (d) None of these

Sol. (b) We know that, log ab = log a + log b

Thus, log 6 + log 5 = log(6 × 5) = log 30

Hence, the correct option is (b).

Example 93. log (1 × 2 × 3) is equal to

(a) log 1 + log 2 + log 3 (b) log 3
(c) log 2 (d) None of these

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Sol. (a) We know that, log ab = log a + log b

Thus, log (1 × 2 × 3) = log 1 + log 2 + log 3

Hence, the correct option is (a).

Example 94. If log x + log y = log (x + y), y can be expressed as

(a) x - 1 (b) x (c) $\frac{x}{x-1}$ (d) None of these

Sol. (c) We have, log x + log y = log (x + y) i.e., log xy = log (x + y)

Thus, xy = x + y

$\Rightarrow$ xy – y = x

$\Rightarrow$ y(x – 1) = x

$\Rightarrow y = \frac{x}{x-1}$

Hence, the correct option is (c).

Example 95. Given log 2 = 0.3010 and log 3 = 0.4771, the value of log 6 is

(a) 0.9030 (b) 0.9542 (c) 0.7781 (d) None of these

Sol. (c) We know that,

log ab = log a + log b

Thus, log 6 = log(2 × 3) = log 2 + log 3

= 0.3010 + 0.4771 = 0.7781

Hence, the correct option is (c).

LAW 2

The logarithm of the quotient of two numbers is equal to the difference of their logarithms to the same base, i.e., $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

E.g.: $\log_4\left(\frac{27}{5}\right) = \log_4(27) - \log_4(5)$

Example 96. $\log\left(\frac{32}{4}\right)$ is equal to

(a) $\frac{\log 32}{\log 4}$ (b) log 32 - log 4

(c) 23 (d) None of these

Sol. (b) Using, $\log\left(\frac{a}{b}\right) = \log a - \log b$

Thus, $\log\left(\frac{32}{4}\right) = \log 32 - \log 4$

Hence, the correct option is (b).

Example 97. $\log 20 - \log 5$ is equal to

(a) $\log 6$ (b) $\log 16$ (c) $\log 4$ (d) None of these

Sol. (c) We know that,

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

Thus, $\log 20 - \log 5 = \log\left(\frac{20}{5}\right) = \log 4$

Hence, the correct option is (c).

Example 98. The simplified value of $\log_{10} 5 + \log_{10} 8 - \log_{10} 4$ is

(a) $\frac{1}{2}$ (b) 4 (c) 1 (d) None of these

Sol. (c) We know that,

$\log_a mn = \log_a m + \log_a n$

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Thus, $\log_{10} 5 + \log_{10} 8 - \log_{10} 4 = \log_{10}\left(\frac{5\times 8}{4}\right)$

$= \log_{10}(10) = 1$

Therefore, $\log_{10} 5 + \log_{10} 8 - \log_{10} 4 = 1$.

Hence, the correct option is (c).

Example 99. Given that $\log_{10} 2 = x$, $\log_{10} 3 = y$, then $\log_{10} 1.2$ is expressed in terms of x and y as

(a) $x + 2y - 1$ (b) $x + y - 1$ (c) $2x + y - 1$ (d) None of these

Sol. (c) We know that,

$\log_a mn = \log_a m + \log_a n$

Thus, $\log_{10} 1{\cdot}2 = \log_{10}\left(\frac{12}{10}\right) = \log_{10}(12) - \log_{10}(10)$

$= \log_{10}(2 \times 2 \times 3) - \log_{10}(10) = \log_{10}(2^2 \times 3) - 1$

$= \log_{10}(2^2) + \log_{10}(3) - 1 = 2\log_{10}(2) + \log_{10}(3) - 1 = 2x + y - 1$

Hence, the correct option is (c).

LAW 3

Logarithm of the number raised to the power is equal to the index of the power multiplied by the logarithm of the number to the same base i.e. $\log_a m^n = n\log_a m$

E.g.: $\log_2 2^4 = 4 \log_2 2$

Example 100. $\log_3 81$ is equal to

(a) 2 (b) 8 (c) 3 (d) None of these

Sol. (d) We have,

$\log_3 81 = \log_3(3^4) = 4\log_3(3) = 4(1) = 4$

Therefore, $\log_3 81 = 4$

Hence, the correct option is (d).

Example 101. The value of log 0.0001 to the base 0.1 is

(a) –4 (b) 4 (c) $\frac{1}{4}$ (d) None of these

Sol. (b) We have,

log 0.0001 to the base 0.1 which can be written as

$= \log_{0.1}(0.0001) = \log_{0.1}(0.1^4) = 4\log_{0.1}(0.1) = 4(1) = 4$

Therefore, the value of log 0.0001 to the base 0.1 is 4.

Hence, the correct option is (b).

Example 102. $\log\left(\frac{1}{81}\right)$ to the base 9 is equal to

(a) 2 (b) $\frac{1}{2}$ (c) –2 (d) None of these

Sol. (c) $\log\left(\frac{1}{81}\right)$ to the base 9 can be written as

$$= \log_9\left(\frac{1}{81}\right) = \frac{\log\frac{1}{81}}{\log 9} = \frac{\log 9^{-2}}{\log 9} = \frac{-2\log 9}{\log 9} = -2$$

Therefore, $\log\left(\frac{1}{81}\right)$ to the base 9 is equal to -2.

Hence, the correct option is (c).

Example 103. The value of $\log_2(\log_2\{\log_3(\log_3 27^3)\})$ is equal to

(a) 1 (b) 2 (c) 0 (d) None of these

Sol. (c) We have,

$\log_2(\log_2\{\log_3(\log_3 27^3)\})$

$= \log_2(\log_2\{\log_3(\log_3 27^3)\}) = \log_2(\log_2\{\log_3(\log_3(3^3)^3)\})$

$= \log_2(\log_2\{\log_3(\log_3 3^9)\}) = \log_2(\log_2\{\log_3(9\log_3 3)\}) = \log_2(\log_2\{\log_3 9\})$

$= \log_2(\log_2\{\log_3 3^2\}) = \log_2(\log_2\{2\log_3 3\}) = \log_2(\log_2 2) = \log_2 1 = 0$

Therefore, the value of $\log_2(\log_2\{\log_3(\log_3 27^3)\})$ is equal to 0.

Hence, the correct option is (c).

Example 104. $\log_{11}\left(1-\frac{1}{3}\right)+\log_{11}\left(1-\frac{1}{4}\right)+\log_{11}\left(1-\frac{1}{5}\right)+\ldots+\log_{11}\left(1-\frac{1}{242}\right)$

(a) 1 (b) 2 (c) –2 (d) None of these

Sol. (c) We have, $\log_{11}\left(1-\frac{1}{3}\right)+\log_{11}\left(1-\frac{1}{4}\right)+\log_{11}\left(1-\frac{1}{5}\right)+...+\log_{11}\left(1-\frac{1}{242}\right)$

$\Rightarrow \log_{11}\left(\frac{3-1}{3}\right)+\log_{11}\left(\frac{4-1}{4}\right)+\log_{11}\left(\frac{5-1}{5}\right)+...\log_{11}\left(\frac{242-1}{242}\right)$

$\Rightarrow \log_{11}\left(\frac{2}{3}\right)+\log_{11}\left(\frac{3}{4}\right)+\log_{11}\left(\frac{4}{5}\right)+\cdots+\log\frac{240}{241}+\log\frac{241}{242}$

$\Rightarrow \log_{11}\left(\frac{2}{242}\right)\Rightarrow \log_{11}\left(\frac{1}{121}\right)\Rightarrow \log_{11}\left(\frac{1}{11^2}\right)\Rightarrow \log_{11}\left(11^{-2}\right)\Rightarrow -2\log_{11}11\Rightarrow -2(1)\Rightarrow -2$

Hence, the correct option is (c).

Example: 105. The value of $\log(1^3 + 2^3 + 3^3 + ... + n^3)$ is equal to

(a) $3 \log 1 + 3 \log 2 + ... + 3 \log n$

(b) $2 \log n + 2 \log(n + 1) - 2 \log 2$

(c) $\log n + \log(n + 1) + \log(2n + 1) - \log 6$

(d) 1

Sol. (b) We know, Sum of cube of 'n' natural numbers $= \left(\frac{n(n+1)}{2}\right)^2$

Thus, $\log (1^3 + 2^3 + ... + n^3)$

$= \log\left(\frac{n(n+1)}{2}\right)^2 = 2\log\left(\frac{(n)(n+1)}{2}\right)$

$= 2(\log n + \log (n + 1) - \log 2 = 2\log n + 2\log(n + 1) - 2\log 2$

CHANGE OF BASE

$$\log_b m = \frac{\log_a m}{\log_a b}$$

E.g.: $\log_3 27 = \frac{\log_{10} 27}{\log_{10} 3} = \frac{\log_{10} 3^3}{\log_{10} 3} = \frac{3\log_{10} 3}{\log_{10} 3} = 3$

Example 106. If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$, these x is equal to

(a) 8 (b) 4 (c) 16 (d) None of these

Sol. (a) Given: $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$

$\Rightarrow \log_2 x + \log_{2^2} x + \log_{2^4} x = \frac{21}{4}$

$\Rightarrow \log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = \frac{21}{4} \quad \left(\because \log_{a^n} x = \frac{1}{n}\log_a x\right)$

$$\Rightarrow \log_2 x \times \left(1+\frac{1}{2}+\frac{1}{4}\right)=\frac{21}{4}$$

$$\Rightarrow \log_2 x \times \frac{7}{4}=\frac{21}{4}$$

$$\Rightarrow \log_2 x = \frac{21}{4}\times\frac{4}{7}=3$$

$\Rightarrow \log_2 x = 3$

$\Rightarrow x = 2^3$ ($\because \log a\ b = c \Rightarrow b = a^c$)

$\Rightarrow x = 8$

Thus, the value of x is 8.

Hence, the correct option is (a).

Example 107. The value of $(\log_b a \times \log_c b \times \log_a c)^3$ is equal to

(a) 3 (b) 0

(c) 1 (d) None of these

Sol. (c) We have, $(\log_b a \times \log_c b \times \log_a c)^3$

We know that, $\log_a b = \frac{\log b}{\log a}$

Thus, $(\log_b a \times \log_c b \times \log_a c)^3 = \left(\frac{\log a}{\log b}\times\frac{\log b}{\log c}\times\frac{\log c}{\log a}\right)^3 = (1)^3 = 1$

Therefore, the value of $(\log_b a \times \log_c b \times \log_a c)^3$ is equal to 1.

Hence, the correct option is (c).

Example 108. If $\log\frac{a-b}{2}=\frac{1}{2}(\log a+\log b)$ then the value of $a^2 + b^2$ is

(a) 6ab (b) 8ab (c) $6a^26^2$ (d) None of these

Sol. (a) $\log\left(\frac{a-b}{2}\right)=\frac{1}{2}\log(ab)$

$$2\log\left(\frac{a-b}{2}\right)=\log(ab)$$

$$\log\left(\frac{a-b}{2}\right)^2=\log(ab)$$

$$\left(\frac{a-b}{2}\right)^2=ab$$

$$\frac{a^2+b^2-2ab}{4}=ab$$

$a^2 + b^2 - 2ab = 4ab$

$a^2 + b^2 = 6ab$

Therefore, the required value is 6ab.

ANTILOGARITHM

- If x is the logarithm of a given number n with a given base then n is called the antilogarithm (antilog) of x to that base. This can be expressed as follows:
- If $\log_a n = x$ then $n = \text{antilog } x$

To find Antilogarithm:

1. Divide the number (m) by 227695
2. Add 1 after step 1
3. Press "multiply & =" button for 19 times

Example 109. Find the antilog of

(i) 2.4523 (ii) -1.0451

Sol. (i) Divide 2.4523 by 227695 and then add 1 to it. Press "multiply & =" button for 19 times, thus we get, antilog (2.4523) = 283.305

(ii) Type 1.0451 and then '+ -' sign in the calculator to get -1.0451 or type '0 - 1.0451'.

Now, divide it by 227695 and then add 1 to it. Press "multiply & =" button for 19 times, thus we get, antilog (-1.0451) = 0.090114

PRACTICE QUESTIONS (PART D)

1. $\log_2 16$ is equal to

(a) 2 (b) 8 (c) 4 (d) None of these

2. $\log_{\sqrt{2}} 64$ is equal to

(a) 12 (b) 6 (c) 1 (d) None of these

3. The value of $\log_3 16 \times \log_4 9$

(a) 1 (b) 2 (c) 4 (d) None of these

4. $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$, then the value of x is _____ **(July 2021)**

(a) 64 (b) 4

(c) 16 (d) 2

5. The simplified value of $\log \sqrt[4]{729\sqrt[3]{9^{-1} \cdot 27^{-4/3}}}$ is

(a) log 3 (b) log 2 (c) log 1/2 (d) None of these

6. $\log_b\left(a^{\frac{1}{2}}\right) \cdot \log_c\left(b^3\right) \cdot \log_a\left(c^{\frac{2}{3}}\right) = ?$

(a) 2 (b) c (c) 1 (d) None of these

7. $\frac{1}{\log_{ab}(abc)}+\frac{1}{\log_{bc}(abc)}+\frac{1}{\log_{ca}(abc)}$ is equal to

(a) 0 (b) 1 (c) 2 (d) –1

8. $\log_b(a) \cdot \log_c(b) \cdot \log_a(c)$ is equal to **(ICAI)**

(a) 0 (b) 1 (c) –1 (d) None of these

9. If logx = m + n & log y = m – n, then find the value of $\log\left(\frac{10x}{y^2}\right)$.

(a) 1 + m – 3n (b) 1 – m + 3n (c) 1 + 3m – n (d) 1 – 3m + n

10. If $x = \log_{24} 12$, $y = \log_{36} 24$, $z = \log_{48} 36$, then xyz + 1 =

(a) 2xy (b) 2xz (c) 2yz (d) 2

Answer Key

1. (c) **2.** (a) **3.** (c) **4.** (c) **5.** (a) **6.** (c) **7.** (c) **8.** (b) **9.** (b) **10.** (c)

FULL CHAPTER PRACTICE QUESTIONS

1. Simplify $\frac{2^n+2^{n-1}}{2^{n+1}-2^n}$

(a) 1/2 (b) 3/2 (c) 2/3 (d) 1/3

2. The value of $\log_4 9 \cdot \log_3 2$ is

(a) 3 (b) 2 (c) 9 (d) 1

3. If $\log\left(\frac{x-y}{2}\right)=\frac{1}{2}(\log x+\log y)$, then the value of $x^2 + y^2$ is

(a) 2xy (b) 4xy (c) $2x^2y^2$ (d) 6xy

4. If $\log_3(\log_4(\log_2 x)) = 0$, then the value of 'x' will be

(a) 4 (b) 8 (c) 16 (d) 32

5. The ratio of the number of ₹5 coins and ₹10 coins is 8 : 15. If the value of ₹5 coins is ₹360, then the number of ₹10 coins will be

(a) 72 (b) 120 (c) 135 (d) 185

6. There are a total of 23 coins of ₹1, ₹2 and ₹5 in a bag. If their value is ₹43 and the ratio of coins of ₹1 and ₹2 is 3 : 2. Then the number of coins of ₹1 is

(a) 12 (b) 5 (c) 10 (d) 14

7. If log 2 = 0.3010 and log 3 = 0.4771, then the value of log 24 is

(a) 1.0791 (b) 1.7323 (c) 1.3801 (d) 1.8301

8. If $3^x = 5^y = 75^z$, then

(a) x + y – z = 0 (b) $\frac{2}{x}+\frac{1}{y}=\frac{1}{z}$ (c) $\frac{1}{x}+\frac{2}{y}=\frac{1}{z}$ (d) $\frac{2}{x}+\frac{1}{z}=\frac{1}{y}$

9. The value of $\frac{1}{\log_3 60}+\frac{1}{\log_4 60}+\frac{1}{\log_5 60}$ is

(a) 0 (b) 1 (c) 5 (d) 60

10. An alloy is to certain copper and zinc in the ratio 9 : 4. The zinc required to melt with 24 kg of copper is

(a) $10\frac{2}{3}$ kg (b) $10\frac{1}{3}$ kg (c) $9\frac{2}{3}$ kg (d) 9 kg

11. A box contains ₹56 in the form of coins of one rupee, 50 paise and 25 paise. The number of 50 paise coins is double the number of 25 paise coins and four times the number of one rupee coins. The numbers of 50 paise coins in the box is

(a) 64 (b) 32 (c) 16 (d) 14

12. If $\log_{10000} x = \frac{-1}{4}$, then x is given by

(a) $\frac{1}{100}$ (b) $\frac{1}{10}$ (c) $\frac{1}{20}$ (d) None of these

13. Ratio of earning of A and B is 4 :7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8 : 7. What is A's earning?

(a) ₹21000 (b) ₹28000 (c) ₹28000 (d) Data inadequate

14. The ratio compounded of duplicate ratio of 4 : 5, triplicate ratio of 3 : 2, sub duplicate ratio 25 : 81 and sub–triplicate ratio of 1000 : 27 is

(a) 1 : 4 (b) 3 : 25 (c) 4 : 1 (d) None of these

15. The third proportional between $(a^2 - b^2)$ and $(a + b)^2$ is

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$ (c) $\frac{(a-b)^2}{a+b}$ (d) $\frac{(a+b)^3}{a-b}$

16. If $2^x - 2^{x-1} = 4$, then x^x is equal to

(a) 7 (b) 3 (c) 27 (d) 9

17. log 144 is equal to

(a) 2 log 4 + 2 log 2 (b) 4 log 2 + 2 log 3

(c) 3 log 2 + 4 log 3 (d) 3 log 2 - 4 log 3

18. The number which when subtracted from each of the terms of the ratio 19 : 31 reducing it to 1 : 4 is **(ICAI)**

(a) 15 (b) 5 (c) 1 (d) None of these

19. Simplify the following: $\frac{a^{p+2} \times b^{p+1}}{a^2 \times b}$

(a) $(ab)^p$ (b) $a^p \times b^p$ (c) Both (a) & (b) (d) None of these

20. If $\log_a(\sqrt{3}) = \frac{1}{6}$, find the value of 'a' **(Dec. 2020)**

(a) 81 (b) 9 (c) 27 (d) 3

21. The value of $\frac{1}{1+a^{y-x}} + \frac{1}{1+a^{x-y}}$ is given by **(ICAI)**

(a) –1 (b) 0 (c) 1 (d) None of these

22. log(1 + 2 + 3) is equal to **(ICAI)**

(a) log1 + log2 + log3 (b) log (1 × 2 × 3)

(c) Both (d) None of these

23. $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$, then find the value of x. **(ICAI)**

(a) 6 (b) 2 (c) 5 (d) 9

24. Find the value of $\frac{3t^{-1}}{t^{-\frac{1}{3}}}$. **(ICAI)**

(a) $t^{\frac{3}{3}}$ (b) $t^{\frac{3}{2}}$ (c) $\frac{3}{t^{\frac{1}{3}}}$ (d) $\frac{3}{t^2}$

25. 4, *, 9, $13\frac{1}{2}$ are in proportion. Then * is **(ICAI)**

(a) 6 (b) 8 (c) 9 (d) None of these

26. The value of $\frac{64(b^4a^3)^6}{4a^4b^2 \times (a^2b)^2}$ is

(a) $16a^{10}b^{20}$ (b) $4a^{20}b^{10}$ (c) $8a^{10}b^{20}$ (d) $4a^{10}b^{20}$

27. P and Q started a business investing ₹85,000 and ₹15,000 respectively. In what ratio the profit earned after 2 years be divided between P and Q respectively? **(ICAI)**

(a) 3 : 4 (b) 3 : 5 (c) 15 : 23 (d) 17 : 23

28. Anand and Deepak started a business investing ₹22,500 and ₹35,000 respectively. Out of a total profit of ₹13,800, Deepak's share is **(ICAI)**

(a) ₹5,400 (b) ₹7,200 (c) ₹8,400 (d) 9,600

29. A and B started a business in partnership by investing ₹20,000 and ₹15,000 respectively. After six months, C joined them with ₹20,000. What will be B;s share in the total profit of ₹20,000 earned at the end of 2 years from the starting of the business?

(a) Rs. 7500 (b) ₹9000 (c) ₹9500 (d) ₹10,000

30. A and B can do a work in 8 days, B and C can do the same work in 12 days. A, B and C together can finish it in 6 days. A and C together will do it in:

(a) 4 days (b) 6 days (c) 8 days (d) 12 days

Answer Key

1. (b)	**2.** (d)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (a)
11. (a)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (c)
21. (c)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (a)	**26.** (a)	**27.** (d)	**28.** **(c)**	**29.** (a)	**30.** (c)

PREVIOUS YEAR QUESTION (PYQ'S)

1. Incomes of R and S are in the ratio 7 : 9 and their expenditures are in the ratio 4 : 5. Their total expenditures is equal to income of R. What is the ratio of their earnings? **(Dec. 2021)**

(a) 23 : 36 (b) 28 : 41 (c) 31 : 43 (d) 35 : 46

2. A bag contains 105 coins containing some 50 paise and 25 paise coins. The ratio of the number of these coins is 4:3. The total value (in Rs) in the bag is **(Dec. 2021)**

(a) 43.25 (b) 41.25 (c) 39.25 (d) 35.25

3. In a department, the number of males and females are in the ratio 3 : 2 . If two males and 5 females joins department, then the ratio becomes 1 : 1, initially the number of females in the department is: **(Dec. 2021)**

(a) 9 (b) 6 (c) 3 (d) 8

4. Let $a=\frac{\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)}$ and $b=\frac{\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)}$. What is the value of $a^2 + b^2$? **(Dec. 2021)**

(a) 64 (b) 62 (c) 60 (d) 254

5. The value of $\frac{6^{n+4}+3^{n+3}\times 2^{n+3}}{5\times 6^n+6^n}$ is: **(Dec. 2021)**

(a) 232 (b) 242 (c) 252 (d) 262

6. If $\left(\frac{3a}{2b}\right)^{2x-4}=\left(\frac{2b}{3a}\right)^{2x-4}$, for some a and b, then the value of x is **(Dec. 2021)**

(a) 8 (b) 6 (c) 4 (d) 2

7. The value of $\left(1-\sqrt[3]{0.027}\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)^2\right)$ is: **(Dec. 2021)**

(a) 11/16 (b) 13/16 (c) 15/16 (d) 1

8. If $\log_{10}3 = x$ and $\log_{10}4 = y$, then the value of $\log_{10}120$ can be expressed as **(Dec. 2021)**

(a) x-y+1 (b) x+y+1 (c) x+y-1 (d) 2x+y-1

9. $\log\left(\frac{p^2}{qr}\right)+\log\left(\frac{q^2}{pr}\right)+\log\left(\frac{r^2}{pq}\right)$ is: **(June 2022)**

(a) pqr. (b) 0 (c) 1 (d) None

10. If x : y = 4 : 6 and 2 : x = 1 : 2, then y = _____ **(June 2022)**

(a) 4 (b) 6 (c) 1/3 (d) 3/2

11. A bag contains 25 paise, 10 paise and 5 paise coins in the ratio 3 : 2 : 1. the total value is ₹40, the number of 5 paise coins is **(June 2022)**

(a) 45 (b) 48 (c) 40 (d) 20

12. If $\log_{\sqrt{3}} a = 6$, then 'a' will be: **(June 2022)**

(a) 27 (b) 36 (c) 15 (d) 1

13. A box contains 25 paise coins and 10 paise coins and 5 paise coins in ratios 3 : 2 : 1 and total money is ₹40. How many 5 paise coins are there? **(June 2022)**

(a) 65 (b) 55 (c) 40 (d) 50

14. If $(\sqrt{3})^{18} = (9)^x$, find x. **(June 2022)**

(a) 18 (b) 9 (c) 8 (d) 19

15. $\log_{\sqrt{2}} 64$ is equal to: **(June 2022)**

(a) 12 (b) 6 (c) 1 (d) 8

16. Find the value of z from $(\sqrt{9})^{-8} \times (\sqrt{3})^{-5} = 3^z$ **(June 2022)**

(a) $\frac{2}{21}$ (b) $\frac{-21}{2}$ (c) $\frac{21}{2}$ (d) $\frac{-2}{21}$

17. Incomes of R and S are in the ratio 7:9 and their expenditures are in the ratio 4:5. Their total expenditure is equal to income of R. What is the ratio of their savings? **(June 2022)**

(a) 23:36 (b) 28:41 (c) 31:43 (d) 35:46

18. A bag contains 105 coins containing some 50 paise, and 25 paise coins. The ratio of the number of these coins is 4:3. The total value (in ₹) in the bag is **(June 2022)**

(a) 43.25 (b) 41.25 (c) 39.25 (d) 35.25

19. If $\log_{10} 3 = x$ and $\log_{10} 4 = y$, then the value of $\log_{10} 120$ can be expressed as **(June 2022)**

(a) x-y+1 (b) x+y+1 (c) x+y-1 (d) 2x+y-1

20. In a department, the number of males and females are in the ratio 3:2. If 2 males and 5 females join the department, then the ratio becomes 1:1. Initially, the number of females in the department is **(June 2022)**

(a) 9 (b) 6 (c) 3 (d) 8

21. Find the value of $\log(x^6)$, if $\log(x) + 2\log(x^2) + 3\log(x^3) = 14$. **(June 2022)**

(a) 3 (b) 4 (c) 5 (d) 6

22. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then the value of $\left(\frac{a+b+c}{3}\right)^3$ is equal to: **(June 2023)**

(a) abc (b) 9abc (c) $\frac{1}{abc}$ (d) $\frac{1}{9abc}$

23. Given that $\log_{10} x = m + n - 1$ and $\log_{10} y = m - n$, the value of $\log_{10}\left(\frac{100x}{y^2}\right)$ expressed in terms of m and n is: **(June 2023)**

(a) 1-m+3n (b) m-1+3n (c) m+3n+1 (d) $m^2 - n^2$

24. The value of $\{\log_6\{3\log_{10} 100\}\}$ **(June 2023)**

(a) 1 (b) 2 (c) 10 (d) 100

25. If $x = y^a$, $y = z^b$, $z = x^c$ then the value of abc is **(June 2023)**

(a) 1 (b) 2 (c) 3 (d) 4

26. If $\log_{10} 2 = y$ and $\log_{10} 3 = x$, then the value of $\log_{10} 15$ is: **(Dec. 2022)**

(a) x-y+1 (b) x+y+1 (c) x-y-1 (d) y-x+1

27. $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9$ equal to: **(Dec. 2022)**

(a) 3 (b) 2 (c) 1 (d) 0

28. A sum of money is to be distributed among A, B, C, D in the proportion of 5:2:4:3. If C gets ₹1,000 more than D, what is B 's share?

(a) ₹2,000 (b) ₹1,500 (c) ₹2,500 (d) ₹1,000

29. A group of 400 soldiers posted at border area had a provision for 31 days. After 28 days 280 soldiers from this group were called back. Find the number of days for which the remaining ration will be sufficient?

(a) 3 (b) 6 (c) 8 (d) 10

30. If $2^x = 4^y = 8^z$ and $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}=\frac{24}{7}$, then the value of z is: **(Dec. 2023)**

(a) $\frac{7}{16}$ (b) $\frac{7}{32}$ (c) $\frac{7}{48}$ (d) $\frac{7}{64}$

31. Divide 27 into two parts, so that 5 times the first and 11 times the second together equal to 195, then the ratio of first and second parts is: **(Dec. 2023)**

(a) 17:10 (b) 15:12 (c) 14:13 (d) 16:11

32. $\frac{9\times 35\times (27)^5}{3\times (81)^4} = 27$, then the value of n is: **(Dec. 2023)**

(a) 2 (b) 0 (c) 3 (d) 4

33. Given $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$. Then find the value of $\frac{1}{x^2}+\frac{1}{y^2}$: **(Dec. 2023)**

(a) 63 (b) 61 (c) 62 (d) 60

34. If the four numbers $\frac{1}{4}, \frac{1}{6}, \frac{1}{10}$ and $\frac{1}{x}$ are proportional, then what is the value of x? **(June 2024)**

(a) 14 (b) 15 (c) 10 (d) $\frac{1}{12}$

35. If $2^x = 4^y = 8^z$ and $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}=\frac{24}{7}$, then the value of z is **(June 2024)**

(a) $\frac{7}{16}$ (b) $\frac{7}{32}$ (c) $\frac{7}{48}$ (d) $\frac{7}{64}$

36. If $\log_a b = 3$ and $\log_b c = 2$, then $\log_a c$ is **(June 2024)**

(a) 5 (b) 6 (c) 10 (d) 4

37. What is the value of $\left(\frac{x^b}{x^c}\right)^{b+c-a}\cdot\left(\frac{x^c}{x^a}\right)^{c+a-b}\cdot\left(\frac{x^a}{x^b}\right)^{a+b-c}$? **(Sept 2024)**

(a) x^{abc} (b) $x^{(a+b+c)}$ (c) –1 (d) 1

38. $\log_2\log_2\log_4 256 + 2\log_{\sqrt{2}} 2$ is equal to **(Sept 2024)**

(a) 2 (b) 3 (c) 5 (d) 7

39. The mean proportional between $12x^2$ and $27y^2$ is; **(Sept 2024)**

(a) 18xy (b) 81xy (c) 8xy (d) 19.5xy

40. The ratio of income of A and B is 5 : 4 and their expenditure is 3 : 2. If at the end of year, each saves ₹1600, then the income of A is **(Sept 2024)**

(a) ₹3,400 (b) ₹3,600 (c) ₹4,000 (d) ₹4,400

41. The ratio of age of two sisters is 5 : 7. One is elder to the other by 8 years. Then the ratio of their ages after 4 years between older to younger is

(a) 4 : 3 (b) 2 : 5 (c) 4 : 5 (d) 3 : 5

42. Three employees A, B and C of a firm receive variable incentives money in the ratio 3 : 4 : 5. Then the management also gave a fixed incentive of ₹4000 to each of them. As a result now the total incentive amount of A, B and C becomes in the ratio 5 : 6 : 7. How much amount did B get as a variable incentive? **(Jan 2025)**

(a) ₹4000 (b) ₹2000 (c) ₹6000 (d) ₹8000

43. If $x = \sqrt{2}+\frac{1}{\sqrt{2}}$ and $y = \sqrt{2}-\frac{1}{\sqrt{2}}$, then $x^2 + y^2$ is **(Jan 2025)**

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 5 (d) 0

44. The simplified value of $\frac{\left[5a^5b^2\times 3\left(ab^3\right)^2\right]}{\left(15a^2b\right)}$ is **(Jan 2025)**

(a) a^7b^7 (b) a^5b^7 (c) a^5b^5 (d) a^7b^5

45. If $\left(\frac{x}{y}\right)^{a^2+4} = \left(x^{-1}y\right)^{-5a}$, then the value of a is **(Jan 2025)**

(a) 4, –1 (b) –4, 1 (c) –4, –1 (d) 4, 1

46. Suppose a father had a sum of ₹3,600 and he decided to divide this amount among his three sons Anil, Sunil and Nimal and such a way that 3 times Anil's share, 6 times Sunil's share and 8 times Nimal's share are all equal, then Anil's share is **(Jan 2025)**

(a) ₹960 (b) ₹1,920 (c) ₹720 (d) ₹1,860

47. The sum of three numbers is 98. If the ratio of the first to second number is 2 : 3 and that of the second to third is 5 : 8, then the second number is **(May 2025)**

(a) 20 (b) 30 (c) 48 (d) 58

48. If $4^x = 5^y = 20^z$, then z is equal to **(May 2025)**

(a) xy (b) $\frac{(x+y)}{xy}$ (c) $\frac{1}{xy}$ (d) $\frac{xy}{(x+y)}$

49. If $\log \frac{a+b}{4} = \frac{1}{2}(\log a + \log b)$, then the value of $\frac{a}{b} + \frac{b}{a}$ will be **(May 2025)**

(a) 12 (b) 14 (c) 16 (d) 8

50. A startup business was initiated by an entrepreneur by investing ₹1, 40, 000. His friend joined him after six months with an amount of ₹2, 10, 000. Thereafter an angel investor joined them with ₹2, 80, 000 after another six months. What should be the ratio of distribution of total earnings, three years since beginning of business among entrepreneur, his friend and angel investor? **(May 2025)**

(a) 7 : 6 : 10 (b) 12 : 15 : 16 (c) 42 : 45 : 56 (d) 2 : 3 : 4

51. The ratio $\frac{1}{2}\sqrt{35} : \frac{1}{3}\sqrt{140}$ is equal to the ratio _____. **(Sept 2025)**

(a) 4 : 3 (b) 2 : 1 (c) 5 : 4 (d) 3 : 4

52. The value of $\log_{\sqrt{a}}\left(\sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}}\right)$ is **(Sept 2025)**

(a) $\frac{7}{8}$ (b) $\frac{15}{16}$ (c) $\frac{15}{8}$ (d) $\frac{3}{4}$

53. The value of x is $\log_x(4) + \log_x(16) + \log_x(64) = 12$ is _____. **(Sept 2025)**

(a) 1 (b) 2 (c) 3 (d) 4

54. XYZ invested ₹1,68,000 in a business. After a few months, MNP joined the business by investing ₹1,12,000 in the business. At the end of year, the total profit between them in the ratio 2 : 1. After how many months did MNP join the business? **(Sept 2025)**

(a) 3 (b) 2 (c) 4 (d) 9

Answer Key

1. (d)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (b)
11. (c)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (b)	**20.** (b)
21. (d)	**22.** (a)	**23.** (a)	**24.** (a)	**25.** (a)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (d)	**30.** (c)

31. (a)	**32.** (b)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** (b)	**37.** (d)	**38.** (c)	**39.** (a)	**40.** (c)
41. (a)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (d)	**46.** (b)	**47.** (b)	**48.** (d)	**49.** (b)	**50.** (b)
51. (d)	**52.** (c)	**53.** (b)	**54.** (a)						

SUMMARY

RATIO

- If a and b are two quantities in the same units, then the fraction $\frac{a}{b}$ is called the ratio of a to b. It is written as a : b.
- The quantities a and b are called the terms of the ratio, a is called the first term or antecedent and b is called the second term or consequent.
- The ratio compounded of the two ratios a : b and c : d is ac : bd.
- A ratio compounded of itself is called its duplicate ratio given by $a^2 : b^2$ and the triplicate ratio of a : b is $a^3 : b^3$.

  Similarly, Sub- duplicate ratio of a : b is $a^{1/2} : b^{1/2}$ and the sub-triplicate ratio of a : b is $a^{1/3} : b^{1/3}$.

  Inverse ratio for any ratio a : b is b : a
- Continued Ratio is the relation between three or more Quantities of the same kind. The continued ratio of three similar quantities a, b, c is written as a : b : c.

PROPORTION

- The quantities a, b, c, d are called terms of the proportion; a, b, c and d are called its first, second, third and fourth terms respectively and a : b = c : d.
- First and fourth terms are called extreme terms, whereas Second and third terms are called means (or middle terms).
- If a : b = c : d are in proportion then $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$ i.e.

  Product of extremes = Product of means. This is called the cross product rule.
- Three quantities a, b, c of the same units are said to be in continuous proportion, then the middle term b is called the mean proportional between a and c, a is the first proportional and c is the third proportional.
- Few properties of proportions :
  - If a : b = b : c i.e., $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$
  - Invertendo: $p : q = r : s \Rightarrow q : p = s : r$
  - Alternendo: $a : b = c : d \Rightarrow a : c = b : d$
  - Componendo: $a : b = c : d \Rightarrow a + b : b = c + d : d$
  - Dividendo: $a : b = c : d \Rightarrow a - b : b = c - d : d$

- Componendo & Dividendo: $a : b = c : d \Rightarrow a + b : a - b = c + d : c - d$
- Addendo: $a : b = c : d = a + c : b + d$
- Subtrahendo: $a : b = c : d = a - c : b - d$
- Generalising: If $a : b = c : d = e : f = \ldots$ each of these ratios $= (a - c - e - \ldots) : (b - d - f - \ldots)$

INDICES: (IF BASE IS SAME)

- $a^m \times a^n = a^{m+n}$
- $a^m / a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^\circ = 1$
- $a^{-m} = \frac{1}{a^m} \& \frac{1}{a^{-m}} = a^m$
- If $a^x = a^y$, then $x = y$
- If $x^a = y^a$, then $x = y$
- $\sqrt[n]{a} = a^{1/n}$
- If $a^x = n$, then $x = \log_a n$
- $\log_a a = 1$

LOGARITHMS

- If base of log is e, then it is a natural logarithm and if base is 10 then it is called common logarithm.

  $\log_a mn = \log_a m + \log_a n$

  $\log_a (m/n) = \log_a m - \log_a n$

  $\log_a m^n = n\log_a m$

  $\log_a 1 = 0$
- $\log_b a \times \log_a b = 1$
- $\log_b a = \frac{\log a}{\log b}$
- $\log_b a \times \log_c b = \log_c a$
- $\log_b a = 1/\log_a b$
- $a^{\log_a x} = x$ (Inverse logarithm)

❑❑❑

CA WALLAH

CHAPTER

02

Equations

If I ask you to buy some bananas and few apples such that their sum is equal to 15. How will you write mathematically?

We can represent it as:

Let the number of bananas be x and the number of apples be y having prices as a and b respectively.

Then, ax + by = 15

WHAT IS EQUATION?

- In simple language, equation is a mathematical statement of equality.
- When we equate '=' variable/variables with constant and find out the value of variable. That will be called an equation.

  E.g.: 2x + 3 = 0, where x is a variable and 3 is a constant.

In this chapter, you have to focus on understanding the questions and forming the equation, if done properly you have solved 50% of the question and 50% is just the steps which you will learn.

CONDITIONAL EQUATION

- In simple language, an equation is a mathematical statement of equality.

  E.g.: $\frac{x+2}{2} = 5$

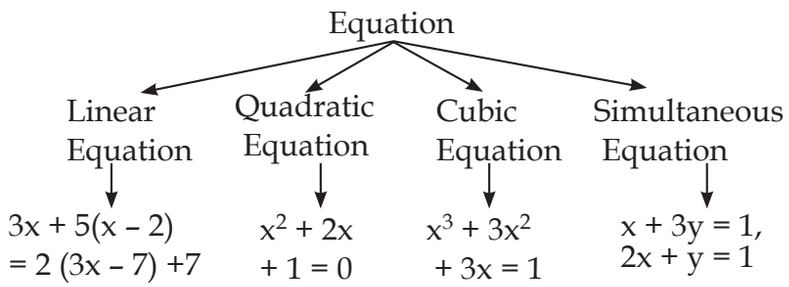
IDENTITY

- If the equality is true for all values of the variable involved, the equation is called an identity.

  E.g.: $\sin^2 x + \cos^2 x = 1$

Solution of the Equation or Root of the Equation

The value of the variable which satisfies the equation will be the solution or root of the equation.



Linear Equation (Simple Equation)

- An equation in the form of $ax + b = 0$ will be called a simple equation or linear equation.

  E.g.: $2x + 3 = 0$ where $a = 2, b = 3$.

Example 1. The equation $2x - 1 = x + 3$ will be satisfied for x equal to

(a) -1 (b) 2 (c) 4 (d) -2

Sol. (c) Given, $2x - 1 = x + 3$

$\Rightarrow 2x - x = 3 + 1$

$\Rightarrow x = 4$

Therefore, the value of x is 4.

Hence, the correct option is (c).

Example 2. The solution of an equation $\frac{x+7}{3} = 4 + \frac{x}{2}$ is

(a) -10 (b) 20 (c) 20 (d) 4

Sol. (a) Given, $\frac{x+7}{3} = 4 + \frac{x}{2}$

$\frac{x+7}{3} - \frac{x}{2} = 4$

$\frac{2\times(x+7)-3\times x}{6} = 4 \Rightarrow 2x + 14 - 3x = 4 \times 6 \Rightarrow -x = 24 - 14 \Rightarrow -x = 10 \Rightarrow x = -10$

Hence, the correct option is (a).

Example 3. 8 is the solution of the equation **(ICAI)**

(a) $\frac{x+4}{4} + \frac{x-5}{3} = 11$ (b) $\frac{x+4}{2} + \frac{x+10}{9} = 8$

(c) $\frac{x+24}{5} = 4 + \frac{x}{4}$ (d) $\frac{x-15}{10} + \frac{x+5}{5} = 4$

Sol. (b) **Go by choices:** Put $x = 8$ in all the equations given in the options.

For option (a):

$\text{LHS} = \frac{8+4}{4} + \frac{8-5}{3} = \frac{12}{4} + 1 = 3 + 1 \neq 11$

Thus, LHS $\neq$ RHS

For option (b):

$\text{LHS } \frac{8+4}{2} + \frac{8+10}{9} = \frac{12}{2} + \frac{18}{9} = 6 + 2 = 8$

Thus, LHS = RHS

For option (c):

$\text{LHS} = \frac{8+24}{5} = \frac{32}{5}$

$$\text{RHS} = 4 + \frac{8}{4} = 4 + 2 = 6$$

Thus, LHS ≠ RHS

For option (d):

$$\text{LHS} = \frac{8-15}{10} + \frac{8+5}{5} = -\frac{7}{10} + \frac{13}{5} = \frac{-7+26}{10} = \frac{19}{10} \neq 4$$

Thus, LHS ≠ RHS

Hence, the correct option is (b).

Example 4. Pick up the correct value x for which $\frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005} = 0$ **(ICAI)**

(a) x = 0 (b) x = 1 (c) x = 10 (d) None of these

Sol. (c) Given equation: $\frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005} = 0$

On multiplying both sides by 0.5, we get

$$0.5 \times \left(\frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005} \right) = 0.5 \times 0$$

$$\Rightarrow \frac{0.5 \times x}{0.5} - \frac{0.5 \times 1}{0.05} + \frac{0.5 \times x}{0.005} - \frac{0.5 \times 1}{0.0005} = 0$$

$\Rightarrow$ x – 10 + 100x – 1000 = 0

$\Rightarrow$ 101x – 1010 = 0

$\Rightarrow$ 101x = 1010

$$\Rightarrow x = \frac{1010}{101}$$

$\Rightarrow$ x = 10

Trick:

Go by choices:

Clearly, for option (c): x = 10

$$\text{LHS} = \frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005}$$

$$= \frac{10}{0.5} - \frac{1}{0.05} + \frac{10}{0.005} - \frac{1}{0.0005}$$

= 20 – 20 2000 – 2000 = 0 = RHS

Hence, the correct answer is option (c) i.e., 10.

PRACTICE QUESTIONS (PART A)

1. The equation –7x + 1 = 5 – 3x will be satisfied for x equal to: **(ICAI)**

(a) 2 (b) – 1 (c) 1 (d) None

2. The equation -x + 1 = 5 – 2x will be satisfied for x equal to

(a) 2 (b) 4 (c) 1 (d) None of these

3. The value of x for which 5x – 11 = 3x + 9 is satisfied

(a) 20 (b) – 2 (c) 10 (d) None

4. The value of y for which the equation 7y – 42 = 6 + 3y is satisfied

(a) 5 (b) 12 (c) 16 (d) None of these

5. What will be the correct value of x for $\frac{x}{10}=\frac{3}{5}$.

(a) –5 (2) 6 (3) 4 (4) –6

6. What will be the correct value x for which $\frac{x}{0.1}-\frac{1}{0.01}+\frac{x}{0.001}-\frac{1}{0.1}=0$

(a) –10 (b) 2 (c) 10 (d) –5

7. The solution of the equation is $\frac{12y+1}{4}=\frac{15y-1}{5}+\frac{2y-5}{3y-1}$

(a) y = 1 (b) y = 2 (c) y = 5 (d) y = 7

Answer Key

1. (b) **2.** (b) **3.** (b) **4.** (b) **5.** (b) **6.** (c) **7.** (d)

STATEMENT BASED

Example 5. The sum of two numbers is 20 and their difference is 4. The numbers are

(a) (12, 15) (b) (12, 8) (c) (2, 8) (d) None of these

Sol. (b) Let two numbers be x and y.

According to the question,

x + y = 20 ….. (i)

x – y = 4 …. (ii)

From equation (i), we get

x = 20 – y

Substituting the value of x in equation (ii),

$\Rightarrow$ 20 – y – y = 4

$\Rightarrow$ 20 – 2y = 4

$\Rightarrow$ –2y = 4 – 20

$\Rightarrow$ –2y = –16

$\Rightarrow y=\frac{16}{2}=8$

Thus, y = 8

Also, x = 20 – y

$\Rightarrow$ x = 20 – 8 = 12

Therefore, (x, y) = (12, 8)

Trick:

Go by choices:

For option (a): (12, 15)

Sum of numbers = 12 + 15 = 27 ≠ 20

For option (b): (12, 8)

Sum of numbers = 12 + 8 = 20, which is true

Difference of numbers = 12 – 8 = 4, which is also true

Therefore, the numbers are 12 and 8.

Hence, the correct option is (b).

Example 6. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and third length is to be twice as the shortest. What is the possible length for the shortest piece?

(a) 22 cm (b) 20 cm (c) 15 cm (d) 18 cm

Sol. (a) Let the shortest piece be of x cm.

According to the question,

Second length = 'x + 3' cm and Third length = '2x' cm

Thus,

$x + x + 3 + 2x = 91$

$\Rightarrow 4x = 91 - 3$

$\Rightarrow 4x = 88$

$\Rightarrow x = 22$

Therefore, the length of shortest piece of board is 22 cm.

Example 7. The value of y for fraction $\frac{x}{y}$ exceeds with x by 5 and if 3 added to both, the fraction becomes $\frac{3}{4}$. Find the fraction.

(a) $\frac{12}{17}$ (b) $\frac{13}{17}$ (c) $-\frac{1}{3}$ (d) None of these

Sol. (a) Since, the value of y exceeds x by 5

$\Rightarrow y = x + 5$

Therefore, the fraction is $\frac{x}{x+5}$

According to question,

$\frac{x+3}{x+8} = \frac{3}{4} \Rightarrow 4x + 12 = 3x + 24 \Rightarrow x = 12$

Therefore, the required fraction = $\frac{12}{12+5} = \frac{12}{17}$

Example 8. If we subtract twice Ram's present age from his age 15 years from the present, the result would be equal to his age after 3 years. Find Ram's present age.

(a) 10 (b) 12 (c) 15 (d) 6

Sol. (d) Let Ram's Present age be x years.

According to question,

$(x + 15) - 2x = x + 3$

$\Rightarrow x - 2x + 15 = x + 3$

$\Rightarrow -x + 15 = x + 3$

$\Rightarrow 15 - 3 = x + x$

$\Rightarrow 12 = 2x$

$\Rightarrow x = 6$

Therefore, the present age of Ram is 6 years.

Hence, the correct option is (d).

Example 9. One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so the student divides the given number by 5. If the answer is 4 short of the correct answer, then the number was

(a) 320 (b) 400 (c) 480 (d) None of these

Sol. (c) Let the required number be x.

According to requirement, the number should be $\frac{1}{6}\times\frac{x}{2}+\frac{1}{4}\times\frac{x}{2}$

The number according to the student is $\frac{x}{5}$

Thus, the relation between the numbers:

$$\frac{x}{5}=\frac{x}{12}+\frac{x}{8}-4$$

$$\frac{x}{12}+\frac{x}{8}-\frac{x}{5}=4$$

LCM of 5, 8, 12 = 120

$$\Rightarrow \frac{(10x+15x-20x)}{120}=4$$

$\Rightarrow x = 120 \times 4$

$\Rightarrow x = 480$

Therefore, the required number is 480.

Hence, the correct option is (c).

Example 10. For a certain commodity, the demand equation giving demand 'd' in kg, for a price 'p' in rupees per kg is d = 50(20 – p). The supply equation gives the supply 's' in kg, for a price 'p' in rupees per kg is s = 75(p – 3). The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.

(a) (9.8, 150) (b) (20, 405) (c) (15, 150) (d) None of these

Sol. (a) Given, Demand equation: d = 50(20 – p)

Supply equation: s = 75(p – 3)

Since, the market price is such at which demand equals supply thus

$d = s \Rightarrow 50(20 - p) = 75(p - 3)$

$\Rightarrow 2(20 - p) = 3(p - 3)$

$\Rightarrow 40 - 2p = 3p - 9 \Rightarrow 3p + 2p = 40 + 9 \Rightarrow 5p = 49 \Rightarrow p = 9.8$

On substituting the value of p in any of the above equation, we get

Quantity, $d = 50(20 - 9.8) = 150$

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART B)

1. If the sum of two numbers is 13 and the sum of their squares is 85 then the numbers are:

(a) 6, 7 (b) 4, 9 (c) 10, 3 (d) 5, 8

2. Divide 65 into two parts such that two times the first part exceeds one fourth of the second by 40. The parts are.

(a) (30, 35) (b) (20, 45) (c) (15, 50) (d) None of these

3. The denominator of a fraction exceeds the numerator by 2. If 5 be added to the numerator the fraction increases by unity. The fraction is.

(a) 3/5 (b) 1/3 (c) 7/9 (d) None of these

4. If a number of which the half is greater than $\frac{1}{5}$ th of the number by 15 then the number is

(a) 50 (b) 40 (c) 80 (d) None of these

5. The ages of three brothers are consecutive even integers. If the sum of their ages is , what is the age of the youngest brother?

(a) 24 years (b) 26 years (c) 28 years (d) 30 years

6. The age of Raj is three times the sum of the ages of his sons and 5 years hence his age will be double the sum of their ages. Find the present age of Raj?

(a) 54 (b) 45 (c) 60 (d) None

Answer Key

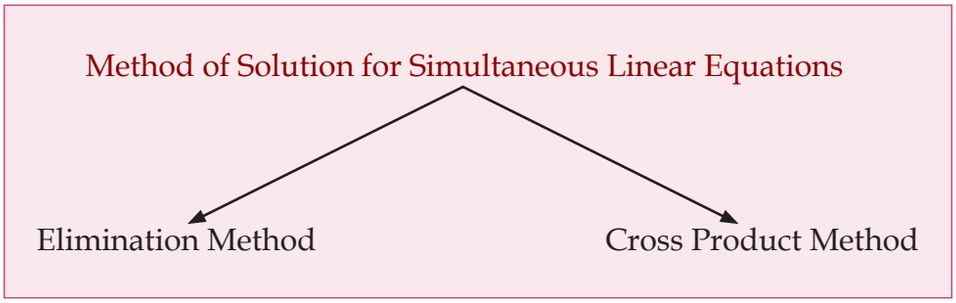
1. (a) **2.** (d) **3.** (a) **4.** (a) **5.** (a) **6.** (b)

SIMULTANEOUS LINEAR EQUATIONS

What will we study here?

- Simultaneous linear equation is basically a method of solving two equations given simultaneously at a time, where we need to find solution for two different variables.
- The general form of that type of equation will be: $ax + by + c = 0$,
  where, x and y are two variables.
- So, let's say two equations possibly are:
  $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

In above equation, we need to find the value (solution or roots) of two linear equation which has two variables x and y.



ELIMINATION METHOD

- In this method two given linear equations are reduced to a linear equation in one variable by eliminating one of the variables and then solving for the other variable.

Example 11. Solve for x and y:

$2x + y = 9$ and $2x - 3y = 5$

Sol. Here, since the coefficient of x are same, thus subtracting second equation from first equation, i.e.,

$2x + y = 9$

$2x - 3y = 5$

$- \quad + \quad -$

$\Rightarrow 2x - 2x + y + 3y = 9 - 5$

$\Rightarrow y = \frac{4}{4} = 1$

Substituting the value of x in first equation, we get

$2x + 1 = 9$

$\Rightarrow 2x = 9 - 1$

$\Rightarrow x = \frac{8}{2} = 4$

Therefore, the solution of given equations is (4, 1).

Example 12. Solve for x and y:

$2x - y = -2$ and $4x - 3y = -12$

(a) (1, 2) (b) (3, 8) (c) (4, 4) (d) (8, 3)

Sol. (b) Given,

$2x - y = -2$ ...(i)

$4x - 3y = -12$ ...(ii)

Multiplying eq. (i) by 2, we get

$4x - 2y = -4$ …. (iii)

On subtracting eq. (iii) from eq. (ii), we get

$4x - 3y - 4x + 2y = -12 + 4$

$\Rightarrow -y = -8$

$\Rightarrow y = 8$

Now, put the value of y in eq. (i), we get

$2x - 8 = -2$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

Therefore, the value of x is 3 and y is 8.

Hence, the correct option is (b) i.e., (3, 8).

Example 13. The solution of the following system of linear equation $2x - 5y + 4 = 0$ and $2x + y - 8 = 0$ will be

(a) (2,-3) (b) (1,-4) (c) (3, 2) (d) (-2, 2)

Sol. (c) Given equations:

$2x - 5y + 4 = 0$ .... (i)

$2x + y - 8 = 0$ .... (ii)

Subtracting both the equations, we get

$-5y - y + 4 + 8 = 0$

$6y = 12$

$y = 2$

Substitute the value of x in eq (i) we get

$2x - 10 + 4 = 0$

$2x = 6$

$x = 3$

Therefore, (x, y) = (3, 2)

Example 14. The solution of the set of equations $3x + 4y = 7$, $4x - y = 3$ is

(a) (1, –1) (b) (1, 1) (c) (2, 1) (d) (1, –2)

Sol. (b) Given equations: $3x + 4y = 7$, $4x - y = 3$

We need to find the value (x, y) so that both the equations are satisfied.

$3x + 4y = 7$ ...(i)

$4x - y = 3$ ...(ii)

Multiplying equation (ii) by 4, we get

$16x - 4y = 12$ ...(iii)

On adding equations (i) and (iii), we get

$(3x + 4y) + (16x - 4y) = 7 + 12$

$\Rightarrow 19x = 19$

$\Rightarrow x = 1$

Now, putting the value of $x = 1$ in equation (i), we have

$3 \times 1 + 4y = 7$

$\Rightarrow 3 + 4y = 7$

$\Rightarrow 4y = 7 - 3$

$\Rightarrow 4y = 4$

$\Rightarrow y = 1$

Therefore, the solution is (1, 1) for the given equations.

Hence, the correct answer is option (b) i.e., (1, 1).

Example 15. The values of x and y satisfying the equations $3x - y = 3$, $x + 2y = 8$ are given by the pair:

(a) (3, 2) (b) (–2, –3) (c) (2, 3) (d) None of these

Sol. (c) Given,

$3x - y = 3$ ...(i)

$x + 2y = 8$ ...(ii)

Multiplying eq. (ii) by 3 on both sides, we get

$3x + 6y = 24$ ...(iii)

On subtracting above eq. (iii) from eq. (i), we get

$3x - y = 3$

$3x + 6y = 24$

$- \quad - \quad -$

$\Rightarrow 3x - 3x - y - 6y = 3 - 24$

$\Rightarrow -7y = -21$

$\Rightarrow y = \frac{-21}{-7} = 3$

Putting value of y in eq. (i), we get

$3x - 3 = 3$

$\Rightarrow 3x = 3 + 3 = 6$

$\Rightarrow x = \frac{6}{3} = 2$

Therefore, (x, y) = (2, 3)

Trick:

Put all the options in given equations, you will see only option (c) will be satisfying both equations, i.e.

For option (c): (2, 3)

Substituting x = 2 and y = 3 in 3x – y = 3,

LHS = 3(2) – 3 = 6 – 3 = 3 = RHS

Now, substituting x = 2 and y = 3 in x + 2y = 8, i.e.

LHS = 2 + 2(3) = 2 + 6 = 8 = RHS

Hence, the correct option is (c).

CROSS MULTIPLICATION METHOD

- Let two equations be:

  $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

  Write coefficients of x and y, also constants in both the equations, likes:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

i.e., $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Example 16. The simultaneous equations 7x – 3y = 31, 9x – 5y = 41 have solutions given by

(a) (–4, –1) (b) (–1, 4) (c) (4, –1) (d) (3, 7)

Sol. (c) Comparing 7x – 3y – 31 = 0 with $a_1x + b_1y + c_1 = 0$

$\Rightarrow a_1 = 7, b_1 = -3, c_1 = -31$

Now, comparing 9x – 5y – 41 = 0 with $a_2x + b_2y + c_2 = 0$

$\Rightarrow a_2 = 9, b_2 = -5, c_2 = -41$

We know that, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$$x = \frac{(-3 \times -41) - (-5 \times -31)}{(7 \times -5) - (9 \times -3)}$$

$$\Rightarrow x = \frac{123 - (155)}{(-35) - (-27)}$$

$$\Rightarrow x = \frac{-32}{-8} = 4$$

Thus,

$$y = \frac{(-31\times 9)-(-41\times 7)}{(7-5)-(9-3)}$$

$$\Rightarrow y = \frac{-279-(-287)}{(-35)-(-27)}$$

$$\Rightarrow y = \frac{8}{-8} = -1$$

$\Rightarrow (x, y) = (4, -1)$

Trick:

Put all the options in given equations, you will see only option (c) will be satisfying both equations.

$(7 \times 4) - (3 \times -1) = 28 + 3 = 31,$

$(4 \times 9) - (5 \times -1) = 36 + 5 = 41$

Hence, the correct option is (c).

Example 17. $1.5x + 3.6y = 2.1$, $2.5(x + 1) = 6y$ have solutions as

(a) (0.2, 0.5) (b) (0.5, 0.2) (c) (2, 5) (d) (–2, –5)

Sol. (a) Given, $1.5x + 3.6y = 2.1$

$\Rightarrow 15x + 36y = 21$

$\Rightarrow 5x + 12y = 7$ ...(i)

Also, $2.5(x + 1) = 6y$

$\Rightarrow 2.5x + 2.5 = 6y$

$\Rightarrow 25x + 25 = 60y$

$\Rightarrow 5x + 5 = 12y$ (on dividing both the sides by 5)

$\Rightarrow 5x - 12y = -5$ ...(ii)

Now, adding eq. (i) and eq. (ii), we get

$(5x + 12y) + (5x - 12y) = 7 + (-5)$

$\Rightarrow 10x = 2$

$\Rightarrow x = 0.2$

Now, putting the value of $x = 0.2$ in equation (i), we get

$5(0.2) + 12y = 7$

$\Rightarrow 1 + 12y = 7$

$\Rightarrow 12y = 7 - 1$

$\Rightarrow 12y = 6$

$$\Rightarrow y = \frac{6}{12}$$

$\Rightarrow y = 0.5$

Thus, the required solution set is $x = 0.2$ and $y = 0.5$.

Hence, the correct answer is option (a).

Example 18. If $2^{x+y} = 2^{2x-y} = \sqrt{8}$ then the respective values of x and y are

(a) $1, \frac{1}{2}$ (b) $\frac{1}{2}, 1$ (c) $\frac{1}{2}, \frac{1}{2}$ (d) None of these

Sol. (a) We have, $2^{x+y} = 2^{2x-y} = \sqrt{8}$

$\Rightarrow 2^{x+y} = 2^{2x-y} = \left(2^3\right)^{\frac{1}{2}}$

$\Rightarrow 2^{x+y} = 2^{2x-y} = 2^{\frac{3}{2}}$

$\Rightarrow x + y = \frac{3}{2}$ & $2x - y = \frac{3}{2}$

On adding both the equations, we get

$(x + y) + (2x - y) = \frac{3}{2} + \frac{3}{2}$

$\Rightarrow 3x = 3$

$\Rightarrow x = 1$

$\Rightarrow y = \frac{3}{2} - x = \frac{3}{2} - 1 = \frac{1}{2}$

Therefore, the values of x and y are 1 and $\frac{1}{2}$ respectively.

METHOD OF SOLVING SIMULTANEOUS LINEAR EQUATION WITH THREE VARIABLES:

$ax + by + cz = d$, where x, y, z are variables, d is a constant and a, b, c are coefficients.

Example 19. $\frac{xy}{x+y} = 20, \frac{yz}{y+z} = 40, \frac{zx}{z+x} = 24$ **(ICAI)**

(a) (120, 60, 30) (b) (60, 30, 120) (c) (30, 120, 60) (d) (30, 60, 120)

Sol. (d) Given: $\frac{xy}{x+y} = 20 \Rightarrow \frac{x+y}{xy} = \frac{1}{20}$

$$\frac{1}{y} + \frac{1}{x} = \frac{1}{20} \quad \ldots.(i)$$

Also, $\frac{yx}{(y+z)} = 40 \Rightarrow \frac{y+z}{yz} = \frac{1}{40}$

$$\frac{1}{z} + \frac{1}{y} = \frac{1}{40} \quad \ldots..(ii)$$

Now, $\frac{zx}{(z+x)} = 24 \Rightarrow \frac{z+x}{zx} = \frac{1}{24}$

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{24} \quad \ldots.(iii)$$

Adding eq (i), (ii) and (iii), we get

$$\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = \frac{1}{20} + \frac{1}{40} + \frac{1}{24}$$

$$\Rightarrow 2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\frac{14}{120}\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{7}{120} \quad \ldots(iv)$$

From (i) and (iv), we get

$$\frac{1}{20}+\frac{1}{z}=\frac{7}{120}\Rightarrow z=120$$

Now, from (ii) and (iv), we get

$$\frac{1}{40}+\frac{1}{x}=\frac{7}{120}\Rightarrow z=30$$

Thus, y = 60

Therefore, the values of (x, y, z) = (30, 60, 120).

Hence, the correct option is (d).

Example 20. Solve for x, y and z, $2x + 3y + 4z = 0$, $x + 2y - 5z = 0$, $10x + 16y - 6z = 0$

(a) (0, 0, 0) (b) (1, –1, 1) (c) (3, 2, –1) (d) (1, 0, 2) **(ICAI)**

Sol. (a) **Detailed Method:**

Given,

$2x + 3y + 4z = 0$ ...(i)

$x + 2y - 5z = 0$ ...(ii)

$10x + 16y - 6z = 0$ ...(iii)

On multiplying equation (ii) by 2, we get

$2x + 4y - 10z = 0$

Thus, subtracting the above equation from eq. (i),

$-y + 14z = 0$

$\Rightarrow y = 14z$

Substitute y value in eq. (i), we get

$2x + 3(14z) + 4z = 0$

$\Rightarrow 2x + 42z + 4z = 0$

$\Rightarrow 2x + 46z = 0$ ...(iv)

Again, substituting y value in eq. (iii), we get

$10x + 16(14z) - 6z = 0$

$\Rightarrow 10x + 224z - 6z = 0$

$\Rightarrow 10x + 218z = 0$ ...(v)

From eq. (iv) and eq. (v), we get z = 0

Also, y = 14z

$\Rightarrow y = 0$

Substituting these values in eq. (i), we get x = 0

Therefore (x, y, z) = (0, 0, 0)

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Trick:

Put all the options in given equations, you will see only option (a) will be satisfying all three equations.

$2 \times 0 + 3 \times 0 + 4 \times 0 = 0$,

$0 + 2 \times 0 - 5 \times 0 = 0$,

$10 \times 0 + 16 \times 0 - 6 \times 0 = 0$

Therefore, the required solution is (0, 0, 0).

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART C)

1. Solve for x and y: $x - 3y = 0$, $x + 2y = 20$

(a) $x = 4, y = 12$ (b) $x = 12, y = 4$

(c) $x = 5, y = 4$ (d) None of these

2. Solve for x and y: $2x + y = -4$ and $5x - 3y = 1$

(a) $x = 1, y = 2$ (b) $x = 2, y = 2$ (c) $x = -1, y = -2$ (d) None of these

3. Solve for x and y: $4x + 2y = 10$ and $5x - y = 4$

(a) $x = \frac{1}{7}, y = \frac{15}{7}$ (b) $x = \frac{1}{9}, y = \frac{17}{7}$

(c) $x = \frac{9}{7}, y = \frac{17}{7}$ (d) None of these

4. $1.5x + 2.4y = 1.8$, $2.5(x + 1)$ have solutions as **(ICAI)**

(a) (0.5, 0.4) (b) (0.4, 0.5) (c) $\left(\frac{1}{2}, \frac{2}{5}\right)$ (d) (2, 45)

5. $\frac{4}{x} - \frac{5}{y} = \frac{x+y}{xy} + \frac{3}{10}$, $3xy = 10(y - x)$ **(ICAI)**

(a) (2, 5) (b) (5, 2) (c) (2, 7) (d) (3, 4)

6. The solution for the pair of $\frac{xy}{y-x} = 110, \frac{yz}{z-y} = 132, \frac{zx}{z+x} = \frac{60}{11}$ is given by **(ICAI)**

(a) (12, 11, 10) (b) (10, 11, 12) (c) (11, 10, 12) (d) (12, 10, 11)

7. Solve for x and y: $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$.

(a) $x = 2, y = -2$ (b) $x = 2, y = 3$ (c) $x = 1, y = 2$ (d) $x = 2, y = -3$

8. The solution to the given set of equations $\frac{x}{5} + \frac{y}{6} + 1 = \frac{x}{6} + \frac{y}{5} = 28$ **(ICAI)**

(a) (6, 9) (b) (9, 6) (c) (60, 90) (d) (90, 60)

9. The solution for the pair of equation $\frac{1}{16x} + \frac{1}{15y} = \frac{9}{20}, \frac{1}{20x} - \frac{1}{27y} = \frac{4}{45}$ is given by,

(a) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (b) $\left(-\frac{1}{2}, \frac{1}{3}\right)$ (c) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (d) $\left(\frac{1}{2}, \frac{1}{4}\right)$ **(ICAI)**

10. The solution for the pair of $\frac{xy}{y-x}=110, \frac{yz}{z-y}=132, \frac{zx}{z+x}=\frac{60}{11}$ **(ICAI)**

(a) (12, 11, 10) (b) (10, 11, 12) (c) (11, 10, 12) (d) (12, 10, 11)

Answer Key

1. (b) **2.** (c) **3.** (c) **4.** (b) **5.** (d) **6.** (b) **7.** (d) **8.** (c) **9.** (c) **10.** (b)

PROBLEMS RELATED TO SIMULTANEOUS EQUATIONS:

Example 21. 4 tables and 3 chairs together cost ₹2,250 and 3 tables and 4 chairs cost ₹1950. Find the cost of 2 chairs and 1 table.

(a) ₹550 (b) ₹1005 (c) ₹750 (d) None of these

Sol. (c) Let the cost of table be x and cost of chair be y, then

$4x + 3y = 2250$ ...(i)

$3x + 4y = 1950$ ...(ii)

Multiplying eq (i) with 3 and eq (ii) with 4 and then subtracting, we get

$12x + 9y (12x + 16y) = 6750-7800$

$- 7y = - 1050$

$y = 150$

Thus, $x = 450$

Therefore, the cost of 2 chairs and 1 table = $2y + x = 2(150) + 450 = ₹750$.

Example 22. A number consists of two digits. The digits in the ten's place is 3 times the digit in the unit's place. If 54 is subtracted from the number, the digits are reversed. The number is

(a) 39 (b) 92 (c) 93 (d) 94

Sol. (c) Let the two-digit number = $10x + y$

The digits in the ten's place is 3 times the digit in the unit's place

$\Rightarrow x = 3y$ ...(i)

If 54 is subtracted from the number, the digits are reversed

$\Rightarrow 10x + y - 54 = 10y + x$

$\Rightarrow 9x = 9y + 54$

$\Rightarrow x = y + 6$

$\Rightarrow 3y = y + 6$ (from i)

$\Rightarrow y = 3$

Thus, $x = 3y$

$\Rightarrow x = 9$

Therefore, the required number is 93.

Trick:

Only for option (c): 93

If 54 is subtracted from the number, the digits are reversed 93 – 54 = 39

Hence, the correct option is (c).

Example 23. The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age. **(ICAI)**

(a) 60 years (b) 52 years (c) 51 years (d) 50 years

Sol. (d) Let the age of man be z years and their sons be x and y.

According to the question,

$z = 2(x + y)$ ...(i)

Five years ago,

Age of father = $z - 5$

Ages of their sons = $x - 5, y - 5$

Thus, the relation five years ago is:

$z - 5 = 3(x - 5 + y - 5)$

$\Rightarrow z - 5 = 3(x + y - 10)$

$\Rightarrow z = 3(x + y) - 25$ ...(ii)

From (i) and (ii), we get

$$z = \frac{3}{2}z - 25$$

$$\Rightarrow z - \frac{3}{2}z = -25$$

$$\Rightarrow -\frac{z}{2} = -25$$

$\Rightarrow z = 50$

Therefore, the age of the person is 50 years.

Hence, the correct option is (d).

Example 24. A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed then find the number. **(ICAI)**

(a) 54 (b) 53 (c) 45 (d) 55

Sol. (c) Let the two-digit number be xy.

Thus, the number be $10x + y$

Sum of its digits = $x + y$

According to the question,

$10x + y = 5(x + y)$

$\Rightarrow 10x + y = 5x + 5y$

$\Rightarrow 10x - 5x = 5y - y$

$\Rightarrow 5x - 4y = 0$ ...(i)

If 9 is added, then the digits are reversed.

$\Rightarrow 10x + y + 9 = 10y + x$

$\Rightarrow 10x - x + 9 = 10y - y$

$\Rightarrow 9x - 9y = -9$

$\Rightarrow x - y = -1$ ...(ii)

Also, $5x - 4y = 0$

Multiplying eq. (ii) with 5, thus

$\Rightarrow 5(x - y) = -5$

$\Rightarrow 5x - 5y = -5$ ...(iii)

Subtracting eq. (iii) from eq. (i), we get

$\Rightarrow (5x - 4y) = -(5x - 5y) = 0 - (-5)$

$\Rightarrow -4y + 5y = 5$

$\Rightarrow y = 5$

On substituting $y = 5$ in $x - y = -1$, we get

$\Rightarrow x = 4$

Therefore, the number is $10x + y = 10(4) + 5 = 45$

Hence, the correct option is (c).

Example 25. The wages of 8 men and 6 boys amount to ₹33. If 4 men earn ₹4.50 more than 5 boys, determine the wages of each man and boy. **(ICAI)**

(a) (₹0.50, ₹3) (b) (₹3, ₹1.50) (c) (₹2.50, ₹2) (d) (₹2, ₹2.50)

Sol. (b) Let the wages of 1 man be ₹x and that of 1 boy be ₹y.

According to the question,

$8x + 6y = 33$ ...(i)

and $4x = 4.50 + 5y$

Or, $4x - 5y = 4.50$ ...(ii)

Multiply eq. (ii) by 2, we get

$8x - 10y = 9$

Subtract it from eq. (i), we get

$16y = 24$

$y = 1.5$

Putting value of y in $8x - 10y = 9$, we get

$x = 3$

Therefore, the wages of each man and boy are ₹3 and ₹1.50 respectively.

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART D)

1. A number consisting of two digits is four times the sum of its digits and if 27 be added to it the digits are reversed. The number is

(a) 63 (b) 35 (c) 36 (d) 60

2. The cost prices of 3 pens and 4 bags is ₹324 and 4 pens and 3 bags is ₹257, then the cost of 1 pen is equal to

(a) ₹16 (b) ₹8 (c) ₹50 (d) ₹75

3. The sum of the digits of a two–digit number is 8. If 13 be subtracted from it the digits in the resulting number will be equal. The number is

(a) 47 (b) 35 (c) 56 (d) None of these

4. Ten years ago, the age of Sanjay was four times that of his son. Ten years hence, his age will be twice of his son, The present ages of Sanjay and his sons are

(a) (58, 20) (b) (60, 20) (c) (50, 20) (d) None of these

5. A fraction becomes $\frac{9}{11}$ if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and denominator, it becomes $\frac{5}{6}$. Find the fraction.

(a) $\frac{1}{4}$ (b) $\frac{3}{10}$ (c) $\frac{7}{9}$ (d) None of these

6. Product of the digits of a two-digit number is 20. If we add 9 to the number, the digits get reversed. Then the original two-digit number is

(a) 54 (b) 45 (c) 20 (d) 63

7. The demand and supply equations for a certain commodity are $4q + 7p = 17$ and $p=\frac{q}{3}+\frac{7}{4}$ respectively where p is the market price and q is the quantity then the equilibrium price and quantity are **(ICAI)**

(a) $2,\frac{3}{4}$ (b) $3,\frac{3}{4}$ (c) $5,\frac{3}{4}$ (d) None of these

Answer Key

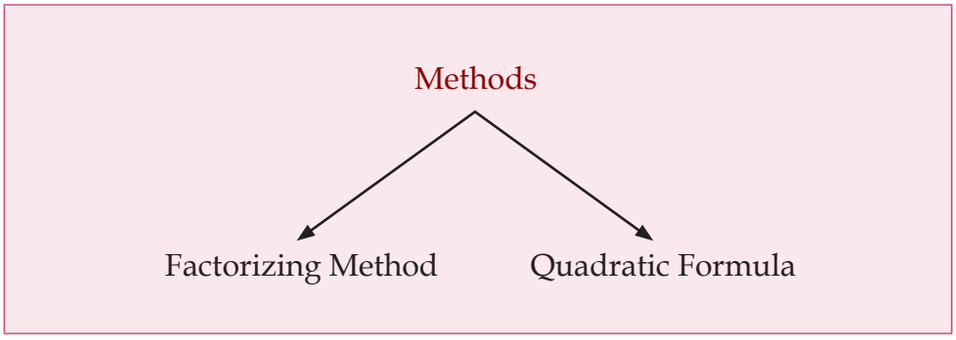
1. (c) **2.** (b) **3.** (b) **4.** (c) **5.** (c) **6.** (b) **7.** (a)

QUADRATIC EQUATION

- An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called a quadratic equation or equation of the second degree.

  E.g.: (i) $3x^2 + 2x + 1 = 0$ (ii) $x^2 - x = 0$

How to find out the roots of a quadratic equation: $ax^2 + bx + c = 0$ $(a \neq 0)$



QUADRATIC FORMULA

For quadratic equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SUM AND PRODUCT OF ROOTS

Let α and β be the roots of equation $ax^2 + bx + c = 0$, then

Sum of roots: $\alpha + \beta = -\frac{b}{a}$

Product of roots: $\alpha\beta = \frac{c}{a}$

How to construct a Quadratic equation:

For the equation $ax^2 + bx + c = 0$ we have

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$$

x^2 – (Sum of the roots) x + Product of the roots = 0

NATURE OF ROOTS

For Quadratic Equation

$ax^2 + bx + c = 0$

We know, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. If $b^2 - 4ac = 0$, then the roots are real and equal.
2. If $b^2 - 4ac > 0$, then the roots are real and unequal (or distinct).
3. If $b^2 - 4ac < 0$, then the roots are imaginary.
4. If $b^2 - 4ac$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct).
5. If $b^2 - 4ac > 0$ but not a perfect square, the roots are real, irrational and unequal.

Example 26. Solve $x^2 - 8x + 16 = 0$ and find the roots.

(a) 4, –4 (b) 4, 0 (c) 4, 4 (d) 1, 4

Sol. (c) Given, $x^2 - 8x + 16 = 0$

Method (1): By factorization method:

$\Rightarrow x^2 - 8x + 16 = 0$

$\Rightarrow x^2 - 4x - 4x + 16 = 0$

$\Rightarrow x(x - 4) - 4(x - 4) = 0$

$\Rightarrow (x - 4)(x - 4) = 0$

$\Rightarrow x - 4 = 0, x - 4 = 0$

$\Rightarrow x = 4, 4$

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Method (2): By Quadratic Formula:

On comparing given equation with $ax^2 + bx + c = 0$, we get

$a = 1, b = -8, c = 16$

By Quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4*1*16}}{2}$$

$$\Rightarrow x = \frac{8 \pm \sqrt{64 - 64}}{2}$$

$$\Rightarrow x = \frac{8}{2} = 4$$

$\Rightarrow x = 4, 4$

Therefore, the roots are 4, 4.

Hence, the correct option is (c).

Example 27. The positive root of the equation $2x^2 - 5x = 3$ is

(a) 5 (b) 3 (c) 4 (d) 1

Sol. (b) Given equation: $2x^2 - 5x = 3$

$\Rightarrow 2x^2 - 5x - 3 = 0$

$\Rightarrow 2x^2 - 6x + x - 3 = 0$

$\Rightarrow 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow (x - 3)(2x + 1) = 0$

$\Rightarrow (x - 3) = 0$ or $(2x + 1) = 0$

$\Rightarrow x = 3$ or $\frac{-1}{2}$

Here, $\frac{-1}{2}$ is not a positive.

$\Rightarrow x = 3$

Therefore, the required positive root is 3.

Hence, option (b) is correct i.e., 3.

Example 28. If the roots of the quadratic equation $2x^2 + 5x - 3 = 0$ are α and β what is the value of $|\alpha - \beta|$?

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{7}{2}$ (d) $\frac{5}{2}$

Sol. (c) Let α and β are roots of the quaratic equation $2x^2 + 5x - 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$

$a = 2; b = 5; c = -3$

Sum of roots $(\alpha + \beta) = \frac{-b}{a} = \frac{-5}{2}$

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$\Rightarrow (\alpha+\beta)=\frac{-5}{2}$ ...(i)

And products of roots $(\alpha\beta) = \frac{c}{a}=\frac{-3}{2}$

$\Rightarrow (\alpha\beta)=\frac{-3}{2}$ ...(ii)

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow (\alpha - \beta)^2 = \left(\frac{-5}{2}\right)^2 - 4\left(\frac{-3}{2}\right)$

$\Rightarrow (\alpha-\beta)^2 = \frac{25}{4}+6=\frac{25+24}{4}=\frac{49}{4}$

$\Rightarrow |\alpha-\beta| = \frac{7}{2}$

Therefore, difference of roots given equation is $\frac{7}{2}$.

Hence, the correct answer is option (c) i.e., $\frac{7}{2}$.

Example 29. If the ratio of the roots of the equation $4x^2 - 6x + p = 0$ is 1 : 2, then the value of p is

(a) 1 (b) 2 (c) –2 (d) –1

Sol. (b) Given equation, $4x^2 - 6x + p = 0$

Let the roots of the equation is α & β, then

$\frac{\alpha}{\beta}=2$

$\Rightarrow 2\alpha = \beta$

Now, sum of roots = $\alpha + \beta = -\frac{b}{a}$

$\Rightarrow \alpha + 2a = \frac{-(-6)}{4}$

$3\alpha = \frac{3}{2}$

$\Rightarrow \alpha = \frac{1}{2}$

$1 \Rightarrow \beta = 2a = 2 \times \frac{1}{2} = 1$

Now, Product of roots = $\alpha\beta$

$\Rightarrow \frac{1}{2}\times 1 = \frac{p}{4}$

$\Rightarrow p = 2$

Example 30. One root of the equation: $x^2 - 2(5 + m) + 3(7 + m) = 0$ is reciprocal of the other. Find the value of m.

(a) $\frac{-20}{3}$ (b) 7 (c) $\frac{1}{7}$ (d) $\frac{1}{17}$

Sol. (a) Given; $x^2 - 2(5 + m) + 3(7 + m) = 0$

Here, $a = 1$, $b = 2(5 + m)$ & $c = 3(7 + m)$

Since, the roots are reciprocal of each other, then the roots of the equation are α and $\frac{1}{\alpha}$. We know that,

$$\alpha \times \frac{1}{\alpha} = \frac{c}{a}$$

$$\Rightarrow 1 = \frac{3(7+m)}{1}$$

$$\Rightarrow 121 + 3m$$

$$\Rightarrow 3m = -20$$

$$\Rightarrow m = -\frac{20}{3}$$

Example 31. If roots of equation $x^2 + x + r = 0$ are α and β and $\alpha^3 + \beta^3 = -6$. Find the value of 'r'.

(a) $-\frac{5}{3}$ (b) $\frac{7}{3}$ (c) $-\frac{4}{3}$ (d) 1

Sol. (a) Given, $x^2 + x + r = 0$

Thus, Sum of roots $= \alpha + \beta = -1$

Product of roots $= \alpha\beta = r$

Also,

$\alpha^3 + \beta^3 = -6$

$\Rightarrow (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -6$

$\Rightarrow (-1)^3 - 3r(-1) = -6$

$\Rightarrow (-1) + 3r = -6$

$\Rightarrow 3r = -5$

$\Rightarrow r = -\frac{5}{3}$

Example 32. If $\alpha + \beta = -2$ and $\alpha\beta = -3$ where α and β are the roots of the equation, which is

(a) $x^2 - 2x - 3 = 0$ (b) $x^2 + 2x - 3 = 0$

(c) $x^2 + 2x + 3 = 0$ (d) $x^2 - 2x + 3 = 0$

Sol. (b) Given roots are,

Sum of roots $(\alpha + \beta) = -2$

Products of roots $(\alpha\beta) = -3$

We know that the quadratic equation is

$x^2 -$ (sum of roots)x + products of roots $= 0$

$\Rightarrow x^2 - (-2)x + (-3) = 0$

Therefore, the required equation is $x^2 + 2x - 3 = 0$

Example 33. If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal, then the value of m is

(a) –3 (b) –1 (c) 1 (d) –2

Sol. (d) We know that,

If roots of equation $ax^2 + bx = c = 0$ are equal, then $b^2 - 4ac = 0$

Given, the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal.

$\Rightarrow b^2 - 4ac = 0$

$8^2 - 4(2)(-m^3) = 0$

$\Rightarrow 64 + 8m^3 = 0$

$\Rightarrow m^3 = -\frac{64}{8} = -8$

$\Rightarrow m = -2$

Therefore, the value of m is – 2.

Hence, the correct option is (d).

Example 34. The difference between the roots of the equation $x^2 - 7x - 9 = 0$ is

(a) 7 (b) $\sqrt{85}$ (c) 9 (d) $2\sqrt{85}$

Sol. (b) Let a and b are roots of the quadratic equation $x^2 - 7x - 9 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$

$a = 1; b = -7; c = -9$

Sum of roots $(\alpha + \beta) = \frac{-b}{a} = \frac{-(-7)}{1} = 7$

$\Rightarrow (\alpha + \beta) = 7$ ...(i)

And products of roots $(\alpha\beta) = \frac{c}{a} = \frac{-9}{1} = -9$

$\Rightarrow (\alpha\beta) = -9$ ...(ii)

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow 7^2 - 4 \times (-9) = 85$

$\Rightarrow \alpha - \beta = \sqrt{85}$

Therefore, difference of roots of given equation is $\sqrt{85}$.

Example 35. For what value of k the given equation has real roots: $x^2 - 10x + k = 0$?

(a) $k \le 25$ (b) $k \ge 25$ (c) $k \le 100$ (d) None of these

Sol. (a) Comparing the equation $x^2 - 10x + k = 0$ with $ax^2 + bx + c = 0$, we get

$a = 1, b = -10$ and $c = k$

Since, the roots are real thus

$b^2 - 4ac \ge 0$

$\Rightarrow (10)^2 - 4(1)(k) \ge 0$

$\Rightarrow 100 - 4k \ge 0$

$\Rightarrow 100 \geq 4k$

$\Rightarrow \frac{100}{4} \geq k$

$\Rightarrow 25 \geq k$ or $k \leq 25$

Hence, the correct option is (a) i.e., $k \leq 25$.

Example 36. The value of p for which the difference between the root of equation $x^2 + px + 8 = 0$ are

(a) ± 2 (b) ± 4 (c) ± 6 (d) ± 8

Sol. (c) Given equation $x^2 + px + 8 = 0$

Suppose α and β are the roots of the equation

Then, sum of the roots $\alpha + \beta$

$\Rightarrow \alpha + \beta = \frac{-b}{a} = \frac{-p}{1}$

$\Rightarrow \alpha + \beta = -p$

Now product of the roots $\alpha\beta = \frac{c}{a}$

$\Rightarrow \alpha\beta = \frac{8}{1} \Rightarrow \alpha\beta = 8$

Now,

$\Rightarrow (\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$

$\Rightarrow p^2 = 2^2 + 4 \times 8$ $[\because \alpha - \beta = 2]$

$\Rightarrow p^2 = 36$

$\Rightarrow p = \pm\sqrt{36}$

$\Rightarrow p = \pm 6$

Example 37. If one root of $5z^2 + 13z + y = 0$ is reciprocal of the other, then the value of y is

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$ (c) -5 (d) 5

Sol. (d) Given equation, $5z^2 + 13z + y = 0$

Let one root of the equation be α, then the other root will be $\frac{1}{\alpha}$.

Comparing the given equation with standard equation $az^2 + bx + c = 0$, we get

$a = 5$, $b = 13$ and $c = y$

Now, product of roots = $\frac{c}{a}$

$\Rightarrow \alpha . \frac{1}{\alpha} = \frac{y}{5}$

$\Rightarrow 1 = \frac{y}{5}$

$\Rightarrow y = 5$

Example 38. If a and b are the roots of the equation $2x^2 - 7x + 3 = 0$ then the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is equal to

(a) $\frac{15}{4}$ (b) $\frac{37}{6}$ (c) $\frac{28}{5}$ (d) None of the above

Sol. (b) Given $2x^2 - 7x + 3 = 0$

We knows that,

If the roots of equation are α and β, then

Sum of roots: $(\alpha+\beta)=\frac{-b}{a}=\frac{-(-7)}{2}=\frac{7}{2}$

Products of roots: $(\alpha\beta)=\frac{3}{2}$

Now, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$=\frac{\left(\frac{7}{2}\right)^2-2\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)}$

$=\frac{(49-12)}{4\times3}=\frac{37}{6}$

Hence, the correct option is (b) i.e. $\frac{37}{6}$

Example 39. Let α and β be the roots of $x^2 + 7x + 12 = 0$, then the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ is

(a) $\frac{49}{144}+\frac{144}{49}$ (b) $\frac{7}{12}+\frac{12}{7}$ (c) $-\frac{91}{12}$ (d) None of the above

Sol. (c) Given: $x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 3x + 4x + 12 = 0$

$\Rightarrow x(x + 3) + 4(x + 3) = 0$

$\Rightarrow (x + 3)(x + 4)$

$\Rightarrow 0\ x + 3 = 0, x + 4 = 0$

$\Rightarrow x = -3, x = -4$

Let $\alpha = -3, \beta = -4$

Now, $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

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$$= \frac{(-3)^2}{(-4)} + \frac{(-4)^2}{(-3)}$$

$$= \frac{9}{-4} + \frac{16}{-3.}$$

$$= -\left(\frac{9}{4} + \frac{16}{3}\right)$$

$$= -\frac{91}{12}$$

Example 40. A solution of the quadratic equation $(a + b - 2c)x^2 + (2a - b - c)x + (c + a - 2b) = 0$ is

(a) $x = 1$ (b) $x = -1$ (c) $x = 2$ (d) $x = -2$

Sol. (b) Given, $(a + b - 2c)x^2 + (2a - b - c)x + (c + a - 2b) = 0$

Go by choices:

For option (b): $x = -1$

Substituting $x = -1$ in the given equation, we get

LHS: $(a + b - 2c) - (2a - b - c) + (c + a - 2b)$

$= a - 2a + a - b + b - 2b - 2c + c + c = 0 = RHS$

Therefore, -1 is the root of the given equation.

Hence, the correct option is (b).

Example 41. If $L + M + N = 0$ and L, M, N are rationals then the roots of the equation: $(M + N - L)x^2 + (N + L - M)x + (L + M - N) = 0$ are

(a) real and irrational (b) real and rational

(c) imaginary and equal (d) real and equal

Sol. (b) It is given that $L + M + N = 0$

$\Rightarrow M + N = -L$ ...(i)

$\Rightarrow N + L = -M$

$\Rightarrow L + M = -N$

Then equation will become

$\Rightarrow (-L - L)x^2 + (-M - M)x + (-N - N) = 0$

$\Rightarrow -2Lx^2 - 2Mx - 2N = 0$

Now, divide both sides by -2, we get

$\Rightarrow Lx^2 + Mx + N = 0$

Now compare with quadratic $ax^2 + bx + c = 0$

We get $a = L$, $b = M$, $c = N$

Now find the discriminant $D = b^2 - 4ac$

$\Rightarrow D = M^2 - 4LN$

$\Rightarrow D = [-(N + L)]^2 - 4LN$

$\Rightarrow D = N^2 + L^2 + 2LN - 4LN$

$\Rightarrow D = N^2 + L^2 - 2LN$

$\Rightarrow D = (N - L)^2$

Here, we can see that D is a perfect square.

So, the roots will be real and rational.

Hence, option (b) is correct, i.e real and rational.

Example 42. If the quadratic equation $x^2 + px + q$ and $x^2 + qx + p$ have a common root then $p + q = ?$

(a) 0 (b) 1 (c) –1 (d) 2

Sol. (c) Here let suppose α be the common roots

$\alpha^2 + p\alpha + q = 0$ .....(i)

$\alpha^2 + q\alpha + p = 0$ ……(ii)

On subtracting (i) and (ii), we get

$\Rightarrow p\alpha - q\alpha + q - p = 0$

$\Rightarrow \alpha(p - q) = p - q$

$\Rightarrow \alpha = 1$ ……(iii)

Now on adding (i) and (ii) we get

$\Rightarrow 2\alpha^2 + (p + q)\alpha + (p + q) = 0$

From (iii) put $\alpha = 1$, Then

$\Rightarrow 2 \times 1^2 + (p + q) \times .1 + (p + q) = 0$

$\Rightarrow 2 + 2(p + q) = 0$

$\Rightarrow 2(p + q) = -2$

$\Rightarrow (p + q) = -1$

Note:

(a) Irrational roots occur in conjugate pairs that is if $(m + \sqrt{n})$ is a root then $(m - \sqrt{n})$ is the other root of the same equation.

(b) If one root is reciprocal to the other root then their product is 1 and so $c/a = 1$ i.e. $c = a$

(c) If one root is equal to another root but opposite in sign then their sum = 0 and so $b/a = 0$. i.e. $b = 0$.

PRACTICE QUESTIONS (PART E)

1. Solving equation $3g^2 - 14g + 16 = 0$, we get roots as

(a) 0 (b) +5 (c) $8 \text{ and } \frac{2}{3}$ (d) $2 \text{ and } \frac{8}{3}$

2. If $2x^2 - (a + 6)2x + 12a = 0$, then roots are

(a) 4 & a^2 (b) 6 & a (c) 3 & 2a (d) 6 & 3a

3. Find the value of k so that x = 2 is a root of the equation $3x^2 - 2kx + 5 = 0$.

(a) $\frac{17}{4}$ (b) $\frac{4}{17}$ (c) $-\frac{17}{4}$ (d) $-\frac{4}{17}$

4. The roots of the equation $x^2 + (2p - 1)x + p^2 = 0$ are real if.

(a) $p \geq 1$ (b) $p \leq 4$ (c) $p \geq \frac{1}{4}$ (d) $p \leq \frac{1}{4}$

5. Find the value of k for which the quadratic equation $2x^2 - kx + 3 = 0$ have two equal roots.

(a) ± 1 (b) ± 2 (c) $\pm 2\sqrt{6}$ (d) $2\sqrt{6}$

6. If p and q are the roots of $x^2 + 2x + 1 = 0$ then the values of $p^3 + q^3$ becomes

(a) 2 (b) –2 (c) 4 (d) –4

7. What will be the value of k, if the roots of the equation $(k - 4)x^2 - 2kx + (k + 5) = 0$ are equal? **(Dec, 2022)**

(a) 18 (b) 20 (c) 19 (d) 21

8. If the roots of the equation x2 – 8x + m = 0 exceeds the other by 4, then the value of m is

(a) m = 10 (b) m = 11 (c) m = 9 (d) m = 12 **(ICAI)**

9. If the sum of two numbers is 11 and the sum of their squares be 85, then the numbers are

(a) 5 and 6 (b) 4 and 7 (c) 2 and 9 (d) None of these

10. If α and β are the roots of $x^2 = x + 1$ then value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is

(a) $2\sqrt{5}$ (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) $-2\sqrt{5}$

11. Solve the equation if $2^{2x+3} - 3^2 \cdot 2^x + 1 = 0$, then the values of x are

(a) 0, 1 (b) 1, 2 (c) 0, 3 (d) 0, –3

12. The values of $2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\cdots}}}}$

(a) $1 \pm \sqrt{2}$ (b) $2 \pm \sqrt{5}$ (c) $1 \pm \sqrt{2}$ (d) None of these

13. If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$ the equation having roots as p and q is

(a) $x^2 - 19x + 3 = 0$ (b) $3x^2 - 19x - 3 = 0$

(c) $3x^2 - 19x + 3 = 0$ (d) None of these

14. The solutions of the equation $\frac{6x}{x+1} + \frac{6(x+1)}{x} = 13$ are

(a) (2, 3) (b) (3, –2) (c) (–2, –3) (d) (2, –3)

15. The satisfying values of x for the equation, $\frac{1}{x+p+q} = \frac{1}{x} + \frac{1}{p} + \frac{1}{q}$ are

(a) (p, q) (b) (–p, –q) (c) (p, –p) (d) (–p, q)

16. The equation $\left(\frac{l-m}{2}\right)x^2-\left(\frac{l+m}{2}\right)x+m=0$ has got two values of x to satisfy the equation given as

(a) $\left(1,\frac{2m}{l-m}\right)$ (b) $\left(1,\frac{m}{l-m}\right)$ (c) $\left(1,\frac{2l}{l-m}\right)$ (d) $\left(1,\frac{l}{l-m}\right)$

17. If a and β are the roots of the equation $x^2 + x + 5 = 0$ then $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ is equal to

(a) $\frac{16}{5}$ (b) 3 (c) $\frac{14}{5}$ (d) 2

Answer Key

1. (d) **2.** (b) **3.** (a) **4.** (d) **5.** (c) **6.** (b) **7.** (b) **8.** (d) **9.** (c) **10.** (d)
11. (d) **12.** (c) **13.** (c) **14.** (d) **15.** (b) **16.** (a) **17.** (c)

PROBLEMS ON QUADRATIC EQUATION

Example 43. Five times of a positive whole number is 3 less than twice the square of the number. The number is

(a) 3 (b) 4 (c) –3 (d) 2

Sol. (a) Let the positive number be x.

According to the question, we have

$2x^2 - 5x = 3$

$\Rightarrow 2x^2 - 5x - 3 = 0$

$\Rightarrow 2x^2 - 6x + x - 3 = 0$

$\Rightarrow 2x(x - 3) + (x - 3) = 0$

$\Rightarrow (x - 3)(2x + 1) = 0$

$\Rightarrow x = 3, -\frac{1}{2}$

Since, the number is positive.

Thus, x = 3

Hence, the correct option is (a).

Example 44. The sum of the two numbers is 8 and the sum of their squares is 34. Taking one number as x form an equation in x and hence find the numbers. The numbers are

(a) (7, 10) (b) (4, 4) (c) (3, 5) (d) (2, 6)

Sol. (c) Let the numbers be x and '8 - x'.

Then, according to the question, we have

$x^2 + (8 - x)^2 = 34$

$\Rightarrow x^2 + 64 + x^2 - 16x = 34$

$\Rightarrow 2x^2 - 16x + 300$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 3x - 5x + 15 = 0$

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$\Rightarrow x(x-3) - 5(x-3) = 0$

$\Rightarrow (x-3)(x-5) = 0$

$\Rightarrow x = 3, x = 5$

Example 45. The sides of an equilateral triangle are shortened by 12 units, 13 units and 14 units respectively and a right–angle triangle is formed. The side of the equilateral triangle is

(a) 17 units (b) 16 units (c) 15 units (d) 18 units

Sol. (a) Let 'x' be the length of each side of the equilateral triangle.

Then, the sides of the right-angled triangle are

$(x-12)$, $(x-13)$ and $(x-14)$

In the above three sides, the side represented by $(x-12)$ is hypotenuse (Because that is the longest side).

Using Pythagorean theorem, we have:

$(x-12)^2 = (x-13)^2 + (x-14)^2$

$\Rightarrow x^2 - 24x + 144 = x^2 - 26x + 169 + x^2 - 28x + 196$

$\Rightarrow x^2 - 30x + 221 = 0$

$\Rightarrow (x-13)(x-17) = 0$

$\Rightarrow x = 13$ or $x = 17$.

Here, $x = 13$ cannot be accepted since in that case sides would be $(12-13, 13-13, 14-13)$ i.e., $-1, 0, 1$ which is not possible since the side cannot be negative or zero.

Therefore, the side of the equilateral triangle is 17 units.

Hence, the correct option is (a).

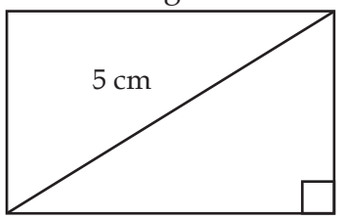
Example 46. The diagonal of a rectangle is 5 cm and one of its sides is 4 cm. Its area is _____ .

(a) 42 sq. cm (b) 12 sq. cm (c) 10 sq. cm (d) 5 sq. cm

Sol. (b) Given, Diagonal of a rectangle = 5 cm

One side of rectangle = 4 cm

Let's assume the other side of the rectangle be 'x' cm



Using Pythagoras theorem,

$4^2 + x^2 = 5^2$

$\Rightarrow 16 + x^2 = 25$

$\Rightarrow x^2 = 25 - 16$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \sqrt{9} \Rightarrow x = 3$ ($\because$ Side cannot be negative)

So, the length and breadth of the rectangle are 3 cm and 4 cm respectively. We know that,

Area = length × breadth

Area = 3 cm × 4 cm

Area = 12 sq.cm

Hence, the correct option is (b).

Example 47. The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as x, form an equation in x and hence find the numbers.

(a) (7, 10) (b) (4, 4) (c) (3, 5) (d) (2, 6)

Sol. (c) Go be options:

For option (a): (7, 10)

Here, sum of roots = $7 + 10 = 17 \neq 18$

For option (b): (4, 4)

Here, sum of roots = $4 + 4 = 8$

Sum of their squares = $4^2 + 4^2 = 16 + 16 = 32 \neq 34$

For option (c): (3, 5)

Here, sum of roots = $3 + 5 = 8$

Sum of their squares = $3^2 + 5^2 = 9 + 25 = 34$

Therefore, both conditions are satisfied.

Hence, the numbers are 3 and 5.

Detailed Method:

Let x and y be the two numbers.

Then, $x + y = 8$

$\Rightarrow y = 8 - x$

Also, $x^2 + y^2 = 34$

$\Rightarrow x^2 + (8 - x)^2 = 34$

$\Rightarrow x^2 + 8^2 + x^2 - 16x = 34$

$\Rightarrow 2x^2 + 64 - 16x = 34$

$\Rightarrow 2x^2 - 16x + 30 = 0$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 3x - 5x + 15 = 0$

$\Rightarrow x(x - 3) - 5(x - 3) = 0$

$\Rightarrow (x - 5)(x - 3) = 0$

$\Rightarrow x = 5$ or $x = 3$

Also, $y = 8 - x$

$y = 8 - 5 = 3$ or $y = 8 - 3 = 5$

Therefore, the two numbers are 3 and 5.

Hence, the correct option is (c).

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PRACTICE QUESTIONS (PART F)

1. There are two consecutive numbers such that the difference of their reciprocals is $\frac{1}{240}$. The numbers are

(a) (15, 16) (b) (17, 18) (c) (13, 14) (d) (12, 13)

2. A distributor of apple Juice has 5000 bottles in the store that it wishes to distribute in a month. From experience it is known that demand D (in number of bottles) is given by $D = -2000p^2 + 2000p + 17000$.

The price per bottle that will result zero inventory is

(a) ₹3 (b) ₹5 (c) ₹2 (d) None of these

3. The sum of two irrational numbers multiplied by the larger one is 70 and their difference is multiplied by the smaller one is 12; the two numbers are

(a) $3\sqrt{2}, 2\sqrt{3}$ (b) $5\sqrt{2}, 3\sqrt{5}$ (c) $2\sqrt{2}, 5\sqrt{2}$ (d) None of these

Answer Key

1. (a) **2.** (a) **3.** (c)

CUBIC EQUATION

An equation of 3rd degree or where the maximum power of x is 3 is called as cubic equation.

Cubic Equation will look like: $ax^3 + bx^2 + cx + d = 0$ where $a \neq 0$ will be called as cubic equation.

E.g.: $x^3 - 6x^2 + 11x - 6 = 0$

For cubic equation, $ax^3 + bx^2 + c + d = 0$

Sum of roots $= -\frac{b}{a}$

Sum of roots taken two at a time $= \frac{c}{a}$

Product of roots $= -\frac{d}{a}$

Example 48. The roots of $x^3 + x^2 - x - 1 = 0$ are

(a) (–1, –1, 1) (b) (1, 1, –1) (c) (–1, –1, –1) (d) (1, 1, 1)

Sol. (a) Given, $x^3 + x^2 - x - 1 = 0$

$\Rightarrow x^2(x + 1) - x - 1 = 0$

$\Rightarrow x^2(x + 1) - (x + 1) = 0$

$\Rightarrow (x^2 - 1)(x + 1+ = 0$ $\{a^2 - b^2 = (a - b)(a + b)\}$

$\Rightarrow (x + 1)(x - 1)(x + 1) = 0$

$\Rightarrow x + 1 = 0, x - 1 = 0, x + 1 = 0$

$\Rightarrow x = -1, x = 1, x = -1$

Therefore, the roots of the equation are –1, –1, 1.

Hence, the correct option is (a).

Example 49. The rational root of the equation $2p^3 - p^2 - 4p + 2 = 0$ is

(a) –2 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Sol. (c) Given equation $2p^3 - p^2 - 4p + 2 = 0$

On solving above equation, we get

$\Rightarrow p^2 (2p - 1) - 2(2p - 1) = 0$

$\Rightarrow (2p - 1)(p^2 - 2) = 0$

For $2p - 1 = 0$,

$\Rightarrow 2p - 1 = 0$

$\Rightarrow p = \frac{1}{2}$, which is rational.

Now, $(p^2 - 2) = 0$

$\Rightarrow p^2 = 2$

$\Rightarrow p = \sqrt{2}$, which is irrational.

Example 50. For what value of 'k', 1 is a root of the following cubic equation $kx^3 - 4x^2 + 3x + 2 = 0$?

(a) 1 (b) –1 (c) 2 (d) $\frac{1}{3}$

Sol. (b) Given cubic equation: $kx^3 - 4x^2 + 3x + 2 = 0$

Since, 1 is a root of the equation, thus it will satisfy the given equation.

$\Rightarrow k(1)^3 - 4(1)^2 + 3(1) + 2 = 0$

$\Rightarrow k - 4 + 3 + 2 = 0$

$\Rightarrow k + 1 = 0$

$\Rightarrow k = -1$

Hence, the correct answer is option (b) i.e., –1.

Example 51. Solve $x^3 - 6x^2 + 5x + 12 = 0$

(a) 1, 3, 4 (b) –1, 3, 4 (c) 1, 6, 2 (d) 1, –6, –2

Sol. (b) Given equation: $x^3 - 6x^2 + 5x + 12 = 0$

We know that, for cubic equation $ax^3 + bx^2 + c + d = 0$

Sum of roots $= -\frac{b}{a}$

Thus, for $x^3 - 6x^2 + 5x + 12 = 0$

Sum of roots $= -\frac{b}{a} = -\frac{-6}{1} = 6$

For option (a): 1, 3, 4

Sum of roots = 1 + 3 + 4 = 7 which is not true

For option (b): –1, 3, 4

Sum of roots = –1 + 3 + 4 = 6 which is true

Also, for other options, it does not hold.

Therefore, x = –1, 3, 4

Hence, the correct option is (b) i.e., –1, 3, 4.

Example 52. If $4x^3 + 8x^2 - x - 2 = 0$, then the value of (2x + 3) is given by

(a) 4, –1, 2 (b) –4, 2, 1 (c) 2, –4, –1 (d) None of these

Sol. (a) Given cubic equation: $4x^3 + 8x^2 - x - 2 = 0$

$\Rightarrow 4x^2(x + 2) - 1(x + 2) = 0$

$\Rightarrow (x + 2)(4x^2 - 1) = 0$

$\Rightarrow (x + 2)[(2x)^2 - 1^2) = 0$

$\Rightarrow (x + 2)(2x + 1)(2x - 1) = 0$ $[\because a^2 - b^2 = (a - b)(a + b)]$

$\Rightarrow (x + 2) = 0$ or $2x + 1 = 0$ or $2x - 1 = 0$

$\Rightarrow x = -2$ or $2x = -1$ or $2x = 1$

$\Rightarrow x = -2, \frac{-1}{2}, \frac{1}{2}$

(i) When $x = -2$, then

$(2x + 3) = 2 \times (-2) + 3 = -4 + 3 = -1$

(ii) When $x = \frac{-1}{2}$, then

$$(2x+3) = 2\times\left(\frac{-1}{2}\right)+3 = -1+3 = 2$$

(iii) When $x = \frac{1}{2}$, then

$$(2x+3) = 2\times\frac{1}{2}+3 = 1+3 = 4$$

Therefore, the required values of (2x + 3) are –1, 2, 4.

Hence, the correct answer is option (a).

PRACTICE QUESTIONS (PART G)

1. The three roots of equation is $x^3 + 9x^2 - x - 9 = 0$ are

(a) 1, –1, –9 (b) 1, –1, 9 (c) 1, 1, 9 (d) –1, –1, –9

2. The roots of the cubic equation $x^3 + 7x^2 - 21x - 27 = 0$ are

(a) (–3, –9, –1) (b) (3, –9, –1) (c) (3, 9, 1) (d) (–3, 9, 1)

3. The roots of equation $y^3 - 4y^2 - 9x + 36 = 0$ are

(a) 1, 3, 4 (b) 3, 3, 4 (c) –3, –3, 4 (d) 3, –3, 4

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4. x, x – 4, x + 5 are the factors of the left–hand side of the equation, find the equation.

(a) $x^3 + 2x^2 - x - 2 = 0$ (b) $x^3 + x^2 - 20x = 0$

(c) $x^3 - 3x^2 - 4x + 12 = 0$ (d) $x^3 - 6x^2 + 11x - 6 = 0$

5. The equation $3x^3 + 5x^2 = 3x + 5$ has got 3 roots and hence the factors of the left–hand side of the equation are

(a) $x-1,\ x-2,\ x-\frac{5}{3}$ (b) x – 1, x + 1, 3x + 5

(c) x + 1, x – 1, 3x – 5 (d) x – 1, x + 1, x – 2

6. The rational root of the equation $2x^3 - x^2 - 4x + 2 = 0$ is

(a) 1 (b) –1 (c) 2 (d) –2

Answer Key

1. (a) **2.** (b) **3.** (d) **4.** (b) **5.** (b) **6.** (a)

FULL CHAPTER PRACTICE QUESTIONS

1. If 2x – 5y = –1, x + 2y = 13, then

(a) x = 7, y = –3 (b) x = –7, y = –3

(c) x = –7, y = 3 (d) x = 7, y = 3

2. If $\frac{3}{x+y}+\frac{2}{x-y}=-1$ and $\frac{1}{x+y}-\frac{1}{x-y}=\frac{4}{3}$ then (x, y) is

(a) (2, 1) (b) (1, 2) (c) (–1, 2) (d) (–2, 1)

3. The root of given equation $x^2 + 36 - 12x = 0$ is

(a) x = 6, 6 (b) x = –6, 6 (c) x = 0, 6 (d) None of these

4. If $2^{x+y} = 2^{2x-y} = \sqrt{8}$, then the respective values of x and y are _____.

(a) $1,\frac{1}{2}$ (b) $\frac{1}{2},1$ (c) $\frac{1}{2},\frac{1}{2}$ (d) None of these

5. The roots of cubic equation $x^3 - 6x^2 + 11x - 6 = 0$ is

(a) x = 1, 2, 3 (b) x = 1, 2, 7 (c) x = 9, 0, –9 (d) x = 1, 5, 9

6. The age of a person is 8 years more than thrice the age of the sum of his two grandsons who were twins. After 8 years his age will be 10 years more than twice the sum of the ages of his grandsons. Then the age of the person when the twins were born is

(a) 86 years (b) 73 years (c) 68 years (d) None of these

7. If $u^{5x} = v^{5y} = w^{5z}$ and $u^2 = vw$, then the value of xy + xz – 2yz will be

(a) 5 (b) 2 (c) 1 (d) 0

8. If the length of a rectangle is 5 cm more than the breadth and if the perimeter of the rectangle is 40 cm, then the length & breadth of the rectangle will be

(a) 7.5 cm, 2.5 cm (b) 10cm, 5cm (c) 2.5 cm, 7.5cm (d) 15.5cm, 10.5cm

9. If α & β are the roots of the equation $x^2 + x + 5 = 0$, then $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ is equal to

(a) $\frac{16}{5}$ (b) 2 (c) 3 (d) $\frac{14}{5}$

10. If the sides of an equilateral triangle are shortened by 3 units, 4 units and 5units respectively and a right triangle is formed, then the side of an equilateral triangle is

(a) 6 units (b) 7 units (c) 8 units (d) 10 units

11. If difference between the roots of the equation $x^2 - kx + 8 = 0$ is 4, then the value of k is

(a) 0 (b) +4 (c) $\pm 8\sqrt{3}$ (d) $\pm 4\sqrt{3}$

12. The roots of the cubic equation $x^3 - 7x + 6 = 0$ are

(a) 1, 2 and 3 (b) 1, –2 and 3 (c) 1, 2 and –3 (d) 1, –2 and –3

13. If α and β be the roots of the quadratic $2x^2 - 4x = 1$, then value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ is _____.

(a) –11 (b) 22 (c) –22 (d) 11

14. The number of students in each section of a school is 36. After admitting 12 new students, four new sections were started. If the total number of students in each section now is 30, then the number of sections initially were

(a) 6 (b) 10 (c) 14 (d) 18

15. The equations $x + 5y = 33$; $\frac{x+y}{x-y}=\frac{13}{5}$ has the solution (x, y) as

(a) (4, 8) (b) (8, 5) (c) (4, 16) (d) (16, 4)

16. One root of the equation: $x^2 - 2(5 + m) + 3(7 + m) = 0$ is reciprocal of the other. Find the value of m.

(a) $\frac{-20}{3}$ (b) 7 (c) $\frac{1}{7}$ (d) $\frac{1}{17}$

17. Find the positive value of k for which the equations: $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots:

(a) 12 (b) 16 (c) 18 (d) 22

18. The sum of two numbers is 75 and their difference is 20. Find the difference of their squares.

(a) 1500 (b) 1600 (c) 1550 (d) None of these

Answer Key

1. (d)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (c)
11. (d)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (a)		

PREVIOUS YEAR QUESTION (PYQ'S)

1. A number consists of two digits such that the digit in one's place is thrice the digit at ten's place. If 36 be added, then the digits are reversed. Find the number. **(June 2019)**

(a) 62 (b) 26 (c) 39 (d) None

2. The value of p for which the difference between the root of equation $x^2 + px + 8 = 0$ is 2 is **(June 2019)**

(a) ±2 (b) ±4 (c) ±6 (d) ±8

3. If the quadratic equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root then p + q = ? **(Jan. 2021)**

(a) 0 (b) 1 (c) –1 (d) 2

4. If a and b are the roots of the equation $2x^2 + 5x + k = 0$, and $4(a^2 + b^2 + ab) = 23$, then which of the following is true? **(July 2021)**

(a) $k^2 + 3k - 2 = 0$ (b) $k^2 - 2k - 2 = 0$

(c) $k^2 - 2k - 3 = 0$ (d) $k^2 - 3k + 2 = 0$

5. The cost of 2 oranges and 3 apples is ₹28. If the cost of an apple is doubled then the cost of 3 oranges and 5 apples is ₹75. The original cost of 7 oranges and 4 apples (in ₹) is **(July 2021)**

(a) 59 (b) 47 (c) 71 (d) 63

6. The value of 'k' is ….. If 2 is a root of the following cubic equation: $x^3 - (k + 1)x + k = 0$

(July 2021)

(a) 2 (b) 6 (c) 1 (d) 4

7. If one root is half of the other of a quadratic equation and the difference in roots is a, then the equation is **(Dec 2021)**

(a) $x^2 + ax + 2a^2 = 0$ (b) $x^2 - 3ax - 2a^2 = 0$

(c) $x^2 - 3ax + 2a^2 = 0$ (d) $x^2 + 3ax - 2a^2 = 0$

8. If the square of a number exceeds twice of the number by 15, then number that satisfies the condition is **(Dec 2021)**

(a) –5 (b) 3 (c) 5 (d) 15

9. If one root of $5z^2 + 13z + y = 0$ be reciprocal of the other, then the value of y is

(June 2022)

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$ (c) 5 (d) –5

10. What will be the value of k, if the roots of the equation $(k - 4)x^2 - 2kx + (k + 5) = 0$ are equal? **(Dec 2022)**

(a) 18 (b) 20 (c) 19 (d) 21

11. A group of 400 soldiers posted at border area had a provision for 31 days. After 28 days, 280 soldiers from this group were called back. Find the number of days for which the remaining ration will be sufficient? **(Dec 2022)**

(a) 3 (b) 6 (c) 8 (d) 10

12. If the roots of the equation $x^3 - px + q = 0$ are in the ratio 2 : 3, then **(Dec 2022)**

(a) $p^2 = 25q$ (b) $p^2 = 6q$ (c) $6p^2 = 5q$ (d) $6p^2 = 25q$

13. The largest side of a triangle is 3 times the shortest side and third side is 4 cm shorter than largest side. If the perimeter of the triangle is at least 59 cm, what is the length of shortest side? **(June 2023)**

(a) less than 7 cm (b) greater than or equal to 7 cm

(c) less than 9 cm (d) greater than or equal to 9 cm

14. The age of a man is four times the sum of the ages of his sons and after 10 years, his age will be double the sum of their ages. The present age of the man must be:

(June 2023)

(a) 56 years (b) 45 years (c) 60 years (d) 64 years

15. If α and β are the roots of the quadratic equation $x^2 - 2x - 3 = 0$, then the equation whose roots are $\alpha + \beta$ and $\alpha - \beta$ is **(June 2023)**

(a) $x^2 - 6x - 8 = 0$ (b) $x^2 - 6x + 8 = 0$

(c) $x^2 + 6x + 8 = 0$ (d) $x^2 + 6x - 8 = 0$

16. If α and β are the roots of the equation $x^2 - (n^2 + 1)\, x + \frac{1}{5}(n^4 + n^2 + 1) = 0$, then the value of $\alpha^2 + \beta^2$ is: **(June 2023)**

(a) $2n$ (b) n^2 (c) $2n^2$ (d) n^3

17. If α and β are the roots of the equation $x^2 - 4x + 1 = 0$, then the value of $\alpha^3 + \beta^3$ will be **(Dec 2023)**

(a) –76 (b) 76 (c) -52 (d) 52

18. The roots of the equation $x^3 + x^2 - x - 1 = 0$ are **(Dec 2023)**

(a) $x = 1, x = -1, x = -1$ (b) $x = 1, x = 1, x = -1$

(c) $x = -1, x = -1, x = -1$ (d) $x = 1, x = 1, x = 1$

19. The solution of linear simultaneous equations $2x - y = 4$ and $3x + 4y = 17$ is

(Dec 2023)

(a) $x = 3, y = 2$ (b) $y = 2, y = 3$ (c) $x = -3, y = -2$ (d) $x = -2, y = -3$

20. The solution of cubic equation $x^3 - 23x^2 + 142x - 120 = 0$ is given by the triplet;

(Dec 2023)

(a) (1, 10, 12) (b) (1, -10, 12) (c) (-1, -10, -12) (d) (1, 10, -12)

21. The roots of the equation $x^2 - 7x + 10 = 0$ are **(June 2024)**

(a) 2 and 5 (b) –2 and –5 (c) 2 and –5 (d) –2 and 5

22. The equation $x^3 - 3x^2 - 4x + 12 = 0$ has three real roots. They are **(June 2024)**

(a) –2, 2, 3 (b) –2, –2, 3 (c) –2, 2, –3 (d) –2, –2, –3

23. If α & β are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $\frac{1}{\alpha}$ & $\frac{1}{\beta}$ is **(June 2024)**

(a) $cx^2 - bx + a = 0$ (b) $cx^2 + bx + a = 0$

(c) $x^2 + bx + a = 0$ (d) $x^2 + bx - a = 0$

24. If α & β are the roots of the equation $x^2 - 8x + 12 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$ **(June 2024)**

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{5}{6}$

25. A fraction becomes 1, when 3 is added to the numerator and 1 is added to the denominator. But when the numerator and denominator are decreased by 2 and 1 respectively, it becomes $\frac{1}{2}$. The denominator of the fraction is **(June 2024)**

(a) 8 (b) 6 (c) 7 (d) 5

26. A person purchased 2 apples and 3 bananas at the cost of ₹90. Later he visited to another shop where shopkeeper told him that if you give me ₹50 and one banana. I can give you 3 apples. He agreed to the deal. What is the cost of one apple and one banana? **(Sept 2024)**

(a) (15, 10) (b) (10, 15) (c) (10, 20) (d) (20, 10)

27. If one root of the equation $x^2 - 3x + k = 0$ is 1, then the value of 'k' is **(Sept 2024)**

(a) 2 (b) 1 (c) –2 (d) –1

28. A number consists of two digits. The digits in ten's place is 3 times the digit in the unit's place. If 54 is subtracted from the number, the digits are reversed. The number is **(Sept 2024)**

(a) 39 (b) 62 (c) 93 (d) 31

29. If one of the root of the cubic equation $3x^3 - 5x^2 - 11x - 3 = 0$ is $-\frac{1}{3}$, then other two roots are: **(Sept 2024)**

(a) 1 & 3 (b) –1 & 3 (c) 1 & –3 (d) –1 & –3

30. What are the values of x and y from the given equation? **(Jan 2025)**

$\frac{x}{2} - \frac{y}{5} = y - x$ and $\frac{x-5}{y-10} = 1$

(a) (20, 25) (b) (15, 20) (c) (25, 30) (d) (30, 35)

31. If α and β are roots of the equation $2x^2 - 4x + 6 = 0$, then the quadratic equation with roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ is **(Jan 2025)**

(a) $3x^2 + 10x + 9 = 0$ (b) $3x^2 - 10x + 9 = 0$

(c) $x^2 - 13x + 3 = 0$ (d) $x^2 + 10x + 9 = 0$

32. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has **(May 2025)**

(a) Two distinct real roots (b) Two equal real roots

(c) No real roots (d) More than two real roots

33. For equation $x^3 - 6x^2 + 5x + 12 = 0$, the product of two roots is 12. Which of the following is correct set of roots of the equation? **(May 2025)**

(a) 1, –3, –4 (b) 1, 6, 2 (c) –1, 3, 4 (d) –1, –6, –2

34. Puru gets on the elevator at the 11th floor of a building and rides up at the rate of 57 floors per minute. At the same time, Ishu gets on an elevator at the 51st floor of the same building and rides down at the rate of 63 floors per minute. If they continue travelling at these rates, then at which floor will their paths cross? **(May 2025)**

(a) 17 (b) 19 (c) 27 (d) 30

35. If α and β are the roots of the quation $x^2 - x - 6 = 0$, then the value of $\alpha^3 + \beta^3 + \alpha^2 + \beta^2 + \alpha + \beta$ is equal to **(Sept 2025)**

(a) 35 (b) 29 (c) 31 (d) 33

36. The roots of the equation $\left(\frac{x}{x-1}\right)^2 - 5\left(\frac{x}{x-1}\right)6 = 0$ are; **(Sept 2025)**

(a) $2, \frac{3}{2}$ (b) $3, \frac{1}{2}$ (c) $2, \frac{1}{3}$ (d) $3, \frac{2}{3}$

37. If α and β are two roots of the equation $2x^2 - 6x + 3 = 0$, then the equation with the roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is _____ **(Sept 2025)**

(a) $x^2 + 4x + 1 = 0$ (b) $x^2 - 4x + 1 = 0$

(c) $x^2 - 4x - 1 = 0$ (d) $x^2 + 4x - 1 = 0$

Answer Key

1. (b)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (b)
11. (d)	**12.** (d)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (a)
21. (a)	**22.** (a)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (d)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (a)
31. (a)	**32.** (c)	**33.** (c)	**34.** (d)	**35.** (d)	**36.** (a)	**37.** (b)			

SUMMARY

- A simple equation in one variable x is in the form $ax + b = 0$, where a, b are known constants and $a \neq 0$.
- The general form of a linear equations in two variables, x and y is $ax + by + c = 0$, where a, b are non-zero coefficients and c is a constant.
- Two such equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ form a pair of simultaneous equations in x and y.
- A value for each unknown which satisfies simultaneously both the equations will give the roots of the equations.
- **Elimination Method**: In this method two given linear equations are reduced to a linear equation in one unknown by eliminating one of the unknowns and then solving for the other unknown.
- **Cross Multiplication Method**: For two equations:

  $a_1x + b_1y + c_1 = 0$

  $a_2x + b_2y + c_2 = 0$

  The solution of the equation are:

  $$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

QUADRATIC EQUATION

- An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called a quadratic equation or equation of the second degree.
- When $b = 0$, the equation is called a pure quadratic equation; and when $b \neq 0$ the equation is called an affected quadratic.

  The value of x that satisfies the given quadratic equation is called as the roots of the equation.

  The roots of a quadratic equation $ax^2 + bx + c = 0$ are

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Sum and Product of the roots** of quadratic equation: $ax^2 + bx + c = 0$

  $$\text{Sum of roots} = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

  $$\text{Product of the roots} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

- To construct a quadratic equation for the equation $ax^2 + bx + c = 0$ we have

  x^2 – (Sum of the roots)x + Product of the roots = 0

Nature of the roots: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(i) If $b^2 - 4ac = 0$, then the roots are real and equal;

(ii) If $b^2 - 4ac > 0$, then the roots are real and unequal (or distinct);

(iii) If $b^2 - 4ac < 0$, then the roots are imaginary;

(iv) If $b^2 - 4ac$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct);

(v) If $b^2 - 4ac > 0$, but not a perfect square the rots are real, irrational and unequal.

Cubic Equation: An equation of 3rd degree is called as a cubic equation

i.e., $ax^3 + bx^2 + cx + d = 0$

For cubic equation, $ax^3 + bx^2 + cx + d = 0$

Sum of roots $= -\frac{b}{a}$

Sum of roots taken two at a time $= \frac{c}{a}$

Product of roots $= -\frac{d}{a}$

❑❑❑

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CHAPTER

03 Linear Inequalities

LINEAR INEQUATION

- If the equality symbol '=' in a linear equation is replaced by an inequality symbol (<, >, ≤, or ≥), then the statement is called linear inequality.

  E.g.: $x \leq 3$, $x + y > 7$ etc.

- Thing to remember for the sign of the inequality:

  1. **Addition/Subtraction:** The sign of inequality doesn't change on adding or subtracting the same quantity to both sides.

     E.g.: Since, $5 < 2$

     $\Rightarrow 5 + 1 < 2 + 1 \Rightarrow 6 < 3$, which is true

     Also, $5 < 2$

     $\Rightarrow 5 - 1 < 2 - 1$

     $\Rightarrow 4 < 1$, which is true

  2. **Multiplication/Division:** The sign of inequality doesn't change while multiplying and dividing with positive integers on both sides. However, the sign changes when both sides gets multiply or divide by a negative integer.

     E.g.:

     - On multiplying and dividing with positive integers.

       Since, $5 < 2$

       - On multiplying both the sides by 2, then

         $5 \times 2 < 2 \times 2 \Rightarrow 10 < 4$

         Also, $12 > 4$

       - On dividing both the sides by 4, then

         $12 \div 4 > 4 \div 4$

         $3 > 1$

       - On multiplying and dividing with negative integers.

         Since, $15 > 3$

       - On multiplying both the sides by –2, then

         $15 \times (-2) < 3 \times (-2)$

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$-30 < -6$

Also, $10 > 6$

- On dividing both the sides by –2, then

  $\Rightarrow 10 \div (-2) > 6 \div (-2)$

  $\Rightarrow -5 < -3$

WHY TO STUDY?

- The weight of all the students in your class is less than 90 kgs.
- Conference room can occupy at most 40 tables or chairs.
- Rajesh has ₹100 and wants to buy some notebooks and pens. The cost of one notebook is ₹40 and that of a pen is ₹20.

  So, whenever there is a limit to something and not an equality for sure then we need to study the inequalities.
- These type of inequalities occur in business whenever there is a limit on supply, demand, sales etc.

  E.g.: Raju wants to produce sofas. But the material needed can be only get on some conditions such that he has to buy cloth of sofa at least for 40 sets of sofas and he can get wood for maximum for 100 sets.

  So in such case he needs to adjust his demands and sales accordingly and for this we need to understand the concept of inequalities.

LINEAR INEQUATION IN ONE VARIABLE

- As the heading says, when there is an inequality on one variable then it will be called as linear inequation in one variable.

  E.g.: $x \geq 5$, $y < 3$

  And the range where the equation satisfies is called as intervals solution space also abbreviated as S.S.

  Intervals (Solution space) of an inequality:

  If $a < b$, and $a < x < b$ means that $a < x$, and $x < b$. That is, x is between a and b.

  The following table will be make you understand it properly:

Interval	Inequality
[a, b]	$a \leq x \leq b$
[a, b)	$a \leq x < b$
(a, b]	$a < x \leq b$
(a, b)	$a < x < b$

E.g.: (-2, 5) is an open interval that includes all the real numbers greater than -2 but less than 5.

[-2, 5) is an interval that includes all the real numbers greater than and equal to -2 but less than 5.

(-2, 5] is an open interval that includes all the real numbers greater than -2 but less than and equal to 5.

[-2, 5] is closed interval that includes all the real numbers from -2 to 5 including the endpoints.

Example 1. The solution of the inequality $8x + 6 < 12x + 14$ is **(Dec 2013)**

(a) $(-2, 2)$ (b) $(0, 2)$ (c) $(2, \infty)$ (d) $(-2, \infty)$

Sol. (d) Given: $8x + 6 < 12x + 14$

$\Rightarrow 6 - 14 < 12x - 8x$

$\Rightarrow -8 < 4x$

$\Rightarrow 4x > -8$

$\Rightarrow x > \frac{-8}{4}$

$\Rightarrow x > -2$

Solution set = $(-2, \infty)$

Hence, the correct option is (d).

Example 2. The solution set of the equations $x + 2 > 0$ and $2x - 6 > 0$ is **(June 2019)**

(a) $(-2, \infty)$ (b) $(3, \infty)$ (c) $(-\infty, -2)$ (d) $(-\infty, -3)$

Sol. (b) Since, $x + 2 > 0$

$\Rightarrow x > -2$

Also, $2x - 6 > 0$

$\Rightarrow 2x > 6$

$\Rightarrow x > 3$ ...(ii)

From (i) and (ii), the common solution set will be:

$x > 3$

i.e., $(3, \infty)$

Hence, the correct option is (b).

Example 3. Find the range of real of x satisfying the inequalities $3x - 2 > 7$ and $4x - 13 > 15$. **(June 2012)**

(a) $x > 3$ (b) $x > 7$ (c) $x < 7$ (d) $x < 3$

Sol. (b) Given, $3x - 2 > 7$

$\Rightarrow 3x > 7 + 2$

$\Rightarrow 3x > 9$

$\Rightarrow x > 3$ ...(i)

Also, $4x - 13 > 15$

$\Rightarrow 4x > 15 + 13$

$\Rightarrow 4x > 28$

$\Rightarrow x > 7$ ...(ii)

From (i) and (ii), we get

$x > 7$

Hence, the correct answer is option (b) i.e., $x > 7$.

Example 4. Solve the inequality: $\frac{(3x-1)}{2} \leq \frac{(x+2)}{4}$.

(a) $x \leq 2$ (b) $x \leq 0.8$ (c) $x \geq 1.5$ (d) $x \geq 2$

Sol. (b) Given, $\frac{(3x-1)}{2} \leq \frac{(x+2)}{4}$

$\Rightarrow \frac{(3x-1)}{1} \leq \frac{(x+2)}{2}$

$\Rightarrow 2(3x - 1) \leq (x + 2)$

$\Rightarrow 6x - 2 \leq x + 2$

$\Rightarrow 6x - x \leq 2 + 2$

$\Rightarrow x \leq \frac{4}{5}$

$\Rightarrow x \leq 0.8$

Hence, the correct option is (b) i.e., $x \leq 0.8$.

Example 5. If $3x + 2 < 2x + 5$ and $4x - 5 \geq 2x - 3$, then x can take the following value

(a) 3 (b) –1 (c) 2 (d) –3 **(Dec 2022)**

Sol. (c) Given, $3x + 2 < 2x + 5$

$\Rightarrow 3x - 2x < 5 - 2$

$\Rightarrow x < 3$ ...(i)

Also, $4x - 5 \geq 2x - 3$

$\Rightarrow 4x - 2x \geq -3 + 5$

$\Rightarrow 2x \geq 2$

$\Rightarrow x \geq 1$ ...(ii)

From (i) and (ii), we get

$1 \leq x < 3$

Thus, out of given options, x can take up the value of 2.

Example 6. Solve for real 'x' if $5x - 3 > 2x + 6$ and $2x - 3 < -3x + 17$.

(a) $3 \leq x < 4$ (b) $3 < x < 4$ (c) $3 \leq x \leq 4$ (d) None of the above

Sol. (b) Given: $5x - 3 > 2x + 6$

$\Rightarrow 5x - 2x > 6 + 3$

$\Rightarrow 3x > 9$

$\Rightarrow x > 3$ ....(i)

Now, $2x-3< -3x+17$

$\Rightarrow 2x + 3x < 3 + 17$

$\Rightarrow 2x + 3x < 3 + 17$

$\Rightarrow 5x < 20$

$\Rightarrow x < 4$ ....(ii)

From (i) and (ii), we get

$3 < x < 4$

Hence, the correct answer is option (b) i.e., $3 < x < 4$.

LINEAR INEQUATION IN TWO VARIABLES

- Again as the heading says, when there is an inequality in two variables then it will be called as linear inequation in two variables. For example: $ax + by \leq c$, $x + 2y < 6$, $x + y \geq 5$, $3x + y > 9$ are some examples of the linear inequality of two variables.

E.g.: Let's plot a graph of $x + 2y \leq 10$

The line of equation corresponding to above inequality is $x + 2y = 10$

When $x = 0$ then $y = 5$

When $y = 0$ then $x = 10$

Thus, the points are (0, 5) and (10, 0).

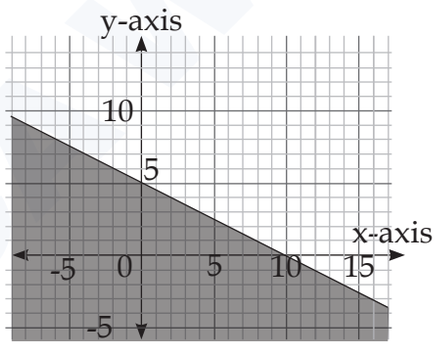
Now, put $x = 0$ and $y = 0$ in given inequality, we get

$0 + 0 \leq 10$

$0 \leq 10$, which is true

Thus, the shaded region will be towards the origin.

Therefore, the required graph is:



Example 7. Rajesh has ₹100 and wants to buy some notebooks and pens. The cost of one notebook is ₹40 and that of a pen is ₹20. Draw the linear inequality

(a) $40x + 20y < 100$ (b) $40x + 20y \leq 100$

(c) $40x - 20y > 100$ (d) $40x + 20y > 100$

Sol. (b) Given, Cost of 1 notebook = ₹40

Cost of 1 pen = ₹20

Let the required number of notebooks be x and the number of pens be y.

According to the question,

$40x + 20y \leq 100$

Hence, the correct option is (b).

Example 8. There are 150 students in a class. If the number of boys are x and number of girls are y, then which of the following inequalities represents the situation?

(a) $x + y \leq 150$ (b) $x + y \geq 150$

(c) $x + y \neq 150$ (d) None of these

Sol. (a) Given, Number of boys = x

Number of girls = y

Since, the number of students are 150, thus

$x + y \leq 150$

Hence, the correct answer is option (a) i.e., $x + y \leq 150$.

Example 9.

I. Fisherman catches x fishes and y crabs for his family to eat on daily basis till the time it reaches a number of minimum 10. Now tell, x and y can be related by the inequality

(a) $x + y \neq 10$ (b) $x + y \leq 10, x \geq 0, y \geq 0$

(c) $x + y \geq 10, x \geq 0, y \geq 0$ (d) None of these

II. An average size fish can feed 4 people while one crab can feed only one people but the employer has to maintain an output of at least 15 people per done. This situation can be expressed as

(a) $4x + y \leq 15$ (b) $4x + y > 15$

(c) $4x + y \geq 15, x \geq 0, y \geq 0$ (d) None of these

III. But he makes sure that he takes home more than 2 fishes to each crab. This situation can be expressed as

(a) $x \geq y/2$ (b) $y \leq x/2$ (c) $y \geq x/2$ (d) $2x > y$

Sol. I. (c) Let the number of fishes be x and the number of crabs be y

Since, the number should be minimum of 10 i.e. the sum of fish and crabs should be equal to or greater than 10.

Also, $x \geq 0, y \geq 0$

Thus, the inequality can be represented as:

$x + y \geq 10 , x \geq 0, y \geq 0$

Hence, the correct option is (c).

II. (c) Let the number of fish be x and the number of crabs be y.

Since, the employer has to maintain an output of at least 15 people per day,

Then according to the problem, it can be represented as

$4x + y \geq 15, x \geq 0, y \geq 0$

Hence, the correct option is (c).

III. (b) Let the number of fishes be x and the number of crabs be y.

Since, for each crab he take more than 2 fishes, then

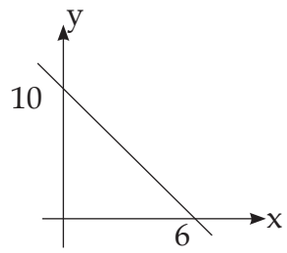
$x \geq 2y$

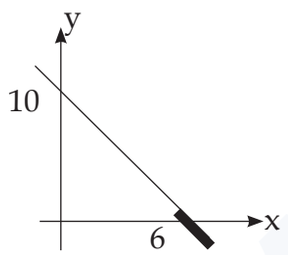
$\Rightarrow \frac{x}{2} \geq y$

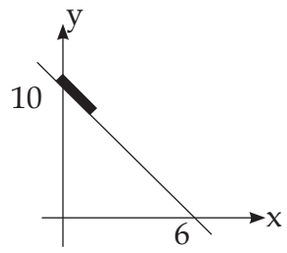
or $y \leq \frac{x}{2}$

Hence, the correct option is (b).

Example 10. The graph to express the inequality $5x + 3y \geq 30$ is **(ICAI)**

(a) 

(b) 

(c) 

(d) None of these

Sol. (c) **Given inequality:** $5x + 3y \geq 30$ is

The line of equation corresponding to given inequality: $5x + 3y = 30$

When $x = 0$, then $y = 10$

When $y = 0$, then $x = 6$

Thus, the coordinates satisfying the equation is (0, 10) and (6, 0).

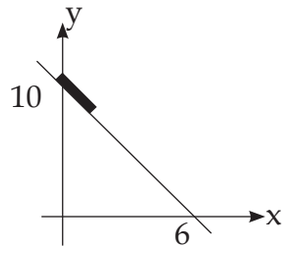
Now, on putting $x = 0$ and $y = 0$ in the above inequality, we get

$5(0) + 3(0) \geq 30$

$\Rightarrow 0 \geq 30$, which is false.

So, the shaded region will be away from the origin.

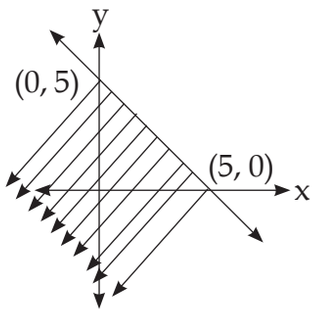
Thus, the required graph is:



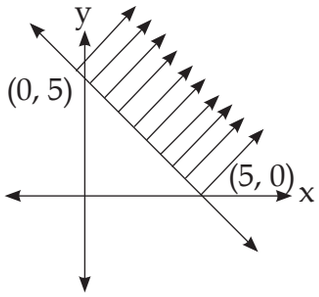
Hence, the correct answer is option (c).

Example 11. The graph to express the inequality $x + y \leq 5$ is

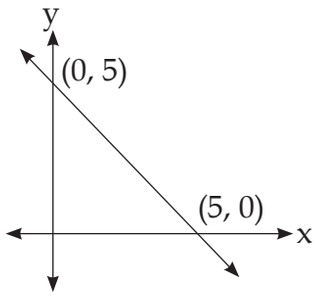
(a)



(b)



(c)



(d) None of these

Sol. (a) Given inequality $x + y \leq 5$

The line of equation corresponding to given inequality: $x + y = 5$

When $x = 0$, then $y = 5$

When $y = 0$, then $x = 5$

Thus, the coordinates satisfying the equation is (0, 5) and (5, 0).

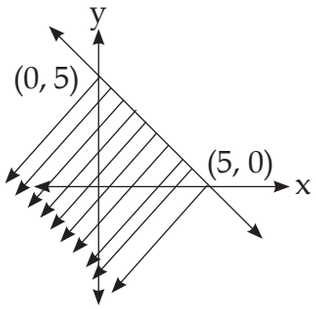
Now, on putting $x = 0$ and $y = 0$ in the above inequality, we get

$0 + 0 \leq 5$

$\Rightarrow 0 \leq 5$, which is true.

So, the shaded region will be towards the origin.

Thus, the required graph is:



Hence, the correct option is (a).

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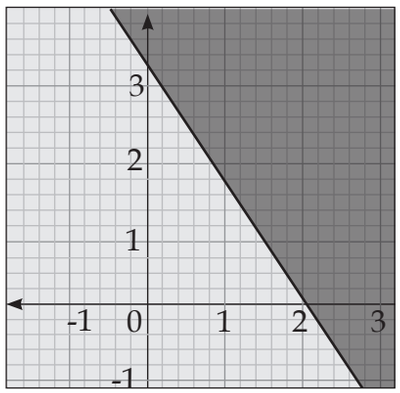
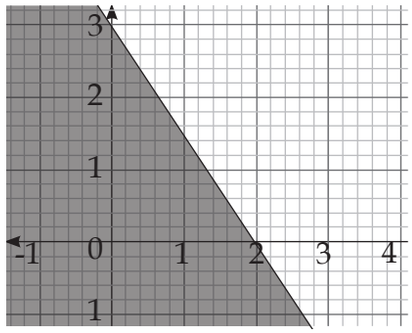
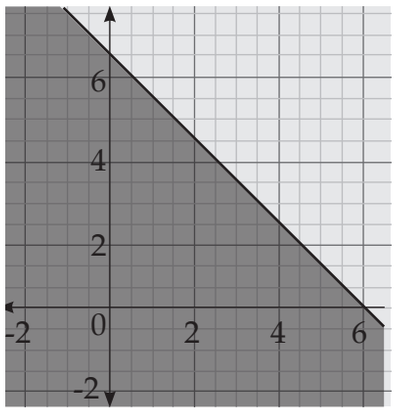
Example 12. The graph to express the inequality $3x + 2y \leq 6$ is

(a) 

(b) 

(c) 

(d) None of these

Sol. (b) Given inequality: $3x + 2y \leq 6$

For line of equation of above inequality: $3x + 2y = 6$

When $x = 0$ then $y = 3$

When $y = 0$ then $x = 2$

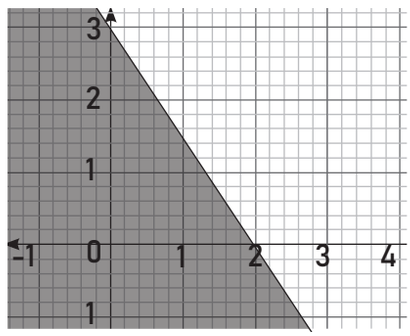
Thus, the coordinates satisfying the equation is (0, 3) and (2, 0).

Now, on putting $x = 0$ and $y = 0$ in the above inequality, we get

$3x + 2y = 3(0) + 2(0) = 0 \leq 6$ which is true

So, the shaded region will be towards the origin.

Thus, the required graph is:



Hence, the correct option is (b).

Example 13. On solving the inequalities $2x+5y \le 20, 3x+2y \le 12, x \ge 0, y \ge 0$, we get the following situation **(ICAI)**

(a) (0, 0), (0, 4), (4, 0) and $\left(\frac{20}{11}, \frac{36}{11}\right)$

(b) (0, 0), (10, 0), (0, 6) and $\left(\frac{20}{11}, \frac{36}{11}\right)$

(c) (0, 0), (0, 4), (4, 0) and (2, 3)

(d) (0, 0), (10, 0), (0, 6) and (2, 3)

Sol. (a) Given, $2x + 5y \le 20$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$

The line of equation corresponding to given inequalities are;

$2x + 5y = 20$ ...(i)

$3x + 2y = 12$ ...(ii)

Multiplying eq. (i) by 3 and eq. (ii) by 2, we get

$6x + 15y = 60$

$6x + 4y = 24$

On subtracting, we get

$11y = 36$

$\Rightarrow y = \frac{36}{11}$

Put value of y in eq. (i), we get

$2x + \frac{180}{11} = 20$

$\Rightarrow 2x = \frac{40}{11}$

$\Rightarrow x = \frac{20}{11}$

Now, for $2x + 5y = 20$, we have

x	0	10
y	4	0

For $3x + 2y = 12$, we have

x	0	4
y	6	0

Therefore, the graph can be plotted as

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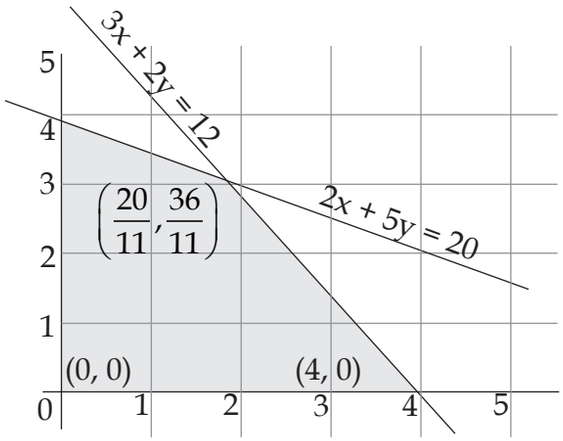


Thus, the grey shaded region represents the solution set and the required solution is: (0, 0), (0, 4), (4, 0) and $\left(\frac{20}{11}, \frac{36}{11}\right)$

Hence, the correct option is (a).

Example 14. A firm makes two types of products: Type A and Type B. The profit on product A is ₹20 each and that on product B is ₹30 each. Both types are processed on three machines M1, M2 and M3. The time required in hours by each product and total time available in hours per week on each machine are as follows: **(ICAI)**

Machine	Product A	Product B	Available Time
M1	3	3	36
M2	5	2	50
M3	2	6	60

The constraints can be formulated taking x_1 = number of units A and x_2 = number of unit of B as

(a) $x_1 + x_2 \le 12$
$5x_1 + 2x_2 \le 50$
$2x_1 + 6x_2 \le 60$
$3x_1 + 3x_2 \le 36$

(b) $3x_1 + 3x_2 \le 36$
$5x_1 + 2x_2 \le 50$
$2x_1 + 6x_2 \le 60;\ x_1 \ge 0, x_2 \ge 0$

(c) $2x_1 + 6x_2 \le 60$
$5x_1 + 2x_2 \le 50;\ x_1 \ge 0, x_2 \ge 0$

(d) None of these

Sol. (c) Given, x_1 = number of units of A and x_2 = number of units of B

Clearly, $x_1 \ge 0$, $x_2 \ge 0$

According to the given data,

The constraints can be formulated as:

$3x_1 + 3x_2 \le 36$

$5x_1 + 2x_2 \le 50$

$2x_1 + 6x_2 \le 60$ such that $x_1 \ge 0$, $x_2 \ge 0$

Hence, the correct answer is option (c).

Example 15. The union forbids employer to employ less than two experienced people (x) to each fresh person (y). This situation can be expressed as **(ICAI)**

(a) $x \le y/2$ (b) $y \le x/2$ (c) $y \ge x/2$ (d) None of these

Sol. (b) We know that each fresh person is denoted by y and experienced person by x.

As the union forbids employers to employ less than 2 experienced person to each fresh person. This means the number of experienced person must be equal to or greater than twice the number of fresh person.

Thus this situation can be denoted as $2y \le x$

or $y \le \frac{x}{2}$

Hence, the correct answer is option (b).

Example 16. On the average an experienced person does 7 units of work while a fresh one works 5 units of work daily but the employer has to maintain an output of at least 35 units of work per day. The situation can be expressed as:

(a) $7x + 5y < 35$ (b) $7x + 5y \le 35$

(c) $7x + 5y > 35$ (d) $7x + 5y \ge 35$

Sol. (d) Let the number of experienced people be x and the number of fresh people be y.

Work done by experienced person per day = 7x

Work done by fresh person = 5y

According to the question,

$7x + 5y \ge 35$

Thus, the situation can be expressed as: $7x + 5y \ge 35$

Hence, the correct option is (d).

Example 17. Solution space of the inequalities $2x + y \le 10$ and $x - y \le 5$:

(i) Include the origin.

(ii) Includes the point (4, 3)

Which one is correct? **(June 2011)**

(a) Only (i) (b) Only (ii)

(c) Both (i) and (ii) (d) None of these

Sol. (a) Given, $2x + y \le 10$ and $x - y \le 5$

(i) For the origin (0, 0):

$2x + y \le 10$

$\Rightarrow 0 + 0 \le 10$ or $0 \le 10$, which is true

$\Rightarrow x - y \le 5$

$\Rightarrow 0 + 0 \le 5$ or $0 \le 5$, which is also true

(ii) For the point (4, 3):

$\Rightarrow 2x + y \le 10$

$\Rightarrow 2(4) + 3 \leq 10$

$\Rightarrow 8 + 3 \leq 10$

$\Rightarrow 11 \leq 10$, which is false

Clearly, (0, 0) satisfies both the inequations.

Hence, the correct option is (a).

Example 18. The linear relationship between two variables in an inequality: **(May 2018)**

(a) $ax + by \leq c$ (b) $axby \leq c$ (c) $axy + by \leq c$ (d) $ax + bxy \leq c$

Sol. (a) We know that,

The standard form of linear equation is $ax + by = c$.

Thus, out of the given options, the linear inequation can be represented as $ax + by \leq c$.

Hence, the correct option is (a).

Example 19. The rules and regulations demand that the employer should empty not more than 5 experienced hands to 1 fresh one and this fact is represented by: (Taking experienced person as x and fresh person as y) **(May 2007)**

(a) $y \geq \frac{x}{5}$ (b) $5y \leq x$ (c) $5y \geq x$ (d) None of these

Sol. (a) According to the question,

The employer should employ not more than 5 experienced hands to 1 fresh one.

This means that 1 fresh can have a maximum of 5 experienced hands.

So, taking experienced person as x and fresh person as y, we get,

$x \leq 5y$ or $y \geq \frac{x}{5}$

Hence, the correct option is (a).

Example 20. If $a > 0$ and $b < 0$, it follows that

(a) $\frac{1}{a} > \frac{1}{b}$ (b) $\frac{1}{a} < \frac{1}{b}$ (c) $\frac{1}{a} = \frac{1}{b}$ (d) None of these

Sol. (a) Since, $a > 0$

$\Rightarrow \frac{1}{a} > 0$ ...(i)

Now, $b < 0$

$\Rightarrow \frac{1}{b} < 0$ .... (ii)

From (i) and (ii), we get

$\frac{1}{a} > \frac{1}{b}$

Hence, the correct answer is option (a).

Example 21. A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per kg of each food is shown below:

	A	B	C	D
Food I:	2	1	1	2
Food II:	1	1	2	3

Assuming x units of food I is to be mixed with y units of food II, the situation can be expressed as

(a) $2x + y \leq 9, x + y \leq 7, x + 2y \leq 10, 2x + 3y \leq 12, x > 0, y > 0$

(b) $2x + y \geq 30, x + y \leq 7, x + 2y \leq 10, x + 3y \geq 12$

(c) $2x + y \geq 9, x + y \geq 7, x + y \leq 10, x + 3y \geq 12$

(d) $2x + y \geq 9, x + y \geq 7, x + 2y \geq 10, 2x + 3y \geq 12, x \geq 0, y \geq 0$

Sol. (d) Given, Quantity of food I = x units

Quantity of food II = y units

Since, the content should be atleast 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D

$\Rightarrow 2x + y \geq 9, x + y \geq 7, x + 2y \geq 10, 2x + 3y \geq 12$

Also, the quantity of food cannot be negative.

$\Rightarrow x \geq 0, y \geq 0$

Hence, the final correct answer is option (d) i.e., $2x + y \geq 9, x + y \geq 7, x + 2y \geq 10, 2x + 3y \geq 12, x \geq 0, y \geq 0$.

Example 22. On solving the inequalities $5x + y \leq 100, x + y \leq 60, x \geq 0, y \geq 0$, we get the following situation: **(Nov 2018)**

(a) (0, 0), (20, 0), (10, 50) & (0, 60)

(b) (0, 0), (60, 0), (10, 50) & (0, 60)

(c) (0, 0), (20, 0), (0, 100) & (10, 50)

(d) None of these

Sol. (a) The inequalities are:

$5x + y \leq 100$

$x + y \leq 60$

The line of equation for inequality: $5x + y \leq 100$

$\Rightarrow 5x + y = 100$

Reference of points:

x	0	20	10
y	100	0	50

The line of equation for inequality: $x + y \leq 60$

$\Rightarrow x + y = 60$

Reference of points:

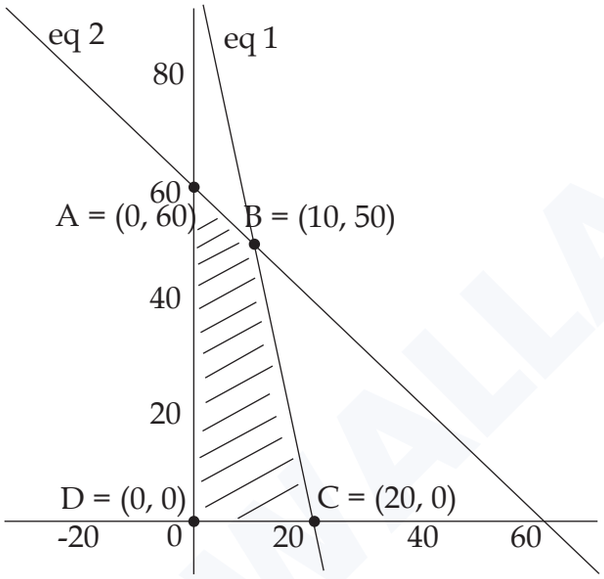
x	0	60	10
y	60	0	50

For the inequalities, the value of x and y will lie either on the lines $x + y = 60$ and $5x + y = 100$ or below them.

It is given $x \geq 0, y \geq 0$.

Thus the value of x and y belongs to 1st quadrant

The graph for the given set of inequalities is as follows:



Thus we get the four points

A = (0, 60); B = (10, 50); C = (20, 0); D = (0, 0)

Hence the correct option is (a).

PRACTICE QUESTIONS

1. $4x > -16$ implies

(a) $x \geq -4$ (b) $x < -4$ (c) $x > -4$ (d) $x \leq -4$

2. Solve for real 'x' if $5x - 2 \geq 2x + 1$ and $2x + 3 < 18 - 3x$

(a) $1 < x < 3$ (b) $-1 > x > -3$ (c) $1 \leq x < 3$ (d) $x = 3$

3. Which of the following value 'x' cannot take in the inequality $4x + 3 < 2x + 5$

(a) 1 (b) 0 (c) –1 (d) –4

4. Find the range of real values of x that satisfy the system of inequalities

$2x + 5 \geq 13$ and $3x - 4 \leq 8$

(a) $x = 3$ (b) $x = 0$ (c) $x = 4$ (d) $x = 5$

5. Solve for real 'x' if $5x - 3 > 2x + 6$ and $2x - 3 < +17$

(a) $3 \leq x < 4$ (b) $3 < x < 4$ (c) $3 \leq x \leq 4$ (d) None of these

6. Raju wants to produce sofas. But the material needed can be only get on some conditions such that he has to buy cloth of sofa at least for 40 sets of sofas (x) and he can get wood (y) for maximum for 100 sets. Form an inequality for it

(a) $x \geq 40, y \leq 100$ (b) $x \leq 40, y \leq 100$

(c) $x \geq 40, y \geq 100$ (d) None of these

7. An employer recruits experienced (x) and fresh workmen (y) under the condition that he cannot employ more than 11 people. x and y can be related by the inequality **(June 2019)**

(a) $x + y = 11$ (b) $x + y \leq 11, x \geq 0, y \geq 0$

(c) $x + y > 11, x \geq 0, y \geq 0$ (d) None of these

8. A company is planning to launch a new product and decides to hire marketing executives and sales executives for the project. If the company cannot employ more than 12 executives, which of the following inequalities correctly relates the number of marketing executives (x) and sales executives (y) that the company can hire?

(a) $x + y \leq 12$ (b) $2x + 3y \leq 12$

(c) $3x + 2y \leq 12$ (d) $4x + 4y \leq 12$

9. If an experienced person gets paid ₹10 per unit of work and a fresh one gets paid ₹6 per unit of work, then the inequality representing the situation where the total cost of labor per day is at most ₹180 is:

(x represents number of experienced people and y represents number of fresh people)

(a) $10x + 6y \leq 180$ (b) $10x + 6y \geq 180$

(c) $6x + 10y \leq 180$ (d) $x + y \leq 18$

10. A factory can produce two materials A and B, with machines in operation for 24 hours a day. Production of A requires 1 hour of processing in machine 1 and 2 hours in machine 2. Production of B requires 2 hours of processing in machine1 and 1 hour in machine2. The manufacturer earns a profit of ₹10 on each unit of A and ₹5 on each unit of B. How many units of each product should be produced in a day in order to achieve maximum profit?

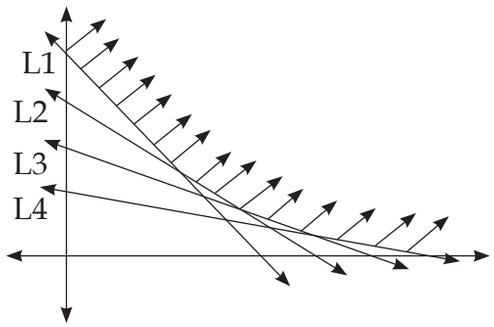
(a) Max: $10x + 5y$
$x + 2y \leq 24$
$2x + y \leq 24$
$x, y \geq 0$

(b) Max: $10x + 5y$
$x + 2y \geq 24$
$2x + y \leq 24$
$x, y \geq 0$

(c) Min: $10x + 5y$
$x + 2y \leq 24$
$2x + y \leq 24$
$x, y \geq 0$

(d) None of these

11. Graphs of the inequations are drawn below:



L1: $2x + y = 2$

L2: $x + y = 2$

L3: $x + 2y = 1$

L4: $x + 3y = 4$

The common region (shaded region) shown in the graph refers to

(a) $\begin{matrix} 2x+y \le 2 \\ x+y \ge 2 \\ x+2y \ge 1 \\ x+3y \ge 4 \end{matrix}$

(b) $\begin{matrix} 2x+y \ge 2 \\ x+y \le 2 \\ x+2y \ge 1 \\ x+3y \ge 4 \end{matrix}$

(c) $\begin{matrix} 2x+y \ge 2 \\ x+y \ge 2 \\ x+2y \ge 1 \\ x+3y \ge 4 \end{matrix}$

(d) None of these

12. The solution of the inequality $\frac{(5-2x)}{3} \le \frac{x}{6} - 5$ is: **(Dec 2020, Jan 2021)**

(a) $x \ge 8$ (b) $x \le 8$ (c) $x = 8$ (d) None of these

13. Which of the following situation do we get on solving the inequalities: $x - y \le 2$, $x + y \le 4$ and $x \ge 1$?

(a) (1, 3), (1, 0), (2, 0), (3, 1) (b) (1, 1), (2, 0), (3, 1), (6, 4)

(c) (1, 0), (2, 0), (3, 1), (7, 5) (d) None of these

14. The solutions of the set of inequalities $2x + y \ge 12$, $5x + 8y \ge 74$, $x + 6y \ge 24$, $x \ge 0$, $y \ge 0$ are **(Nov 2019)**

(a) $(24, 0), \left(\frac{126}{11}, \frac{23}{11}\right), (2, 8), (0, 12)$ (b) $(0, 24), (2, 8), (0, 12), \left(\frac{126}{11}, \frac{23}{11}\right)$

(c) (8, 4), (2, 8), (0, 12), (0, 24) (d) (8, 4), (0, 0), (0, 6), (2, 0)

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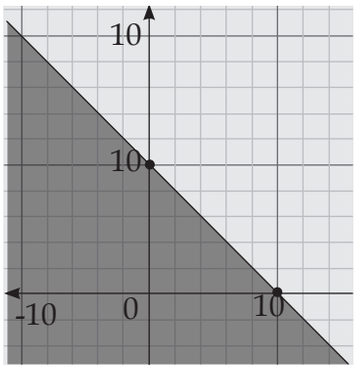
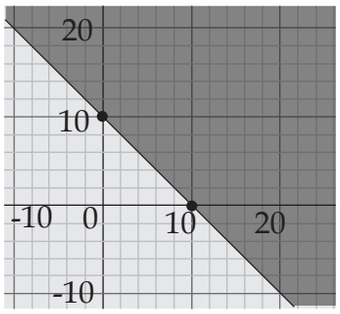
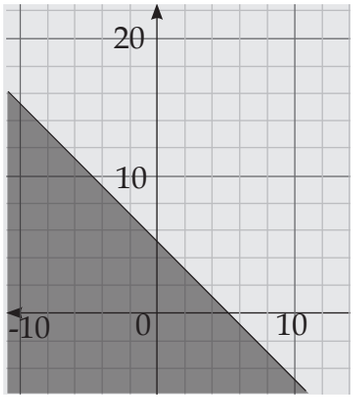
15. Which of the following graph represents the inequality $x + y \leq 10$?

(a) 

(b) 

(c) 

(d) None of these

16. On solving the inequalities $6x + y \geq 18$, $x + 4y \geq 12$, $2x + y \geq 10$, we get the following situation

(a) (0, 18), (12, 0), (4, 2) & (2, 6)
(b) (3, 0), (0, 3), (4, 2) & (7, 6)
(c) (5, 0), (0, 10), (4, 2) & (7, 6)
(d) (0, 18), (12, 0), (4, 2), (0, 0) & (7, 6)

17. A small manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry, and then sent to another machine for finishing. The number of man-hours for the firm available per week are as follows:

	Foundry	Machine-shop
A	10	5
B	6	4
Capacity per week (man hours)	1000	600

Let the firm manufacture x units of A and y units of B.

The constraints are:

(a) $10x + 6y \leq 1000$, $5x + 4y \geq 600$, $x \geq 0$; $y \leq 0$
(b) $10x + 6y \leq 1000$, $5x + 4y \leq 600$, $x \geq 0$; $y \geq 0$
(c) $10x + 6y \geq 1000$, $5x + 4y \leq 600$, $x \leq 0$; $y \geq 0$
(d) $10x + 6y \geq 1000$, $5x + 4y \geq 600$, $x \leq 0$; $y \leq 0$

18. Vitamin A and B are found in food F_1 and F_2. One unit of F_1 contains 20 units of vitamin A and 30 units of vitamin B. One unit of F_2 contains 60 units of vitamin A and 40 units of vitamin B. Cost per unit of F_1 and F_2 are ₹3 and ₹4 respectively. The minimum daily requirement of vitamin A and B is 80 and 100 units respectively.

Problem is to determine the mixture of food F_1 and F_2, which meets the requirement at minimum cost by assuming that x_1 units of food F_1 and x_2 units of food F_2 are required to fulfill the need of vitamins. The constraints are

(a) $20x_1 + 60x_2 \leq 80$, $30x_1 + 40x_2 \leq 40$, $x_1 \leq 0$; $x_2 \leq 0$

(b) $20x_1 + 60x_2 \geq 80$, $30x_1 + 40x_2 \leq 40$, $x_1 \geq 0$; $x_2 \leq 0$

(c) $20x_1 + 60x_2 \geq 80$, $30x_1 + 40x_2 \geq 40$, $x_1 \geq 0$; $x_2 \geq 0$

(d) $20x_1 + 60x_2 \leq 80$, $30x_1 + 40x_2 \geq 40$, $x_1 \leq 0$; $x_2 \geq 0$

19. A fertilizer company produces two types of fertilizers called grade I (x) and grade II (y). Each of these types is processed through two critical chemical plant units. Plant A has a maximum of 120 hours available in a week and plant B has a maximum of 180 hours available in a week. Manufacturing one bag of grade I fertilizer requires 6 hours in plant A and 4 hours in plant B. Manufacturing one bag of grade II fertilizer requires 3 hours in plant A and 10 hours in plant B. Express this using linear inequalities.

(a) $6x + 3y \leq 120$, $4x + 10 = 180$ (b) $6x + 3y = 120$, $4x + 10y > 180$

(c) $6x + 3y \leq 120$, $4x + 10y \leq 180$ (d) $6x + 3y < 120$, $4x + 10y < 180$

20. A car manufacturing company manufacturers cars of two types A and B. Model A requires 150 man-hours for assembling, 50 man-hours for painting and 10 man-hours for checking and testing. Model B requires 60 man-hours for assembling, 40 man-hours for painting and 20 man-hours for checking and testing. There are available 30 thousand man-hours for assembling,13 thousand man-hours for painting and 5 thousand man-hours for checking and testing. Express the above situation using linear inequalities. Let the company manufacture x units of type A model of car and y units of type B model of car. **(May 2007)**

Then the inequalities are:

(a) $5x + 2y \geq 1000$, $5x + 4y \geq 1300$, $x + 2y \leq 500$, $x \geq 0$, $y \geq 0 \leq \geq$

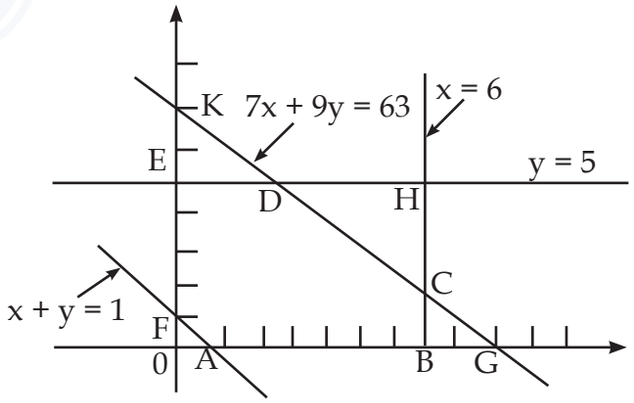
(b) $5x + 2y \leq 1000$, $5x + 4y \leq 1300$, $x + 2y \leq 500$, $x \geq 0$, $y \geq 0$

(c) $5x + 2y \leq 1000$, $5x + 4y \leq 1300$, $x + 2y \leq 500$, $x \geq 0$, $y \geq 0$

(d) $5x + 2y = 1000$, $5x + 4y \geq 1300$, $x + 2y = 500$, $x \geq 0$, $y \geq 0$

21. Graph of the following linear inequalities :

$x + y \geq 1$, $y \leq 5$, $x \leq 6$, $7x + 9y \leq 63$, $x \geq 0$, $y \geq 0$ is given below;



(a) DCHAD (b) BCGB (c) ABCDEFA (d) EDKE

22. The time required to produce a unit of product A is 3 hours and that for product B is 5 hours. The total available time is 220 hours. If x and y are the number of units of A and B that are produced then

(a) $3x + 2y = 220$ (b) $3x + 2y \geq 220, x \geq 0, y \geq 0$

(c) $3x + 2y \leq 220, x \geq 0, y \geq 0$ (d) $5x + 2y \geq 220, x \geq 0, y \geq 0$

23. A company produces two types of leather belts, say A and B. Belt A is of superior quality and belt B is of lower quality. Each belt of type A requires twice as much as required by a belt of type B. If all belts were of type B, the company would produce 1000 belts per day. But the supply of leather is sufficient only for 800 belts per day. Belt A requires fancy buckles and only 400 fancy buckles are available per day. For belt of type B only 700 buckles are available per day.

Constraints can be formulated by assuming that the company produce x units of belt A and y units of belt B as:

(a) $x + 2y \leq 1000, x + y \geq 800, x \leq 400, y \leq 700, x \geq 0, y \geq 0$

(b) $x + 2y \leq 1000, x + y \leq 800, x \leq 400, y \leq 700, x \geq 0, y \geq 0$

(c) $x + 2y \geq 1000, x + y \leq 800, x \leq 400, y \leq 700, x \geq 0, y \geq 0$

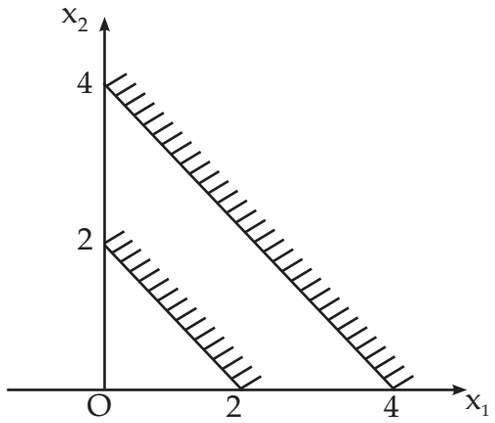
(d) None of these

Answer Key

1. (c)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (a)
11. (c)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (c)	**20.** (b)
21. (c)	**22.** (c)	**23.** (b)							

PREVIOUS YEAR QUESTION (PYQ'S)

1. The region indicated by the shading in the graph is expressed by the inequalities:



(a) $x_1 + x_2 \leq 2, x_1 + x_2 \geq 4, x_1 \geq 0, x_2 \geq 0$ (b) $x_1 + x_2 \leq 2, x_1 + x_2 \leq 4, x_1 \geq 0, x_2 \geq 0$

(c) $x_1 + x_2 \geq 2, x_1 + x_2 \geq 4, x_1 \geq 0, x_2 \geq 0$ (d) $x_1 + x_2 \leq 2, x_1 + x_2 > 4, x_1 \geq 0, x_2 \geq 0$

2. XYZ company has a policy for its recruitment as it should not recruit more than eight men (x) to three women (y). How can this fact be expressed in inequality?

(Dec 2021)

(a) $3y \geq 8x$ (b) $3y \leq \frac{x}{8}$ (c) $8y \geq 3x$ (d) $8y \leq 3x$

3. A labour can be paid under two methods as given below:

(i) ₹600 fixed and ₹50 per hour.

(ii) ₹170 per hour

If a particular job work takes 'x' hours to complete, find out the value of x for which the method (ii) gives the labour gets the better wages. **(June 2022)**

(a) $x = 6$ (b) $x = 4$ (c) $x = 3$ (d) $x = 2$

4. If $2x + 5 > 3x + 2$ and $2x - 3 \leq 4x - 5$, then 'x' can take the following values

(Dec 2022)

(a) 4 (b) –4 (c) 2 (d) –2

5. The largest side of a triangle is 3 times the shortest side and third side is 4 cm shorter than largest side. If the perimeter of the triangle is at least 59 cm, what is the length of shortest side?

(June 2023)

(a) less than 7 cm (b) greater than or equal to 7 cm

(c) less than 9 cm (d) greater than or equal to 9 cm

6. The solution of the inequality $\frac{5-2x}{3} \leq \frac{x}{6}$ is: **(Dec 2023)**

(a) $x \geq 8$ (b) $x \geq 7$ (c) $x \leq \frac{80}{3}$ (d) $x \geq \frac{40}{3}$

7. A software company should recruit more than or equal to 10 employees at a time in their recruitment drive. Under these conditions, company recruits experienced (x) and freshers (y) employees. The values of x and y can be related by the following inequality: **(Dec 2023)**

(a) $x + y \leq 10, x \geq 0, y \geq 0$ (b) $x + y \geq 10, x \geq 0, y \geq 0$

(c) $x + y \neq 10, x \geq 0, y \geq 0$ (d) $x + y \geq 10, x \leq 0, y \leq 0$

8. In a garment factory, an average experienced tailor can stitch 5 shirts while a fresh tailor can stitch 3 shirts daily, but the employer has to maintain an output of at least 30 shirts stitched per day. This can be formulated as: **(June 2023)**

(a) $5x + 3y \leq 30$ (b) $5x + 3y > 30$

(c) $5x + 3y \geq 30, x \geq 0, y \geq 0$ (d) $5x + 3y \leq 30, x \geq 0, y \geq 0$

9. A fertilizer company produces two types of fertilizers called grade I and grade II. Each of these types is processed through a critical chemical plant unit. The plant has a maximum of 180 hours available in a week. Manufacturing one bag of grade I fertilizer requires 4 hours in the plant. Manufacturing one bag of grade II fertilizer requires 10 hours in that plant. Express this using linear inequalities: **(June 2023)**

(a) $2x_1 + 5x_2 \leq 180$ (b) $4x_1 + 10x_2 > 180$

(c) $2x_1 + 5x_2 > 180$ (d) $4x_1 + 10x_2 \leq 180$

10. A senior typist can type five reports and a junior typist can type three reports per day. But the management needs to complete at least 30 reports in a day. If S and J denote the number of senior and junior typists assigned for the work, which of the following inequality represents the constraint? **(June 2024)**

(a) $5S + 3J \leq 30$ (b) $3S + 5J < 30$ (c) $5S + 3J \geq 30$ (d) $35 + 5J > 30$

11. Given the constraints, $x \leq 3$, $y \leq 4$ and $4x + 3y \leq 12$, the point _____ is in the feasible region. **(June 2024)**

(a) (3, 4) (b) (2, 4) (c) (2, 2) (d) (1, 1)

12. A dietician recommends mixture of two kinds of foods to a person so that mixture contains at least 45 units of carbs, 25 units of protein, 15 units of fat and 15 units of fibre. The above contents of nutrients are available in the foods as below:

	Carbs	Protein	Fat	Fibre
Food-1	20	5	3	2
Food-2	10	2	4	5

If 'x' units of food-1 is mixed with 'y' units of food-2, how dietician recommendation can be expressed? **(Sept 2024)**

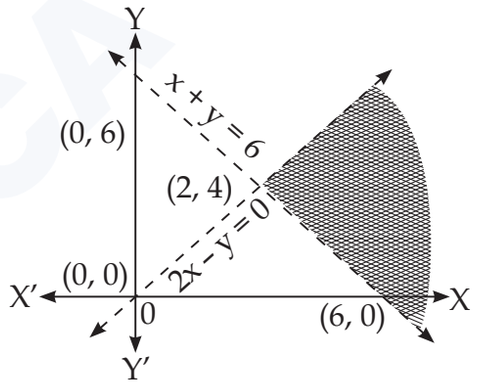
(a) $20x + 10y \leq 45; 5x + 2y \geq 25; 3x + 4y \leq 15; 2x + 5y \geq 15; x \geq 0; y \geq 0$

(b) $20x + 10y \leq 25; 5x + 2y \geq 45; 3x + 4y \leq 15; 2x + 5y \geq 15; x \geq 0; y \geq 0$

(c) $20x + 10y \geq 45; 5x + 2y \geq 25; 3x + 4y \geq 15; 2x + 5y \geq 15; x \geq 0; y \geq 0$

(d) $20x + 10y \leq 45; 5x + 2y \leq 25; 3x + 4y \leq 15; 2x + 5y \leq 15; x \geq 0; y \geq 0$

13. The shaded area is represented by which of the following option? **(Sept 2024)**



(a) $x + y > 6; 2x - y > 0; x > 0$ (b) $x + y < 6; 2x - y > 0; x < 0$

(c) $x + y > 6; 2x - y < 0; x > 0$ (d) $x + y > 6; 2x - y > 0; x < 0$

14. A company produce two type of product A & B which require processing in two machines. First machine can be used up to 15 hrs. and second can be used at most 12 hrs. in a day. The product A requires 2 hrs. on machine 1 & 3 hrs. on machine 2. The product B requires 3 hrs. on machine 1 & 1 hour on machine 2. This can be expressed as: **(Jan 2025)**

(a) $2x_1 + 3x_2 \leq 15, 3x_1 + x_2 \leq 15$ (b) $2x_1 + 3x_2 \leq 15, 3x_1 + x_2 \leq 12$

(c) $3x_1 + 2x_2 \leq 15, 2x_1 + x_2 \leq 12$ (d) $2x_1 + 3x_2 \leq 12, 3x_1 + x_2 \leq 15$

15. A manufacturer produces two products A and B. The profit on product A is ₹8 on each unit, and the profit on product B is ₹13 on each unit. Then the objective function is: **(Jan 2025)**

(a) Maximize $Z = 8x_1 + 13x_2$ (b) Minimize $Z = 8x_1 + 13x_2$

(c) Minimize $Z = 13x_1 + 8x_2$ (d) Maximize $Z = 13x_1 + 8x_2$

16. On solving the inequalities $6x + y \geq 18$, $x + 4y \geq 12$, $2x + y \geq 10$; which of the following are correct solutions? **(May 2025)**

(a) (0, 18), (12, 0), (4, 2) and (2, 6) (b) (3, 0), (0, 3), (4, 2) and (7, 6)

(c) (5, 0), (0, 10), (2, 4) and (2, 6) (d) (0, 18), (12, 0), (4, 2) and (0, 7)

17. The longest side of a triangle is 2 times the shortest side and the third side is 4 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side? **(May 2025)**

(a) 7 cm (b) 9 cm (c) 11 cm (d) 13 cm

18. Which of the following is a solution of the inequality $\frac{5x}{3} \leq \frac{x}{6} - 5$? **(Sept 2025)**

(a) $\left(-\infty, -\frac{10}{3}\right]$ (b) $\left(-\infty, -\frac{10}{3}\right)$ (c) $\left(-\infty, -\frac{8}{3}\right]$ (d) $\left(-\infty, -\frac{8}{3}\right)$

19. The number of solutions of $\frac{5-2x}{4} < \frac{x}{8} - 5 > 3 - 2x$ are _____, where x is a real number.

(Sept 2025)

(a) Infinitely many (b) Only two (c) Exactly one (d) No solution

20. The region specified by the inequalities $10x + 29y \geq 40$ and $15x - 4y \leq 25$ includes the point _____ . **(Sept 2025)**

(a) (1, 1.25) (b) (3, 2.5) (c) (2.5, 2.5) (d) (4, 1.25)

Answer Key

1. (a)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (c)
11. (d)	**12.** (c)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (a)

SUMMARY

- The chapter on inequalities explores mathematical relationships between expressions or numbers that are not equal. Inequalities are represented using symbols such as " < " (less than), " >" (greater than), "≤" (less than or equal to), and "≥" (greater than or equal to).
- **Inequality Symbols:** Inequalities are expressed using symbols such as " < ", " > ", "≤" and "≥" to compare two quantities or expressions.

- **Solving Linear Inequalities:** Solving linear inequalities involves finding the range of values for two variables that satisfy the given inequality. This often includes algebraic manipulation, such as adding, subtracting, multiplying, or dividing both sides of the inequality by constants.
- **Graphical Representation:** One of the fundamental aspects of this chapter is the graphical representation of linear inequalities. These inequalities can be visualized on a coordinate plane as shaded regions. The region below or above a line (depending on the inequality symbol) represents the set of points that satisfy the inequality.
- **Word Problems:** Inequality concepts are applied to real-world situations in word problems. These problems require translating verbal descriptions into mathematical inequalities and solving for the unknown quantities.
- Overall, understanding linear inequalities and their graphical representation is crucial in mathematics and real-life applications. It helps describe relationships where quantities are not necessarily equal but can fall within a range of values. Solving linear inequalities is a fundamental skill in algebra and has numerous applications in fields such as economics, science, and engineering.

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CA WALLAH

CHAPTER

04 Mathematics of Finance

WHAT IS FINANCE ?

Basically Finance is the management of money which involves investing, borrowing , lending, saving and forecasting. In this chapter we will try to understand the basic mathematical rules related to finance

1. PERSONAL FINANCE

- How individuals manage their income, savings, expenses, investments, insurance, loans, retirement planning, etc.
- Example: Making a budget, investing in SIPs, taking loans, or planning EMIs.

2. CORPORATE FINANCE

- How companies raise capital, invest it, and manage profits or losses.
- Involves decisions like:
  - Which project is profitable?
  - Is it better to raise funds through debt or equity?
  - Should profits be distributed as dividends or reinvested in the business?
- How the government manages revenue and expenditure?
- Taxes, subsidies, fiscal deficit, and budgeting are all part of public finance.

3. PUBLIC FINANCE

- How government manages revenue and expenditure.
- Taxes, subsidies, fiscal deficit, budgeting - all are part of public finance.

WHAT IS TIME VALUE OF MONEY AND WHY IT IS SO IMPORTANT?

Time Value of Money (TVM) is an important concept that money in hand today is worth more than a money promised in the future. Basically, it is a fundamental financial concept that recognizes the worth of money over time. It emphasizes the principle that a sum of money available today is worth more than the same amount of money in the future. This is primarily because money has the potential to earn returns or interest when invested or utilized immediately. Basically Value of money decreases over time due to plenty of reasons. This concept is directly related to purchasing power of customer , as value of money decreases, customers have to pay more maintain same level of consumption of goods and services

Example - 10 Rs in 1995 - 10 samosa

10 Rs in 2024 - 1 Samosa

(Purchasing power of customer is decreased)

CA WALLAH

WHY IS INTEREST PAID?

Interest compensates for the time value of money, recognizing that receiving money earlier is generally more beneficial than receiving it later. By paying interest, the borrower compensates the lender for the temporary use of their funds.

- **Opportunity Cost:** When individuals or organizations lend money, they forgo potential alternative uses of those funds. The interest paid serves as compensation for the opportunity cost of not investing or using the money elsewhere.
- **Inflation:** Inflation erodes the purchasing power of money over time. Lenders charge interest to account for the expected decrease in the value of money due to inflation. This ensures that the lender's real purchasing power is maintained or increased.
- **Liquidity Preference:** Lenders may require interest as compensation for the lack of immediate access to their funds. By lending money, they forgo the ability to use that money for their own immediate needs or expenses.
- **Risk Factor:** Interest also accounts for the risk associated with lending money. Lenders assess the creditworthiness of borrowers and charge interest rates based on the perceived risk of default. Higher-risk borrowers are charged higher interest rates to compensate for the increased probability of non-repayment.

  These factors collectively determine the interest rate charged on loans, bonds, or other financial instruments, allowing lenders to earn a return on their capital while borrowers gain access to funds for various purposes.

DEFINITIONS

Interest: Fee paid for the use of another party's money. To the borrower it is the cost of renting money, to the lender the income from lending it.

E.g.: The principal amount invested by Anita is ₹10,000. After 1 year, she received the accumulated amount or balance, including both the principal and the interest, as ₹10,500.

Thus, the difference between ₹10,500 and ₹10,000 which is ₹500 is the interest she earned in a year.

Principal: The total amount of money borrowed (or invested), not including any interest.

E.g.: Sarah wants to buy a new laptop and decides to take a loan of ₹30,000 from a finance company. The ₹30,000 is the principal amount she borrows, not including any interest.

Rate of Interest: The interest rate (rate of interest) is the rate at which interest is charged on the principal amount which is landed or borrowed. Rate of interest is usually expressed as percentages.

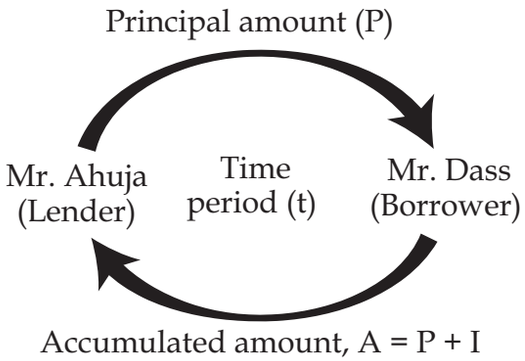
E.g.: Mark wants to invest his savings in a fixed deposit with a bank. The bank offers an annual interest rate of 6% on fixed deposits. The 6% is the rate of interest at which Mark's money will earn interest over time.

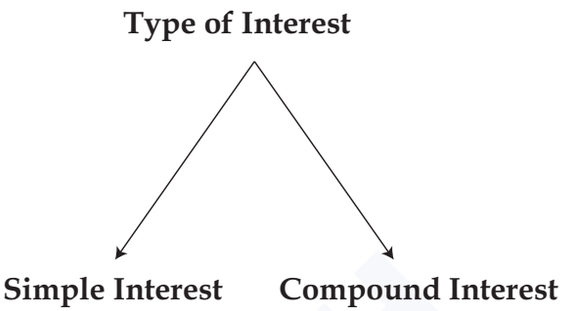
Time Period: The time period refers to the duration for which the principal amount is borrowed or invested, and during which interest is charged or earned. It is the length of time over which the financial transaction takes place.

E.g.: If John borrows ₹50,000 from a bank at an annual interest rate of 8% for a time period of 2 years, then the time period in this case is 2 years.

Accumulated amount (or Balance): The amount which you receive after the due period including the principal and the interest in called as accumulated amount.

E.g.: Ankita invested ₹20,000 in a savings account that earns an annual interest rate of 4%. After three years, she checked her account balance and found it to be ₹22,408. The ₹22,408 is the accumulated amount or balance, including the principal and the interest earned over the three-year period.





SIMPLE INTEREST

- Simple interest is the way when we take interest at a simple rate on the principal amount over a period of time.
- Simple interest increases (or decreases) if Principal amount increases (or decreases) or if interest rate increases (or decreases) or if the time period increases (or decreases).
- It means Simple interest(I) is directly proportional to Principal Amount (P), Interest rate (R%), Time period (T), Therefore,

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

Example 1. S.I. on ₹3,500 for 3 years at 12% per annum is **(ICAI)**

(a) ₹1,200 (b) ₹1,260 (c) ₹2,260 (d) None of these

Sol.(b) Given, Principal (P) = ₹3,500

Time (T) = 3 years

Rate (R) = 12% p.a.

We know that,

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$\text{S.I.} = \frac{3500 \times 12 \times 3}{100} = 1260$$

Therefore, the simple interest on ₹3,500 for 3 years at 12% per annum is ₹1,260.

Calculator trick: I = P × T × R% = 3500 × 3 × 12% = 1,260

Hence, the correct option is (b).

Example 2. Sania deposited ₹50,000 in a bank for two years with the interest rate of 5.5% p.a. How much interest would she earn?

(a) ₹2,750 (b) ₹5,500 (c) ₹55,500 (d) None of these

Sol.(b) Given, Amount deposited in the bank (P) = ₹50,000

Rate of interest (R) = 5.5% p.a.

Time (T) = 2 years

We know that,

$$S.I. = \frac{P \times R \times T}{100}$$

$$= \frac{50,000 \times 5.5 \times 2}{100} = 500 \times 5.5 \times 2 = 5,500$$

Calculator trick: I = 50,000 × 2 × 5.5% = 5500

Therefore, she earned ₹5,500 as interest.

Hence, the correct option is (b).

Example 3. How much interest will be earned on ₹500 at 10% simple interest for 5 years and what will be the accumulated amount?

(a) ₹125, ₹625 (b) ₹250, ₹500

(c) ₹250, ₹750 (d) None of these

Sol.(c) Given, Principal (P) = ₹500, Rate of interest (R) = 10% p.a. and Time (T) = 5 years

We know that,

$$S.I. = \frac{P \times R \times T}{100}$$

$$= \frac{500 \times 10 \times 5}{100} = ₹250$$

Thus, the interest earned is ₹250.

Now, accumulated amount (A) = P + I

A = 500 + 250

A = ₹750

Therefore, the interest earned is ₹250 and the accumulated amount is ₹750.

Calculator trick: I = 500 × 5 × 10% = 250

A = P + I = 500 + 250 = 750

Hence, the correct option is (c).

Example 4. If P = ₹5,000, T = 1 year, I = ₹300, R will be **(ICAI)**

(a) 5% (b) 4% (c) 6% (d) None of these

Sol.(c) Given, P = ₹5,000, T = 1 year, I = ₹300

We know that,

$$\Rightarrow S.I. = \frac{P \times R \times T}{100}$$

$$\Rightarrow 300 = \frac{5,000 \times R \times 1}{100}$$

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$\Rightarrow R = \frac{300}{50}$

$\Rightarrow R = 6\%$

Therefore, the rate of interest is 6%

Hence, the correct option is (c).

Example 5. P = ₹12,000, A = ₹16,500, T = $2\frac{1}{2}$ years. Rate percent per annum simple interest will be **(ICAI)**

(a) 15% (b) 12% (c) 10% (d) None of these

Sol.(a) Given that:

P = ₹12,000, A = ₹16,500 and T = $2\frac{1}{2} = \frac{5}{2}$ years

We know that,

$I = A - P$ and $I = \frac{P \times R \times T}{100}$

where,

A = Accumulated amount

P = Principal

T = Time in years

R = Rate of interest per annum

Thus, I = 16,500 – 12,000 = ₹4,500

We have, $I = \frac{P \times R \times T}{100}$

$\Rightarrow 4500 = \frac{12000 \times R \times 5}{2 \times 100}$

$\Rightarrow 4500 = 60 \times R \times 5$

$\Rightarrow R = \frac{4500}{5 \times 60} = 15\%$

Therefore, the rate percent per annum is 15%.

'OR'

We know,

$A = P\left(1 + \frac{RT}{100}\right)$

$16500 = 12000\left(1 + \frac{R(2.5)}{(100)}\right)$

R = 15%

Calculator trick: Steps to follow:

1. 16500 ÷ 12000
2. –1
3. ×100
4. ÷2.5 which gives the required result i.e., 15%.

Hence, the correct option is (a).

Example 6. Find the rate of interest if the amount owned after 3 months is ₹525, borrowed amount being ₹500.

(a) 25% (b) 20% (c) 30% (d) None of these

Sol.(b) Given, Amount (A) = ₹525

Principal (P) = ₹500

Time (T) = 3 months = $\frac{3}{12} = \frac{1}{4}$ years

We know that,

Amount (A) = Principal (P) + Interest (I)

$$A = P + \frac{P \times R \times T}{100}$$

$$\Rightarrow 525 = 500 + \frac{500 \times R \times 1}{4 \times 100}$$

$$\Rightarrow 525 - 500 = \frac{5 \times R}{4}$$

$$\Rightarrow 25 = \frac{5R}{4}$$

$$\Rightarrow 5R = 100$$

$$\Rightarrow R = 20\%$$

Therefore, the required rate of interest is 20%.

Hence, the correct option is (b).

Example 7. A sum of money amounts to ₹6,200 in 2 years and ₹7,400 in 3 years. The principal and rate of interest are **(ICAI)**

(a) ₹3,800, 31.58% (b) ₹3,000, 20%

(c) ₹3,500, 15% (d) None of these

Sol.(a) **Detailed:**

Given, A_1 = ₹6200 and t_1 = 2 years

A_2 = ₹7400 and t_2 = 3 years

We know that,

A = P(1 + rt)

$\Rightarrow A_1 = P(1 + rt_1)$

$\Rightarrow 6200 = P(1 + 2r)$ ...(i)

Also $A_2 = P(1 + rt_2)$

$\Rightarrow 7400 = P(1 + 3r)$ ...(ii)

Now divide equation (i) by (ii), we get

$$\Rightarrow \frac{6200}{7400} = \frac{P(1+2r)}{P(1+3r)}$$

$$\Rightarrow \frac{31}{37} = \frac{1+2r}{1+3r}$$

By cross multiplication, we get

$\Rightarrow 31 \times (1 + 3r) = 37 \times (1 + 2r)$

$\Rightarrow 31 + 93r = 37 + 74r$

$\Rightarrow 93r - 74r = 37 - 31$

$\Rightarrow 19r = 6$

$$\Rightarrow r = \frac{6}{19} = 0.315789 \times 100\% = 31.58\%$$

Trick:

when t_1, t_2 & A_1, A_2 are given

$$\text{then } r = \left[\frac{A_2 - A_1}{A_1 t_2 - A_2 t_1}\right] \times 100$$

$$r = \frac{7400 - 6200}{6200(3) - 74(2)} \times 100$$

$r = 31.58\%$

Now, put $r = \frac{6}{19}$ in equation (i) we get

$$\Rightarrow 6200 = P\left(1 + 2 \times \frac{6}{19}\right)$$

$$\Rightarrow 6200 = P\left(1 + \frac{12}{19}\right)$$

$$\Rightarrow 6200 = P \times \frac{31}{19}$$

$$\Rightarrow P = \frac{6200 \times 19}{31}$$

$\Rightarrow P = ₹3,800$

Therefore, the principal and rate of interest are ₹3800 and 31.57% respectively.

Alternate method:

If simple interest for 1 year be S.I then

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Amount in 2 years = A_2 =P + 2 (S.I)

Amount in 3 years = A_3 =P + 3 (S.I)

Thus $A_3 - A_2$ = S.I

S.I. = 7400 – 6200 = 1200

$\Rightarrow \frac{(P \times R \times 1)}{100} = 1200$

$\Rightarrow$ PR = 120,000

Go by choices:

Option (a): ₹3,800, 31.58%

PR = 3800 × 31.58 = 120004 ≈ 120000

Option (b): ₹3,000, 20%

PR = 3000 × 20 = 60000 ≠ 120000

Option (c): ₹3,500, 15%

PR = 3500 × 15 = 52500 ≠ 120000

Hence, the correct option is (a).

Example 8. P = ₹8,500, A = ₹10,200, R = $12\frac{1}{2}$ % SI, T will be **(ICAI)**

(a) 1 year 7 months (b) 2 years

(c) $1\frac{1}{2}$ years (d) None of these

Sol.(a) Given that:

P = ₹8,500, A = ₹10,200 and R = $12\frac{1}{2}$ % = $\frac{25}{2}$ % years

We know that,

I = A – P and I = $\frac{P \times R \times T}{100}$

Thus, I = 10,200 – 8,500

$\Rightarrow$ I = ₹1,700

Now, I = $\frac{P \times R \times T}{100}$

$\Rightarrow 1700 = \frac{8500 \times 25 \times T}{2 \times 100}$

$\Rightarrow 1700 = \frac{85 \times 25 \times T}{2}$

$\Rightarrow$ 1700 × 2 = 85 × 25 × T

$\Rightarrow T = \frac{1700 \times 2}{85 \times 25} = 1\frac{3}{5}$ years

Therefore, the time period is $1\frac{3}{5}$ years = 1 year 7 months (approx).

Hence, the correct option is (a).

Example 9. A sum of money doubles itself in 10 years. The number of years it would triple itself is **(ICAI)**

(a) 25 years (b) 15 years (c) 20 years (d) None of these

Sol. (c) Given, T = 10 years

Let the Principal be P.

Now, according to the question

Accumulated amount = 2 × Principal

i.e., A = 2P

We know that,

I = A – P, where I = Interest

$\Rightarrow$ I = 2P – P

$\Rightarrow$ I = P

Also, $I = \frac{P \times R \times T}{100}$

$\Rightarrow P = \frac{P \times R \times 10}{100}$

$\Rightarrow$ R = 10%

Now, when A = 3P, then

I = A – P = 3P – P

$\Rightarrow$ I = 2P

$\therefore I = \frac{P \times R \times T}{100}$

$\Rightarrow 2P = \frac{P \times 10 \times T}{100}$

$\Rightarrow$ T = 20 years

Therefore, the sum of money will triple in 20 years.

Hence, the correct option is (c).

Trick:

In Simple interest

For double amount	For triple amount
$r = \frac{1}{t}$ and $t = \frac{1}{r}$	$r = \frac{2}{t}$ and $t = \frac{2}{r}$

where r is given in decimals

CA WALLAH

Sum doubles in 10 years	for triple amount
$r = \frac{1}{10}$ r = 0.01 or 1%	$t = \frac{2}{r}$ $= \frac{2}{0.01}$ = 20 year

Example 10. Rohit invested ₹50,000 in a bank at the rate of 5.5% p.a. simple interest rate. He received ₹69,250 after the end of term. Find out the period for which the sum was invested by Rohit.

(a) 7 years (b) 5 years (c) 10 years (d) none of these

Sol.(a) Given, Money invested (P) = ₹50,000

Amount received (A) = ₹69,250

Rate of interest = 5.5%

We know that,

Amount (A) = Principal (P) + Interest (I)

$$A = P + \frac{P \times R \times T}{100}$$

$$\Rightarrow 69,250 = 50,000 + \frac{50,000 \times 5.5 \times T}{100}$$

$$\Rightarrow 69,250 = 50,000\left(1 + \frac{55 \times T}{1000}\right)$$

$$\Rightarrow \frac{69,250}{50,000} = 1 + \frac{55T}{1000}$$

$$\Rightarrow 1.385 - 1 = \frac{55T}{1000}$$

$$\Rightarrow 0.385 = \frac{55T}{1000}$$

$$\Rightarrow T = \frac{385}{55} \Rightarrow T = 7$$

Therefore, Rohit made the investment for 7 years.

Hence, the correct option is (a).

Example 11. Kiran took some amount from Suraj for $6\frac{1}{2}$ years at the rate of 4% p.a. simple interest and Kiran after that time has to pay ₹31,500 at the end of the term. What amount did Kiran took from Suraj?

(a) ₹25,240.40 (b) ₹30,000 (c) ₹25,000 (d) ₹21,550

Sol.(c) Given, Amount to be paid (A) = ₹31,500, Rate of interest = 4% and Time period (T)

$= 6\frac{1}{2} = 6.5$ years

We know that,

Amount (A) = Principal (P) + Interest (I)

$A = P + \frac{P \times R \times T}{100}$

$\Rightarrow A = P\left(1 + \frac{R \times T}{100}\right)$

$\Rightarrow 31{,}500 = P\left(1 + \frac{R \times T}{100}\right)$

$\Rightarrow 31{,}500 = P\left(1 + \frac{4 \times 6.5}{100}\right)$

$\Rightarrow 31{,}500 = P\,(1 + 0.26)$

$\Rightarrow 31{,}500 = P(1.26)$

$\Rightarrow P = \frac{31{,}500}{1.26} \Rightarrow 25{,}000$

Therefore, Kiran borrowed ₹25,000 from Suraj.

Hence, the correct option is (c).

Example 12. A farmer borrowed ₹3600 at the rate of 15% simple interest per annum. At the end of 4 years, he cleared this account by paying ₹4000 and a cow. The cost of the cow is **(Dec 2022)**

(a) ₹1,000 (b) ₹1,200 (c) ₹1550 (d) ₹1,760

Sol.(d) Given, P = ₹3600, R = 15% and T = 4 years

We know that,

$\text{S.I.} = \frac{P \times R \times T}{100}$

$\Rightarrow \text{S.I.} = \frac{3600 \times 15 \times 4}{100}$

$\Rightarrow$ S.I. = 2160

Thus, A = P + S.I. = 3600 + 2160 = ₹5760

Since, the farmer cleared the account by paying ₹4000 and a cow.

Thus, the cost of cow = ₹5760- ₹4000= ₹1760

Therefore, the cost of a cow is ₹1760.

Hence, the correct option is (d)

PRACTICE QUESTIONS (PART A)

1. If P = ₹3,600, A = ₹6,400, then Simple interest i.e. I will be

(a) ₹1,800 (b) ₹2800 (c) ₹1600 (d) ₹9000

2. P = ₹12,000; R = 5%; T = 5 years using $I=\frac{PTR}{100}$ then I will be

(a) ₹2,000 (b) ₹3,000

(c) ₹3,700 (d) None of these

3. ₹80,000 is invested to earn a monthly interest of ₹1200 at the rate of _____ p.a. simple interest.

(a) 12% (b) 14% (c) 16% (d) 18%

4. Find the rate of interest if the amount owed after 6 months is ₹1050 borrowed amount being ₹1000.

(a) 9% (b) 10% (c) 11% (d) 12%

5. ₹8,829 is invested into three different sectors in such a way that their amounts at 4% p.a. S.I. after 5 years; 6 and 8 years are equal. Find each part of the sum.

(a) 3069, 2970, 2790 (b)3089, 2970, 2790

(c) 3609, 2970, 2790 (d)3069, 2960, 2760

6. Raju borrowed ₹10000 from Gopal at the interest rate of 7% for 3 years. How much interest Raju needs to pay in 3 years and how much in total will Gopal receive?

(a) ₹1,700; ₹11,700 (b) ₹2,000; ₹12,000

(c) ₹2,100; ₹12,100 (d) ₹2,300; ₹12,300

7. A certain sum of money was put at S.I. for 2.5 years at a certain rate of S.I. p.a. Had it been put at 4% higher rate, it would have fetched ₹500 more. Find the sum of money.

(a) ₹4000 (b) ₹5000

(c) ₹6000 (d) None of these

8. ₹850 becomes ₹1250 in 5 years at a simple rate of interest. How much would be its amount after 4 years if rate of interest is increased by 5% ?

(a) ₹1230 (b) ₹1320 (c) ₹1340 (d) None of these

9. A certain sum of money becomes six times at 5% S.I. p.a. At what rate % it will become 12 times.

(a) 15% (b) 13% (c) 11% (d) None of these

10. How long will it take for a sum to quadruple itself at 10% per year simple interest?

(a) 13 years (b) 15 years (c) 25 years (d) 30 years

11. A person invests ₹1000 in a savings bank paying 6% simple interest. What is the balance or amount of the savings account after 6 months?

(a) ₹1030 (b) ₹1060 (c) ₹30 (d) None of these

Answer Key

1. (b) **2.** (b) **3.** (d) **4.** (b) **5.** (d) **6.** (c) **7.** (b) **8.** (c) **9.** (c) **10.** (d)
11. (a)

COMPOUND INTEREST

The interest calculated on the initial principal, which also includes all of the accumulated interest of previous periods of a deposit or loan.

SYSTEM USED BY BANKS

For example: Suppose you deposit ₹1,000,000 in CITI bank for 3 years at 5% p.a. compounded annually. To calculate the Interest, the concept of compound interest is used.

Now, what will be the formula for it?

$$A = P\left(1+\frac{r}{m}\right)^{t\times m}$$

$$C.I. = P\left(1+\frac{r}{m}\right)^{t\times m} - P$$

Where P = Principal

t = time period

A = Accumulated amount

m = conversion period

CONVERSION PERIOD

The time period according to which it is going to be compounded.

E.g.: we can take a compounding period–annually, semi-annually, quarterly, monthly or daily.

TYPICAL CONVERSION PERIOD

Conversion Period	Description	Number of Conversion period in a year (m)
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded Semi Annually	2
12 months	Compounded Annually	1

Example 13. Raju took a loan of ₹10,000 from bank at the compound interest rate of 7% annually for 3 years. How much Raju needs to pay bank now?

(a) ₹12,250.43 (b) ₹22,250.43 (c) ₹2,250.43 (d) None of these

Sol.(a) **Method 1:**

Given, Loan amount (P) = ₹10,000

Time period (t) = 3 years

Rate of interest (r) = 7% p.a.

Since, the interest is compounded annually, thus m = 1

Therefore, $A = P\left(1+\frac{r}{m}\right)^{t\times m}$

$= 10000\left(1+\frac{7}{100}\right)^{3}$

$= 10000(1.07)^3 = 12{,}250.43$

Calculation trick: x^n

(i) Type "x" on screen of calculator

(ii) Press "×"

(iii) Press "=" (n-1) times

Now, to find 1.07^3

(i) Type 1.07 on screen of calculator

(ii) Press "×"

(iii) Press "=" 2 times

Therefore, Raju needs to pay ₹12,250.43 to the bank.

Method 2:

A = 10000 + 7% + 7% + 7% = 12250.43

Hence, the correct option is (a).

Example 14. If P = ₹1,000, R = 5% p.a, n = 4; What is Amount and C.I.? **(ICAI)**

(a) ₹1,215.50, ₹215.50 (b) ₹1,125, ₹125

(c) ₹2,115, ₹115 (d) None of these

Sol. (a) Given that :

P = ₹1,000, R = 5% p.a. n (time period) = 4

We know that,

$A = P\left(1+\frac{R}{100}\right)^{n}$

$\Rightarrow A = 1000\left(1+\frac{5}{100}\right)^{4}$

$\Rightarrow A = 1000\ (1 + 0.05)^4$

$\Rightarrow A = 1000 \times 1.21550625$

$\Rightarrow A =$ ₹1215.50 (approx.)

Now, C.I = A – P

$\Rightarrow$ C.I = 1215.50 – 1000

$\Rightarrow$ C.I = ₹215.50

Calculation trick:

1.05 × = = =

Then × 1000 which gives 1215.50 as amount

Subtract 1000 from it to get the value of C.I.

Therefore, the amount and C.I are ₹1215.50 and ₹215.50 respectively.

Hence, the correct option is (a).

Example 15. ₹100 will become after 20 years at 5%, p.a compound interest of **(ICAI)**

(a) ₹250 (b) ₹205 (c) ₹165.33 (d) None of these

Sol.(c) It is given that P = ₹100, r = 5%, t = 20 and m = 1

We know that,

$$\text{C.I.} = P\left(1+\frac{r}{m}\right)^{t\times m} - P$$

$$\Rightarrow \text{C.I} = 100\left(1+\frac{5}{100}\right)^{20} - 100$$

$\Rightarrow$ C.I = 100 × 2.65329 – 100

$\Rightarrow$ C.I = ₹165.33

Therefore, the required sum of money is ₹165.33

Hence, the correct option is (c).

Example 16. ₹5,000 is invested at annual rate of interest of 5%. What is the amount after two years if compounding is done]

(I) Annually

(II) Semi-annually

(III) Quarterly

(IV) monthly

Sol. Given: Amount invested (P) = ₹5,000

Rate of interest (r) = 5%

Time (t) = 2 years

We know that,

$$\text{Amount, } A = P\left(1+\frac{r}{m}\right)^{tm}$$

(I) When the compounding is annually i.e., m = 1

$$\text{Thus, Amount (A)} = P\left(1+\frac{r}{100}\right)^{t}$$

$$A = 5000\left(1+\frac{5}{100}\right)^{2}$$

$= 5000\ (1.05)^2 = ₹5,512.5$

(II) When the compounding is semi-annually i.e., m = 2

Thus, Amount $A = P\left(1+\frac{r}{m}\right)^{t\times m}$

$A = 5000\left(1+\frac{5}{200}\right)^{4}$

$= 5000\ (1.025)^4 = ₹5,519.064$

(III) When the compounding is done quarterly i.e., m = 4

Thus, Amount (A) $= P\left(1+\frac{r}{4\times 100}\right)^{4t}$

$A = 5000\left(1+\frac{5}{400}\right)^{8} = 5000\ (1.0125)^8 = ₹5,522.43$

(IV) When the compounding is done monthly i.e., m = 12

Thus, Amount (A) $= P\left(1+\frac{r}{12\times 100}\right)^{12t}$

$A = 5000\left(1+\frac{5}{1200}\right)^{24}$

$= 5000\ (1.00141666) = ₹5,524.706$

FORMULAS FOR COMPOUND INTEREST

- $A_n = P(1 + i)^n$
- Compound Interest (C.I.) $= P[(1 + i)^n - 1]$

where, A_n = Amount accumulated after nth period

i = Annual rate of interest

P = Principal amount

n is total conversion i.e. time period * total no. of conversions

Example 17. The C.I on ₹16000 for $1\frac{1}{2}$ years at 10% p.a. payable half–yearly is **(ICAI)**

(a) ₹2,222 (b) ₹2,522 (c) ₹2,500 (d) None of these

Sol. (b) Given, Principal (P) = ₹16000

Time (t) $= 1\frac{1}{2} = \frac{3}{2}$ years

Rate of interest = 10%

m = 2 (interest is payable half yearly)

Compound interest is given as,

$C.I = P\left(1+\frac{r}{m}\right)^{t\times m} - P$

$= 16000\left(1+\frac{10}{2\times100}\right)^{2\times\frac{2}{3}} - 16000$

$= 16000\ (1 + 0.06)^3 - 16000$

$= (16000 \times 1.157625) - 16000$

$= ₹18522 - ₹16000 = ₹2522$

Hence, the correct option is (b).

Example 18. In what time will ₹4,000 amount to ₹4,410 at 10% per annum interest compounded half-yearly?

(a) 1 year (b) 2 years

(c) 2.5 years (d) Cannot be determined

Sol.(a) Given, P = ₹4000

A = ₹4,410

r = 10%

m = 2

Thus, Amount (A) = $P\left(1+\frac{r}{2\times100}\right)^{2t}$

$\Rightarrow 4410 = 4000\left(1+\frac{10}{200}\right)^{2t}$

$\Rightarrow \frac{4410}{4000} = (1.05)^{2t}$

$\Rightarrow 1.1025 = (1.05)^{2t}$

$\Rightarrow (1.05)^2 = (1.05)^{2t}$

On comparing, we get

2t = 2

$\Rightarrow$ t = 1 year

Hence, the correct option is (a).

Example 19. The difference between the S.I and the C.I on ₹2,400 for 2 years at 5% p.a. is

(a) ₹5 (b) ₹10 (c) ₹16 (d) ₹6 **(ICAI)**

Sol.(d) Given, Principal (P) = ₹2,400

Time (t) = 2 years

Rate of interest (R) = 5% p.a

Compound Interest (C.I) = $P\left(1+\frac{R}{100}\right)^t - P$

Simple Interest (S.I.) = $\frac{P\times R\times t}{100}$

According to the question,

C.I – S.I

$$= \left[P\left(1+\frac{R}{100}\right)^{t} - P\right] - \left(\frac{P \times R \times t}{100}\right)$$

$$= \left[2400\left(1+\frac{5}{100}\right)^{2} - 2400\right] - \left(\frac{2400 \times 5 \times 2}{100}\right) = [2400\ (1 + 0.05)^2 - 2400] - 240$$

$= [2400\ (1.1025) - 2400] - 240 = 246 - 240 = ₹6$

Therefore, the required difference is ₹6.

Hence, the correct option is (d).

Trick:

Difference b/w S.I. & C.I.

For two years	For Three years
$= Pr^2$	$= Pr^2\ (r + 3)$

Therefore, the difference between S.I. and C.I. for 2 years is Pr^2 i.e,.

Difference = $2400\ (0.05)^2$

= 6

Example 20. The difference between C.I. and S.I. on a certain sum of money invested for 3 years at 6% p.a is ₹110.16. The Principal is **(ICAI)**

(a) ₹3,000 (b) ₹3,700 (c) ₹12,000 (d) ₹10,000

Sol.(d) Given, Time (T) = 3 years

Rate (R) = 6% p.a

Difference between C.I and S.I. = ₹110.16

m = 3

We know that,

Compound interest is given as,

$$C.I = P\left(1+\frac{R}{100}\right)^{n} - P$$

Simple interest is given as, $S.I = \frac{P \times R \times T}{100}$

According to the question,

C.I. – S.I = ₹110.16

$$\Rightarrow P\left(\left(1+\frac{6}{100}\right)^{3} - 1\right) - \frac{P \times 6 \times 3}{100} = ₹110.16$$

$\Rightarrow 0.191\ P - 0.18\ P = ₹110.16$

$\Rightarrow 0.011\ P = ₹110.16$

$\Rightarrow P = \frac{110.16}{0.011}$

$\Rightarrow P = ₹10,000$ (approx.)

Hence, the correct option is (d).

Trick:

Difference = $Pr^2 (r + 3)$

$110.15 = P (0.06)^2 (0.06 + 3)$

$P = 10,000$

Other useful tricks of calculator

To find the nth root = $x^{(1/n)}$:

(i) Write the number.

(ii) Press √….. 12 times

(iii) -1

(iv) ÷ n

(v) + 1

(vi) × = …. 12 times

CALCULATOR TRICK TO FIND OUT LOGARITHM:

To find $\log_{10}$ m:

1. Type number (m)
2. Press root for 19 times.
3. Subtract 1
4. Multiply by 227695

We can get the required value of $\log_{10}$ m.

Another Calculator Trick to find out Logarithm:

1. Type number (m) and press root for 13 times.
2. Subtract 1
3. Multiply by 3558

To find Antilogarithm:

1. Divide the number (m) by 227695
2. Add 1 after step 1
3. Press "multiply & =" button for 19 times

PRACTICE QUESTIONS (PART B)

1. The time in which a sum of money will be double at 5% p.a. C.I is

(a) 10 years (b) 12 years (c) 14.2 years (d) None of these

2. If P = ₹8,000, R = 5% p.a., A = ₹8,820, then find the time period if the interest is compounded annually.

(a) 3.1 years (b) 3 years (c) 2.5 years (d) 2 years

3. Mr. X borrowed ₹5,120 at 12.5% p.a. C.I. At the end 3 years, the money was repaid along with the interest accrued. The amount of interest paid by him is.

(a) ₹2100 (b) ₹2170 (c) ₹2000 (d) None of these

4. If ₹64 amount to ₹83.20 in 2 years, what will ₹86 amount to in 4 years at the same rate percent per annum?

(a) 127.60 (b) 147.60 (c) 145.34 (d) 117.60

5. In how many years a sum of money triples at 5% pa. compound interest payable on a half-yearly basis?

(a) 15 years 7 months (b) 18 years 6 months

(c) 18 years 8 months (d) 22 years 3 months

6. Find the compound interest of ₹700 invested for 15 years at 8% compounded semiannually.

(a) ₹1570.37 (b) ₹2270.37 (c) ₹1800.75 (d) None of these

7. How long will it take for ₹5000 amount to ₹7000 if it is invested at 8% compounded quarterly?

(a) $4\frac{1}{4}$ years (b) $3\frac{1}{2}$ years (c) $5\frac{1}{4}$ years (d) None of these

8. At what rate per annum compounded annually will ₹20,000 amount to ₹23,254 in 2 years?

(a) 10% (b) 8% (c) 6.07% (d) 5.6%

9. The difference between the simple and compound interest on a certain sum for 2 years at 4% p.a. is ₹600. The sum is

(a) ₹3,50,000 (b) ₹3,75,000 (c) ₹3,27,500 (d) ₹2,97,550

10. A person opened an account on Jan 2019 with a deposit of ₹500. The account paid 5% interest compounded quarterly. In July 2019 he closed the account and added enough additional money to invest in a 6-months time deposit for ₹800, earning 5% compounded monthly.

(I) How much additional amount did the person invest on July 1?

(II) What was the maturity value of his time deposit on Jan 1, 2020?

(III) How much total interest was earned?

(a) ₹512.58, ₹820.21, ₹32.79 (b) ₹512.58, ₹820.21, ₹12.58

(c) ₹237.42, ₹820.21, ₹32.79 (d) ₹1037.42, ₹820.21, ₹32.79

11. The annual birth and death rates per 1,000 are 30 and 10 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is

(a) 35 years (b) 30 years (c) 25 years (d) none of these

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Answer Key

1. (c) **2.** (d) **3.** (b) **4.** (c) **5.** (d) **6.** (a) **7.** (a) **8.** (c) **9.** (b) **10.** (c)
11. (a)

EFFECTIVE RATE OF INTEREST

- If interest is compounded more than once a year, the effective interest rate for a year exceeds the per annum interest rate.
- For example: If we invest ₹1000 for a year at the compound interest rate of 5% quarterly. The effective rate of interest will be more when compounded in comparison to 5%.

  So, to find out effective interest rate:

  Effective Rate of Interest (E) is given as

$$E = \left(\left(1+\frac{r}{m}\right)^{m} - 1\right)$$

To convert this into percentage we can multiply it by 100

Example 21. The effective rate of interest corresponding to a nominal rate 4% p.a. payable half yearly is

(a) 4% (b) 4.04% (c) 4.45% (d) None of these

Sol.(b) Given, Rate of interest (R) = 4% p.a.

Interest is payable half yearly, i.e., m = 2

r = 4%

Effective Rate of Interest (E) is given as $\left(\left(1+\frac{r}{m}\right)^{m} - 1\right)$

$$\Rightarrow E = \left(\left(1+\frac{4}{200}\right)^{2} - 1\right) \times 100\%$$

$= ((1.02)^2 - 1) \times 100\% = (1.0404 - 1) \times 100\% = 0.0404 \times 100\% = 4.04\%$

Hence, the correct option is (b).

Example 22. Find the compound interest and effective rate of interest if an amount of ₹10,000 is deposited in a bank for one year at the rate of 4% per annum compounded quarterly.

(a) ₹406, 4% (b) ₹406, 4.06% (c) ₹400, 4.50% (d) None of these

Sol.(b) Given, P = ₹10,000

r = 4%, m = 4

t = 1 year

Thus, C.I. $= \left[P\left(1+\frac{r}{4\times 100}\right)^{4t} - 1\right]$

$= 10{,}000 \left[\left(1+\frac{4}{400}\right)^{4\times1} - 1\right] = 10{,}000\ [(1.01)^4 - 1]$

$= 10000(0.04060401) = 406$ (approx.)

Now, effective rate of interest can be calculated as:

$\text{E.R.I.} = \left[\left(1+\frac{4}{4\times100}\right)^{4\times1}\right]\times100 = [(1.01)^4 - 1] \times 100$

$= [1.04060401 - 1] \times 100 = 4.06\ \%$ (approx.)

Hence, the correct option is (b).

Example 23. Which is a better investment: 7% per year compounded monthly or 7.5% per year compound interest?

(a) 7% compound interest (b) 7.5% compound interest

(c) Both are same (d) Cannot be determined

Sol.(b) Given that,

Number of conversion period, m = 12

Nominal rate of interest, r = 7%

Therefore, effective rate of interest $E = \left[\left(1+\frac{r}{m}\right)^{m} - 1\right]$

$\Rightarrow E = \left[\left(1+\frac{7}{1200}\right)^{12} - 1\right]\times100$

$\Rightarrow E = [1.072290 - 1] \times 100$

$\Rightarrow E = 0.072290 \times 100$

$\Rightarrow E = 7.229\%$ (approx.)

As we know, $7.5\% > 7.229\%$

Thus, 7.5% per year compound interest is a better investment than 7% per year compounded monthly.

Hence, option (b) is the correct answer.

Example 24. A machine is depreciated at the rate of 10% on reducing balance. The original cost of the machine was ₹50,000 and its ultimate scrap value was ₹15,000. The effective life of the machine is

(a) 10 years (b) 11.4 years (c) 12.5 years (d) None

Sol.(b) Given, Depreciation Rate = 10%

Principal (Initial Value) = ₹50,000

Amount (Net Value) = ₹15,000

$\therefore i = \frac{\text{Rate of Depreciation}}{100} = \frac{10}{100} = 0.10$

We know that, Net value = Initial value × $(1 - i)^n$

$\Rightarrow 15000 = 50000(1 - 0.1)^n$

$\Rightarrow (0.3) = (0.9)^n$

Taking log on both sides, we get

$\log 0.3 = \log (0.9)^n$

$\Rightarrow \log 0.3 = n \log 0.9$ [$\because \log am = m \log a$]

$\Rightarrow n = \dfrac{\log 0.3}{\log 0.9} \Rightarrow n = 11.4$ years (appx.)

Hence, the correct answer is option (b).

PRACTICE QUESTIONS (PART C)

1. The effective rate of interest corresponding to a nominal rate 3% p.a. payable half yearly is

(a) 3.2% p.a. (b) 3.25% p.a. (c) 3.0225% p.a. (d) None of these

2. The effective annual rate of interest corresponding to a nominal rate of 6.5% p.a. compounded quarterly is

(a) 6.0% (b) 6.25% (c) 6.50% (d) 6.66%

3. An investment earns 5% interest per annum, compounded half yearly. What is the effective annual rate of interest?

(a) 5.0650% (b) 5.0925% (c) 5.0625% (d) 5.1525%

4. Which is the better investment, 9% compounded quarterly or 9.2% per year simple interest?

(a) 9% compounded (b) 9.2% compound interest

(c) Both are same (d) Cannot be determined

5. How long will ₹8,000 take to amount to ₹9,261 at 20% per annum, if the interest is compounded quarterly?

(a) 6 months (b) 8 months (c) 9 months (d) 1 year

6. A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹1,00,000 and its ultimate scrap value was ₹30,000. The effective life of the machine is

(a) 4.5 years (appx.) (b) 5.4 years (appx.)

(c) 5 years (appx.) (d) None of these

7. A machine worth ₹4,90,740 is depreciated at 15% on its opening value each year. When it value would reduce to ₹2,00,750?

(a) 5 year 5 months (b) 5 year 6 months

(c) 5 year 7 months (d) 5 year 8 months

Answer Key

1. (c) **2.** (d) **3.** (c) **4.** (a) **5.** (c) **6.** (b) **7.** (b)

ANNUITY

In many cases, individuals have regular financial obligations where they make equal payments at fixed intervals, such as monthly or yearly. Examples of such payments include life insurance premiums, rent for a rented house, housing loan payments, vehicle loan instalments, etc. This consistent payment of a fixed amount over a specified time period is known as an annuity.

An annuity can be defined as a sequence of periodic payments or receipts made regularly over a specific duration. It applies to situations where individuals receive a fixed amount regularly, like pension payments or rental income from a property.

TAKING THREE CASES

Case 1: When the payment or receipt amount is not the same.

Year end	Payment (₹)
2014	1,000
2015	2,000
2016	3,000
2017	4,000

Case 2: When Payment (or received) amount is same, but the period is not constant.

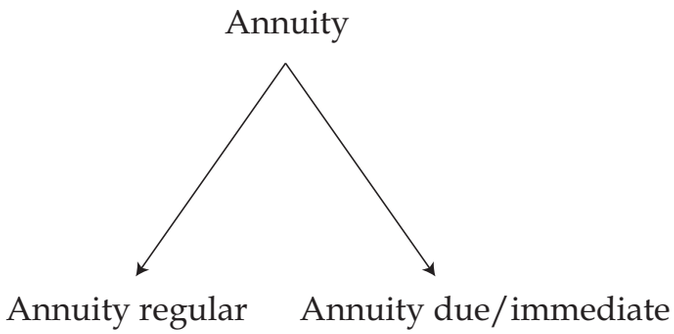
Year end	Payment (₹)
2014	1,000
2015	1,000
2018	1,000
2020	1,000

Case 3: When payment and periodic period are same.

Year end	Payment (₹)
2014	2,000
2015	2,000
2016	2,000
2017	2,000

Here, case 1 and 2 cannot be an annuity whereas case 3 can be an annuity since the payment and the time interval between the consecutive payments are same.

There are two types of Annuity:



REGULAR ANNUITY

In a regular annuity, the first payment or receipt occurs at the end of the first period. For example, consider the following table:

Year end	Payment (₹)
2014	2,000
2015	2,000
2016	2,000
2017	2,000

In this case, the first payment or receipt takes place at the end of the first year, making it a regular annuity.

ANNUITY DUE OR ANNUITY IMMEDIATE

In an annuity due or annuity immediate, the first payment or receipt is made today, at the beginning of the annuity. For example, consider the following table:

In the beginning of	Payment (₹)
2014	2,000
2015	2,000
2016	2,000
2017	2,000

In this case, the first payment or receipt is made at the beginning of the first year. This type of annuity is referred to as annuity due or annuity immediate.

FUTURE VALUE

Future value refers to the monetary worth of an investment or sum of money at a specific point in the future. It represents the value that today's money will grow to over time when compounded at a certain interest rate.

We know that,

$$A_n = P(1 + i)^n$$

Future value of a single cash flow can be computed by the above formula. Replace A by future value (F) and P by single cash flow (C.F.), therefore

$$F = C.F.\ (1 + i)^n$$

Let's consider an example to illustrate the concept of future value. Suppose you invest ₹10,000 in a fixed deposit that offers an annual interest rate of 5%. At the end of the first year, your investment will grow to ₹10,500. This amount includes the original principal of ₹10,000 and the interest earned of ₹500. Therefore, we can say that ₹10,000 invested today is expected to be worth ₹10,500 in one year's time, assuming an interest rate of 5%.

FUTURE VALUE OF AN ANNUITY REGULAR

Example 25. You invest ₹1000 in a three year investment that pays you 6 % per annum. Calculate the future value of the investment.

(a) ₹1191.1 (b) ₹1200.5 (c) ₹900.8 (d) ₹1000

Sol.(a) Given, Single cash flow (C.F.) = ₹1000

Rate of interest (i) = 6% p.a. = 0.06 p.a.

Time period = 3 years

We know that,

$F = C.F.\ (1 + i)^n$

Where, $i = \frac{r}{100 \times c}$ and n = t.c.

Thus, $F = C.F.\ (1 + i)^n$

where, $i = \frac{r}{100 \times c}$ and n = t.c

Thus, $F = C.F.\ (1 + i)^n$

$= 1000\ (1 + 0.06)^3 = 1000\ (1.06)^3 = 1191.1$ (approx.)

Calculator trick:

1. 1.06 × = =

2. Then × 1000

Therefore, the future value of the investment is ₹1191.1.

Hence, the correct option is (a).

CALCULATING THE FORMULA FOR FUTURE VALUE OF ANNUITY

If A be the periodic payments, the future value A(n, i) of the annuity is given by

$$A(n, i) = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Example 26. Find the future value of an annuity of ₹10000 made annually for 5 years at an interest rate of 12% compounded annually.

Given that $(1.12)^5 = 1.7623$.

(a) ₹52500.64 (b) ₹60000.50 (c) ₹63528.47 (d) None of these

Sol. (c) Given, Annuity amount (R) = ₹10,000

Time in years (n) = 5×1 = 5

Rate (i) = 12% = $\frac{12}{100}$ = 0.12 p.a.

Future value FV is given as,

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right] = \frac{10000\ \{(1+0.12)^5 - 1\}}{0.12}$$

$$= \frac{10000\ \{(1.12)^5 - 1\}}{0.12} = \frac{10000\ (1.7623 - 1)}{0.12}$$

$$= \frac{10000\ (0.7623)}{0.12} = ₹63528.47$$

Calculator trick:

1. 1.12 × = = = =

2. –1

3. × 10000

4. ÷0.12

Hence, the correct answer is option (c).

Example 27. ₹500 is invested at the end of each month in an account paying interest 12% per year compounded monthly. What is the future value of this annuity after the 9th payment?

(a) ₹4000 (b) ₹4684.36 (c) ₹5526.64 (d) None of these

Sol. (b) Given, Annuity amount (R) = ₹500

c = 12

n = 1 × 9 = 9

Rate (i) = 12% = $\frac{12}{100 \times 12} = 0.01$ (monthly interest rate)

Future Value is given as,

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right] = \frac{500\left\{(1+0.01)^9 - 1\right\}}{0.01} = \frac{500\left\{(1.01)^9 - 1\right\}}{0.01} = ₹4684.36$$

Calculator trick:

1. 1.01 × = = = = = = = = =

2. –1

3. ÷0.01

4. ×500

Hence, the correct option is (b).

FUTURE VALUE OF ANNUITY DUE OR ANNUITY IMMEDIATE

To calculate the future value of an Annuity due/Annuity immediate:

Step 1: Calculate the future value as though it is an ordinary annuity.

Step 2: Multiply the result by (1 + i)

Example 28. Rahul invests ₹10,000 every year starting from today for next 10 years. Suppose interest rate is 8% per annum compounded annually. Calculate future value of the annuity.

Given that $(1 + 0.08)^{10} = 2.15892500$

(a) ₹1,56,455 (b) ₹1,60,855 (c) ₹1,90,865 (d) ₹2,00,505

Sol.(a) Given, R = ₹10,000, n = 10 × 1 = 10 and i =8% = 0.08

Amount of Annuity is given as,

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= \frac{10,000((1+0.08)^{10} - 1)}{0.08} = 1,44,865.625$$

Now, multiply the above result by (1 + i) to get the required result i.e.,

= 1,44,865.625(1 + 0.08) = 1,44,865.625(1.08) = 1,56,454.875

Therefore, the future value of the annuity is ₹1,56,454.875.

Hence, the correct option is (a).

Example 29. The amount of an annuity certain of ₹150 for 12 years at 3.5% p.a C.I is

(a) ₹2,190.28 (b) ₹1,290.28 (c) ₹2,180.28 (d) None of these

Sol.(a) Given, Annuity Amount (R) = ₹150

Time (n) = 12 × 1 = 12

Interest (i) = 3.5% = 0.035

Amount of Annuity is given as,

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right] = \frac{150\ ((1+0.035)^{12} - 1)}{0.035}$$

$$= \frac{150(1.511 - 1)}{0.035} = \frac{76.65}{0.035} = ₹2190.28 \text{ (approx.)}$$

Therefore, the required amount is ₹2190.28.

Hence, the correct option is (a).

Example 30. If the amount of an annuity after 25 years at 5% p.a. C.I is ₹50,000, the annuity will be

(a) ₹1,406.90 (b) ₹1,047.62 (c) ₹1,146.90 (d) None of these

Sol.(b) Given, Future Value (FV) = ₹50000

Time in years (n) = 25 × 1 = 25

Interest rate (i) = 5% = $\frac{5}{100\times1} = 0.05$

Future Value FV is given as,

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$50000 = R\left[\frac{(1+0.05)^{25} - 1}{0.05}\right]$$

$$50000 = R\left[\frac{2.386}{0.05}\right]$$

$$R = \frac{50000\times0.05}{2.386}$$

R = ₹1047.62

Therefore, the required annuity will be ₹1047.62.

Hence, the correct option is (b).

Example 31. A person invests ₹500 at the end of each year with a bank which pays interest at 10% p.a. C.I. annually. The amount standing to his credit one year after he has made his yearly investment for the 12th time is

(a) ₹11,761.36 (b) ₹10,000

(c) ₹12,000 (d) none of these

Sol. (a) Given, Annuity amount (R) = ₹500

Time (n) = 12

Interest rate (i) = 10% = $\frac{10}{100\times1} = 0.1$

Future Value is given as,

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]\times(1+i) = \frac{500\{(1+0.1)^{12} - 1\}}{0.1}\times(1+0.1)$$

$$= \frac{500\{2.1384283767\}}{0.1}\times(1.1) = (10690.1418835)\ (1.1) = ₹11761.36 \text{ (approx.)}$$

Therefore, the required amount is ₹11761.36.

Hence, the correct answer is option (a).

PRACTICE QUESTIONS (PART D)

1. Find the future value of annuity of ₹6,000 made annually for 8 years at interest rate of 12% compounded annually.

(Given that $(1.12)^8 = 2.47596$

(a) ₹73798 (b) ₹53655 (c) ₹87561 (d) ₹98934

2. At an interest rate of 14% compounded annually, how much should be invested at the end of each year for a period of 7 years to accumulate a total amount of ₹11,000 by the end of the 7th year?

(a) ₹950 (b) ₹1400 (c) ₹1030 (d) ₹2000

3. Sahil decided to invest ₹50,000 every year starting from today for next 12 years. If the interest rate is 7.5% per annum compounded annually, then calculate the future value of the annuity.

[Given that $(1 + 0.075)^{12} = 2.3817796$]

(a) ₹990,275 (b) ₹900,298 (c) ₹890,275 (d) None of these

4. Raju invests ₹20,000 every year in a deposit scheme starting from today for next 12 years. Assuming that interest rate on this deposit is 7% per annum compounded annually. What will be the future value of this annuity? Given that $(1 + 0.07)^{12} = 2.25219150$

(a) ₹540,576 (b) ₹382,813 (c) ₹643,483 (d) ₹357,769

5. Sonu's mother decide to gift him ₹50,000 every year on his birthday. If Sonu deposits this amount in a bank which provides interest 5% Compounded annually. How much Sonu will receive after 10 years?

(a) 607,306 (b) 6,28,894 (c) 6,30,435 (d) 6,45,205

Answer Key

1. (a) **2.** (c) **3.** (a) **4.** (a) **5.** (b)

PRESENT VALUE

WE UNDERSTOOD WHAT THE FUTURE VALUE IS!

- The value of today's money in the future, right!!
- Now, present value means: Value of future money in the present.

  Present Value refers to the current value of a future sum of money, considering the time value of money and the expected rate of return. It represents the amount of money that would need to be invested today to equal the future amount at a given interest rate.

  So, formula for Present value for amount A_n, time period n period at the rate of interest per period i can easily be obtained from the below equation:

$$A_n = P(1 + i)^n$$

$$\text{i.e., } P = \frac{A_n}{(1+i)^n}$$

PRESENT VALUE OF ANNUITY REGULAR

The present value of an annuity regular refers to the current worth of a series of periodic payments or receipts over a specified time period. It helps determine the value of the annuity in today's terms.

To calculate the present value of an annuity regular, we can use the following formula:

$$P(n,i) = R\frac{(1-(1+i)^{-n})}{i}$$

One more formula is $A = \frac{V}{P(n,i)}$ and it uses amortization, where amortization refers to the process of gradually paying off a debt over a specific period of time through regular instalments. It involves the systematic reduction of the principal amount owed, along with the payment of interest, until the debt is fully repaid.

In case of loan if you pay your annuity properly i.e. principal amount + interest, instalment are same and paid over equal period of time then it is said that loan is amortized.

For example, let's consider an annuity regular where you receive ₹5,000 at the end of each year for five years, and the interest rate is 6% per annum. Using the formula, we can calculate the present value of this annuity regular.

Example 32. The present value of an annuity of ₹80 a year for 20 years at 5% p.a is

(a) ₹997 (b) ₹900

(c) ₹1,000 (d) none of these

Sol.(a) $P.V. = R\left[\frac{\{1-(1+i)^{-n}}{i}\right]$, where

R = value of each payment

i = rate of interest per period

n = number of periods

Given,

R = ₹80,

i = 5% p.a. = 0.05 p.a.

n = 20 × 1 = 20

Thus, $P.V. = 80\frac{\{1-(1+0.05)^{-20}\}}{0.05}$

$P.V. = 80\left[\frac{\{1-(1.05)^{-20}\}}{0.05}\right]$

P.V. = 996.97

P.V. = 997 (approx.)

Calculator trick:

1. Find $(1.05)^{-20}$

2. 1.05 on calculator

3. Press ÷

4. Press = 20 times

5. Press GT

6. × 80

Therefore, the present value of an annuity is ₹997.

Hence, the correct answer is option (a) i.e. ₹997.

Example 33. A person bought a house paying ₹20,000 cash down and ₹4,000 at the end of each year for 25 years at 5% p.a. C.I. The cash down price is **(ICAI)**

(a) ₹75,000 (b) ₹76,000 (c) ₹76,370.80 (d) None of these

Sol.(c) Given, ₹20,000 is paid at the beginning as cash down and the rest ₹4,000 is made in the installments.

Thus, the present value is calculated by:

$$P.V. = R\left[\frac{\{1-(1+i)^{-n}}{i}\right], \text{ where}$$

R = ₹4000, n = 25 × 1 = 25 and $i = 5\% = \frac{5}{100\times 1} = 0.05$

$$\Rightarrow P.V. = 4000\left[\frac{1-(1+0.05)^{-25}}{0.05}\right]$$

$$\Rightarrow P.V. = 4000\left[\frac{1-(1.05)^{-25}}{0.05}\right]$$

$\Rightarrow$ P.V. = 56375.5

Therefore, the cash down price = 20000 + 56375.5 = 76375.5

Hence, the correct answer is option (c) i.e., ₹76375.5.

PRESENT VALUE OF ANNUITY DUE OR ANNUITY IMMEDIATE

Steps to calculate Present Value for Annuity Immediate/Due

1. Calculate the present value for (n) period as annuity regular
2. Multiply the final answer by (1 + i)

Annuity Due

- Future value = $R\left[\frac{(1+i)^n-1}{i}\right](1+i)$
- Present Value = $R\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$

Example 34. Your Papa wants to give you ₹10000 every year starting from today for next 5 years as a gift. So, being a smart father, he invested at the interest rate of 15% in mutual funds today morning. What should be the exact amount to be invested such that it exactly gives the money to be gifted?

(a) ₹50,000.0 (b) ₹28,549.8 (c) ₹38,549.8 (d) None of these

Sol.(c) Since, it is an annuity immediate. Thus,

R = ₹10000

n = 4 years

i = 15% = 0.15

Therefore, $P.V. = R\left[\frac{\{1-(1+i)^{-n}}{i}\right]$

$$P.V. = 10000\left[\frac{1-(1+15)^{-5}}{0.15}\right](1+0.15)$$

= 33521.55 × 1.15

= 38550

Now, the total amount to be invested = 28,549.8 + 10,000 = ₹38,549.8

Hence, the correct option is (c).

Example 35. Suppose you bought a flat worth ₹40,00,000 at the interest rate of 5% where you need to pay an equal 10 instalments at the end of every year where initially you gave to the bank ₹10,00,000. So, what will be the amount of every instalment?

(a) ₹97,128.25 (b) ₹2,85,900.800

(c) ₹9,71,000.0 (d) ₹3,88,513.725

Sol.(d) According to the given information,

Cost of the flat = ₹40,00,000

Interest rate = 5%

Number of instalments = 10

Since, the initial payment of ₹10,00,000 is already made, thus the remaining amount is ₹30,00,000.

P.V. = ₹30,00,000

n = 10

i = 5% = 0.05

Therefore, $P.V. = R\left[\frac{\{1-(1+i)^{-n}}{i}\right]$

$$30,00,000 = R\left[\frac{\{1-(1+i)^{-n}}{0.05}\right]$$

$$R = \left[\frac{30,00,000\times 0.05}{(1-(1.05)^{-10})}\right]$$

R = ₹3,88,513.725

Therefore, the amount of every instalment is ₹3,88,513.725.

Example 36. If Aisha borrows ₹6,00,000 from City Bank at the interest rate of 12% for 5 years. What will be the equal annual instalment?

(a) ₹2,66,000.750 (b) ₹1,66,445.839

(c) ₹5,54,655.395 (d) None of these

Sol.(b) According to the given information,

Amount borrowed = ₹6,00,000

Interest rate = 12%

Number of instalments = 5

P.V. = ₹6,00,000

n = 5

i = 12% = 0.12

We know that,

$$P.V. = R\left[\frac{\{1-(1+i)^{-n}}{i}\right]$$

$$6,00,000 = R\left[\frac{\{1-(1+0.12)^{5}\}}{0.12}\right]$$

$$6,00,000 = R\left[\frac{\{1-(1.12)^{-5}\}}{1.12}\right]$$

R = ₹1,66,445.839

Therefore, the amount of every instalment is ₹1,66,445.839.

Hence, the correct option is (b).

Example 37. Mr. X retires in the age of 55. His company promised him an annual pension of Rs 60,000 for next 10 years starting from today. If money is worth 8% compounded annually. How much money he should get today in lumpsum?

(a) 3,47,850 (b) 4,34,813 (c) 6,45,842 (d) 3,58,740

Sol.(b) Given; R = ₹60,000, $i = 8\% = \frac{8}{100} = 0.08$, n = 10

Since it is starting from today thus it is the case of annuity immediate.

Therefore, present value is given by

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right] \times (1 + i)$$

$$PV = 60,000\left[\frac{1-(1+0.08)^{-10}}{0.08}\right](1+0.08)$$

= 60,000 (6.71)(1.08)

= 4,34, 813

PRACTICE QUESTIONS (PART E)

1. What is the present value of ₹1 to be received after two years compounded annually at 10% interest rate?

(a) ₹1 (b) ₹0.83 (c) ₹1.5 (d) ₹2

2. Find the present value of ₹10,000 to be required after 5 years if the interest rate be 9%?

(a) ₹6,500 (b) ₹6,000 (c) ₹8,500 (d) None of these

3. Find the present value of ₹5,00,000 be required after 6 years if the rate of interest is 7.5%. (Given that $(1.075)^6 = 1.5433$)

(a) ₹303,981 (b) ₹323,981 (c) ₹300,781 (d) ₹360,900

4. Find the present value of ₹1000 due at the end of 4 years if money is 7% effective.

(a) ₹616.56 (b) ₹812.75 (c) ₹762.89 (d) None of these

5. Calculate the present value of an annuity of ₹2,000 payable at the end of each year for a period of 5 years. The rate of interest is 8% compounded annually.

(a) ₹7,985 (approx) (b) ₹8,360 (approx)

(c) ₹12,000 (approx) (d) None of these

6. A loan of ₹1,02,000 is to be paid back in two equal annual instalments. If the rate of interest is 4% p.a. compounded annually, then the total interest charged under this instalment plan is

(a) ₹6,160 (b) ₹8,120 (c) ₹5,980 (d) ₹7,560

7. Sachin purchases a bike for ₹3,50,000. He decided to get a loan of ₹3,00,000 at 15% p.a. from a bank and remaining balance of ₹50,000 he pays at the time of purchase. He has to pay the whole amount of loan in 12 equal monthly installments with interest starting from the end of the first month. The money he has to pay at the end of every month is

[Given: $(1.0125)^{12} = 1.16075452$]

(a) ₹27077.50 (b) ₹37798.50 (c) ₹54000.50 (d) None of these

Answer Key

1. (b) **2.** (a) **3.** (b) **4.** (c) **5.** (a) **6.** (a) **7.** (a)

FEW MORE USE OF TIME VALUE OF MONEY

- Sinking funds
- Leasing
- Capital expenditure
- Valuation of bond
- Perpetuity
- Net present value technique (NPV)
- Compound Annual Growth Rate (CAGR)

SINKING FUNDS

Sinking funds refer to setting aside money over a period of time to accumulate a specific amount needed to repay a debt or fund a future obligation like debentures, renew items etc.

It involves making regular payments into an account or investment to ensure sufficient funds are available when needed.

Example: A homeowner setting aside a certain amount of money each month in a savings account to accumulate funds for a future major home repair, such as replacing the roof or renovating the kitchen.

Thus, using formula A = P.A. (n, i); where $A(n,i) = R\left[\frac{(1+i)^n - 1}{i}\right]$

Example 38. Company Alpha needs to pay ₹10,00,000 to banks after 5 years for which they started investing every year at the end with equal instalment with an interest rate of 10%.

(a) ₹1,00,000 (b) ₹2,63,785.64 (c) ₹1,63,797.48 (d) None of these

Sol.(c) According to the given information,

Future value = ₹10,00,000

n = 5 × 1 = 5

i = 10% = 0.10

We know that,

$$F.V. = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 10,00,000 = R\left[\frac{(1+0.10)^5 - 1}{0.10}\right]$$

$$\Rightarrow 10,00,000 = R\left[\frac{(1.10)^5 - 1}{0.10}\right]$$

$$\Rightarrow R\left[\frac{10,00,000 \times 0.10}{(1.10)^5 - 1}\right]$$

⇒ A = ₹1,63,797.48

Therefore, the required equal instalment is ₹1,63,797.48

Hence, the correct option is (c).

Example 39. A company creates a sinking fund of ₹2,00,000 in a bank account for 15 years for which the bank offers interest rate 6% per annum. The yearly payment to be paid by company is approximately

(a) ₹8,945 (b) ₹8,145 (c) ₹8,593 (d) ₹9,645

Sol.(c) According to the given information,

Future value = ₹2,00,000

n = 15 × 1 = 15

i = 6% = 0.06

We know that,

$$F.V. = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 2,00,000 = R\left[\frac{(1+0.06)^{15}-1}{0.06}\right]$$

$$\Rightarrow 2,00,000 = R\left[\frac{(1.06)^{15}-1}{0.06}\right]$$

$\Rightarrow$ R = ₹8,592.55

$\Rightarrow$ R = ₹8,593 (approx)

Therefore, the required yearly payment to be paid by company is ₹8,593.

Hence, the correct option is (c).

Example 40. A machine costs ₹5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹25000. What amount should be set aside every year if the sinking fund investment accumulate at 3.5% compound interest p.a.?

(a) ₹16,000 (b) ₹16,500 (c) ₹16,050 (d) ₹16,005

Sol.(c) Given, Cost of machine = ₹5,20,000

Time (n) = 25 years

m = 1

r = 3.5%

$$\Rightarrow i = \frac{3.5}{100} = 0.035$$

Since, cost of new model is 25% higher, after 25 years with scrap value realization of ₹25000 thus

Total cost = 5,20,000 + 25% of 5,20,000 – 25000 = ₹6,25,000

If 'R' be the amount which is kept aside then

$$625000 = R\frac{\left[(1+0.035)^{25}-1\right]}{0.035}$$

$\Rightarrow$ R = ₹16046.27 = 16,050 (approx.)

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART F)

1. A sinking fund is created for redeeming debentures worth ₹5 lakhs at the end of 25 year. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?

(a) ₹12,006 (b) ₹12,040 (c) ₹12,039 (d) ₹12,035

2. Sinking fund factor is the reciprocal of:

(a) Present value interest factor of a single cash flow

(b) Present value interest factor of an annuity

(c) Future value interest factor of an annuity

(d) Future value interest factor of a single cash flow

Answer Key

1. (a) **2.** (c)

LEASING

Leasing is a financial arrangement in which the owner of an asset, known as the lessor, grants the right to use the asset to another party, known as the lessee, for a specified period of time. The lessee pays a periodic fee, known as the lease rental, to the lessor for the use of the asset.

In a lease agreement, the lessor retains ownership of the asset while providing the lessee with the right to use it. The asset can be tangible, such as equipment, machinery, vehicles, or real estate, or intangible, such as intellectual property or software.

REMEMBER:

- Always use present value of regular annuity formula
- Find ideal rent of asset using its given cost
- Actual cost of asset = Present value of all future ideal rent
- Then compare ideal rent with actual rent

Example 41. A machine is available in ₹100,000 or by leasing if for 8 years at an annual rate of ₹17,000. If If money can be borrowed at 12% per annum, What should be the course of action ?

(a) Purchase (b) Lease

(c) Both are same (d) None

Sol.(b) Here, Purchase value = ₹1,00,000 or Rent = ₹17,000 for 8 years

Thus, to find the annuity amount on ₹1,00,000 for 8 years at the interest rate of 12% per annum.

i.e., P.V = ₹1,00,000, i = 12% = $\frac{12}{100}$ = 0.12, n = 8

Thus, P.V. = $R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$\Rightarrow$ 1,00,000 = $R\left[\frac{1-(1+0.12)^{-8}}{0.12}\right]$

$\Rightarrow$ R = 20,130

Clearly, leasing is favorable to the lessor.

Hence, the correct option is (b).

Example 42. ABC Ltd. wants to lease out an asset costing ₹3,60,000 for a five year period. It has fixed a rental of ₹1,05,000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favourable to the company?

(a) The agreement is favourable (b) The agreement is not favourable

(c) No difference (d) Cannot be determined

Sol.(a) To find the present value of annuity of ₹1,05,000 for five years at the interest rate of 14% per annum.

i.e., A = ₹1,05,000, $i = 14\% = \frac{14}{100} = 0.14, n = 5 \times 1 = 5$

Thus, P.V. = $R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$360{,}000 = R\left[\frac{1-(1.14)^{-5}}{0.14}\right]$

360,000 = R (3.4330)

R = 1,04,864

Here, the ideal rent is ₹1,04,864

But lessor is receiving ₹1,05,000 for the lessor which is higher than the ideal rent

Thus, leasing is favorable to the lessor.

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART G)

1. A company is considering of buying two electronics either by making full payment of ₹17,000 or by leasing it for five years at an annual rate of ₹5000. Which course of action is preferable if the company can borrow money at 14% compounded annually?

(a) Buying the electronics by making a full payment of ₹17,000 is preferable.

(b) Leasing the electronics for five years at an annual rate of ₹5000 is preferable.

(c) Both options are equally preferable.

(d) It cannot be determined based on the given information.

2. A company is considering buying a piece of equipment for ₹75,000 or leasing it for 5 years at an annual rent of ₹18,000. If the company's cost of capital is 12%, which option is more financially attractive?"

(a) Leasing the equipment is more financially attractive

(b) Buying the equipment is more financially attractive

(c) Both options have the same cost

(d) Cannot be determine

Answer Key

1. (a) **2.** (a)

CAPITAL EXPENDITURE

Capital expenditure refers to the funds spent by a company or organization to acquire or improve long-term assets that are expected to generate benefits over an extended period. It involves investing in assets that have a useful life beyond the current accounting period and are not intended for immediate resale.

The purpose of capital expenditure is to enhance the productive capacity, efficiency, or competitiveness of a business. These expenditures are aimed at acquiring or upgrading assets such as property, plant, equipment, machinery, technology, or infrastructure that will support the company's operations and contribute to future revenue generation.

E.g.: A manufacturing company may invest in new machinery to increase production capacity and improve efficiency. A technology company may invest in research and development to develop innovative products or acquire intellectual property rights. A real estate developer may invest in acquiring land and constructing new properties.

Example 43. A cloth making machine can be purchased for ₹200000. Machine will contribute ₹50000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether the machine should be purchased or not.

(a) Leasing is preferable (b) Should be purchased

(c) No difference (d) Cannot be determined

Sol.(a) Given, Annuity amount (R) = ₹50,000

Rate of interest (i) = 10 % = $\frac{10}{100} = 0.10$

Time (n) = 5 × 1 = 5 years

We know that,

If the present value is less than the purchase value, then the leasing is preferable.

Present value (P.V.) is given as,

$$\text{P.V.} = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 50,000\left[\frac{1-(1+0.10)^{-5}}{0.10}\right] = 50,000\left[\frac{1-(1.10)^{-5}}{0.10}\right] = ₹1,89,539.338$$

We have concluded that leasing is preferable.

Hence, the correct answer is option (a) i.e., Leasing is preferable.

Example 44. Juicer A with a useful life of seven years costs ₹15,000 while another Juicer B with a useful life of five years costs ₹12,000. The first juicer saves labour expenses of ₹3,000 annually and the second one saves labour expenses of ₹3,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

(a) Buying Juicer A is the preferred course of action.

(b) Buying Juicer B is the preferred course of action.

(c) Both options are equally preferred.

(d) It cannot be determined based on the given information.

Sol.(b) Cost of juicer A = ₹15,000

Cost of juicer B = ₹12,000

Now, finding the present value of savings of both the juicers.

For juicer A:

R = ₹3,000

$i = 10\% = \frac{10}{100} = 0.10$

$n = 7 \times 1 = 7$

$$\text{Thus, P.V.} = R\left[\frac{1-(1+i)^{-n}}{i}\right] = 3000\left[\frac{1-(1+0.10)^{-5}}{0.10}\right]$$

$$= 3000\left[\frac{(1.10)^5-1}{0.10(1.10)^5}\right] = ₹14{,}605 \text{ (approx).}$$

For juicer B:

A = ₹3,200

$i = 10\% = \frac{10}{100} = 0.10$

$n = 5 \times 1 = 5$

$$\text{Thus, P.V.} = A\left[\frac{1-(1+i)^{-n}}{i}\right] = 3200\left[\frac{1-(1+0.10)^{-5}}{0.10}\right] = ₹12{,}130 \text{ (approx.)}$$

Since, juicer A costing ₹15,000 saves ₹14,605 whereas juicer B costing ₹12,000 saves ₹12,130 thus juicer B is the preferred course of action.

Hence, the correct option is (b).

VALUATION OF BOND

Valuation of a bond refers to the process of determining the fair value or price at which a bond should be bought or sold in the market. Bonds are debt securities issued by governments, municipalities, or corporations to raise capital. When an investor buys a bond, they are essentially lending money to the issuer in exchange for periodic interest payments and the return of the principal amount at maturity.

When we say that bonds are "issued," it means that the bond issuer creates and offers the bonds to the market for purchase by investors. The issuer, which can be a government entity, a local municipality, or a corporate entity, issues the bonds as a means of borrowing money from investors. By selling bonds, the issuer receives an upfront sum of money from investors, which it agrees to repay at a future date, along with periodic interest payments.

Example 45. Rajesh intends purchasing a three year ₹1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and Rajesh requires a rate of return of 20%?

(a) ₹900.35 (b) ₹789.35 (c) ₹1000 (d) None of these

Sol.(b) Here, 10% of ₹1,000 = $\frac{10}{100}\times 1000 = ₹100$

Thus, Present value of bond $= \frac{100}{(1+0.20)}+\frac{100}{(1+0.20)^2}+\frac{100}{(1+0.20)^3}+\frac{1000}{(1+0.20)^3}$

$= \frac{100}{(1.2)}+\frac{100}{(1.2)^2}+\frac{100}{(1.2)^3}+\frac{1000}{(1.2)^3} = 789.35$

Therefore, the purchase value of the bond ₹789.35.

'OR'

Valuation of bond = Present value of A for n periods + Present value of B for n periods

$= A\left[\frac{1-(1+i)^{-n}}{i}\right]+\frac{B}{(1+i)^n}$

Here, A = 10% of 1000 = 100

r = 20%, m = 1

n = 3

B = 1000

Thus, Valuation of bond $= 100\left[\frac{1-(1+0.20)^{-3}}{0.20}\right]+\frac{1000}{(1+0.20)^3}$

= 210.65 + 578.70 = 789.35

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART H)

1. An investor is considering purchasing a four-year bond with a face value of ₹10,000 and a nominal interest rate of 6%. The investor requires a rate of return of 8%. What is the maximum price the investor should be willing to pay for the bond?

(a) ₹9000 (b) ₹9500 (c) ₹10000 (d) ₹10500

2. An investor intends purchasing a three year ₹1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

(a) ₹1000 (b) ₹887.152 (c) ₹907.125 (d) None of these

Answer Key

1. (b) **2.** (c)

PERPETUITY

When you get an annuity for unlimited time, it becomes Perpetuity.

We need to understand two basic things:

1. When we talk about money far in the future, the present value of it becomes today extremely low.

2. And principal is never repaid, that means we do not have to think about the principal.

Then $PVA_{\infty} = \frac{R}{i}$

where, R = the payment or receipt each period

i = the interest rate

Example 46. Hari wants to retire and receive ₹15,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 7% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

(a) ₹25,71,428 (b) ₹2,14,285 (c) ₹1,80,000 (d) None of these

Sol.(a) Given, R = ₹15,000, m = 12 and r = 7%

$\Rightarrow i = \frac{7\%}{12} = \frac{7}{1200}$

We know that,

$PVA \infty = \frac{R}{i} = \frac{15000}{\frac{7}{1200}}$

$= \frac{15000}{7} \times 1200$ = ₹25,71,428 (approx.)

Therefore, Hari need ₹25,71,428 to set aside to achieve his perpetuity goal.

Hence, the correct option is (a).

CALCULATING GROWING PERPETUITY

Growing perpetuity means the periodic instalment is increasing with fixed interest rate

$PVA = \frac{A}{i-g}$

Example 47. Assuming that the discount rate is 8% per annum, how much would you pay to receive ₹1000, growing at 4%, annually, forever?

(a) ₹25,000 (b) ₹2,500 (c) ₹1,00,000 (d) None of these

Sol.(a) Given, A = ₹1000

r = 8%

$g = 4\% = \frac{4}{100} = 0.04$

Here, c = 1

$\Rightarrow i = 8\% = \frac{8}{100} = 0.08$

Therefore, $PVA = \frac{R}{i-g}$

$= \frac{1000}{0.08-0.04} = \frac{1000}{0.04} = 25,000$

Therefore, the correct option is (a).

PRACTICE QUESTIONS (PART I)

1. Calculate the present value of a perpetual payment stream of ₹30,000 per month at an annual interest rate of 15%.

(a) ₹23,60,000 (b) ₹30,00,000 (c) ₹24,00,000 (d) ₹10,80,000

2. A person wants to save for his retirement and plans to make annual deposits of ₹50,000 at the end of each year for perpetuity. If the rate of interest is 12%. What is the present value of his retirement fund?

(a) ₹500,000 (b) ₹416,667 (c) ₹600,000 (d) ₹450,000

3. Ramesh wants to retire and receive ₹3,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 8% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

(a) ₹4,00,000 (b) ₹4,49,775 (c) ₹5,59,775.25 (d) None of these

4. If the discount rate is 16% per annum, then how much a company has to pay to receive ₹360 growing at 10% annually forever?

(a) ₹6,000 (b) ₹5,800 (c) ₹5,400 (d) ₹5,200

5. At a discount rate of 8% per year, what would be the present value of receiving ₹300 growing at an annual rate of 6% indefinitely?

(a) ₹11,500 (b) ₹12,000 (c) ₹15,000 (d) ₹10,000

6. If you wish to receive ₹85,000 annually for indefinite period with a annual growing rate of 4%, How much should be invested now if discount rate is 9% CI compounded annually

(a) 17,00,000 (b) 60,00,000 (c) 8,00,000 (d) None

Answer Key

1. (c) **2.** (b) **3.** (b) **4.** (a) **5.** (c) **6.** (a)

RATE OF RETURN

Net Present Value Technique (NPV)

Net present value = Present value of cash inflow – Present value of cash outflow

Example 48. Compute the net present value for a project with a net investment of ₹1,00,000 and net cash flows year one is ₹55,000 for year two is ₹80,000 and for year three is ₹15,000. Further, the company's cost of capital is 10%.

(a) ₹50,005 (b) ₹49,995 (c) ₹27,384 (d) ₹1,27,340

Sol.(c) According to the question, we have

As per the definition,

Net present value = Present value of cash inflow – Present value of cash outflow

Given, Present value of cash outflow = ₹1,00,000

O	1 year	2 year	3 year
(1 lakh)	55000	80,000	15000

P.V. of a Cash flow

$$NPV = \frac{Amount\ (cf)}{(1+r)^t}$$

$$NPV = \frac{55000}{(1.10)^1} + \frac{80,000}{(1.10)^2} + \frac{15000}{(1.10)^3} - 100000$$

Hence, the correct option is (c).

DECISION RULE

If NPV > 0, Accept the Proposal

If NPV < 0, Reject the Proposal

COMPOUND ANNUAL GROWTH RATE (CAGR)

It is the method to find the smoothed annualized gain of an investment over a given time period.

$$CAGR(t_0, t_n) = \left(\frac{V(t_n)}{V(t_0)}\right)^{\frac{1}{t_n - t_0}} - 1$$

Where $V(t_0)$ = Beginning Period; $V(t_n)$ = End Period

Trick:

Steps to find n^{th} root of any number:

1. Write the number.
2. Press √ ….. 12 times
3. –1
4. ÷n
5. +1
6. ×= …. 12 times

Example 49. A company recorded its annual revenues over a five-year period as follows:

Year	2001	2002	2003	2004	2005
Revenue (in thousand ₹)	100	110	121	133	146.41

What is the compound annual growth rate (CAGR) of the company's revenue over this five-year period?

(a) 10% (b) 20% (c) 30% (d) 40%

Sol.(a) We know that,

$$CAGR(t_0, t_n) = \left(\frac{V(t_n)}{V(t_0)}\right)^{\frac{1}{t_n - t_0}} - 1$$

where $V(t_0)$ is beginning period and $V(t_n)$ is end period

According to the question,

Beginning Value, $V(t_0)$ = ₹100,000

Ending Value, $V(t_n)$ = ₹146,410

Here, $t_n - t_0 = 2005 - 2001 = 4$

$\therefore$ CAGR for year 5 with respect to year 1 is given by:

$$\left(\left(\frac{146410}{100000}\right)^{\frac{1}{4}} - 1\right) \times 100\%$$

$= (1.1 - 1) \times 100\% = 0.1 \times 100\% = 10\ \%$

Calculation trick:

To find 4th root

1. 146410 ÷ 100000

2. Press root 12 times

3. –1

4. ÷4 (because of fourth root)

5. +1

6. ×= ….. 12 times

Therefore, the required compound annual growth rate is 10%.

Hence, the correct option is (a).

NOMINAL RATE OF INTEREST

Nominal Interest Rate = Real Interest Rate + Inflation

Example 50. If the nominal rate of growth is 17% and inflation is 9% for five years. Let P be the Gross Domestic Product (GDP) amount at the present year then the projected real GDP after 6 years is

(a) 1.587 P (b) 1.921 P (c) 1.403 P (d) 2.51 P

Sol. (a) Given, Nominal rate of return = 17% p.a.

Inflation = 9% p.a.

Effective Interest = 17% – 9% = 8% p.a.

$= \frac{8}{100} = 0.08$ p.a.

Let P be the Gross Domestic Product i.e. Present value,

Future Value is given by the formula,

$F.V = P.V(1 + i)^n = P(1 + 0.08)^6 = P\ (1.5868) \approx 1.587\ P$

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART J)

1. If P = ₹12,000, T = 3 years and I = ₹1,800, then R will be

(a) 2.5% (b) 5% (c) 7.5% (d) 10%

2. If the discount rate is 14% per annum, then how much a company has to pay to receive ₹280 growing at 9% annually forever?

(a) ₹5600 (b) ₹2,800 (c) ₹1,400 (d) ₹4,200

3. Let the operating profit of a manufacturer for five years is given as:

Year	1	2	3	4	5	6
Operating profit (in lakh ₹)	90	100	106.4	107.14	120.24	157.35

Then operating profit of compound annual growth rate (CAGR) for year 6 with respect to year 2 is given at

(a) 9% (b) 12% (c) 11% (d) 13%

4. The profit (in lakh ₹) of a company for the past five years are as follows:

Year	1	2	3	4	5
Profit	20	25	30	35	40

Find the compound annual growth rate (CAGR) of the profits for the period from year 1 to year 5.

(a) 15% (b) 19% (c) 25% (d) 35%

Answer Key

1. (b) **2.** (a) **3.** (b) **4.** (b)

FULL CHAPTER PRACTICE QUESTIONS

1. If P = ₹5,000 R = 15%, T = $4\frac{1}{2}$ years, using I = $\frac{PTR}{100}$, I will be

(a) ₹3,375 (b) ₹3,300 (c) ₹3,735 (d) ₹3,300

2. A sum was invested for 3 years as per C.I. and the rate of interest for first year is 9%, 2nd year is 6% and 3rd year is 3% p.a. respectively. Find the sum if the amount in the three years is ₹550?

(a) ₹250 (b) ₹300 (c) ₹462.16 (d) ₹350

3. A sum of money amount to ₹6,200 in 2 years and ₹7400 in 3 years, then the principal is **(ICAI)**

(a) ₹3000 (b) ₹3,500 (c) ₹3800 (d) none

4. The certain sum of money became ₹692 in 2 years and ₹800 in 5 years, then the principal amount is

(a) ₹520 (b) ₹620 (c) ₹720 (d) ₹820

5. How much will ₹25000 amount to in 2 years at compound interest if the rates for the successive years are 4% and 5% per year?

(a) ₹27300 (b) ₹27000 (c) ₹27500 (d) ₹27900

6. Assuming that the discount rate is 8% per annum, how much would an investor be willing to pay to receive ₹500 growing at a rate of 4% annually forever?

(a) ₹12,500 (b) ₹15,000 (c) ₹10,000 (d) ₹8,000

7. If the difference between the compound interest compounded annually and simple interest on a certain amount at 10% per annum for two years is ₹372, then the principal amount is

(a) ₹37200 (b) ₹37000 (c) ₹37500 (d) None of these

8. A certain sum of money Q was deposited for 5 year and 4 months at 4.5% simple interest and amounted to ₹248, then the value of Q is

(a) ₹200 (b) ₹210 (c) ₹220 (d) ₹240

9. A man deposited ₹8000 in a bank for 3 years at 5% per annum compound interest, after 3 years he will get

(a) ₹8800 (b) ₹9261 (c) ₹9,200 (d) ₹9000

10. A certain money doubles itself in 10 years when deposited on simple interest. It would triple itself in

(a) 20 years (b) 15 years (c) 25 years (d) 30 years

11. If ₹10,000 is invested at 8% per year compounded quarterly, then the value of the investment after 2 years is **(Given: $(1 + 0.2)^8 = 1.171659$)**

(a) ₹11, 716.59 (b) ₹10,716.59 (c) ₹117.16 59 (d) None of these

12. A bank pays 10% rate of interest, interest being calculated half yearly. A sum of ₹400 is deposited in the bank. The amount at the end of 1 year will be

(a) ₹440 (b) ₹439 (c) ₹441 (d) ₹442

13. A person borrows ₹5000 for 2 years at 4% per annual simple interest. He immediately lends to person at $6\frac{1}{4}$ % per annum for 2 years, find his gain in the transaction per year.

(a) ₹112.50 (b) ₹225 (c) ₹125 (d) ₹107.50

14. Mr. X invests ₹10000 every year starting from today for the next 10 years. Suppose interest rate is 8 % per year compounded annually. Calculate future value of the annuity.

(a) ₹1,56,454.88 (b) ₹1,56,554.88 (c) ₹1,44,865.625 (d) None of these

15. If ₹1000 be invested at interest rate of 5% and the interest be added to the principal every 10 year, than the number of years in which it will amount to ₹2000 is:

(a) $16\frac{2}{3}$ years (b) $6\frac{1}{2}$ years (c) 16 years (d) $6\frac{2}{3}$ years

16. If compounded interest on any sum at the rate of 5% for two years is ₹512.50, then the sum would be:

(a) ₹3000 (b) ₹4000 (c) ₹5000 (d) ₹6000

17. In how many years will the sum of money become four times at 12% p.a. simple interest?

(a) 18 years (b) 21 years (c) 25 years (d) 28 years

18. How much amount is required to be invested every year to accumulate ₹7,96,870 at the end of 10 years, if interest is compounded annually at 10%?

(a) ₹40,000 (b) ₹45,000 (c) ₹48,000 (d) ₹50,000

19. A person lends ₹6000 for 4 years and ₹8000 for 3 years at simple interest. If he gets ₹2,400 as total interest, then the rate of interest is:

(a) 5% (b) 4% (c) 6% (d) 7%

20. If a person bought a house by paying ₹45,00,000 down payment and ₹80,000 at the end of each year till the perpetuity, assuming the rate of interest as 16%, the present value of house (in ₹) is given as

(a) ₹47,00,000 (b) ₹45,00,000 (c) ₹57,80,000 (d) ₹50,00,000

21. The future value of annuity of ₹2,000 for 5 years at 5% compounded annually is given (in nearest ₹) as

(a) ₹51,051 (b) ₹21,021 (c) ₹11,051 (d) ₹61,254

22. Assuming that the discount rate is 7% p.a. How much would they pay to receive ₹200 growing at 5% annually forever?

(a) ₹2,500 (b) ₹5,000 (c) ₹7,500 (d) ₹10,000

23. ₹800 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annually after 10th payment?

(a) ₹4,444 (b) ₹8,756 (c) ₹3,491 (d) ₹8,182

24. Rajan is planning for his retirement and desires to receive a monthly payment of ₹7,500. He wishes to ensure that this monthly payment continues to be passed on to future generations even after his demise. With an expected interest rate of 5% compounded annually, how much does Rahul need to set aside to achieve his perpetuity goal?

(a) ₹21,00,000 (b) ₹15,00,000

(c) ₹24,00,000 (d) ₹18,00,000

25. A person wants to lease out a machine costing ₹5,00,000 for a 10 year period. It has fixed a rental of ₹51,272 per annum payable annually starting from the end of first year. Suppose rate of interest is 10% per annum, compounded annually on which money can be invested. To whom this agreement is favourable?

(a) Favour for lessee (b) Favour for lessor

(c) Not for both (d) Can't be determined

Answer Key

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (a)
11. (a)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (d)	**19.** (a)	**20.** (d)
21. (c)	**22.** (d)	**23.** (d)	**24.** (d)	**25.** (a)					

PREVIOUS YEAR QUESTION (PYQ'S)

1. An investor is saving to pay off an obligation of ₹15,250 which will be due in seven years., if the investor is earning 7.5% simple interest rate per annum, he must deposit ₹_____ to meet the obligation. **(June 2022)**

(a) ₹8000 (b) ₹9000 (c) ₹10,000 (d) ₹11,000

2. The annual rate of simple interest is 12.5%. In how many years does the principal double? **(June 2022)**

(a) 11 years (b) 9 years (c) 8 years (d) 7 years

3. An investment is earning compound interest, ₹100 invested in the year 2 accumulated to ₹105 by year 4. If ₹500 invested in the year 5, will become ₹_____ by year 10. **(June 2022)**

(a) ₹364.80 (b) ₹564.80 (c) ₹464.80 (c) ₹664.80

4. There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest of ₹12,000 after 3 years at the same rate? **(June 2022)**

(a) ₹3972 (b) ₹2160 (c) ₹3120 (d) ₹3742

5. The present value of ₹2000 after 8 years at the rate of 6% per annum, is _____.

[Given; $(1.06)^8 = 1.59385$] **(June 2022)**

(a) ₹1054 (b) ₹1254 (c) ₹3054 (d) ₹2054

6. ₹800 is invested at the end of each month in an account of ₹1000 made annually for 7 years at interest rate 14% compounded annually. Given that $(1.14)^7 = 2.5023$ **(June 2022)**

(a) ₹10,730.71 (b) ₹5365.35 (c) ₹8756 (d) ₹9892.34

7. Assuming that the discount rate is 7% p.a. How much would you pay to receive ₹200 growing at 5% annually forever? **(June 2022)**

(a) ₹2500 (b) ₹5000 (c) ₹7500 (d) ₹10,000

8. ₹2500 is paid every year for 10 years to pay off a loan. What is the loan amount if the interest rate is 14% p.a. compounded annually? **(June 2022)**

(a) ₹15,847.90 (b) ₹13,040.27 (c) ₹14,674.21 (d) ₹16,345.11

9. Raj made an investment of ₹15,000 in a scheme and at the time of maturity the amount was ₹25,000. If Compound Annual Growth Rate (CAGR) for this investment is 8.88%. Calculate the approximate number of years for which he has invested the amount. **(June 2022)**

(a) 6 (b) 7.7 (c) 5.5 (d) 7

10. Madhu took a loan of ₹50,000 from XYZ bank. The rate of interest is 10% per annum. The first instalment will be paid at the end of year 5. Determine the amount (in ₹) of equal instalments, if Madhu wishes to repay the amount in five instalments.

(June 2022)

(a) ₹19,510 (b) ₹19,430 (c) ₹19,310 (d) ₹19,630

11. Ramesh invests ₹20,000 per year in a stock index fund, which earns 9% per year for the next ten years. What would be the closest value of the accumulated value of the investment upon payment of the last installment? **(June 2022)**

Given, $(1.09)^{10} = 2.36736$

(a) ₹3,88,764.968 (b) ₹3,03,858.594

(c) ₹2,68,728.484 (d) ₹4,08,718.364

12. A machine worth ₹4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹2,000,750? **(Dec 2022)**

(a) 5 years 5 months (b) 5 years 6 months

(c) 5 years 7 months (d) 5 years 8 months

13. If ₹64 amount to ₹83.20 in 2 years, what will ₹86 amount to in 4 years at the same rate per cent per annum? **(Dec 2022)**

(a) ₹127.60 (b) ₹147.60 (c) ₹145.34 (d) ₹117.60

14. The effective annual rate of interest corresponding to a normal rate of 6% per annum payable half yearly is: **(Dec 2022)**

(a) 6.06% (b) 6.07% (c) 6.08% (d) 6.09%

15. 10 years ago, the Earning per share (EPS) of ABC Ltd. was ₹5 share.Its EPS for this year is ₹22. Compute at what rate EPS of the company grow annually? **(Dec 2022)**

(a) 15.97% (b) 16.77% (c) 18.64% (d) 14.79%

16. Mr. Prakash invested money in two schemes 'A' and 'B' offering compound interest at the rate of 8% and 9% per annum respectively. If the total amount of interest accrued through these two schemes together in two years was ₹4818.30 and total amount invested was ₹27,000. What was the amount invested in scheme 'A'? **(Dec 2022)**

(a) ₹12,000 (b) ₹12,500 (c) ₹13,000 (d) ₹13,500

17. A sum of money invested of compound interest doubles itself in four years. In how many years it becomes 32 times of itself at the same rate of compound interest?

(Dec 2022)

(a) 12 years (b) 16 years (c) 20 years (d) 24 years

18. The difference between compound interest and simple interest on an amount of ₹15,000 for 2 years is ₹96. What is the rate of interest per annum? **(Dec 2022)**

(a) 9% (b) 8% (c) 11% (d) 10%

19. A sum of money doubles itself in 4 years at a certain compound interest rate. In how many years this sum will become 8 times at the same compound interest rate?

(Dec 2022)

(a) 12 years (b) 14 years (c) 16 years (d) 18 years

20. A farmer borrowed ₹3600 at the rate of 15% simple interest per annum. At the end of 4 years, he cleared this account by paying ₹4000 and a cow. The cost of the cow is;

(Dec 2022)

(a) ₹1000 (b) ₹1200 (c) ₹1550 (d) ₹1760

21. A company created a sinking fund of ₹2,00,000 in a bank account for 15 years bank offers interest rate 6% per annum the yearly payment to be paid by company is approximately is

(Dec 2022)

(If need, use $(1.06)^{14} = 2.209$)

(a) ₹8,945 (b) ₹8,145 (c) ₹9,345 (d) ₹9,645

22. How much amount is required to be invested every year so as to accumulate ₹5,00,000 at the end of 12 years if interest is compounded annually at 10%? **(Dec 2022)**

(where, A(12,0.1) = 21.384284)

(a) ₹23381.65 (b) ₹24385.85 (c) ₹26381.65 (d) ₹28362.75

23. Raju invests ₹20,000 every year in a deposit scheme starting from today for next 12 years. Assuming that interest rate on this deposit is 7% per annum compounded annually. What will be the future value of this annuity? **(Dec 2022)**

Given that, $(1 + 0.07)^{12} = 2.25219159$

(a) ₹5,40,526 (b) ₹3,82,813 (c) ₹6,43,483 (d) ₹3,57,769

24. Mr A invested ₹10,000 every year for next 3 years at the interest rate of 8 per cent per annum compounded annually. What is the future value of the annuity?

(Dec 2022)

(a) ₹32,644 (b) ₹32,464 (c) ₹34,426 (d) ₹36,442

25. ₹5,000 is invested every month end in an account paying interest at the rate of 12% per annum compounded monthly. What is the future value of this annuity just after making 11th payment ?

(Dec 2022)

(Given that, $(1.01)^{11} = 1.1156$)

(a) ₹57,800 (b) ₹56,100 (c) ₹56,800 (d) ₹57,100

26. Sinking fund factor is the reciprocal of **(Dec 2022)**

(a) Present value interest factor of a single cash flow

(b) Present value interest factor of an annuity

(c) Future value interest factor of an annuity

(d) Future value interest factor of a single cash flow

27. Suppose you have decided to make a Systematic Investment Plan (SIP) in a mutual fund with ₹1,00,000 every year from today for next 10 years where you get return at the rate of 10% per annum compounded annually. What is the future value of this annuity? **(June 2023)**

(Given that, $(1.1)^{10} = 2.59374$)

(a) ₹17,35,114 (b) ₹17,53,411 (c) ₹17,35,411 (d) ₹17,53,114

28. Ms. Paul invested ₹1,00,000 in a mutual fund scheme in January 2018. After one year in January, 2019 she got a divided amounting to ₹10,000 for first year, ₹12,000 for second year, ₹16,000 for third year, ₹18,000 for fourth year and ₹21,000 for fifth year in January 2023. What is Compounded Annual Growth Rate (CAGR) of dividend return? **(June 2023)**

Given; $(1.2038)^4 = 2.1$)

(a) 20.38% (b) 18.59% (c) 16.36% (d) 15.89%

29. A company want to replace its existing tool room machine at the end of 10 years, the expected cost of the machine would be ₹10,00,000. If management of the company creates a sinking fund, how much provision needs to be made out of revenue each year which can earn at the rate of 10% compounded annually? **(June 2023)**

Given A(10, 0.10 = 15.937425)

(a) ₹74,625 (b) ₹72,514 (c) ₹62,745 (d) ₹67,245

30. A car is available for ₹4,98,200 cash payment or ₹60,000 cash down payment followed by three equal annual instalments. If the rate of interest charged is 14% per annum compounded yearly, then total interest charged in the instalment plan is **(June 2023)**

(Given P(3, 0.14 = 2.32163)

(a) ₹1,46,314 (b) ₹1,46,137 (c) ₹1,28,040 (d) ₹1,58,040

31. Govinda's mother decides to gift him ₹50,000 every year starting from today for the next five years. Govinda deposits this amount in a bank as and when he receives and gets 10% per annum interest rate, compounded annually. What is the present value of this annuity? **(June 2023)**

(a) ₹2,80,493.5 (b) ₹2,08,493.5 (c) ₹2,08,943.5 (d) ₹2,58,493.5

32. If the discount rate is 10% per annum, how much amount would you pay to receive ₹2,500 growing at 8% annually forever? **(June 2023)**

(a) ₹1,25,000 (b) ₹2,50,000 (c) ₹1,50,000 (d) ₹2,00,000

33. Mr Shahad got his retirement benefits amounting to ₹50,00,000. He want to receive a fixed monthly sum of amount for his rest of life, starting after one month and thereafter he want to pass on the same to future generations. He expects to earn an interest of 9% compounded annually. He expects to earn an interest of 9% compounded annually. Determine how much perpetuity amount he will receive every month? **(June 2023)**

(a) ₹39,500 (b) ₹38,500 (c) ₹37,500 (d) ₹36,600

34. Jonny wants to have ₹2,00,000 in his saving account after three years. The rate of interest offered by bank is 8% per annum compounded annually. How much should he invest today to achieve his target amount ? **(June 2023)**

(a) ₹1,47,489.10 (b) ₹1,58,766.44

(c) ₹1,71,035.59 (d) ₹1,84,417.96

35. Mr. Ram invested a total of ₹1,00,000 in two different banks for a fixed period. The first bank yields an interest of 9% per annum and second, 11% per annum. If the total interest at the end of one year is 9.75% per annum, then the amount invested in these banks are respectively. **(Dec 2023)**

(a) ₹52,500; ₹47,500 (b) ₹62,500; ₹37,500

(c) ₹57,500; ₹42,500 (d) ₹67,500; ₹32,500

36. Manoj invests ₹12,000 at 6% per annum simple interest to obtain a total amount of ₹14,880. What is the time for which the amount was invested? **(Dec 2023)**

(a) 3 years (b) 4 years (c) 2 years (d) 5 years

37. A machine costing 1,00,000 has useful life of 10 years. If the rate of depreciation is 12%. What is the value of the machine at the end of its life? **(Dec 2023)**

[Given $(0.88)^{10} = 0.27850$]

(a) 25,850 (b) 26,850 (c) 27,850 (d) 28,850

38. Mr XYZ invested ₹60,000 in a nationalized bank in the form of fixed deposit at the rate of 7.5% per annum simple interest rate. He received ₹73,500 after the end of the term of fixed deposit. Calculate the period for which ₹60,000 was invested in fixed deposit. **(Dec 2023)**

(a) 3 years (b) 3.5 years (c) 4 years (d) 4.5 years

39. The nominal rate of interest is 10% per annum. The interest is compounded quarterly. The effective rate of interest per annum will be: **(Dec 2023)**

(a) 10% (b) 10.10% (c) 10.25% (d) 10.38%

40. A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realised at the time of sale being ₹23,240 and ₹9,000 respectively. Approximately, for how many years the machine is put to use? **(Dec 2023)**

(a) 7 (b) 8 (c) 9 (d) 10

41. The population of a town increases every year by 2% of the population at the beginning of that year. The approximate number of years, by which the total increase of population will be 40% is _____ . **(Dec 2023)**

[Given; $(1.02)^8 = 1.17166$]

(a) 15 (b) 17 (c) 19 (d) 20

42. The compound interest on ₹15,625 for 9 months at 16% per annum compounded quarterly is: **(Dec 2023)**

(a) ₹1851 (b) ₹1941 (c) ₹1951 (d) ₹1961

43. Mr. X makes a deposit of ₹50,000 in a bank for a period of $2\frac{1}{2}$ years. If the rate of interest is 12% per annum compounded half yearly, then the maturity value of the money deposited by Mr X is; **(Dec 2023)**

[where, $(1.06)^5 = 1.3382$]

(a) ₹66910 (b) ₹66123 (c) ₹67925 (d) ₹66550

44. What is the effective rate of interest when principal amount of ₹50,000 deposited in a nationalized bank for one year, corresponding to a nominal rate of interest 8% per annum compounded quarterly. **(Dec 2023)**

[Given, $(1.02)^4 = 1.0824$]

(a) 10.38% (b) 8.08% (c) 8.16% (d) 8.24%

45. Calculate the present value of ₹2000 to be required after 10 years compounded annually at 5% per annum. **(Dec 2023)**

[Given, $(1.05)^{10} = 1.62889$]

(a) ₹1227.82 (b) ₹1282.48 (c) ₹1328.35 (d) ₹1822.65

46. How much amount is required to be invested every year so as to accumulate ₹30,000 at the end of 10 years if the interest compounded annually at 10%. **(Dec 2023)**

[Given A(10,0.1) = 15.9374]

(a) ₹1882.36 (b) ₹1828.30 (c) ₹1832.65 (d) ₹1853.65

47. Mr. X invests in an annuity immediately that promises annual payments of ₹50,000 for the next 16 years. If the interest rate is 6% compounded annually then the approximate present value of annuity is: **(Dec 2023)**

[where, $(1.06)^{15} = 2.3965$]

(a) ₹5,51,217.75 (b) ₹5,75,900.00

(c) ₹5,05,288.08 (d) ₹5,35,612.45

48. If the initial investment of ₹4,00,000 become ₹6,00,000 in 24 months, then the Compound Annual Growth Rate (CAGR) is: **(Dec 2023)**

(a) 30.33% (b) 22.4% (c) 19.46% (d) 14.47%

49. Compute the compound interest on ₹6000 for $1\frac{1}{4}$ years at 8% p.a. interest compounded quarterly. **(Dec 2023)**

(a) ₹642 (b) ₹630.78 (c) ₹634.68 (d) ₹624.48

50. What will be future value of an annuity ₹2500 made annually for 12 years at an interest rate of 5% compounded annually? **(Dec 2023)**

[$(1.05)^{12} = 1.7958$]

(a) ₹37588.58 (b) ₹39790.00 (c) ₹40873.13 (d) ₹42603.68

51. A person wants to open a shop have two options to acquire a commercial space either by leasing for 10 years at annual rent of ₹2,00,000 or by purchasing the space for ₹12,00,000. If person can borrow money at 14% compounded per annum, Which alternate is most suitable? **(Dec 2023)**

(Given P(10,0.14 = 5.21611)

(a) Leasing (b) Purchase (c) Can't say (d) Data insufficient

52. Suppose Mr X invested ₹5000 every year starting from today in mutual fund for next 10 years. Assuming that average return compounded annually is at 18% per annum. What is the future value? **(Dec 2023)**

(a) ₹1,83,677.68 (b) ₹1,38,678.85

(c) ₹1,83,776.53 (d) ₹1,38,774.55

53. If a loan of ₹30,000 is to be paid in 5 annual installments with interest rate of 14% per annum, then the equal annual instalment will be? Given P (5,0.14) = 3.43308 **(June 2024)**

(a) ₹7,400 (b) ₹8,100 (c) ₹8,738 (d) ₹8,322

54. Find the future value of an annuity of ₹5,000 made annually for 6 years at interest rate of 12% compounded annually if $(1 + 0.12)^6 = 1.9738$ **(June 2024)**

(a) ₹45,575 (b) ₹40,575 (c) ₹40,755 (d) ₹45,755

55. If the interest rate on a loan is 1% per month, the effective rate of interest is **(June 2024)**

(a) 12% (b) 12.36% (c) 12.68% (d) 12.84%

56. Ram borrowed ₹5,000 at 12.5% per annum compound interest. The money was repaid after 3 years. The total interest paid by him approximately is? $(1 + 0.125)^3 = 1.4238$ **(June 2024)**

(a) ₹2,119 (b) ₹2,100 (c) ₹2,200 (d) ₹2,300

57. A person invests in a fund that pays 4% per annum for four years. The future value of current ₹4,000 would be ₹_____. (Use if needed, $(1.04)^4 = 1.1698$, $\frac{1}{(1.04)^4} = 0.8545$, $(1.04)^5 = 1.2160$ and $\frac{1}{(1.04)^5} = 0.8219$) **(June 2024)**

(a) ₹4,419 (b) ₹4,679 (c) ₹4,688 (d) ₹3,768

58. What is the present value of ₹5,000 to be obtained after six years if the interest rate is 5% per annum? (Use the following if needed: $\frac{1}{(1.05)^n} = 0.74621, 0.71068, 0.67686$ and 0.64462 for n = 6, 7, 8 and 9 respectively) **(June 2024)**

(a) ₹3,731 (b) ₹3,384 (c) ₹3,223 (d) ₹3,553

59. Find the effective rate of interest if an amount of ₹40,000 deposited in a bank for 1 year at the rate of 10% compounded semi-annually. **(June 2024)**

(a) 10.05% (b) 10.10% (c) 10.20% (d) 10.25%

60. The difference between the compound interest amount and simple interest amount for a period of two years, at same rate r, is **(June 2024)**

(a) $P \times r^2$ (b) $P \times \frac{r}{2}$ (c) $P \times 2 \times r$ (d) $P^2 \times r$

61. An amount ₹4,500 becomes ₹7,200 in two years at a simple interest rate of **(June 2024)**

(a) 15% (b) 25% (c) 30% (d) 40%

62. You bought a painting 10 years ago as an investment. You originally paid ₹85,000 for it. If you sold it for ₹4,84,050, what was your annual return of investment? **(June 2024)**

(a) 4.7% (b) 12.8% (c) 19% (d) 47%

63. What is the present value of an investment that pays ₹400 at the end of three years and ₹500 at the end of 6 years? **(June 2024)**

(a) ₹320 (b) ₹280 (c) ₹335 (d) ₹340

64. At 8% compounded annually, how long will it take ₹750 to double? **(June 2024)**

(a) 48 months (b) 6.5 years (c) 9 years (d) 12 years

65. You are considering two investments: Investment A yields 10% compounded quarterly. Investment B yields r% compounded semi-annually. Both investments have equal annual yields. Find r **(June 2024)**

(a) 10.875% (b) 10% (c) 10.38% (d) 10.125%

66. The sum required to earn a monthly interest on ₹1,200 at 18% per annum simple interest is: **(Sept 2024)**

(a) ₹50,000 (b) ₹60,000 (c) ₹80,000 (d) ₹66,000

67. The compound interest on ₹40,000 at 12% per annum compounded quarterly for 6 months is **(Sept 2024)**

(a) ₹2,643 (b) ₹2,463 (c) ₹2,364 (d) ₹2,436

68. At a certain rate of interest per annum, the difference between the compound interest and simple interest on ₹3,00,000 for two years is ₹480, then the rate of interest per annum is; **(Sept 2024)**

(a) 2% (b) 4% (c) 6% (d) 8%

69. The value of a machine depreciates every year at the rate of 10% per annum on its value at the beginning of that year. If the present value of the machine is ₹72,900 , then machine's worth 3 years ago was: **(Sept 2024)**

(a) ₹94,710 (b) ₹80,000 (c) ₹1,00,000 (d) ₹75,087

70. What is the effective rate of interest when the principal amount of ₹50,000 deposited in a nationalized bank for one year, corresponding to a nominal rate of interest 6% per annum payable half yearly? **(Sept 2024)**

(a) 6.06% (b) 6.07% (c) 6.08% (d) 6.09%

71. Kanta wants to accumulate ₹4,91,300 in her savings account after three years. The rate of interest offered by bank is $6\frac{1}{4}\%$ per annum compounded annually. How much amount should she invest today to achieve her target amount? **(Sept 2024)**

(a) ₹4,09,600 (b) ₹4,37,500 (c) ₹46,900 (d) ₹49,600

72. Mr X makes a deposit of ₹12,000 in a bank where the amount doubles at compound interest of 5 years, then what will be the total amount he will have after twenty years? **(Sept 2024)**

(a) ₹96,000 (b) ₹1,20,000 (c) ₹1,24,000 (d) ₹1,92,000

73. The Earning Per Share (EPS) of a company for five years is given below: **(Sept 2024)**

Year	2019	2020	2021	2022	2023
EPS	40	25	40	60	90

Calculate the Compounded Annual Growth Rate (CAGR) of EPS.

(a) 24.47% (b) 23.47% (c) 22.47% (d) 21.47%

74. In an account paying interest @9% per year compounded monthly, ₹200 is invested at the end of each month. What is the future value of this annuity after 10th payment? [Where, $(1.0075)^{10} = 1.0775$] **(Sept 2024)**

(a) ₹1,022 (b) ₹2,066 (c) ₹2,044 (d) ₹2,155

75. What is the present value of ₹1,000 to be received after two years compounded annually at 10% interest rate? **(Sept 2024)**

(a) ₹800 (b) ₹826 (c) ₹836 (d) ₹835

76. What is the annual contribution required by an organisation to accumulate ₹20,00,000 in ten years for the construction of a new manufacturing plant, utilizing a sinking fund with an annual interest rate of 6% compounded annually? {where, A(10, 0.06) = 13.180785} **(Sept 2024)**

(a) ₹1,51,736.03 (b) ₹1,67,440.90 (c) ₹1,75,433.60 (d) ₹1,83,714.28

77. An investor intends to purchase a three year ₹1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now, if it matures at par and the interest required a rate of return of 14%? **(Sept 2024)**

(a) ₹904 (b) ₹907.125 (c) ₹905.25 (d) ₹909

78. The loan of ₹16,550 is to be paid in three equal instalments at compound interest. The value of annual instalment, if the rate of interest is 10% per annum is: **(Sept 2024)**

(a) ₹6,655 (b) ₹1,243 (c) ₹6,565 (d) ₹1,343

79. A perpetuity has a cash flow of ₹625 and a required rate of return of 8%. If the cash flow is expected to grow at a constant rate of 4% per year, then the intrinsic value of this perpetuity (present value of growing perpetuity) is; **(Sept 2024)**

(a) ₹15,625 (b) ₹13,000 (c) ₹14,250 (d) ₹16,667

80. Anil deposited a certain amount in a bank at the rate of 10% per annum compounded semi-annually. At the end of one year Anil received a sum of ₹13,230. Then the sum deposited in the bank is **(Jan 2025)**

(a) ₹1,200 (b) ₹13,000 (c) ₹12,000 (d) ₹5,000

81. A certain amount at a rate of simple interest x, doubles in 5 years. At another rate of simple interest y, it becomes three times in 8 years. Then the difference between these two interest rates is **(Jan 2025)**

(a) 8% (b) 5% (c) 3% (d) 4%

82. John borrows a loan of 10,000 from a bank and he agreed to pay back in 24 equal instalments at the rate of 10% compound interest per annum. Then each instalment amount is (Given that $(1.1)^{24} = 9.84973$) **(Jan 2025)**

(a) ₹1,112.99 (b) ₹1,200.35 (c) ₹1,211.99 (d) ₹1,321.56

83. What is the present value of ₹8,000 to be required after 10 years if the interest rate be 6%? (Given that $(1.06)^{10} = 1.7908$) **(Jan 2025)**

(a) ₹4,467.28 (b) ₹6,499.87 (c) ₹5,867.32 (d) ₹1,790.86

84. The effective rate of interest corresponding to a nominal rate of 8% per annum payable quarterly is (Given that $(1.02)^4 = 1.08243216$) **(Jan 2025)**

(a) 5.38% (b) 6.24% (c) 8.24% (d) 82.4%

85. Sunil plans to save for his higher studies. He wants to accumulate a sum of ₹5,00,000 at the end of 10 years. How much amount should he invest every year if the interest rate is 10% compounded annually? (Given that $(1.1)^{10} = 2.593742$) **(Jan 2025)**

(a) ₹3,137.27 (b) ₹31,372.71 (c) ₹31,312.71 (d) ₹3,000.32

86. The future value of an annuity of ₹7,200 made annually for 5 years at the rate of 12% compounded annually is (Given that $(1.12)^5 = 1.76234$) **(Jan 2025)**

(a) ₹4,574.50 (b) ₹45,740.40 (c) ₹54,740.50 (d) ₹2,400.50

87. A certain amount is invested in a bank. What annual rate of interest compounded annually becomes 8 times of this investment in 5 years? (Given that $8^{1/5} = 1.515716$) **(Jan 2025)**

(a) 5.15% (b) 51.57% (c) 15.15% (d) 1.51%

88. If the compound interest on a certain sum for 2 years at 5% per annum is ₹246, then the simple interest on the same sum for double the time and double the rate per annum is: **(Jan 2025)**

(a) ₹960 (b) ₹900 (c) ₹1,000 (d) ₹1,100

89. Sam invested ₹12,000 for 10 years in a financial company. At the end of 10th year his investment value is ₹18,000. Then the Compound Annual Growth Rate (CAGR) is if $(x)^{1/n} = 1.0413$ **(Jan 2025)**

(a) 4.13% (b) 41.40% (c) 11.56% (d) 12.06%

90. Mr. A invested ₹20,000 in a bank at the rate of 4.5% p.a. He received ₹27,500 after end of term. Find out the period? **(Jan 2025)**

(a) 8.34 Yrs (b) 4.50 Yrs (c) 6.50 Yrs (d) 8.10 Yrs

91. ₹1,500 is paid every year for 10 years to pay a loan. What is the loan amount, if rate of interest 5% p.a.? If $(1.05)^{10} = 1.6288$ **(Jan 2025)**

(a) 11,505.50 (b) 11,581.53 (c) 11,903.38 (d) 12,503.48

92. Madhu deposits ₹100 in a Bank at the beginning of every year for 20 years at 10% interest rate compounded annually, how much would she earn after 20 years? [Given that $(1.1)^{20} = 6.7275$]

(Jan 2025)

(a) ₹6,500.45 (b) ₹6,300.25 (c) ₹5,600.25 (d) ₹6,250.35

93. How much amount is required to be invested every year so as to accumulate ₹15,00,000 at the end of 20 years if interest is compounded annually at 10%? [Given A(n,i) = 57.274999]

(Jan 2025)

(a) ₹29,190.35 (b) ₹26,189.44 (c) ₹24,155.35 (d) ₹30,698.44

94. There is 60% increase in amount in 6 years at simple interest. What will be the compound interest of ₹12,000 after three years at the same rate? **(May 2025)**

(a) ₹2,160 (b) ₹3,120 (c) ₹3,972 (d) ₹6,240

95. The effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half-yearly is **(May 2025)**

(a) 6.06% (b) 6.07% (c) 6.08% (d) 6.09%

96. The compound interest on a certain sum for 2 years at 10% per annum is ₹525. The simple interest on the same sum for double the time at half the rate percent per annum is

(May 2025)

(a) ₹400 (b) ₹500 (c) ₹600 (d) ₹800

97. Find the future value of an investment of ₹7,000 compounded quarterly at 10% per annum for 3 years. [Given that $(1.025)^{12} = 1.34489$] **(May 2025)**

(a) ₹9,414.20 (b) ₹7,435.73 (c) ₹7,941.42 (d) ₹8,000.00

98. A sum of ₹725 is lent in the beginning of a year at a certain rate of simple interest. After 8 months, a sum of ₹362.50 more is lent but at the rate twice the former. At the end of the year, ₹33.50 is earned as interest from both the loans. What was the original rate of interest?

(May 2025)

(a) 3.6% (b) 4.54% (c) 3.46% (d) 4.12%

99. Shiv deposits ₹10,000 annually in a bank for 5 years, at 10 percent annual compounding interest rate. Calculate the approximate value of this series of deposits at the end of five years, if each deposit occurs at the beginning of the year. **(May 2025)**

(a) ₹61,050 (b) ₹67,156 (c) ₹71,050 (d) ₹77,160

100. If you deposit ₹4,000 into an account paying 6% annual interest compounded quarterly, how much approximate money will be in the account after 5 years? [Given that $(1.015)^{20} = 1.34489$]

(May 2025)

(a) ₹3387.42 (b) ₹4387.42 (c) ₹5387.42 (d) ₹6387.42

101. Relationship between annual nominal rate of interest and annual effective rate of interest, if frequency of compounding is greater than one **(May 2025)**

(a) Effective rate < Nominal rate (b) Effective rate > Nominal rate

(c) Effective rate = Nominal rate (d) Effective rate = 0.9 times Nominal rate

102. Madhu invests ₹15,000 in a scheme and at the time of maturity the amount became ₹25,000. If CAGR for this investment is 8.88%, calculate the approximate number of years for which she has invested the amount. [Given that log(1.667) = 0.2219 and log(1.089) = 0.037]

(May 2025)

(a) 6 years (b) 7.7 years (c) 5.5 years (d) 7 years

103. Raju will pay instalments of ₹3,150 per month for the next 3 years towards his loan at an interest rate 12.4%, discounted monthly, what was the approximate amount of loan taken initially? [Given that $(1.01033)^{36} = 1.448$] **(May 2025)**

(a) ₹13,683.60 (b) ₹9,742.29 (c) ₹94,345.17 (d) ₹74,158.24

104. Dinesh received a cash bonus of ₹1,00,000 which he deposited in a bank which pays 10 percent interest compounded annually. How much approximate -equal amount can Dinesh withdraw annually for a period of 10 years? [Given that $(1.1)^{10} = 2.59374$] **(May 2025)**

(a) ₹16,273 (b) ₹38,554 (c) ₹62,745 (d) ₹32,474

105. Find the approximate future value of an annuity due of ₹500 per quarter for 8 years and 9 months at the interest rate of 6% compounded quarterly. [Given that $(1.015)^{35} = 1.6839$]

(May 2025)

(a) ₹13,740.86 (b) ₹29,428.23 (c) ₹56,971.95 (d) ₹22,796.66

106. A project is expected to provide cash inflows as follows for 3 years: **(May 2025)**

Year	1	2	3
Cash inflows (₹)	40,000	50,000	30,000

(a) ₹99,240 (b) ₹1,02,840 (c) ₹1,12,640 (d) ₹92,315

107. How much approximate amount should you save annually to accumulate ₹20,00,000 by the end of 12 years, if the saving earns an interest of 14 percent compound annually?
[Given that $(1.14)^{12} = 4.8179$]

(a) ₹4,15,118 (b) ₹5,23,848 (c) ₹73,339 (d) ₹1,11,200

108. The difference between the compound interest and the simple interest on a certain sum at 12% per annum for 2 years is ₹90, when the interest is compounded annually. Then the sum is ₹_____. **(Sept 2025)**

(a) ₹6150 (b) ₹6050 (c) ₹6350 (d) ₹6250

109. If Mr. XYZ is investing ₹86,000 in a bank fixed deposit scheme where interest will be payable at 12% per annum, compounded half-yearly, what will be the effective rate of interest in a year? **(Sept 2025)**

(a) 12.36% (b) 12.24% (c) 12.12% (d) 12.48%

110. Mohan invests ₹25, 000 every year starting from today for next 5 years. Interest rate is 7% per annum compounded annually. The future value of the annuity is _____.
[Given: $(1 + 0.07)^5 = 1.40255$] **(Sept 2025)**
(a) ₹1,46,768 (b) ₹1,53,832 (c) ₹1,45,768 (d) ₹1,44,768

111. A machine is depreciated at the rate of 15% on reducing balance. The original cost of the machine was ₹2,00,000. In approximately how many years, the value of the machine was 54,500? (Given; $(0.85)^4 = 0.522$) **(Sept 2025)**
(a) 9 (b) 6 (c) 8 (d) 7

112. Mr. X borrowed ₹6,000 from Mr. Y at 10% per annum simple interest. After two years Mr. X wanted to repay this amount, Mr. Y insisted on paying the amount at compound interest at the same rate compounded annually. How much extra does Mr. X have to pay? **(Sept 2025)**
(a) ₹60 (b) ₹1,260 (c) ₹1,200 (d) ₹80

113. The compound interest on ₹1,00,000, compounded quarterly, for 9 months at the rate of 4% per annum is _____. **(Sept 2025)**
(a) 3,010.1 (b) 3,030.1 (c) 3,330.1 (d) 3,003.1

114. The difference between the compound interest and the simple interest on a certain sum at 12% per annum for 2 years is ₹90, when the interest is compounded annually. Then the sum is ₹_____. **(Sept 2025)**
(a) 6,150 (b) 6,050 (c) 6,350 (d) 6,250

115. Ms. Y invested ₹2,00,000 in a mutual fund equity scheme. She redeemed entire investment after 96 months and received ₹6,00,000 after redemption. What was the Compound Annual Growth Rate (CAGR) in percentage? (Given: $1.1472^4 = 1.732$) **(Sept 2025)**
(a) 14.72 (b) 15.72 (c) 13.72 (d) 12.72

116. A loan of ₹5,000 is lent for three years at the rate of 10% per annum, compounded semi-annually. The future value of the money is ₹_____.(Given: $1.05^6 = 1.34$) **(Sept 2025)**
(a) 6,500 (b) 6,600 (c) 6,700 (d) 6,800

117. If ₹2,470 is obtained as an interest in 4 years and 4 months at the rate of 3% per annum simple interest rate in bank deposit, how much amount was deposited in ₹ ? **(Sept 2025)**
(a) 17,000 (b) 18,000 (c) 19,000 (d) 20,000

118. The compound interest of ₹4,900 is ₹1661 for 2 years at a certain rate of interest, compounded annually. What is the rate of interest per annum in percentage? **(Sept 2025)**
(a) 19.71 (b) 17.71 (c) 13.71 (d) 15.71

119. You are interested in an investment of ₹5,000 in a fund that promises ₹50 at the end of each year, forever. What is the annual interest rate on this investment? **(Sept 2025)**
(a) 1% (b) 2% (c) 1.5% (d) 2.5%

120. An investment was priced at ₹100 per share in year 0, priced at ₹150 per share in the end of the first year, and priced ₹200 per share in the end of the second year. What is the Compound Annual Growth Rate (CAGR) of the investment? **(Sept 2025)**
(a) 21.42% (b) 31.42% (c) 41.42% (d) 51.42%

121. One person wants to have ₹20,000 at the end of six years. Hence, he deposits ₹_____ (rounded in rupee) in a fund that pays 3% per annum, compounded annually. (Given: $1.03^6 = 1.194$)

(Sept 2025)

(a) 17,000 (b) 17,250 (c) 17,750 (d) 16,750

122. Suppose you deposit ₹1,000 today, ₹2,000 after one year from today and ₹3,000 after two years from today, in a deposit that pays 10% per annum, compounded annually. What is the balance in the deposit at the end of two years i.e. just after deposit of ₹3,000? **(Sept 2025)**

(a) 6,000 (b) 6,410 (c) 6,600 (d) 6,800

Answer Key

1. (c)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (c)
11. (b)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (d)
21. (a)	**22.** (a)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (c)
31. (b)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (b)	**37.** (c)	**38.** (a)	**39.** (d)	**40.** (c)
41. (b)	**42.** (c)	**43.** (a)	**44.** (d)	**45.** (a)	**46.** (a)	**47.** (d)	**48.** (b)	**49.** (d)	**50.** (b)
51. (a)	**52.** (d)	**53.** (c)	**54.** (b)	**55.** (c)	**56.** (a)	**57.** (b)	**58.** (a)	**59.** (d)	**60.** (a)
61. (c)	**62.** (c)	**63.** (a)	**64.** (c)	**65.** (d)	**66.** (c)	**67.** (d)	**68.** (b)	**69.** (c)	**70.** (d)
71. (a)	**72.** (d)	**73.** (c)	**74.** (b)	**75.** (b)	**76.** (a)	**77.** (b)	**78.** (a)	**79.** (a)	**80.** (c)
81. (b)	**82.** (a)	**83.** (a)	**84.** (c)	**85.** (b)	**86.** (b)	**87.** (b)	**88.** (a)	**89.** (a)	**90.** (a)
91. (b)	**92.** (b)	**93.** (b)	**94.** (c)	**95.** (d)	**96.** (b)	**97.** (a)	**98.** (c)	**99.** (b)	**100** (c)
101 (b)	**102** (a)	**103** (c)	**104** (a)	**105** (d)	**106** (d)	**107** (c)	**108** (d)	**109** (a)	**110** (b)
111 (c)	**112** (a)	**113** (b)	**114** (d)	**115** (a)	**116** (c)	**117** (c)	**118** (d)	**119** (a)	**120** (c)
121 (d)	**122** (b)								

SUMMARY

- The sum of money received in the future is less valuable than it is today.
- **Interest:** Price paid by the borrower for the use of lender's money, Or we say if we borrow some money from a person for a certain time period then the extra amount paid than the initial amount is called interest.
- **Simple interest:** It is interest computed on principal for entire period of borrowing

  Formula : $S.I. = \frac{P \times R \times T}{100}$

  $A = P + S.I.$

  where,

  A = Final value of an investment (accumulated amount)

  P = Initial value of an investment (Principal amount)

  R = rate of interest in decimal

  S.I. = amount of interest

  T = time in years

- **Compound interest:** Interest that occurs when earning for each specified time period is added to the principal amount.

  Formula: $C.I = P\left(1+\frac{r}{100}\right)^n - P$

  $A = P\left(1+\frac{r}{100}\right)^n$

  where,

  P = Initial value of an investment (Principal Amount)

  r = Annual rate of interest

  n = t × no. of conversions per year (c)

- **Effective rate of interest:** Effective interest rate is calculated by formula

  $E = \left(\left(1+\frac{r}{100c}\right)^c - 1\right) \times 100\%$

- **Annuity** can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.
  - Annuity may be of two types:

  (i) Annuity Regular: In annuity regular first payment/receipt takes place at the end of first period.

  (ii) Annuity Due or Annuity Immediate: When the first receipt or payment is made today (at the beginning of the annuity) it is called annuity due or annuity immediate.

- **Future Value of Annuity Regular:** If A be the periodic payments, the future value A (n,i) of the annuity is given by

  $A(n,i) = A\left[\frac{(1+i)^n - 1}{i}\right]$

- **Future value of an Annuity Due/Annuity Immediate** = Future value of annuity regular × (1 + i) where i is the interest rate in decimal

- **Present value of an Annuity Regular:** The present value P of the Annuity A due at the end of n period at the rate of r per interest period may be obtained by

  $P.V. = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

- **Present Value of an Annuity Due or Annuity Immediate:**

  (i) Calculate the present value for (n – 1) period as annuity regular.

  (ii) Add the initial payment (receipt) in the above.

- **Capital Expenditure (investment decision):** Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment.

- **Valuation of Bond:** A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issued for a fixed term longer than one year.
- **Perpetuity:** When you get an annuity for unlimited time, it becomes Perpetuity.

$$PVA_{\infty} = \frac{R}{i}$$

- **Calculating Growing Perpetuity:**

Growing perpetuity means the periodic instalment is increasing with fixed interest rate.

$$PVA = \frac{A}{i-g}$$

- **Rate of Return:**

Net present value = Present value of cash inflow – Present value of cash outflow

- **Compound Annual Growth Rate (CAGR):**

It is the method to find the smoothed annualized gain of an investment over a given time period

$$CAGR(t_0, t_n) = \left(\frac{V(t_n)}{V(t_0)}\right)^{\frac{1}{t_n - t_0}} - 1$$

where, V (t_0) = Beginning Period and V (t_n) = End Period

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CA WALLAH

CHAPTER

05

Basic Concepts of Permutations and Combinations

Permutation and combination are essential concepts that help solve complex counting problems. For instance, let's consider the scenario of arranging members of the Lok Sabha in different seating arrangements.

FUNDAMENTAL PRINCIPLE OF COUNTING

- **Multiplication Rule:** If one task can be done in m ways and another task which is independent of the first task can be done in n ways, after the first task has been performed, then the number of possible ways in which both the tasks can be done simultaneously is m × n.

  E.g.: Let's say Task A is choosing a shirt from a collection of 5 different shirts, and Task B is selecting a pair of pants from a collection of 3 different pants. Using the Multiplication Rule:

  Number of ways to choose a shirt = 5

  Number of ways to select a pair of pants = 3

  Total number of ways to choose a shirt and pants simultaneously = 5 × 3 = 15

Example 1. Raghav has 3 different types of shirts and 2 different types of trousers. Whenever he goes out, he likes to wear a shirt and a trouser. In how many ways can he decide what to wear?

(a) 5 (b) 4 (c) 6 (d) 8

Sol. (c) Given, Number of shirts = 3

Number of trousers = 2

Total ways = 3 × 2 = 6

Hence, the correct option is (c).

- **Addition Rule:** If one task can be done in m ways and another task which is independent of the first task can be done in n ways, then the total number of ways either of them can perform is m + n.

  E.g.: Task A: Choosing a dessert from a menu with 4 options.

  Task B: Selecting a drink from a menu with 3 options.

  Number of ways to choose a dessert = 4

  Number of ways to select a drink = 3

  Total number of ways to either choose a dessert or select a drink = 4 + 3 = 7

Example 2. Rani has 3 different types of shoes and 2 different types of sandals. Whenever she goes out, she likes to wear either a shoe or a sandal. In how many ways can she decide what to wear?

(a) 3 (b) 2 (c) 5 (d) 6

Sol. (c) Given, Number of shoes = 3

Number of sandals = 2

Since, she likes to wear either a shoe or a sandal, thus

Total required ways = 3 + 2 = 5

Hence, the correct option is (c).

What we understand is that, if "And" comes in a statement use MULTIPLICATION RULE, when "Or" comes in a statement use ADDITION RULE.

Example 3. A room has 6 doors. In how many ways can a man enter the room through one door and come out through a different door?

(a) 30 (b) 36 (c) 64 (d) 80

Sol. (a) Given, Total doors = 6

If entered from one door then there are 6 options available and for exit, there are 5 doors.

Total ways = 5 × 6 = 30

Hence, the correct option is (a).

Example 4. In a class there are 27 boys and 14 girls. The teacher wants to select 1 boy or 1 girl to represent the class in a function. In how many ways can the teacher make this selection?

(a) 41 (b) 224 (c) 278 (d) 378

Sol. (a) Given, Total boys = 27

Total girls = 14

Since, the teacher wants to select 1 boy or 1 girl as class representative, thus

Total required ways = 27 + 14 = 41

Hence, the correct option is (a).

Example 5. Given 4 flags of different colors, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

(a) 5 (b) 4 (c) 12 (d) 8

Sol. (c) Given, There are 4 flags of different colors.

Since, a signal requires the use of 2 flags one below the other.

Thus, the selection of the first signal can be done in 4 ways and that of the second can be done in 3 ways.

Using, multiplication rule,

Total required signals = 4 × 3 = 12

Hence, the correct option is (c).

Example 6. In a monthly test, the teacher decides that there will be three questions, one from each of Exercise 7, 8 and 9 of the textbooks. If there are 12 questions in Exercise 7, 18 in Exercise 8 and 9 in Exercise 9, in how many ways can three questions be selected?

(a) 1214 (b) 1234 (c) 6000 (d) 1944

Sol. (d) Given, Number of questions in Exercise 7 = 12,
Number of questions in Exercise 8 = 18,
Number of questions in Exercise 9 = 9
Total ways of selecting three questions = 12 × 18 × 9 = 1944
Hence, the correct option is (d).

Example 7. In how many ways can 5 letters be posted in 4 letter boxes?

(a) 120 (b) 1024 (c) 625 (d) None of these

Sol. (b) Total number of letters = 5
Total number of letter boxes = 4
∴ For each letter, there are 4 options (letter boxes) where it can be posted.
Thus, total number of ways = 4 × 4 × 4 × 4 × 4 = 1024
Hence, the correct option is (b).

Example 8. Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, when

(I) the repetition of the letters is not allowed.
(II) the repetition of the letters is allowed.

(a) 16, 186 (b) 24, 256 (c) 24, 172 (d) None of these

Sol. (b) Given word = ROSE
Total letters = 4
(I) repetition of letters is not allowed = 4 × 3 × 2 × 1 = 24
(II) repetition of letters is allowed = 4 × 4 × 4 × 4 = 4^4 = 256
Hence, the correct option is (b).

PRACTICE QUESTIONS (PART A)

1. There are 10 trains running between Calcutta and Delhi. The number of ways in which a person can go from Calcutta to Delhi and return by a different train is

(a) 99 (b) 90 (c) 80 (d) None of these

2. You have four different colors of socks, and you want to wear one sock of each color. In how many ways can you choose which socks to wear?

(a) 8 ways (b) 12 ways (c) 16 ways (d) 24 ways

3. A person can go from place 'A' to 'B' by 11 different modes of transport but is allowed to return back to 'A' by any mode other than the one earlier. The number of different ways, the entire journey can be complete is

(a) 110 (b) 10^{10} (c) 9^5 (d) 10^9

4. There are ten flights operating between city A and city B. The number of ways in which a person can travel from city A to city B and return by different flight is

(a) 90 (b) 95 (c) 80 (d) 78

5. Consider the word "TRAIN". How many distinct 5-letter words can be formed from its letters without repeating any letter?

(a) 120 (b) 24 (c) 360 (d) 30

6. How many 3 digit odd numbers can be formed using the digits 5, 6, 7, 8, 9, if the digits can be repeated?

(a) 55 (b) 75 (c) 65 (d) 36

7. In how many ways can 3 letters be posted in 4 letter boxes?

(a) 24 (b) 27 (c) 64 (d) None of these

8. How many 4-letter codes can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

(a) 10,000 (b) 5040 (c) 2520 (d) None of the above

9. There are 4 routes between Delhi and Patna. In how many ways can a man go from Delhi to Patna and return if for returning

I. Any of the route is taken

II. The same route is taken

III. The same route is not taken?

(a) 16, 4, 12 (b) 16, 4, 16 (c) 4, 4, 12 (d) None of these

10. How many numbers are there between 100 and 1000 in which all the digits are distinct?

(a) 1000 (b) 900 (c) 648 (d) 504

Answer Key

1. (b) **2.** (b) **3.** (a) **4.** (a) **5.** (a) **6.** (b) **7.** (c) **8.** (b) **9.** (a) **10.** (c)

THE FACTORIAL

If we take a number 'n', then n! or ⌊n is called the factorial of 'n' and the value of n! is equal to multiplication of 1 to n i.e. n! = 1 × 2 × 3 × 4 ×... × n or we can write,

n! = n(n – 1)(n – 2)... 3 × 2 × 1

Note: 0! = 1

Example 9. Find the value of

(I) 8! (II) 5! (III) $\frac{10!}{4!}$

(a) 5040, 720, 75600 (b) 20,160, 120, 151200

(c) 40,320, 120, 151200 (d) 40,320, 520, 151100

Sol. (c) We know that,

n! = n(n – 1)(n – 2)... 3 × 2 × 1

(I) 8! = 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 40,320

(II) 5! = 5 × 4 × 3 × 2 × 1 = 120

(III) $\frac{10!}{4!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1} = 10\times9\times8\times7\times6\times5 = 1,51,200$

Hence, the correct option is (c).

Example 10. Find x if $\frac{1}{6!}+\frac{1}{5!}=\frac{x}{7!}$.

(a) 49 (b) 81 (c) 88 (d) 91

Sol. (a) Given,

$$\frac{1}{6!}+\frac{1}{5!}=\frac{x}{7!} \Rightarrow \frac{1}{6!}+\frac{6}{6\times 5!}=\frac{x}{7!} \Rightarrow \frac{1}{6!}+\frac{6}{6!}=\frac{x}{7!}$$

$$\Rightarrow \frac{1+6}{6!}=\frac{x}{7!} \Rightarrow \frac{7}{6!}=\frac{x}{7!} \Rightarrow x=\frac{7!\times 7}{6!}$$

$\Rightarrow x = 7 \times 7 = 49$

Hence, the answer is option (a).

Example 11. Find the value of n if (n + 1)! = 30(n – 1)!.

(a) 6 (b) 5 (c) 7 (d) None of these

Sol. (b) Given, (n + 1)! = 30(n – 1)!

$\Rightarrow$ (n + 1) × n × (n – 1)! = 30(n – 1)!

$\Rightarrow$ (n + 1) × n = 30

$\Rightarrow$ (n + 1) × n = 6 × 5 $\Rightarrow$ n + 1 = 6

$\Rightarrow$ n = 5

Hence, the correct option is (b).

Example 12. Justify 0! = 1.

Sol. We know that,

n! = n(n – 1)(n – 2) ×... × 3 × 2 × 1

Let us consider for n = 3

3! = 3 × 2 × 1

$$\Rightarrow \frac{3!}{3}=2!$$

$$\text{or } 2!=\frac{3!}{3}$$

In general, $(n-1)!=\frac{n!}{n}$

Put n = 1 in above equation, we get

$$(1-1)!=\frac{1!}{1} \Rightarrow 0!=\frac{1}{1} \Rightarrow 0!=1$$

PRACTICE QUESTIONS (PART B)

1. Find the value of 6!.

(a) 720 (b) 360 (c) 120 (d) None of these

2. The value of $\frac{9!}{5!}$ is

(a) 362,880 (b) 15,120 (c) 3024 (d) None of these

3. The value of $\frac{10!}{6!4!}$ is

(a) 210 (b) 360 (c) 720 (d) None of these

4. The value of $\frac{5!}{10!}$ is

(a) $\frac{1}{30240}$ (b) 30240 (c) 120 (d) 36,28,800

5. Find n if (n + 2)! = 12(n)!.

(a) 4 (b) 3 (c) 6 (d) None of these

6. Find the value of x if $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$.

(a) 11 (b) 121 (c) 150 (d) None of these

7. Find n if (n + 1)! = 12 x (n-1).

(a) 3 (b) 4 (c) 7 (d) None

8. If $\frac{n!}{2!(n-2)!}$ & $\frac{n!}{4!(n-4)!}$ are in the ratio 2:1. Find the value of n.

(a) 2 (b) 3 (c) 4 (d) 5

Answer Key

1. (a) **2.** (c) **3.** (a) **4.** (a) **5.** (a) **6.** (b) **7.** (a) **8.** (d)

LET US UNDERSTAND THE TWO WORDS

1. PERMUTATION: In Permutation, we consider the arrangement of objects in a specific order. The order of the arrangement matters.

For example, let's consider the football team selection scenario from your class of 30 students. The coach wants to select 11 students for the football team and arrange them in specific positions: forward player, midfielder, backward player, and goalkeeper.

In this case, the order of selection and arrangement of the players is important.

2. COMBINATION: In Combination, we consider the selection of objects without any specific order. The order of the selection does not matter.

Continuing with the football team example, if we only want to select 11 students for the team without considering their specific positions, we are dealing with combinations. Here, the order in which the students are selected does not matter.

In case of Permutation: We do arrangements, and we say order matters.

In case of Combination: We do selection, and we say order doesn't matter.

PERMUTATIONS

A permutation determines the number of possible arrangements in a set when the order of the arrangements matters. It can be calculated using the formula:

- $^{n}P_{r} = \frac{n!}{(n-r)!}$, where $r \leq n$

where n is the total number of objects and r is the number of objects to be arranged.

E.g., If we have three different boxes: one Yellow, one Green, and one Red and we want to arrange them on a table, the number of possible arrangements (permutations) would be:

$$^{3}P_{3} = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 6$$

In the scenario where all n students are winners, the permutation formula becomes:

$$^{n}P_{n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

Example 13. Evaluate each of the following:

(I) $^{4}P_{2}$ (II) $^{7}P_{3}$ (III) $^{10}P_{6}$

Sol. (I) $^{4}P_{2} = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4\times3\times2!}{2!} = 4 \times 3 = 12$

(II) $^{7}P_{3} = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7\times6\times5\times4!}{4!} = 210$

(III) $^{10}P_{6} = \frac{10!}{(10-6)!} = \frac{10\times9\times8\times7\times6\times5\times4!}{4!} = 151200$

Example 14. In $^{n}P_{r}$, the restriction is **(ICAI)**

(a) $n > r$ (b) $n \geq r$ (c) $n \leq r$ (d) None of these

Sol. (b) $^{n}P_{r}$ stands for the permutation of n objects taken r at a time. In other words, it is the number of ways in which r objects can be selected from n distinct objects, where the order of selection matters.

The restriction in $^{n}P_{r}$ is that we can select only r objects out of n objects, and the order of selection matters. This means that we cannot select more than r objects or less than r objects, and we have to select them in a specific order.

Therefore, the correct answer is option 'b', which states that the restriction in $^{n}P_{r}$ is $n \geq r$. This means that we need at least r objects to select from a total of n objects.

Hence, the answer is option (b).

Example 15. If $^{n}P_{4} = 12 \times {}^{n}P_{2}$, then n is equal to **(ICAI)**

(a) –1 (b) 6 (c) 5 (d) None of these

Sol. (b) **Detailed method:**

Given, ${}^{n}P_4 = 12 \times {}^{n}P_2$

$$\Rightarrow \frac{n!}{(n-4)!} = 12\frac{n!}{(n-2)!}$$

$$\Rightarrow \frac{[n(n-1)(n-2)(n-3)(n-4)!]}{(n-4)!} = 12\frac{n(n-1)(n-2)!}{(n-2)!}$$

$\Rightarrow n(n-1)(n-2)(n-3) = 12n(n-1)$

$\Rightarrow n(n-1)[(n-2)(n-3) - 12] = 0$

$\Rightarrow n(n-1)(n^2 - 5n + 6 - 12) = 0$

$\Rightarrow n(n-1)(n^2 - 5n - 6) = 0$

$\Rightarrow n(n-1)[n^2 - (6-1)n - 6] = 0$

$\Rightarrow n(n-1)[n^2 - 6n + n - 6] = 0$

$\Rightarrow n(n-1)[n(n-6) + 1(n-6)] = 0$

$\Rightarrow n(n-1)(n-6)(n+1) = 0$

$\Rightarrow n = 0$ or $n - 1 = 0$ or $n - 6 = 0$ or $n + 1 = 0$

$\Rightarrow n = 0$ or $n = 1$ or $n = 6$ or $n = -1$

Since, n cannot be negative.

Therefore, $n \neq -1$

If n = 0 or 1, then in those cases (n – 2) and (n – 3) are not positive integers.

Thus, n = 6 is the correct answer.

Go by choices:

Given: ${}^{n}P_4 = 12 \times {}^{n}P_2$

We know that,

For ${}^{n}P_r$, $n \geq r$

Also, n and r are positive.

Thus, n cannot be –1.

For option (b): n = 6

$$\Rightarrow \text{LHS: } {}^{6}P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 6\times5\times4\times3 = 360$$

$$\Rightarrow \text{RHS: } 12 \times {}^{6}P_2 = 12 \times \frac{6!}{(6-2)!} = 12\times\frac{6!}{4!} = 12\times6\times5 = 360$$

Clearly, LHS = RHS

Therefore, the required value of n is 6.

Hence, the answer is option (b).

Example 16. If ${}^{5}P_r = 60$, then the value of r is **(ICAI)**

(a) 3 (b) 2 (c) 4 (d) None of these

Sol. (a) $\Rightarrow \frac{5!}{(5-r)!} = 60 \Rightarrow \frac{5\times4\times3\times2}{(5-r)!} = 60 \Rightarrow \frac{120}{(5-r)!} = 60$

$\Rightarrow (5 - r)! = 2$

Since, $(2)! = 2$

Therefore, $(5 - r) = 2$

$r = 5 - 2 = 3$

Hence, the answer is option (a).

Example 17. If ${}^{n_1+n_2}P_2 = 132$ and ${}^{n_1-n_2}P_2 = 30$, then **(ICAI)**

(a) $n_1 = 6, n_2 = 6$ (b) $n_1 = 10, n_2 = 2$

(c) $n_1 = 9, n_2 = 3$ (d) None of these

Sol. (c) Given, ${}^{n_1+n_2}P_2 = 132 \Rightarrow \frac{(n_1+n_2)!}{(n_1+n_2-2)!} = 132$ ...(i)

Also ${}^{n_1-n_2}P_2 = 30$

$\Rightarrow \frac{(n_1-n_2)!}{(n_1-n_2-2)!} = 30$ ...(ii)

Go by choices:

For option (c): $n_1 = 9, n_2 = 3$

$\Rightarrow \frac{(9-3)!}{(9-3-2)!} = 30$ (eq. ii)

$\Rightarrow \frac{(6)!}{(4)!} = 30 \Rightarrow \frac{6\times5\times4!}{(4)!} = 30$

$\Rightarrow 30 = 30$

$\Rightarrow$ LHS = RHS

Therefore, the required values are $n_1 = 9, n_2 = 3$.

Hence, the answer is option (c).

Example 18. How many 4 letter words can be formed from 'COMPUTER'?

(a) 1223 (b) 1680 (c) 7880 (d) 7200

Sol. (b) The word "COMPUTER" has 8 different letters and we have to form words using any 4 letters from the 8 given letters. Therefore, total numbers of words that can be formed:

${}^8P_4 = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8\times7\times6\times5\times4!}{4!} = 8\times7\times6\times5 = 1680$

Hence, the answer is option (b).

Example 19. The number of arrangements of the letters in the word `FAILURE', so that vowels are always coming together is **(ICAI)**

(a) 576 (b) 575 (c) 570 (d) None of these

Sol. (a) Given word: FAILURE

Since, all the 4 vowels should be coming together thus we will assume all 4 vowels as one letter i.e. letters would be F, L, R and AEIU

Now, the possible arrangement of above 4 letters = 4!

But vowels can also rearrange their positions (AEIU, AIUE, AUIE, etc.)

Total possible arrangement of vowels = 4!

Therefore, total arrangements will be 4! × 4! = 24 × 24 = 576

Hence, option (a) is correct.

Example 20. 10 examination papers are arranged in such a way that the best and worst papers never come together. The number of arrangements is **(ICAI)**

(a) 9 × 8! (b) 10! (c) 8 × 9! (d) None of these

Sol. (c) Number of ways in which 10 papers can be arranged is 10!

When worst and best paper come together come together consider them as one paper

We will have only 9 papers, so 9 papers can be arranged in 9! ways

And two papers can arrange themself in 2! ways

So, the number of arrangements when the worst and best don't come together

= 10! – 9! × 2!

= 10 × 9! – 9! × 2!

= 9! × (10 – 2) = 8 × 9!

Hence, option (c) is correct.

Example 21. 3-digit numbers to be formed out of the figures 0, 1, 2, 3, 4, 7, 8, 9 (no digit is repeated) then number of such numbers is

(a) 336 (b) 294 (c) 1050 (d) None of these

Sol. (b) Given digits: 0, 1, 2, 3, 4, 7, 8, 9

We can't use 0 for the first place, so there are 7 ways to fill the first place.

Now, second place can be filled in 7 ways since at second place 0 can be filled.

Now, for third place there are 6 ways to fill it.

Thus, there are 7 × 7 × 6 = 294 ways to form a 3–digit number using the given digits.

Hence, the correct option is (b).

Example 22. If 19 states teams are participating in a national singing contest then the number of ways the first, second and third positions may be won is

(a) 5814 (b) 93024 (c) 342 (d) None of these

Sol. (a) Given, Number of teams participating = 19

So, number of ways first, second and third positions may be won is ${}^{19}P_3$

$$\Rightarrow \frac{19!}{(19-3)!} = \frac{19!}{16!}$$

$\Rightarrow 19 \times 18 \times 17 = 5814$

Hence, the answer is option (a).

Example 23. In how many ways among 8 students – (5 boys and 3 girls), can school select School Prefect, Head boy and Head Girl, if no member can hold two positions and each boy and girl is eligible for School Prefect too?

(a) 90 (b) 150 (c) 60 (d) None of these

Sol. (a) Given,

Total students = 8

Number of boys = 5

Number of girls = 3

Now, if a girl is selected as a prefect, then,

Number of ways of choosing head girl and a girl prefect = ${}^3P_2 = 6$

Number of ways of choosing head boy is ${}^5P_1 = 5$

Similarly, if a boy is selected as a prefect, then

Number of ways of selecting head girl = ${}^3P_1 = 3$

Number of ways of choosing head boy and a boy prefect is ${}^5P_2 = 20$

Therefore, total no. of ways = $6 \times 5 + 3 \times 20$

$= 30 + 60 = 90$

Therefore, there are a total 90 required ways.

Hence, the correct option is (a).

Example 24. The number of ways the letters of the word `TRIANGLE' to be arranged so that the word 'ANGLE' will be always present is

(a) 20 (b) 60 (c) 24 (d) 32

Sol. (c) Given, In the word 'TRIANGLE' the word 'ANGLE' should always be present.

Thus, consider 'ANGLE' as one letter so the letters will be T, R, I and 'ANGLE'.

Therefore, there are 4 letters which can be arranged in 4! i.e. 24 ways.

Hence, option (c) is correct.

CIRCULAR PERMUTATION

Circular Permutation refers to arrangements where the objects or individuals are arranged in a circular manner. Unlike linear permutation, where the arrangement is in a straight line, circular permutation involves arranging objects in a circular form.

DIFFERENCE BETWEEN LINEAR PERMUTATION AND CIRCULAR PERMUTATION

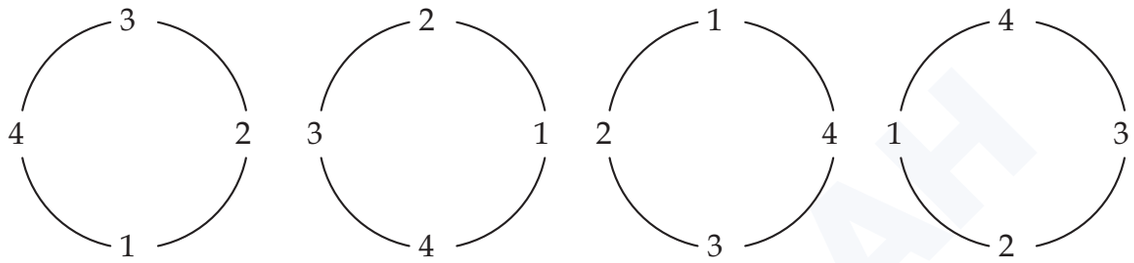
Let's consider two scenarios: the sitting arrangement of a langar (community meal) and the sitting arrangement of a circular table party.

In the case of a langar, where people sit in a line, it follows the concept of linear permutation. Each person has a distinct seat, and the order in which they sit matters. For example, if there are 4 people, the number of linear permutations would be 4!.

On the other hand, in the case of a circular table party, the arrangement is circular. The seats are arranged in a circle, and the order in which individuals sit becomes significant. Circular permutation takes into account that the arrangement repeats after one full revolution. To calculate the number of circular permutations, we use the formula (n – 1)!, where n represents the number of objects or individuals.

For example, if there are 4 people at the circular table, the number of circular permutations would be 3!.

Let's say there are 4 people sitting in circular arrangement as shown in the below figure:



Clearly, all the four arrangements are similar to each other.

i.e., these 4-people permutations equal to one in circular.

Thus, n ordinary permutations are equal to one permutation.

Hence, there are ${}^{n}P_{n}$ / n ways in which n things can be arranged in circular permutations which is equal to (n – 1)!.

Example 25. The number of ways in which 6 boys form a ring is

(a) 100 (b) 110 (c) 120 (d) None of these

Sol. (c) We know that, the number of circular permutations of n different things chosen at a time (n – 1)!

Thus, the number of ways in which 6 boys form a ring is (6 – 1)! =120

Hence, option (c) is correct i.e., 120.

Example 26. The number of ways in which 7 boys sit around a table so that two particular boys may sit together is **(ICAI)**

(a) 240 (b) 200 (c) 120 (d) None of these

Sol. (a) Consider two boys as one, so we have 6 boys.

We know that, the number of circular permutation of n different things chosen at a time (n – 1)!.

So, the number of ways in which 6 boys form a ring is (6 – 1)! =120

But two boys can arrange themselves in 2! ways.

So, the number of ways in which 7 boys sit around a table so that two particular boys may sit together is

2! × 120 = 240.

Hence, option (a) is correct i.e., 240.

Example 27. 3 ladies and 3 gents can be seated at a round table so that any two and only two of the ladies sit together. The number of ways is **(ICAI)**

(a) 70 (b) 27 (c) 72 (d) None of these

Sol. (c) We have 3 ladies and 3 gents.

We can select 2 ladies out of 3 in ${}^3P_2 = 6$ ways

Also, only two ladies sitting together means the remaining lady can't sit adjacent to selected, which leaves us with the option that selected ladies are surrounded by gents.

Now, 4 seats remain and the remaining lady can't take the adjacent one, which leaves 2 ways for her.

Gents can be arranged in 3! ways on the remaining seats.

So, the total number of ways = $6 \times 2 \times 3! = 72$

Hence, option (c) is correct.

Example 28. 5 persons are sitting in a round table in such way that tallest person is always on the right–side of the shortest person; the number of such arrangements is **(ICAI)**

(a) 6 (b) 8 (c) 24 (d) None of these

Sol. (a) Assume the tallest and shortest person as one.

Now, we have a total of 4 persons.

We know that, the number of circular permutations of n different things chosen at a time $(n-1)!$.

Therefore, the number of such arrangements is $(4-1)! = 3! = 6$

Hence, option (a) is correct i.e., 6.

Example 29. If 50 different jewels can be set to form a necklace then the number of ways is

(a) $\frac{50!}{2}$ (b) $\frac{49!}{2}$ (c) 49! (d) None of these **(ICAI)**

Sol. (b) We know that,

Since, in forming a necklace or a garland there is no difference between a clockwise and anti-clockwise direction because we can simply turn it over so that clockwise becomes anti clockwise and vice versa.

Thus, number of necklaces that can be formed with n beads of different colours is $\frac{(n-1)!}{2}$

So, if 50 different jewels can be set to form a necklace then the number of ways is $\frac{(50-1)!}{2} = \frac{49!}{2}$.

Hence, option (b) is correct i.e. $\frac{49!}{2}$.

PERMUTATION WITH RESTRICTIONS

Theorem 1: Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is ${}^{n-1}P_r$.

Example 30. How many ways 5 glasses of Coca – cola can be served to 10 people if one says that he does not drink coca cola?

(a) 10000 (b) 27506 (c) 6290 (d) 15120

Sol. (d) Since Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is ${}^{n-1}P_r$

So, here n = 10, r = 5

Total ways: ${}^{9}P_5$

$\Rightarrow \frac{9!}{4!} = 9 \times 8 \times 7 \times 6 \times 5$

$\Rightarrow 15120$

Hence, the correct option is (d).

Theorem 2: Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is: r. ${}^{n-1}P_{r-1}$

Example 31. The number of arrangements of 6 different things taken 3 at a time in which one particular thing always occurs is:

(a) 60 (b) 25 (c) 30 (d) None of these

Sol. (a) We know that, number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is r. ${}^{n-1}P_{r-1}$.

So, the number of arrangements of 6 different things taken 3 at a time in which one particular thing always occurs is: $3 \times {}^{6-1}P_{3-1} = {}^{5}P_2 \times 3$

$\Rightarrow 5 \times 4 \times 3 = 60$

Hence, option (a) is correct.

Example 32. How many four digit numbers greater than 5000 can be formed out of the digits 0, 1, 2, 3, 5, 7, 8, 9 if no digit is repeated in any number?

(a) 330 (b) 840 (c) 460 (d) None of these

Sol. (b) Given digits: 0, 1, 2, 3, 5, 7, 8, 9

Four digits number greater than 5000 that can be formed out of the given digits can begin with 5, 7, 8 or 9

Thus, possible ways for thousands place = 4

So, rest 3 digits can be chosen in 7 × 6 × 5

Therefore, the required ways:

$\Rightarrow 4 \times 7 \times 6 \times 5 = 840$

Hence, option (b) is correct.

Example 33. The sum of all 4-digit number containing the digits 2, 4, 6, 8, without repetitions is

(a) 1,33,330 (b) 1,22,220 (c) 2,13,330 (d) 1,33,320

Sol. (d) (Hint, Formula: (n – 1)! × Sum of digits × (11111... n times)

Here, n = 4

Thus, (n – 1)! = (4 – 1)! = 3! = 6

Sum of digits = 2 + 4 + 6 + 8 = 20

Therefore, the required sum = 6 × 20 × 1111 = 133,320

Hence, the correct option is (d).

PRACTICE QUESTIONS (PART C)

1. Evaluate ${}^5P_3$

(a) 30 (b) 60 (c) 120 (d) None of these

2. How many numbers divisible by 5 of 6 digits can be made from the digit 2, 3, 4, 5, 6, 7?

(a) 120 (b) 600 (c) 240 (d) None of these

3. Find the value of n if ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$.

(a) 3 (b) 9 (c) 10 (d) None of these

4. In how many different ways can five persons stand in a line for a group photograph?

(a) 120 (b) 60 (c) 20 (d) None of these

5. If there are 6 books on Accounts, 3 on Business Mathematics and 2 on Economics. In how many ways can we place them if the books on the same subject are to be together?

(a) 5,184 (b) 8,640 (c) 25,920 (d) 51,840

6. The number of ways in which the letters of the word 'DOGMATIC' can be arranged is:

(a) 40319 (b) 40320 (c) 40321 (d) None of these

7. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

(a) 40320 (b) 5040 (c) 6720 (d) None of these

8. The total number of ways in which six '+' and four '–' sign can be arranged in a line such that no two '–' sign occur together is **(ICAI)**

(a) $\frac{7!}{3!}$ (b) $\frac{6! \times 7!}{3!}$ (c) 35 (d) None of these

9. How many numbers of seven digit numbers which can be formed from the digits 3, 4, 5, 6, 7, 8, 9 no digits being repeated are not divisible by 5?

(a) 4320 (b) 4690 (c) 3900 (d) 3890

10. In how many ways can the letters of the word 'STRANGE' be arranged so that the vowels come together?

(a) 120 (b) 720 (c) 1440 (d) None of these

11. The number of ways in which 7 girls form a ring is

(a) 120 (b) 710 (c) 720 (d) None of these

12. How many arrangements can be made out of the letters of the word 'RAJESH', the vowels never beings separated?

(a) 120 (b) 240 (c) 360 (d) None of these

13. How can we arrange 10 people on a circular table such that two people do not want to sit together?

(a) $7 \times 8!$ (b) $10 \times 9!$ (c) $10!$ (d) None of these

14. There are 6 varieties of Rice, 3 of Breads and 2 of sweets. In how many ways can these be placed on the table if the same types of dishes to be are to be together?

(a) 15,000 (b) 20,450 (c) 51,840 (d) None of these

15. How many different numbers can be formed by using any five out of six digits 1, 2, 3, 4, 5, 6, no digit being repeated in any number?

(a) 1 (b) 36 (c) 120 (d) 720

16. A group of 8 students is forming a circle for a game. In how many ways can they arrange themselves if rotations of the same arrangement are considered the same?

(a) 5760 (b) 1680 (c) 5040 (d) 720

17. The number of numbers lying between 100 and 1000 can be formed with the digits 1, 2, 3, 4, 5, 6, 7 is **(ICAI)**

(a) 210 (b) 200 (c) 110 (c) None of these

18. The number of arrangements in which the letters of the word 'TABLE' be arranged so that the words thus formed begin with T and do not end with E is

(a) 18 (b) 120 (c) 24 (d) None of these

19. Find the sum of all the 4 digit numbers that can be formed with the digits 3,4,5 and 6

(a) 119988 (b) 11988 (c) 191988 (d) None of these

20. The number of arrangements of 10 different things taken 4 at a time in which one particular thing always occurs is

(a) 2015 (b) 2016 (c) 2014 (d) None of these

21. The value of $\Sigma_{r=1}^{9} r \cdot {}^{r}P_{r}$ is

(a) 10! (b) 10! - 1 (c) 11! (d) 9! - 1

22. The number of 4 digit numbers greater than 5,000 can be formed out of the digits 3,4,5,6 and 7 (no digit is repeated). The number of such is

(a) 72 (b) 27 (c) 70 (d) None of these

23. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

(i) 4 letters are used at a time,

(ii) all letters are used at a time,

(iii) All letters are used but the first letter is a vowel?

(a) 240, 720, 240 (b) 360, 720, 240

(c) 360, 720, 240 (d) None of these

24. How many words can be formed by using all the letters of the word "ALLAHABAD"? In how many of them both L do not come together?

(a) 3,628,800; 5880 (b) 40,320; 5880

(c) 7560, 5880 (d) None of these

25. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

(a) 33810 (b) 16905 (c) 8620 (d) None of these

Answer Key

1. (b)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (c)
11. (c)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (b)
21. (b)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (a)					

COMBINATION

The number of ways in which selection is done where order does not matter can be calculated as ${}^{n}C_{r}$ where ${}^{n}C_{r} = \frac{n!}{r! \times (n-r)!}$.

Example 34. ${}^{12}C_{8}$ =

(a) 215 (b) 495 (c) 745 (d) None of these

Sol. (b) We know that, ${}^{n}C_{r} = \frac{n!}{r! \times (n-r)!}$

Thus, ${}^{12}C_{8} = \frac{12!}{8! \times (12-8)!}$

$= \frac{12 \times 11 \times 10 \times 9 \times 8!}{4 \times 3 \times 2 \times 1 \times 8!}$

$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = 495$

So, the value of ${}^{12}C_{8}$ is 495.

Hence, option (b) is correct i.e., 495.

Example 35. In how many ways can I select 5 cards from a pack of 52?

(a) 2598960 (b) 2598830 (c) 2600480 (d) None of these

Sol. (a) Total number of ways 5 cards can be selected from 52 pack of cards is:

$\Rightarrow {}^{52}C_{5} = \frac{52!}{5!(52-5)!}$

$\Rightarrow \frac{52!}{5! \times 47!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2} = 2598960$

Hence, the correct option is (a).

Example 36. The number of straight lines obtained by joining 16 points on a plane, no three of them being on the same line is

(a) 120 (b) 110 (c) 210 (d) None of these

Sol. (a) Total points = 16

We know, to form a straight line, 2 points are required.

Since, no three points are in same line thus the number of straight line that can be formed from 16 points

$$= {}^{16}C_2 = \frac{16!}{14! \times 2!} = 8 \times 15 = 120$$

Therefore, the required number of straight lines is 120.

Hence, the correct option is (a).

Example 37. Out of 7 boys and 4 girls, a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is

(a) 429 (b) 439 (c) 419 (d) 441

Sol. (d) Given,

Number of boys = 7

Number of girls = 4

Number of teams with at least one girl = Total number of teams - Number of teams with no girl = ${}^{11}C_5 - {}^{7}C_5$

$$= \frac{11!}{6! \times 5!} - \frac{7!}{2! \times 5!} = 462 - 21 = 441$$

Hence, the correct option is (d) i.e. 441.

Example 38. If there are 40 guests in a party. If each guest takes a shake hand with all the remaining guests. Then the total number of hands shake is

(a) 780 (b) 840 (c) 1560 (d) 1600

Sol. (a) No. of guests = 40

As we know for the shake hands, 2 persons are required.

Thus, total number of hand shakes = ${}^{40}C_2$

$$= \frac{40!}{(40-2)! \times 2!} = \frac{40!}{38! \times 2!} = \frac{40 \times 39}{2} = 780$$

Hence, the correct answer is option (a) i.e. 780.

Example 39. The number of diagonals in a decagon is **(ICAI)**

(a) 30 (b) 35 (c) 45 (d) None of these

Sol. (b) We know that, decagon have 10 sides.

The number of diagonals in a polygon having n sides is ${}^{n}C_2 - n$ or $\frac{1}{2}n(n-3)$

Here, n = 10

Thus, number of diagonals

$= \frac{1}{2} \times 10 \times (10-3) = 5 \times 7 = 35$

Therefore, the number of diagonals in decagon are 35.
Hence, option (b) is correct i.e., 35.

SOME RESULTS OF COMBINATION TO TAKEN CARE

- Formula: ${}^{n}C_{r} = \frac{n!}{r! \times (n-r)!}$
- If r = n then, ${}^{n}C_{r} = {}^{n}C_{n} = 1$
- Similarly, if r = 0, then ${}^{n}C_{r} = {}^{n}C_{0} = 1$
- Also, note that $0 \le r \le n$, then only ${}^{n}C_{r}$ exists. Similarly, ${}^{n}C_{n-r}$ exist only when $0 \le n - r \le n$
- ${}^{n}C_{r} = {}^{n}C_{n-r}$

Two more important results:

1. ${}^{n+1}C_{r} = {}^{n}C_{r} + {}^{n}C_{r-1}$
2. ${}^{n}P_{r} = {}^{n-1}P_{r} + r.\,{}^{n-1}P_{r-1}$

Example 40. The value of ${}^{11}C_{2} + {}^{11}C_{3}$ is

(a) 210 (b) 200 (c) 220 (d) None of these

Sol. (c) Since ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

$${}^{11}C_{2} + {}^{11}C_{3} = \frac{11!}{2!(11-2)!} + \frac{11!}{3!(11-3)!}$$

$$\Rightarrow \frac{11!}{2! \times 9!} + \frac{11!}{3! \times 8!} = \frac{11 \times 10}{2} + \frac{11 \times 10 \times 9}{3 \times 2}$$

$$\Rightarrow \frac{110}{2}(1 + 3) = 110 \times 2 = 220$$

Hence, the correct option is (c).

PERMUTATIONS WHEN SOME OF THE THINGS ARE ALIKE, TAKEN ALL AT A TIME

Let us take the case, where there are n things in which n_1 things are alike of one kind, n_2 things alike of second kind and n_3 things are alike of third kind. Then the number of possible permutations when all n things taken at a time is: $P = \frac{n!}{n_1! n_2! n_3!}$

PERMUTATIONS WHEN EACH THING MAY BE REPEATED ONCE, TWICE,... UPTO TIMES IN ANY ARRANGEMENT

The number of permutations of n things taken r at time when each thing may be repeated r times in any arrangement = n^r.

- Combinations of n different things taking some or all of n things at a time = $2^n - 1$.
- Combination of n things taken some or all at a time when n_1 things are alike of one kind, n_2 things are alike of second kind & n_3 things are alike of third kind:
  $= (n_1 + 1) \times (n_2 + 1) \times (n_3 + 1) - 1$
- If we have to select the combination such that r_1 things to be selected from n_1 and r_2 things to be selected from n_2 then, total selections are: ${}^{n_1}C_{r_1} \times {}^{n_2}C_{r_2}$

Example 41. How many different permutations are possible from the letters of the word 'MATHEMATICS'?

(a) 11! (b) $\frac{11!}{2! \times 2! \times 2!}$ (c) $\frac{11!}{2! \times 2!}$ (d) None of these

Sol. (b) The word given is 'MATHEMATICS'

Total number of letters in 'MATHEMATICS' is 11.

Number of 'T' = 2

Number of 'M' = 2

Number of 'A' = 2

Therefore number of arrangements $= \frac{11!}{2! \times 2! \times 2!}$

Hence, option (b) is correct.

Example 42. Rajesh is planning Christmas party, in how many ways he can invite his 11 friends for the party?

(a) 11! (b) 2048 (c) 2047 (d) None of these

Sol. (c) We know that,

Combinations of n different things taking some or all of n things at a time = $2^n - 1$

Thus, the number of ways Rajesh can invite his 11 friends = $2^{11} - 1 = 2048 - 1 = 2047$

Hence, the correct option is (c).

Example 43. By how many different ways you can take 10 Donuts, 6 Waffles and 8 pastries from your pantry for the picnic?

(a) 692 (b) 693 (c) 480 (d) None of these

Sol. (a) Given,

Number of Donuts (n_1) = 10

Number of Waffles (n_2) = 6

Number of Pastries (n_3) = 8

Therefore, the required ways = $(n_1 + 1) \times (n_2 + 1) \times (n_3 + 1) - 1$

$= (10 + 1) \times (6 + 1) \times (8 + 1) - 1$

$= 11 \times 7 \times 9 - 1 = 693 - 1 = 692$

Hence, the correct option is (a).

Example 44. The number of ways in which 9 things can be divided into twice groups containing 2, 3, and 4 things respectively is **(ICAI)**

(a) 1250 (b) 1260 (c) 1200 (d) None of these

Sol. (b) Total number of things = 9

The first group can be formed by choosing 2 things out of 9 things

Number of ways = ${}^9C_2$

Now, second group can be formed by choosing 3 things out of 7

Number of ways = ${}^7C_3$

Now, second group can be formed by choosing 4 things out of 4

Number of ways = ${}^4C_4$

Therefore, total number of ways = ${}^9C_2 \times {}^7C_3 \times {}^4C_4 = 1260$ ways.

Hence, option (b) is correct.

Example 45. The number of ways a person can contribute to a fund out of 1 ten–rupee note, 1 five–rupee note, 1 two–rupee and 1 one-rupee note is

(a) 15 (b) 25 (c) 10 (d) None of these

Sol. (a) A person can contribute to a fund out of 1 ten rupees note, 1 five–rupees, 1 two–rupee and 1 one rupee note.

As a person has 4 notes, he can contribute either one note or two notes or three notes.

The number of different contributions can be

$= 2 \times 2 \times 2 \times 2 - 1 = 16 - 1 = 15$

Hence, option (a) is correct.

Example 46. ${}^{(n-1)}P_r + r.\ {}^{(n-1)}P_{(r-1)}$ is equal to

(a) ${}^nC_r$ (b) $\dfrac{n}{r!(n-r)!}$ (c) ${}^nP_r$ (d) None of these

Sol. (c) By property, we have

${}^{(n-1)}P_r + r.\ {}^{(n-1)}P_{(r-1)} = {}^nP_r$

Hence, option (c) is correct.

Example 47. ${}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \ldots + {}^nC_n$ equals

(a) $2^n - 1$ (b) 2^n (c) $2n + 1$ (d) None of these

Sol. (a) We know that,

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \ldots + {}^nC_nx^n$

Now, put x = 1 in above equation, we get

$(1 + x)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n$

${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n - 1$

Hence, option (a) is correct i.e., $2^n - 1$.

Example 48. The number of ways in which 12 students can be equally divided into three groups is

(a) 5775 (b) 7575 (c) 7755 (d) None of these

Sol. (a) Given, Total students = 12

Since, 12 students are equally divided into three groups thus

$$\text{Possible ways} = \frac{{}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4}{3!}$$

$$= \frac{12!}{(4!)^3(3!)} = 5775$$

Hence, the correct option is (a).

Example 49. The Number of ways in which 15 mangoes can be equally divided among 2 students is

(a) $\frac{15}{(5!)^4}$ (b) $\frac{15}{(5!)^3}$ (c) $\frac{15}{(5!)^2}$ (d) None of these

Sol. (b) Given, Total number of mangoes = 15

Number of students = 3

$$\Rightarrow \frac{15}{3} = 5 \text{ mangoes}$$

So, the required number of ways in which 15 mangoes may be equally distributed among 3 students is $= \frac{15!}{5! \times 5! \times 5!} = \frac{15!}{(5!)^3}$

Hence, option (b) is correct.

PRACTICE QUESTIONS (PART D)

1. A CA needs three accountants and ten men apply. In how many ways can these selections take place?

(a) 6,04,800 (b) 36,28,800 (c) 6 (d) 120

2. Army General wishes to simultaneously promote 3 of its 5 Captains to Majors. In how many ways can these promotions take place?

(a) 10 (b) 20 (c) 40 (d) None of these

3. Find the value of n in $4.{}^{n}C_2 = {}^{n+2}C_3$

(a) 2 (b) 7

(c) Both (a) and (b) (d) None of these

4. If ${}^{8}P_r = 6720$ and ${}^{8}C_r = 56$; find the value of r.

(a) 2 (b) 5 (c) 4 (c) None of these

5. A committee of 7 members is to be chosen from 6 CA, 4 CS and 5 CS. In how many ways can this be done if in the committee, there must be at least one member from each group and at least 3 Chartered Accountants?

(a) 2570 (b) 1200 (c) 3570 (d) None of these

6. Out of 6 boys & 4 girls, find the number of ways for selecting 5 members for a committee in which there are exactly two girls?

(a) 120 (b) 1440 (c) 720 (d) 71

7. The letters of the word 'BOOKKEEPER' are rearranged to form different arrangements. How many different arrangements can be formed considering the repetitions of the letters?

(a) 10! (b) $\frac{10!}{2!\times 2!\times 2!\times 3!}$

(c) $\frac{10!}{2!\times 3!}$ (d) None of the above

8. If 3 books on computer, 3 books on commerce, and 5 books on economics are arranged in such away that the books of same subject are kept together, then the number of ways in which this can be done are:

(a) 4320 (b) 35820 (c) 35920 (d) 25920

9. The number of triangle that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line is:

(a) 185 (b) 175 (c) 105 (d) 115

10. The number of numbers between 1000 and 10000, which can be formed by the digits 1, 2, 3,4,5,6 without repetition is:

(a) 720 (b) 180 (c) 360 (d) 540

11. The number of ways in which 4 person can occupy 9 vacant seats is:

(a) 6048 (b) 3024 (c) 1512 (d) 4536

12. How many different permutations are possible from the letters of the word 'CALCULUS'?

(a) 40,320 (b) 20,160 (c) 10,080 (d) 5,040

13. If $^{10}C_3 + 2\cdot{}^{10}C_4 = {}^{n}C_5$ then value of n is:

(a) 10 (b) 11 (c) 12 (d) 13

14. The maximum number of points of intersection of 10 circles of will be:

(a) 2 (b) 20 (c) 90 (d) 180

15. If $^{n+1}C_{r+1} : {}^{n}C_r : {}^{n-1}C_{r-1} = 8 : 3 : 1$, then n is equal to:

(a) 22 (b) 16 (c) 10 (d) 15

16. There are 6 men and women in a group, then the number of ways in which a committee of 5 persons can be formed of them, if the committee is to include at least 2 women are:

(a) 180 (b) 186 (c) 120 (d) 105

17. If $^{n}P_r = 720$ and $^{n}C_r = 120$, then value of 'r' is:

(a) 4 (b) 5 (c) 6 (d) 3

18. A student has three books on computer, three books on economics and five books on commerce. If these books are to be arranged subject wise, then these can be placed on shelf in the number of ways:

(a) 25290 (b) 25920 (c) 4230 (d) 4320

19. A person has ten friends of whom six are relatives. If he invites five guests such that three of them are his relatives, then the total number of ways in which he can invite them are:

(a) 30 (b) 60 (c) 120 (d) 75

20. 5 men and 4 women to sit in a row in such a manner that the women always occupy the even places. The number of such arrangement will be:

(a) 126 (b) 1056 (c) 2080 (d) 2880

21. There are 5 books on English, 4 books on Tamil and 3 books on Hindi. In how many ways can these books be placed on a shell if the books on the same subjects are to the together?

(a) 1,36,800 (b) 1,83,600 (c) 1,03,680 (d) 1,63,800

22. If six times the number of permutations of 'n' items taken 3 at a time is equal to seven times the number of permutation of (items taken at a time, then the value of 'n' will be:

(a) 7 (b) 9 (c) 13 (d) 21

23. If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$ then value of x will be:

(a) 999 (b) 998 (c) 997 (d) None of these

24. How many different words can be formed with the letters of the word 'LIBERTY'?

(a) 4050 (b) 5040 (c) 5400 (d) 4500

25. In how many ways can a family consist of 3 children have different birthdays in a leap year?

(a) ${}^{365}C_{365}$ (b) ${}^{366}C_{3}$

(c) 366 × 365 × 364 (d) None of these

26. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, then 'r' is equal is

(a) 2 (b) 3 (c) 4 (d) 5

27. Number of ways of shaking hands in a group of 10 persons shaking hands to each other are:

(a) 45 (b) 54 (c) 90 (d) 10

28. If ${}^{n}P_{4} = 20\ ({}^{n}P_{2})$ then the value of 'n' is

(a) –2 (b) 7 (c) –2 and 7 both (d) None of these

29. If ${}^{n}P_{r} = 3024$ and ${}^{n}C_{r} = 126$, then find n and r. **(Dec, 2022)**

(a) 9, 4 (b) 10, 3 (c) 12, 4 (c) 11, 4

30. The number of permutations of the word 'ACCOUNTANT, is

(a) $10! \div (2!)^4$ (b) $10! \div (2!)^3$ (c) 10! (c) None of these

31. In how many ways that the crew of an eight be arranged so that if 3 of erew can row only on a stroke side and 2 row on the other side is **(July 2019)**

(a) 1728 (b) 256 (c) 164 (c) 126

32. The number of ways in which 5 boys and 3 girls are to be seated together such that no two girls are together is **(Dec 2019)**

(a) 14,400 (b) 2400 (c) 720 (c) None of these

33. Out of 6 Boys & 4 girls, find the number of ways for selecting 5 members committee in which there is exactly two girls?

(a) 120 (b) 1440 (c) 720 (c) 71

34. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

(a) 52600 (b) 50400 (c) 44100 (d) None of these

35. In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

(a) 420 (b) 540 (c) 680 (d) 720

Answer Key

1. (d)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (a)	**10.** (c)
11. (b)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (d)
21. (c)	**22.** (d)	**23.** (a)	**24.** (b)	**25.** (c)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (a)	**30.** (a)
31. (a)	**32.** (a)	**33.** (a)	**34.** (b)	**35.** (a)					

PREVIOUS YEAR QUESTION (PYQ'S)

1. There are ten flights operating between city A and city B. The number of ways in which a person can travel from city A to city B and return by different flight is **(Jan 2021)**

(a) 90 (b) 95 (c) 85 (d) 78

2. How many odd numbers of four digits can be formed with digits 0, 1, 2, 3, 4, 7 and 8? **(Jan 2021)**

(a) 150 (b) 300 (c) 120 (d) 210

3. In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy the odd numbered positions? **(Jan 2021)**

(a) 32 (b) 36 (c) 48 (d) 60

4. ${}^{n}C_{p} + 2{}^{n}C_{p-1} + {}^{n}C_{p-2} =$ **(Jan 2021)**

(a) ${}^{n}C_{p}$ (b) ${}^{n+2}C_{p}$ (c) ${}^{n+1}C_{p+1}$ (d) ${}^{n+2}C_{p-1}$

5. A business house wishes to simultaneously elevate two of its six branch heads. In how many ways can these elevations take place? **(Jan 2021)**

(a) 12 (b) 3 (c) 6 (d) 15

6. 'n' locks and 'n' corresponding keys are available but the actual combination is not known. The maximum number of trials that are needed to assigns the keys to the corresponding locks is **(Jan 2021)**

(a) ${}^{n-1}C_{2}$ (b) ${}^{n+1}C_{2}$ (c) $\sum_{k=2}^{n} k$ (d) $\sum_{k=2}^{n} (k-1)$

7. If ${}^{n}P_{6} = 20\ {}^{n}P_{4}$, then the value of n is given by; **(July 2021)**

(a) n = 5 (b) n = 3 (c) n = 9 (d) n = 8

8. How many seven digit numbers can be formed from the digits 3, 4, 5, 6, 7, 8, 9 such that no digits being repeated and are not divisible by 5? **(July 2021)**

(a) 4320 (b) 4690 (c) 3900 (d) 3890

9. A person can go from place 'A' to 'B' by 11 different modes of transport but is allowed to return back to 'A' by any mode other than one earlier. The number of different ways the entire journey can be complete is **(July 2021)**

(a) 110 (b) 10^{10} (c) 9^5 (d) 10^9

10. The number of ways 5 boys and 5 girls can be seated at a round table, so no two boys are adjacent is **(July 2021)**

(a) 2550 (b) 2880 (c) 625 (d) 2476

11. The number of four letter words can be formed using the letters of the word DICTIONARY is **(Dec 2021)**

(a) 5040 (b) 720 (c) 90 (d) 30240

12. The number of words that can be formed using the letters of "PETROL" such that the words do not have "P" in the first position is **(Dec 2021)**

(a) 720 (b) 120 (c) 600 (d) 540

13. If ${}^{n}P_{2} = 12$, then the value of n is **(Dec 2021)**

(a) 2 (b) 3 (c) 4 (d) 6

14. Six points are marked on a straight line and five points are marked on another line which is parallel to the first line. How many straight lines, including the first two, can be formed with these points? **(June 2022)**

(a) 28 (b) 30 (c) 32 (d) 34

15. How many 4 letter words with or without meaning, can be formed out of letters of the word, 'LOGARITHMS', if repetition of letters is not allowed? **(June 2022)**

(a) 7020 (b) 5040 (c) 1480 (d) 2520

16. Out of 7 boys and 4 girls, a team of a debate club of 5 is chosen. The number of teams such that each team includes at least one girl is **(June 2022)**

(a) 4439 (b) 429 (c) 419 (d) 441

17. The total number of sitting arrangements of 8 members of a board on a row with the president and the vice-president occupying central places is **(June 2022)**

(a) 7! × 2! (b) 6! × 2! (c) 6! (d) 7!

18. The solution for $\frac{n!}{10} = {}^{n-1}P_{n-3}$, then n = **(June 2022)**

(a) 5 (b) 6 (c) 7 (d) 8

19. A multiple-choice test contains five questions and each has four possible options. How many different answer keys are possible? **(June 2022)**

(a) 20 (b) 120 (c) 256 (d) 1024

20. There are 20 points in a plane area. How many triangles can be formed by these points if 5 points are collinear? **(June 2022)**

(a) 550 (b) 560 (c) 1130 (d) 1140

21. The number of ways 4 boys and 3 girls can be seated in a row so that they are alternate is **(Dec 2022)**

(a) 12 (b) 288 (c) 144 (d) 256

22. If ${}^{n}P_{r} = 3024$ and ${}^{n}C_{r} = 126$, then find n and r. **(Dec 2022)**

(a) 9, 4 (b) 10, 3 (c) 12, 4 (d) 11, 4

23. How many 3 digits odd numbers can be formed using the digits 5, 6, 7, 8, 9 if the digits can be repeated? **(Dec 2022)**

(a) 55 (b) 75 (c) 65 (d) 85

24. A committee of 3 women and 4 men is to be formed out of 8 women and 7 men. Mrs. Kajal refuses to serve in a committee in which Mr Yash is a member. The number of such committee can be **(June 2023)**

(a) 1530 (b) 1500 (c) 1520 (d) 1540

25. If ${}^{6}P_{2} = 12 \times {}^{6}P_{r}$, then r is equal to

(a) 1 (b) 2 (c) 3 (d) 4

26. In how many different ways can the letters of the word 'SOFTWARE' be arranged so that the vowels always come together?

(a) 720 (b) 144 (c) 2880 (d) 4320

27. In the next world cup of cricket, there will be 12 teams divided equally into two equal groups. Team of each group will play a match against other teams of the group. From each group, 3 top teams will qualify for next round. In this round, each team will play against each other. Four top teams of this round will qualify for semi-finals and play against each other and then two top teams will go to final, where they play the best of three matches. How much minimum number of matches in the next world cup will be? **(June 2023)**

(a) 54 (b) 53 (c) 38 (d) 43

28. Find 'n' if ${}^{n}P_{2} = 72$. **(Dec 2023)**

(a) 12 (b) 36 (c) 24 (d) 9

29. If ${}^{15}C_{3} = {}^{15}C_{r+3}$, then 'r' is equal to **(Dec 2023)**

(a) 5 (b) 4 (c) 3 (d) 2

30. In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together? **(Dec 2023)**

(a) 810 (b) 1440 (c) 25200 (d) 50400

31. In how many ways can 5 boys and 5 girls sit in a row so that no girls are together? **(June 2024)**

(a) 14,400 (b) 14,100 (c) 13,400 (d) 12,400

32. In how many ways can the letters of the word 'STADIUM' be arranged in such a way that the vowels all occur together? **(June 2024)**

(a) 7! × 3! (b) 5! × 4! (c) 5! × 3! (d) 6! × 3!

33. How many ways can 5 different trophies be arranged on a shelf if one particular trophy must always be in the middle? **(June 2024)**

(a) 24 (b) 48 (c) 120 (d) 144

34. A user wants to create a password using 4 lowercase letters (a-z) and 2 uppercase letters (A-Z). No letter can be repeated in any form. In how many ways can the password be created if the password must start with an uppercase letter? **(June 2024)**

(a) 26 × 25 × 24 × 23 × 22 × 5 × 21 (b) 26 × 25 × 24 × 23 × 22 × 2 × 21

(c) 26 × 5 × 25 × 24 × 23 × 22 × 2 × 21 (d) 6 × 26 × 25 × 24 × 23 × 22 × 21

35. In a class of 4 boys and 3 girls, they are required to sit in a row in such a way that no two girls sit together. Compute, in how many different ways they can sit together. **(Sept 2024)**

(a) 60 (b) 480 (c) 720 (d) 1440

36. How many total combinations can be formed of 8 different counters marked as 1, 2, 3, 4, 5, 6, 7 & 8, taking 4 counters at a time and there being at least one odd and one even numbered counter in each combination? **(Sept 2024)**

(a) 68 (b) 66 (c) 64 (d) 62

37. In a party, every person shakes hands with every other person. If these are 105 handshakes in total, find the number of persons in the party. **(Sept 2024)**

(a) 14 (b) 15 (c) 21 (d) 22

38. A selection is to be made for one post of Principal and two posts of Vice-Principal. Amongst the six candidates called for the interview, only two are eligible for the post of Principal, while they all six are eligible for the post of Vice-Principal. The number of possible combinations for the selection is; **(Sept 2024)**

(a) 4 (b) 12 (c) 18 (d) 20

39. In how many ways can 5 doctors, 4 professors and 6 Auditors be seated in a row so that all person of the same profession sit together? **(Jan 2025)**

(a) 3! × 5! × 4! (b) 3! × 5! (c) 3! × 5! × 4! × 6! (d) 3! × 5! × 6!

40. A panel has total of 11 members including 5 males and 6 females. Find out the number of ways of picking 2 males and 3 females from the given panel team. **(Jan 2025)**

(a) 200 (b) 110 (c) 220 (d) 350

41. A committee of 3 members is formed from 5 women and 3 men in such a way that it consists of at least 2 members who are women. In how many different ways can it be done?

(Jan 2025]

(a) 50 (b) 40 (c) 60 (d) 30

42. A committee of 3 members is formed from 5 women and 3 men in such a way that it consists of at least 2 members who are women. In how many different ways can it be done? **(Jan 2025)**

(a) 50 (b) 40 (c) 60 (d) 30

43. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed? **(May 2025)**

(a) 210 (b) 1050 (c) 25200 (d) 21400

44. Seema, Bharti, Priyanka, Khushboo and Lalita are 5 speakers. The number of ways in which Seema will always speak before Bharti shall be **(May 2025)**

(a) 24 (b) $4! \times 2!$ (c) $5!$ (d) 12

45. A team of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done if these teams consist of 1 man and 2 women? **(May 2025)**

(a) 10 (b) 6 (c) 16 (d) 8

46. In how many of distinct permutations of the letters in "MISSISSIPPI" when four I's do not come together? **(May 2025)**

(a) 34650 (b) 40320 (c) 840 (d) 33810

47. Different words are made with rearrangement of letters of the word 'TROPICAL' in a way that the vowels occupy odd places, when counted from left. How many such words are there? **(Sept 2025)**

(a) 720 (b) 1440 (c) 2880 (d) 2160

48. How many different words from the letters of the word 'MATHEMATICS' can be formed so that all the vowels always come together in any word? **(Sept 2025)**

(a) 10,080 (b) 1,20,960 (c) 49,89,600 (d) 20,160

49. Find n if ${}^{n}P_{5} = 20\ {}^{n}P_{3}$. **(Sept 2025)**

(a) 7 (b) 8 (c) 9 (d) 10

50. In a school, for a class monitor selection, there are 6 candidates, and students need to choose up to 3 monitors. A student can vote for 1 or 2 or 3 candidates. In how many ways a student can vote? **(Sept 2025)**

(a) 41 (b) 42 (c) 43 (d) 44

Answer Key

1. (a)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (b)
11. (a)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (c)
21. (c)	**22.** (a)	**23.** (b)	**24.** (d)	**25.** (b)	**26.** (d)	**27.** (b)	**28.** (d)	**29.** (c)	**30.** (d)
31. (a)	**32.** (c)	**33.** (a)	**34.** (a)	**35.** (d)	**36.** (a)	**37.** (b)	**38.** (d)	**39.** (c)	**40.** (a)
41. (c)	**42.** (b)	**43.** (c)	**44.** (a)	**45.** (b)	**46.** (d)	**47.** (c)	**48.** (b)	**49.** (b)	**50.** (a)

SUMMARY

- **Multiplication rule:** If a certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n' different ways, then total number of ways of doing both things simultaneously = m × n.
- **Addition rule:** If there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectively then either of two jobs can be done in (m + n) ways.
- **Factorial:** The factorial n, written as n! or ⌊n, represents the product of all integers from 1 to n both inclusive i.e. n! = n(n – 1)(n – 2)... 3. 2. 1 and note that 0! = 1.
- **Permutation:** The ways of arranging or selecting a smaller or equal number of objects from a collection of objects with due regard being paid to the order of arrangement or selection, are called permutations.
- The number of permutations of n things chosen r at a time is given by: ${}^{n}P_{r} = \frac{n!}{(n-r)!}$.
- **Circular Permutation:**
  - (i) Arranging n things in circular arrangement is given by: $\frac{{}^{n}P_{n}}{n} = (n-1)!$ ways in which all the n things can be arranged in a circle.
  - (ii) When clockwise and anticlockwise arrangement are not different, then the number of circular permutation of n different things $= \frac{(n-1)!}{2}$
  - (ii) Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is ${}^{n-1}P_{r}$.
  - (iii) Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is r. ${}^{n-1}P_{r-1}$.
- **Combinations:** The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important, are called combinations given by: ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$.
- Permutations when some of the things are alike, taken all at a time is $\frac{n!}{n_1!.n_2!.n_3!....}$.
- Permutations of r things out of n when each thing may be repeated once, twice, …upto r times in any arrangement n^r.
- The total number of ways in which it is possible to form groups by taking some or all of n things $2^n - 1$. The total, number of ways in which it is possible to make groups by taking some or all out of n (= $n_1 + n_2 + n_3 + ...$) things, where n_1 things are alike of one kind and so on, is given by $\{(n_1 + 1)(n_2 + 1)(n_3 + 1) \ldots\} - 1$.
- The combinations of selecting r_1 things from a set having n_1 objects and r_2 things from a set having n_2 objects where combination of r_1 things, r_2 things are independent is given by: ${}^{n_1}C_{r_1} \times {}^{n_2}C_{r_2}$.

❑❑❑

CHAPTER

06

Sequence and Series– Arithmetic and Geometric Progressions

Why to study sequence and series?

- When you start doing a job, then you will be working on a lot of data files and then you need to identify a particular pattern in between to understand or analyse it.

WHAT IS SEQUENCE?

When numbers are separated by "," and all numbers follow a particular law or some definite rule then numbers are said to be in sequence and the number is called an element.

Generally, the sequence is written as $a_1, a_2, a_3, a_4, a_n, \ldots\ldots\ldots$ where $a_1, a_2, a_3, a_4, \ldots, a_n$ are its terms and n is the natural number and represents the number of terms or we can say n^{th} term is the number at the nth position of the particular sequence and is denoted by a_n.

E.g.: Tell which one is in the sequence?

(i) 1, 14, 3, 5, 8 (ii) 1, 2, 5, 15, 20, 30

(iii) 1, 2, 3, 4, 5, 6, 7 (iv) 50, 45, 30, 15, 3, 1

We can see that,

In (i), the numbers do not obey any rule or law, thus it is not a sequence.

In (ii), the numbers are in ascending order but it does not obey any rule.

In (iii), the difference between the consecutive numbers is 1 or by adding 1 to each number, we get the next number, thus it forms a sequence.

In (iv), the numbers are in descending order, but it does not obey any rule.

Thus, the numbers in (iii) only form a sequence.

Some of the sequences:

1. Sequence of natural numbers (N): 1, 2, 3, 4, 5, …..
2. Sequence of square numbers (n^2): 1, 4, 9, 16, 25, …..
3. Sequence of $\frac{1}{n}$: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\ldots$
4. Sequence of odd positive numbers: 1, 3, 5, 7, 9, …
5. Sequence of even positive numbers: 2, 4, 6, 8, 10, …
6. Sequence of $\frac{1}{(n+2)}$: $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots\ldots\ldots\ldots$
7. Sequence of $\frac{n}{(n+2)}$: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\ldots\ldots$

Example 1. Complete the sequence: 1, 4, 7, 10, 13, ?

(a) 21 (b) 17 (c) 16 (d) None of these

Sol. (c) Given sequence: 1, 4, 7, 10, 13, ?

Then, $a_1 = 1$, $a_2 = 4$, $a_3 = 7$, $a_4 = 10$, $a_5 = 13$

The given sequence is following a pattern i.e., each successive number is increasing by 3 i.e.,

$1 + 3 = 4$

$4 + 3 = 7$

$7 + 3 = 10$

$10 + 3 = 13$

Thus, the next number is $13 + 3 = 16$

Hence, the correct option is (c).

Example 2. Tell the next number in the sequence 1, 8, 27, 64, ___ .

(a) 125 (b) 256 (c) 111 (d) None of these

Sol. (a) Given sequence: 1, 8, 27, 64, ?

The given sequence is in the form of cube of the natural numbers i.e., 1^3, 2^3, 3^3, 4^3

Then, the 5th term in sequence will be $5^3 = 5 \times 5 \times 5 = 125$

Hence, the correct option is (a).

Example 3. Prime numbers less than 20 are

(a) 1, 2, 3, 11, 13, 19 (b) 2, 3, 5, 7, 11, 13, 17, 19

(c) 2, 5, 11, 13, 19 (d) None of these

Sol. (b) Prime numbers are those numbers which have exactly 2 factors i.e., 1 and the number itself.

Then, the prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.

Hence the correct option is (b).

Example 4. The first three terms of sequence when n^{th} term t_n is $n^2 - 2n$ are **(ICAI)**

(a) –1, 0, 3 (b) 1, 0, 2 (c) –1, 0, –3 (d) None of these

Sol. (a) Given that, $t_n = n^2 - 2n$

Thus, the first three terms of the sequence can be obtained by substituting the values of n as 1, 2 and 3 i.e.,

$t_1 = (1)^2 - 2(1) = 1 - 2 = -1$

$t_2 = (2)^2 - 2(2) = 4 - 4 = 0$

$t_3 = (3)^2 - 2(3) = 9 - 6 = 3$

Therefore, the first three terms of sequence are –1, 0, 3.

Hence the correct option is (a).

SERIES

When we add the terms of the sequence, then it is called a series.

Let $a_1, a_2, a_3, ..., a_n$, be the sequence then the series can be written as

$a_1 + a_2 + a_3 + ... + a_n + ...$

E.g.: If the sequence is 1, 2, 3, 4, 5, 6, ….. then the series is 1 + 2 + 3 + 4 + ...

TYPES OF SERIES

- **Finite series:** A finite series is a series that has a specific number of terms. The addition of the terms in the series stops after a certain point.

  E.g.: the series 1 + 2 + 3 + 4 + 5 is a finite series as it has a definite end.

- **Infinite series:** An infinite series is a series that continues indefinitely, without an end. The addition of terms in the series goes on indefinitely.

  E.g.: the series 1 + 2 + 3 + 4 + 5 + ... is an infinite series as there is no specific endpoint.

Example 5. If $u_1 = 2, u_2 = 4, u_3 = 6, u_4 = 8$, then S_4 is:

(a) 20 (b) 15 (c) 5 (d) None of these

Sol. (a) Given that, $u_1 = 2, u_2 = 4, u_3 = 6, u_4 = 8$

We know that, $S_n = \sum_{r=1}^{n} u_r$

Thus, $S_4 = \sum_{r=1}^{4} u_r = u_1 + u_2 + u_3 + u_4 = 2 + 4 + 6 + 8 = 20$

Hence, the correct option is (a).

Example 6. The nth element of the sequence –1, 2, –4, 8, ... is

(a) $(-1)^n (2)^{n-1}$ (b) 2^{n+1} (c) 2^n (d) 2^{n-1}

Sol. (a) Given sequence, -1, 2, -4, 8, ...

From the above sequence, we see that the terms are negative and positive alternatively and the successive term is double of the previous term.

i.e., the terms are

–1 which is of the form $(-1)^1 2^{1-1}$

$-1 \times -2 = 2$ which is of the form $(-1)^2 2^{2-1}$

$2 \times -2 = -4$ which is of the form $(-1)^3 2^{3-1}$

$-4 \times -2 = 8$ which is of the form $(-1)^4 2^{4-1}$

$8 \times -2 = -16$ which is of the form $(-1)^5 2^{5-1}$

Therefore, the nth element of the sequence = $(-1)^n 2^{n-1}$

Hence, the correct option is (a).

Example 7. The sum to ∞ of the series –5, 25, –125, 625 … can be written as **(ICAI)**

(a) $\sum_{k=1}^{\infty}(-5)^k$ (b) $\sum_{k=1}^{\infty}5^k$ (c) $\sum_{k=1}^{\infty}-5^k$ (d) None of these

Sol. (a) Given sequence: –5, 25, –125, 625, …

The above sequence can be written as,

$\Rightarrow$ – 5, 25, –125, 625, …

$\Rightarrow (-5)^1, (-5)^2, (-5)^3, (-5)^4$...[$\because (-a)^n = a^n$ when n is even, $(-a)^n = -a^n$ when n is odd]

$\Rightarrow \sum_{k=1}^{\infty}(-5)^k$

$$\left[\therefore (-a)^1 + (-a)^2 + (-a)^3 ... = \sum_{k=1}^{\infty}(-a)^k\right]$$

Hence, the correct option is (a).

Example 8. $\sum_{i=4}^{7}\sqrt{2i-1}$ can be written as **(ICAI)**

(a) $\sqrt{7}+\sqrt{9}+\sqrt{11}+\sqrt{13}$ (b) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$

(c) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$ (d) None of these

Sol. (a) Given that $\sum_{i=4}^{7}\sqrt{2i-1}$

We know that, $\sum_{i=1}^{n}a_i = a_1 + a_2 + a_3 \ ... + a_n$

$\Rightarrow \sum_{i=4}^{7}\sqrt{2(i)-1} = \sqrt{2(4)-1} + \sqrt{2(5)-1} + \sqrt{2(6)-1} + \sqrt{2(7)-1}$

$\Rightarrow \sum_{i=4}^{7}\sqrt{2(i)-1} = \sqrt{8-1} + \sqrt{10-1} + \sqrt{12-1} + \sqrt{14-1}$

$\Rightarrow \sum_{i=4}^{7}\sqrt{2(i)-1} = \sqrt{7} + \sqrt{9} + \sqrt{11} + \sqrt{13}$

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART A)

1. The nth element of the sequence 5, 7, 9, 11,..... is

(a) 3n + 2 (b) n + 4 (c) 2n + 3 (d) none of these

2. The pth element of the sequence –1, 2, –4, 8,.....is

(a) $(-1)^p 2^{p-1}$ (b) 5.2^{p+1} (b) 3.2^p (d) None of these

3. –5, 25, –125, 265,..... to 100 terms can be written as

(a) $\sum_{i=1}^{100}(-5)^i$ (b) $\sum_{i=1}^{100}(5)^i$ (c) $\sum_{i=1}^{100}-5^i$ (d) None of these

4. $\sum_{i=1}^{4}\sqrt{5-i}$ can be written as

(a) $2+\sqrt{2}+\sqrt{3}$ (b) $3+\sqrt{2}+\sqrt{3}$

(c) $\sqrt{2}+\sqrt{3}+\sqrt{5}+1$ (d) None of these

5. The first three terms of a sequence when n^{th} term (t_n) is $n^3 - n$ are:

(a) 6, 24, 25 (b) 0, 6, 24 (c) 1, 2, 5 (d) None of these

Answer Key

1. (c) **2.** (a) **3.** (a) **4.** (b) **5.** (b)

ARITHMETIC PROGRESSION (A.P.)

- If the common difference (d) between consecutive terms is same then the sequence is called Arithmetic Progression (A.P.).
- We can also say that $a_1, a_2, a_3, a_4, \ldots\ldots\ldots\ldots, a_n$ is in Arithmetic Progression (A.P.) when

  $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots\ldots\ldots = a_n - a_{n-1}$

  E.g.: 1, 4, 7, 10, 13, ...

In the given sequence, the common difference between consecutive terms is 3. Therefore, it is an Arithmetic Progression (A.P.) with a common difference (d) of 3.

In an Arithmetic Progression (A.P.), each term is obtained by adding a constant value (d) to the previous term.

Thus, if the first term is 'a' and the common difference is 'd', then the nth term will be:

$a_n = a + (n - 1)d$

Now in general, an A.P. series can be written as: a, a + d, a + 2d, a + 3d, ... a + (n – 1)d

Example 9. If the sequence 20, 15, 10, 5, 0, –5, –10 is in A.P., then the common difference is

(a) 5 (b) – 5 (c) 10 (d) None of these

Sol. (b) Given sequence: 20, 15, 10, 5, 0, –5, –10

Since, it is in A.P then the common difference, $d = a_2 - a_1 = a_3 - a_2 = \ldots$

According to the question,

First term, $a_1 = 20$

Second term, $a_2 = 15$

Thus, $a_2 - a_1 = 15 - 20 = -5$

Therefore, the common difference (d) = –5

Hence, the correct option is (b).

Example 10. If the first term of an A.P. is 15 and the common difference is 4, then the required A.P. will be

(a) 15, 19, 21, 23, ... (b) 15, 19, 23, 27, ...

(c) 11, 15, 19, 25, ... (d) None of these

Sol. (b) Given, a = 15, d = 4

Thus,

First term = $a = a_1 = 15$

Second term = $a_2 = a_1 + d = 15 + 4 = 19$

Third term $a_3 = a_2 + d = 19 + 4 = 23$

Fourth term $a_4 = a_3 + d = 23 + 4 = 27$

and so on

Therefore, the required A.P. will be 15, 19, 23, 27, ...

Hence, the correct option is (b).

Example 11. The 6th term of the progression 1, 3, 5, 7 ... is

(a) 5 (b) 6 (c) 11 (d) None of these

Sol. (c) Given sequence: 1, 3, 5, 7, …

We know that,

n^{th} term of an A.P. is given by $a_n + a + (n - 1)d$

According to the given sequence,

First term, $a_1 = 1$

Second term, $a_2 = 3$

Thus, common difference (d) = $a_2 - a_1 = 3 - 1 = 2$

Here, n = 6

Therefore, 6th term = $1 + (6 - 1)2 = 1 + 5(2) = 1 + 10 = 11$

Hence, the correct option is (c).

Example 12. The 11th term of the series, S = 7 + 10 + 13 + 16 +, ... is

(a) 35 (b) 37 (c) 47 (d) 65

Sol. (b) Given series, S = 7 + 10 + 13 + 16 +,.....

Here, a = 7, d = 10 – 7 = 3 and n = 11

Therefore, 11th term of the series is given by

$a_{11} = a + (11 - 1)d$

$= 7 + 10(3)$

$= 37$

Hence, the correct option is (b).

Example 13. Which term of the progression –1, –3, –5, ... is –39? **(ICAI)**

(a) 21st (b) 20th (c) 19th (d) None of these

Sol. (b) Given progression, –1, –3, –5, ...

Here, common difference (d) is the same. So, given progression is in A.P.

We know that,

n^{th} term of an A.P. is given by $a_n = a + (n - 1)d$

where,

a = –1

d = –3 – (–1) = – 3 + 1 = – 2

Let the nth term of an A.P. is –39.

So, $a_n = -39$

$\Rightarrow a + (n - 1)d = -39$

$\Rightarrow (-1) + (n - 1)(-2) = -39$

$\Rightarrow -1 - 2n + 2 = -39$

$\Rightarrow 1 - 2n = -39$

$\Rightarrow -2n = -40$

$\therefore n = 20$

Thus, the correct option is (b) i.e., 20th.

Example 14. The first term of an A.P. is 5, the common difference is 3, and the last term is 80; find the number of terms.

(a) 78 (b) 26 (c) 15 (d) None of these

Sol. (b) Given, $a = 5$, $d = 3$ and a_n or $l = 80$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 80 = 5 + (n - 1)3$

$\Rightarrow 80 = 5 + 3n - 3$

$\Rightarrow 80 = 2 + 3n$

$\Rightarrow 78 = 3n$

$\Rightarrow n = 26$

Therefore, the number of terms is 26.

Hence, the correct option is (b).

Example 15. $(x + 1)$, $3x$, $(4x + 2)$ are in A.P., then the value of x is

(a) 2 (b) 3 (c) 4 (d) 5

Sol. (b) Given, $(x + 1)$, $3x$, $(4x + 2)$ are in A.P.

Thus, $a_2 - a_1 = a_3 - a_2$

$\Rightarrow 3x - (x + 1) = (4x + 2) - 3x$

$\Rightarrow 2x - 1 = x + 2$

$\Rightarrow x = 3$

Therefore, the value of x is 3.

Hence, the correct option is (b).

Example 16. Which term of the progression $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\ldots$ is $\frac{19}{2}$?

(a) 5 (b) 10 (c) 15 (d) None of these

Sol. (b) Given, sequence $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

The sequence $a_1, a_2, a_3, \ldots a_n$ is in Arithmetic Progression (AP),

when $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$

Then,

First term (a) $= \frac{1}{2}$

Second term $a_2 = \frac{3}{2}$

Third term $a_3 = \frac{5}{2}$

Then, the common difference (d) = $\frac{3}{2} - \frac{1}{2} = \frac{3-1}{2} = 1$

We know that, $t_n = a + (n-1)d = \frac{19}{2}$

$\Rightarrow \frac{1}{2} + (n-1)1 = \frac{19}{2}$

$\Rightarrow (n-1)1 = \frac{19}{2} - \frac{1}{2} = \frac{19-1}{2} = \frac{18}{2} = 9$

$\Rightarrow (n - 1) = 9$

$\Rightarrow n = 9 + 1 = 10$

Hence, the correct option is (b).

Example 17. The 20th term of arithmetic progression whose 6th term is 38 and 10th term is 66 is **(Dec 2020)**

(a) 136 (b) 118 (c) 178 (d) 210

Sol. (a) Given, $a_6 = 38$ and $a_{10} = 66$

Thus,

$a + 5d = 38$

$a + 9d = 66$

On subtracting, we get

$4d = 28$

$d = 7$

$a = 38 - 5d = 38 - 35 = 3$

Therefore, 20th term = $a + 19d = 3 + 19(7) = 136$

Hence, the correct answer is option (a) i.e. 136

Example 18. If the pth term of an A.P. is q and the qth term is q then what will be the value of the (p + q)th term? **(Dec 2022)**

(a) 0 (b) 1 (c) p + q – 1 (d) 2(p + q – 1)

Sol. (a) We know that,

n^{th} term of an A.P. is given by,

$a_n = a + (n - 1)d$

Given that,

$a_p = q$ and $a_q = p$

As per question,

$a + (p - 1)d = q$ ...(i)

$a + (q - 1)d = p$ ...(ii)

Subtracting eq (ii) from eq. (i),

$(p - q)d = q - p$

$d = -1$

Putting in eq. (i),

$\Rightarrow a + (p - 1)(-1) = q$

$\Rightarrow a - p + 1 = q$

$\Rightarrow a = p + q - 1$

Thus, (p + q) term is given by,

$\Rightarrow a_{p+q} = a + (p + q - 1)d$

$\Rightarrow a_{p+q} = p + q - 1 + (p + q - 1)(-1)$

$\Rightarrow a_{p+q} = p + q - 1 - p - q + 1$

$\therefore a_{p+q} = 0$

Hence, the correct option is (a) i.e., 0.

Example 19. If 2nd term = 7 and 7th term = 27, then what is the required A.P.?

(a) 3, 7, 11, 15, ... (b) 3, 5, 7, 9, ... (c) 5, 6, 7, 8, ... (d) None of these

Sol. (a) Given that, $t_2 = 7$, $t_7 = 27$

We know that, $t_n = a + (n - 1)d$

Then,

$\Rightarrow t_2 = a + (2 - 1)d \Rightarrow 7 = a + d$ ...(i)

$\Rightarrow t_7 = a + (7 - 1)d \Rightarrow 27 = a + 6d$ ...(ii)

On subtracting the above equations, we get

$27 - 7 = 6d - d \Rightarrow 20 = 5d \Rightarrow d = 4$

From the equation (i), we get

$a + d = 7 \Rightarrow a + 4 = 7 \Rightarrow a = 7 - 4 \Rightarrow a = 3$

Then, the sequence is a, a + d, a + 2d, a + 3d, ...

Therefore, sequence = 3, (3 + 4), (3 + 2(4)), (3 + 3(4)), ... = 3, 7, 11, 15...

Hence, the correct option is (a).

SELECTION OF TERMS OF AN ARITHMETIC PROGRESSION

When sum of terms of an A.P. is given

Number of Terms	Terms	Common Difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d

Number of Terms	Terms	Common Difference
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d
7	a– 3d, a – 2d, a – d, a, a + d, a + 2d, a + 3d	d

Example 20. Divide 24 into three parts which are in arithmetic progression (A.P.) and such that the product of the first two parts is 48. Find the three parts.

(a) 4, 8, 12 (b) 6, 12, 18 (c) 3, 6, 9 (d) 6, 8, 10

Sol. (d) **Detailed method:**

Let the three parts of A.P. be a – d, a and a + d.

Given that the product of the first two parts is 48, we have

$(a - d) \times a = 48$

$\Rightarrow a^2 - da = 48$ ...(i)

Also, the sum of the three parts is 24

$\Rightarrow (a - d) + a + (a + d) = 24$

$\Rightarrow 3a = 24$

$\Rightarrow a = 8$

Substituting the value of 'a' in equation (i), we get:

$8^2 - 8d = 48$

$\Rightarrow 64 - 8d = 48$

$\Rightarrow 8d = 16$

$\Rightarrow d = 2$

So, the three parts are:

$a - d = 8 - 2 = 6$

$a = 8$,

$a + d = 8 + 2 = 10$

Go by choices:

Option (a): 4, 8, 12

Sum of terms = 4 + 8 + 12 = 24

Product of the first two parts = $4 \times 8 = 32 \neq 48$

Option (b): 6, 12, 18

Sum of terms = 6 + 12 + 18 = $36 \neq 24$

Option (c): 3, 6, 9

Sum of terms = 3 + 6 + 9 = $18 \neq 24$

Option (d): 6, 8, 10

Sum of terms = 6 + 8 + 10 = 24

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Product of the first two parts = 6 × 8 = 48

Thus, option (d) satisfies both the conditions.

Therefore, the required terms are 6, 8, 10.

Hence, the correct option is (d) i.e., 6, 8, 10.

Example 21. Divide 36 into three parts which are in A.P. and are such that the product of the first two parts is 108.

(a) 9,12,15 (b) 1,2,3 (c) 11,12,15 (d) None of these

Sol. (a) **Detailed method:**

Let the three parts be a – d, a, a + d

Then,

$\Rightarrow$ a – d + a + a + d = 36

$\Rightarrow$ 3a = 36

$a = \frac{36}{3} = 12$

Product of the first two parts is 108, then

$\Rightarrow$ a(a – d) = 108

$\Rightarrow$ 12(12 – d) = 108

$\Rightarrow$ 144 – 12d =108

$\Rightarrow$ – 12d = –36

$\Rightarrow d = \frac{36}{12} = 3$

Therefore, the three parts be a – d, a, a + d = 12 – 3, 12, 12 + 3 = 9, 12, 15

Go by choices:

Option (a): 9, 12, 15

Sum of terms = 9 + 12 + 15 = 36

Product of the first two parts = 9 × 12 = 108

Thus, option (a) satisfies both the conditions.

Therefore, the required terms are 9, 12, 15.

Hence, the correct option is (a).

Example 22. Four numbers form an arithmetic progression. The sum of the first and last terms is 8, and the product of the two middle terms is 15. Find the smallest number in the series.

(a) 4 (b) 3 (c) 1 (d) None of these

Sol. (c) Let the four terms be a – 3d, a – d, a + d, a + 3d

Since, the sum of the first and last terms is 8 i.e., (a – 3d) + (a + 3d) = 8

$\Rightarrow$ 2a = 8 $\Rightarrow$ a = 4

Now, the product of the two middle terms is 15

$\Rightarrow (a - d)(a + d) = 15$

$\Rightarrow (4 - d)(4 + d) = 15$

$\Rightarrow 16 - d^2 = 15 \Rightarrow d^2 = 1$

$\Rightarrow d = +1$ or -1

When a = 4 and d = 1, then the terms are 1, 3, 5, 7

When a = 4 and d = – 1, then the terms are 7, 5, 3, 1

Clearly, the smallest number is 1.

Hence, the correct option is (c).

Example 23. Divide 12.50 into five parts in A.P. such that the first part and the last part are in the ratio of 2 : 3.

(a) 2, 2.25, 2.5, 2.75, 3

(b) –2, –2.25, –2.5, 2.75, –3

(c) 4, 4.5, 5, 5.5, 6

(d) –4, –4.5, –5, –5.5, –6

Sol. (a) Given that,

Ratio of first and last part = 2 : 3

i.e., $\frac{\text{first part}}{\text{last part}} = \frac{2}{3} = x$

Let first part (a) = 2x & last part = 3x

Let the first terms of A.P. be:

= 2x, 2x + d, 2x + + 2d, 2x + 3d, 3x

Since, the sum of these terms = 12.50

$\Rightarrow 2x + 2x + d + 2x + 2d + 2x + 3d + 3x = 12.5$

$\Rightarrow 11x + 6d = 12.5$ ...(i)

Also, 5th term = 2x + 4d = 3x

$\Rightarrow x = 4d$ ...(ii)

Putting the value of x in (i),

$\Rightarrow 11 \times 4d + 6d = 12.5$

$\Rightarrow 50d = 12.5$

$\Rightarrow d = 0.25$

From eq (ii),

$\Rightarrow x = 4 \times 0.25 = 1$

So, 5 parts are

2x = 2

2x + d = 2 + 0.25 = 2.25

2x + 2d = 2 + 0.5 = 2.5

2x + 3d = 2 + 0.75 = 2.75

3x = 3

Hence, the correct option is (a) i.e., 2, 2.25, 2.5, 2.75, 3.

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SUM OF THE FIRST N TERMS

Let S be the sum, a be the 1st term and l be the last term of an A.P.

If the number of terms are n, then $l = a_n$ and d be the common difference of the A.P.

Thus, formula for Arithmetic Progressions are:

Sum of n terms of an A.P., $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{n}{2}[a + a + (n-1)d]$$

$$= \frac{n}{2}[a + a_n]$$

or we can say

Sum of n terms of an A.P., $S_n = \frac{n(a+l)}{2}$, where $l = a_n$

Example 24. The sum of the series 1, 3, 5, 7, 9, ... to 100 terms is

(a) 1000 (b) 10,000 (c) 20,000 (d) None of these

Sol. (b) Given sequence 1, 3, 5, 7, 9, ... to 100 terms

Then, $a_1 = 1$, $a_2 = 3$, $a_3 = 5$, ...

Here, the common difference (d) = $a_2 - a_1 = 3 - 1 = 2$

We know that,

Sum of n terms of A.P $(S_n) = \frac{n}{2}(2a + (n-1)d)$

$\Rightarrow (S_{100}) = \frac{100}{2}(2(1) + (100-1)2)$

$\Rightarrow (S_{100}) = 50(2 + (99)2) = 50(2 + 198) = 50(200) = 10000$

Therefore, the sum of 100 terms $(S_{100}) = 10{,}000$

Hence, the correct option is (b).

Example 25. The number of terms of the series: 5 + 7 + 9... must be taken so that the sum is 480. **(July 2021)**

(a) 20 (b) 10 (c) 15 (d) 25

Sol. (a) Given: 5 + 7 + 9 + ... = 480

Since the common difference is same thus it is an A.P. with a = 5, d = 2

$\Rightarrow \frac{n}{2}\{2a + (n-1)d\} = 480$

$\Rightarrow \frac{n}{2}\{10 + (n-1)2\} = 480$

$\Rightarrow n(2n + 8) = 960$

$\Rightarrow 2n^2 + 8n - 960 = 0$

$\Rightarrow n^2 + 4n - 480 = 0$

$\Rightarrow n^2 + 24n - 20n - 480 = 0$

$\Rightarrow (n + 24)(n - 20) = 0$

$\Rightarrow n = 20$ (since, n cannot be negative)

Hence, the answer is option (a) i.e. 20.

Example 26. The first and the last term of an AP are –4 and 146 respectively. The sum of the terms is 7171. The number of terms is **(ICAI)**

(a) 101 (b) 100 (c) 99 (d) None of these

Sol. (a) Given that,

$a = -4$

$l = 146$

$S_n = 7171$

Sum of first n terms of an A.P. is given by,

$$S_n = \frac{n}{2}[a + l]$$

$$\Rightarrow 7171 = \frac{n}{2}[-4 + 146]$$

$$\Rightarrow 7171 = \frac{n(142)}{2}$$

$\therefore n = 101$

Therefore, the correct option is (a) i.e., 101.

Example 27. If the sum of 'n' terms of an AP is $2n^2$, the fifth term is **(July 2021)**

(a) 20 (b) 50 (c) 18 (d) 25

Sol. (c) Given: Sum of 'n' terms of an AP

We know,

$S_1 = a = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$S_4 = a_1 + a_2 + a_3 + a_4$

$S_5 = a_1 + a_2 + a_3 + a_4 + a_5$

$\Rightarrow$ Fifth term $= a_5 = S_5 - S_4 = 2(5)^2 = 2(4)^2 = 50 - 32 = 18$

Hence, the correct answer is option (c) i.e. 18.

Example 28. Find the sum of the following arithmetic progression: $3, \frac{9}{2}, 6, \frac{15}{2}$, ... to 25 terms

(a) 450 (b) 500 (c) 525 (d) 800

Sol. (c) According to the question,

$a = 3$

$d = \frac{9}{2} - 3 = \frac{9-6}{2} = \frac{3}{2}$

$n = 25$

We know that,

Sum of an A.P., $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{25}{2}\left[6 + (25-1)\frac{3}{2}\right] = \frac{25}{2}\left[6 + (24)\frac{3}{2}\right]$

$= \frac{25}{2}[6+36] = \frac{25}{2}[42] = 525$

Hence, the correct option is (c).

Example 29. The sum of five terms of AP is 75, find the 3rd term. **(Dec 2019)**

(a) 20 (b) 30 (c) 15 (d) None of these

Sol. (c) Given: Sum of five terms of AP = 75

$\frac{5}{2}\{2a + (5-1)d\} = 75$

$2a + 4d = 30$

$a + 2d = 15$

i.e. 3rd term = 15

Hence, the correct option is (c) i.e. 15.

Example 30. The sum of a certain number of terms of an A.P. series –8, –6, –4 ... is 52, then the number of terms are

(a) 20 (b) –4 (c) 13 (d) None of these

Sol. (c) Given sequence: –8, –6, –4,...

So, $a_1 = -8$, $a_2 = -6$, $a_3 = -4$

Common difference (d) = $a_2 - a_1 = -6 - (-8) = -6 + 8 = 2$

We know that,

Sum of n terms (S_n) of an A.P

$(S_n) = \frac{n}{2}(2a + (n-1)d)$

$\Rightarrow 52 = \frac{n}{2}\left(2(-8) + (n-1)2\right)$

$\Rightarrow 52 = \frac{n}{2}(-16 + 2n - 2)$

$\Rightarrow 52 = \frac{n}{2}(-18 + 2n)$

$\Rightarrow 52 = n(-9 + n) = n^2 - 9n$

$\Rightarrow n^2 - 9n - 52 = 0$

$\Rightarrow n^2 - 13n + 4n - 52 = 0$

$\Rightarrow n(n - 13) + 4(n - 13) = 0$

$\Rightarrow (n - 13)(n + 4) = 0$

$\Rightarrow n = 13, -4$

Since, the number of terms cannot be negative.

Therefore, number of terms (n) = 13

Hence, the correct option is (c).

Example 31. The sum of all natural numbers between 200 and 400 which are divisible by 7 is **(ICAI)**

(a) 7,730 (b) 8,729 (c) 7,729 (d) 8,730

Sol. (b) The numbers lying between 200 and 400 which are divisible by 7 are 203, 210, 217, ..., 399

This is an A.P. with first term is 203 & common difference is 7

Let the number of terms of the A.P. be n.

We know that,

n^{th} term of an A.P. is given by,

$a_n = a + (n - 1)d$

$\Rightarrow 399 = 203 + (n - 1)7 \Rightarrow 7(n - 1) = 196$

$\Rightarrow n - 1 = 28$

$\therefore n = 29$

Sum of first n terms of an A.P. is given by,

$S_n = \frac{n}{2}[a + l]$

where, l = last term

$\Rightarrow S_n = \frac{29}{2}[203 + 399]$

$\therefore S_n = 8729$

Hence, the correct option is (b) i.e., 8,729.

Example 32. The first and the last term of an AP are –10 and 70 where the common difference is 10, then the numbers of terms in the AP are

(a) 8 (b) 9 (c) 11 (d) None of these

Sol. (b) Given,

First term (a_1) = –10

Last term (a_n) = 70

Here, the common difference (d) = 10

We know that, nth term of an A.P. $(t_n) = a + (n - 1)d$

$\Rightarrow 90 = 10 + (n - 1)10$

$\Rightarrow 90 = 10 + 10n - 10$

$\Rightarrow 90 = 10n$

$\Rightarrow n = 9$

Therefore, number of terms (n) = 9

Hence, the correct option is (b).

Example 33. A person pays ₹975 by monthly instalment each less than the former by ₹5. The first instalment is ₹100. The time by which the entire amount will be paid is **(ICAI)**

(a) 10 months (b) 15 months (c) 14 months (d) none of these

Sol. (b) Given that,

First term (a) = 100

Common difference (d) = – 5

Sum of n terms (Sn) = 975

We know that,

The sum of n terms $(S_n) = \frac{n}{2}\left(2a + (n-1)d\right)$

$\Rightarrow 975 = \frac{n}{2}\left(2(100) + (n-1)(-5)\right)$

$\Rightarrow 975 = \frac{n}{2}\left(200 - 5n + 5\right)$

$\Rightarrow 1950 = 200n - 5n^2 + 5n$

$\Rightarrow 5n^2 - 205n + 1950 = 0$

$\Rightarrow n^2 - 41n + 390 = 0$

$\Rightarrow n^2 - 15n - 26n + 390 = 0$

$\Rightarrow n(n - 15) - 26(n-15) = 0$

$\Rightarrow (n - 15)(n - 26) = 0$

$\Rightarrow n = 15, 26$

Therefore, the required time by which the entire amount will be paid is 15 months.

Hence, the correct option is (b).

Example 34. A man saved ₹16500 in ten years. In each year after the first, he saved ₹100 more than he did in the preceding year. How much did he save in the first year?

(a) ₹1000 (b) ₹1100 (c) ₹1200 (d) ₹1500

Sol. (c) Let the amount saved in the first year be ₹x.

According to the question,

In the second year, he saves 'x + 100'

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In the third year, he saves x + 100 + 100 = x + 200

And so on.

Thus, a = x, d = 100, n = 10 and Sn = 16,500

We know that,

$\Rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 16500 = \frac{10}{2}[2x + (10-1)100]$

$\Rightarrow 16500 = 5\ (2x + 900)$

$\Rightarrow 3300 = 2x + 900$

$\Rightarrow 2400 = 2x$

$\Rightarrow x = 1200$

Therefore, the man saved ₹1200 in the first year.

Hence, the correct option is (c).

Example 35. The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign. The 3rd term of the AP is **(ICAI)**

(a) $6\frac{4}{11}$ (b) 6 (c) $\frac{4}{11}$ (d) None of these

Sol. (a) Given, First term $(a_1) = 14$

where, the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign.

$S_{10} = -S_5$

$\Rightarrow \frac{n}{2}(2a + (n-1)d) = -\left(\frac{n}{2}(2a + (n-1)d)\right)$

$\Rightarrow \frac{10}{2}(2(14) + (10-1)d) = -\left(\frac{5}{2}(2(14) + (5-1)d)\right)$

$\Rightarrow \frac{10}{2}(28 + 9d) = -\left(\frac{5}{2}(28 + 4d)\right)$

$2(28 + 9d) = -(28 + 4d)$

$56 + 18d = -28 - 4d$

$22d = -84$

$\Rightarrow d = \frac{-42}{11}$

We know that, $(t_3) = a + 2d$

Therefore, $(t_3) = a + 2d = 14 + 2\left(\frac{-42}{11}\right) = 14 - \frac{84}{11} = \frac{154 - 84}{11} = \frac{70}{11} = 6\frac{4}{11}$

Hence the correct option is (a).

ARITHMETIC MEAN

If a, b, c are in A.P., then b is called an arithmetic mean between a and c, such that $b = \frac{a+c}{2}$

E.g.: If the numbers 2, 6 and 10 are in A.P., then 6 is the arithmetic mean between 2 and 10 since

$\frac{2+10}{2} = 10$

Now, if 'k' arithmetic mean need to be inserted then

$A.M_k = a + kd$

i.e., $A.M_1 = a + d$

$A.M_2 = a + 2d$

$A.M_3 = a + 3d...$

$A.M_m = a + md$

Example 36. The arithmetic mean between 33 and 77 is **(ICAI)**

(a) 50 (b) 45 (c) 55 (d) None of these

Sol. (c) We know that,

A.M. of a & b = $\frac{a+b}{2}$

Thus, A.M. between 33 and 77 is given by,

$A.M. = \frac{33+77}{2}$

$A.M. = \frac{110}{2}$

A.M. = 55

Therefore, the arithmetic mean between 33 and 77 is 55.

Hence, the correct option is (c) i.e., 55.

Example 37. Find the 3 arithmetic means between 15 and 35.

(a) 16, 18, 20 (b) 10, 15, 20 (c) 20, 25, 30 (d) None of these

Sol. (c) Let A_1, A_2, A_3 are 3 arithmetic means between 15 and 35

Then, first term a = 15, a_5 = 35

We know that, $a_n = a + (n - 1)d$

Then, $a_5 = 15 + (5 - 1)d$

$\Rightarrow 35 = 15 + (4)d$

$\Rightarrow 35 - 15 = 4d$

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$\Rightarrow 20 = 4d$

$\Rightarrow d = 5$

Thus,

$A_1 = a_2 = a + d = 15 + 5 = 20$

$A_2 = a_3 = a_2 + d = 20 + 5 = 25$

$A_3 = a_4 = a_3 + d = 25 + 5 = 30$

Therefore, the three arithmetic means A_1, A_2, A_3 are 20, 25, 30.

Hence, the correct option is (c).

Example 38. The 4 arithmetic means between –2 and 23 are **(ICAI)**

(a) 3, 13, 8, 18 (b) 18, 3, 8, 13

(c) 3, 8, 13, 18 (d) None of these

Sol. (c) Given,

First term, a = –2

Last term = 23

In general an A.P. series can be written as

a, a + d, a + 2d, a + 3d, a + 4d, a + 5d,...

Let, 4 arithmetic means between –2 are 23 are $A.M_1$, $A.M_2$, $A.M_3$, $A.M_4$

Now, series can be written as

–2, $A.M_1$, $A.M_2$, $A.M_3$, $A.M_4$, 23

We know that, n^{th} term of an A.P. is given by,

$a_n = a + (n - 1)d$

As per question,

$23 = -2 + (6 - 1)d$

$\Rightarrow 23 = -2 + 5d$

$\Rightarrow 5d = 25$

$d = 5$

So,

$A.M_1 = a + d = -2 + 5 = 3$

$A.M_2 = a + 2d = -2 + 10 = 8$

$A.M_3 = a + 3d = -2 + 15 = 13$

$A.M_4 = a + 4d = -2 + 20 = 18$

Therefore, the correct option is (c) i.e., 3, 8, 13, 18.

- Sum of 1st n natural or counting numbers: 1, 2, 3, 4, 5, 6, 7, ..., n

$$S=\left(\frac{n(n+1)}{2}\right)$$

- Sum of the Squares of the first, n natural numbers:

$$S = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

- Sum of Cubes of the first, n natural numbers:

$$S = 1^3 + 2^3 + 3^3 + ... + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

Example 39. Find out the sum of 1st n natural odd numbers.

(a) n^{2n} (b) 2n (c) n^2 (d) None of these

Sol. (c) 1st n natural odd numbers are 1, 3, 5, 7, 9, ..., n

Thus, first term $a_1 = 1$ and common difference (d) = $a_2 - a_1 = 3 - 1 = 2$

We know that, sum of terms in a sequence is $S_n = \frac{n}{2}(2a+(n-1)d)$

$$\Rightarrow (S_n)=\frac{n}{2}(2(1)+(n-1)2)$$

$$\Rightarrow (S_n)=\frac{n}{2}(2+2n-2)$$

$$\Rightarrow (S_n)=\frac{n}{2}(2n)$$

$$\Rightarrow (S_n) = n^2$$

Therefore, the sum of 1st n odd natural numbers $(S_n) = n^2$

Hence the correct option is (c).

Example 40. The number of numbers between 74 and 25,556 divisible by 5 is

(a) 5,090 (b) 5,097 (c) 5,095 (d) None of these

Sol. (b) The numbers between 74 and 25,556 which are divisible by 5 are:

75, 80, 85, 90, ..., 25550, 25555

Thus,

First term $(a_1) = 75$

Common difference (d) = 5

Last term $(a_n) = 25555$

We know that, $a_n = a + (n - 1)d$

$\Rightarrow 25555 = 75 + (n - 1)5$

$\Rightarrow 25555 = 75 + 5n - 5$

$\Rightarrow 25555 = 70 + 5n$

$\Rightarrow 25555 - 70 = 5n$

$\Rightarrow 25485 = 5n$

$\Rightarrow n = 5097$

Therefore, the number of numbers between 74 and 25556 divisible by 5 is 5097.

Hence, the correct option is (b).

Example 41. The sum of n terms of an AP is $3n^2 + 5n$. The series is

(a) 8, 14, 20, 26 (b) 8, 22, 42, 68

(c) 22, 68, 114,... (d) None of these

Sol. (a) Given that,

The sum of n terms of an AP $(S_n) = 3n^2 + 5n$

We know that,

First term $= t_1 = S_1$

$\Rightarrow S_2 = t_1 + t_2 \Rightarrow t_2 = S_2 - t_1$

$\Rightarrow S_3 = t_1 + t_2 + t_3 \Rightarrow t_3 = S_3 - S_2$

Thus, $t_n = S_n - S_{n-1}$

$t_1 = S_1 = 3(1)^2 + 5(1) = 3 + 5 = 8$

$\Rightarrow t_2 = S_2 - S_{2-1} = S_2 - S_1 = (3(2)^2 + 5(2)) - 8 = (3(4) + 10) - 8 = 12 + 10 - 8 = 14$

$\Rightarrow t_3 = S_3 - S_{3-1} = S_3 - S_2 = (3(3)^2 + 5(3)) - 22 = (3(9) + 15) - 22 = 27 + 15 - 22 = 20$

$\Rightarrow t_4 = S_4 - S_{4-1} = S_4 - S_3 = (3(4)^2 + 5(4)) - 42 = (3(16) + 20) - 42 = 68 - 42 = 26$

Therefore, the series is $t_1, t_2, t_3, t_4, ..., t_n = 8, 14, 20, 26, ...$

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART B)

1. The first term and the common difference of the A.P.: 3, 1, –1, –3, ... is

(a) 3, –1 (b) 3, –2 (c) 1, –2 (d) None of these

2. The 20th term of the A.P: 3, 5, 7, 9, ... is

(a) 39 (b) 40 (c) 41 (d) 45

3. Which term of the A.P: 21, 18, 15, ... is –81?

(a) 30 (b) 32 (c) 34 (d) 35

4. The 11th term of the A.P.: –3, $-\frac{1}{2}$, 2, ... is

(a) 28 (b) 22 (c) –38 (d) $-48\frac{1}{2}$

5. The number of the terms in the A.P.: 7, 13, 19, ..., 205 are

(a) 34 (b) 39 (c) 35 (d) 33

6. The value of x such that 8x + 4, 6x – 2, 2x + 7 will form an A.P. is **(ICAI)**

(a) 15 (b) 2 (c) $\frac{15}{2}$ (d) None of these

7. The value of K for which the terms 7K + 3, 4K – 5, 2K + 10 are in A.P. is

(a) -13 (b) -23 (c) 13 (d) 23 **(Nov 2018)**

8. The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term a and common difference .

(a) a = 3, d = 2 (b) a = 4, d = 3 (c) a = 5, d = 4 (d) a = 6, d = 5

9. Divide 69 into 3 parts which are in A.P and are such that the product of first two parts is 460 **(Dec 2020)**

(a) 20, 23, 26 (b) 21, 23, 25 (c) 19, 23, 27 (d) 22, 23, 24

10. Divide 144 into three parts which are in A.P. and such that the largest is twice the smallest, the smallest of three numbers will be:

(a) 48 (b) 36 (c) 13 (d) 32

11. Divide 96 into four parts which are in A.P. and the ratio between product of their means to the product of their extremes is 15 : 7 and find the largest term of the series.

(a) 6 (b) 18 (c) 30 (d) 42

12. Divide 20 into five parts in A.P. such that the first and last parts are in the ratio 2 : 3.

(a) $\frac{24}{5}, \frac{29}{5}, 6, \frac{33}{5}, \frac{36}{5}$ (b) $6, \frac{36}{5}, \frac{33}{5}, \frac{24}{5}, \frac{27}{5}$

(c) $\frac{27}{5}, \frac{24}{4}, \frac{36}{5}, \frac{33}{5}, 6$ (d) None of these

13. Find the sum of the series, 201, 203, 205, ..., 299.

(a) 11,000 (b) 15,400 (c) 12,500 (d) 25,750

14. Insert two arithmetic means between 68 and 260.

(a) 132, 196 (b) 130, 194 (c) 70, 258 (d) None of these

15. The sum of the first 22 terms of the A.P.: 8, 3, –2, ...

(a) 979 (b) –979 (c) 875 (d) None of these

16. Insert 4 arithmetic means between 4 and 324.

(a) 64, 132, 164, 260 (b) 68, 132, 196, 260

(c) 32, 68, 132, 196 (d) None of these

17. Insert 4 A.M's between 3 and 18:

(a) 12, 15, 9, 6 (b) 6, 9, 12, 15 (c) 9, 6, 12, 15 (d) 15, 12, 9, 6

18. The sum of first 51 terms of an A.P. whose 2nd and 3th terms are 14 and 18 respectively.

(a) 5110 (b) 5120 (c) 5550 (d) 5610

19. Find the sum of the series, 201, 203, 205, ..., 299.

(a) 11,000 (b) 15,400 (c) 12,500 (d) 25,750

20. The sum of the series $3\frac{1}{2}+7+10\frac{1}{2}+14+$ ..... to 17 terms is **(ICAI)**

(a) 530 (b) 535 (c) $535\frac{1}{2}$ (d) None of these

21. The first term of an A.P. is 5, the last term is 45 and the sum of its terms is 1000, then the number of terms is

(a) 20 (b) 30 (c) 35 (d) 40

22. Find the sum of first 20 terms of an A.P. whose first term is 3 and the last term is 57.

(a) 300 (b) 350 (c) 500 (d) 600

23. The sum of the first 22 terms of an A.P. whose d = 7 and 22nd term is 149 is

(a) 1551 (b) 1661 (c) 1775 (d) None of these

24. The sum of all natural numbers from 100 and 300, which are divisible by 4, is _____.

(a) 10,200 (b) 30,000 (c) 8,200 (d) 2,200

25. The sum of all natural numbers between 400 and 900 which are divisible by 13 is

(a) 22504 (b) 29405 (c) 25402 (d) 25,350

Answer Key

1. (b)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (d)
11. (d)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (c)	**20.** (c)
21. (d)	**22.** (d)	**23.** (b)	**24.** (a)	**25.** (d)					

GEOMETRIC PROGRESSION (G.P.)

- If a sequence is such that the one term is the multiple of the previous term. It means that there is a common multiplier called a common ratio which is the ratio of a term to its previous term.

E.g.:

(1) 1, 3, 9, 27, 81 … (2) 1, $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}$, …. (3) 1, –4, 16, –64, 256 …

In (1), the common ratio = $\frac{3}{1}$ = 3

In (2), the common ratio = $\frac{\frac{1}{2}}{1}=\frac{1}{2}$

In (3), the common ratio = $-\frac{4}{1}$ = –4

Now, if we need to generalize the Geometric progression, where first term = a, common ratio = r and number of terms = n, then

n^{th} term of G.P. $t_n = ar^{n-1}$

Common ratio: $r = \frac{\text{Any term}}{\text{Preceding term}} = \frac{t_n}{t_{n-1}} = r$

Geometric Progression series = $a + ar + ar^2 + ar^3 + ar^4 + ar^5 + \ldots$ such that $r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \ldots$

Example 42. If 5, 25, 125 … is a G.P, then what is the common ratio and its 15th term?

(a) $10, 5^{15}$ (b) $5, 5^{10}$ (c) $5, 5^{15}$ (d) None of these

Sol. (c) Given series: 5, 25,125 ….

First term $(a_1) = 5$

Second term$(a_2) = 25$

We know that,

The sequence $a_1, a_2, a_3, \ldots, a_n$ is a Geometric Progression (G.P.),

when $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \ldots = \frac{a_n}{a_{n-1}} = r$ and r is the common ratio.

$\Rightarrow r = \frac{a_2}{a_1} = \frac{25}{5} = 5$

We know that, n^{th}term in the sequence $t_r = ar^{n-1}$

$\Rightarrow T_r = ar^{n-1} = 5(5)^{15} - 1 = 51 + 15 - 1 = 5^{15}$

Therefore, the common ratio is 5 and its 15th term is 5^{15}

Hence, the correct option is (c).

Example 43. t_{12} of the series – 128, 64, -32, ... is **(ICAI)**

(a) $\frac{-1}{6}$ (b) 16 (c) $\frac{1}{16}$ (d) None of these

Sol. (c) Given series is in G.P. since the common ratio is same.

We know that, general term of G.P. is given by,

$t_n = ar^{n-1}$

where,

$a = -128$

$r = \frac{64}{-128} = -\frac{1}{2}$

So, the 12th term of series is,

$t_{12} = ar^{12-1}$

$\Rightarrow t_{12} = (-128) \times (-0.5)^{11}$

$\Rightarrow t_{12} = -(2)^7 \times \left(-\frac{1}{2}\right)^{11}$

$\Rightarrow t_{12} = \frac{1}{(2)^4}$

$\Rightarrow t_{12} = \frac{1}{16}$

Therefore, the correct option is (c) i.e., $\frac{1}{16}$.

Example 44. In the series 25, 5, 1, ..., $\frac{1}{3125}$, then which term is $\frac{1}{3125}$? **(Dec 2019)**

(a) 8th term (b) 9th term (c) 15th term (d) None of these

Sol. (a) Given series: 25, 5, 1, ..., $\frac{1}{3125}$

Since, the common ratio is same, thus it is a G.P.

Here, ratio = $\frac{1}{5}$

We know that,

$t_n = ar^{n-1}$

$\Rightarrow 25(\frac{1}{5})^{n-1} = \frac{1}{3125}$

$\Rightarrow 5^2 \times \frac{1}{5^{n-1}} = \frac{1}{3125}$

$\Rightarrow \frac{1}{5^{n-3}} = \frac{1}{5^5}$

$\Rightarrow n - 3 = 5$

$\Rightarrow n = 8$

Thus, $\frac{1}{3125}$ is 8th term.

Hence, the correct answer is option (a) i.e. 8th term.

Example 45. Which term of series 3, 9, 27,… is 243?

(a) 5 (b) 6 (c) 7 (d) None of these

Sol. (a) Given series: 3, 9, 27,…

First term $(a_1) = 3$

Second term $(a_2) = 9$

The sequence $a_1, a_2, a_3 \ldots a_n$ is in Geometric Progression (G.P.),

when $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \ldots = \frac{a_n}{a_{n-1}} = r$

where r is the common ratio.

$\Rightarrow r = \frac{a_2}{a_1} = \frac{9}{3} = 3$

We know that, n^{th}term in the sequence

$t_n = ar^{n-1}$

$\Rightarrow 243 = (3)3^{n-1} \Rightarrow 243 = 3^{1+n-1}$

$\Rightarrow 3^5 = 3^n \Rightarrow n = 5$

Therefore, 243 is the 5th term

Hence the correct option is (a).

Example 46. In a GP. if the fourth term is 3, then the product of first seven terms is

(a) 3^5 (b) 3^7 (c) 3^6 (d) 3^8 **(June 2019)**

Sol. (b) Given: Fourth term = 3

i.e., $ar^3 = 3$

Now, Product of first seven terms

$= a(ar)\,(ar^2)\,(ar^3)\,(ar^4)\,(ar^5)\,(ar^6) = a^7r^{0+1+2+\ldots+6} = a^7r^{21} = (ar^3)^7 = 3^7$

Hence, the correct answer is option (b) i.e. 3^7.

Example 47. Which term of the G.P. $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ is 729?

(a) 9th (b) 10th (c) 12th (d) None of these

Sol. (c) Given: $\sqrt{3}, 3, 3\sqrt{3}, \ldots 729$

Here, $a = \sqrt{3}$ and $r = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = \sqrt{3}$

Let 729 be n^{th} term of G.P., then

$t_n = ar^{n-1}$

$\Rightarrow 729 = \sqrt{3}(\sqrt{3})^{n-1} \Rightarrow 729 = (\sqrt{3})^n$

$\Rightarrow 3^6 = (\sqrt{3})^n \Rightarrow (\sqrt{3})^{12} = (\sqrt{3})^n$

On comparing, we get

$n = 12$

Therefore, 729 is 12th term of G.P.

Hence, the correct option is (c).

Example 48. The last term of the series x^2, x, 1, ... to 31 terms is **(ICAI)**

(a) x^{28} (b) $\dfrac{1}{x}$ (c) $\dfrac{1}{x^{28}}$ (d) None of these

Sol. (c) We have, x^2, x, 1, ...

Here, the common ratio $\left(r = \dfrac{1}{2}\right)$ is same. So, the given series is in G.P.

We know that,

n^{th} term of an G.P. is, $t_n = ar^{n-1}$

Therefore, the last or term of G.P. is given by,

$a_{31} = (x^2)\left(\frac{1}{x}\right)^{31-1}$

$\Rightarrow a_{31} = (x^2)\left(\frac{1}{x}\right)^{30} \Rightarrow a_{31} = x^{2-30} \Rightarrow a_{31} = x^{-28}$

$\therefore a_{31} = \frac{1}{x^{28}}$

Hence, the correct option is (c) i.e., $\frac{1}{x^{28}}$.

Example 49. Find the G.P where 3rd term is 9 and 7th term is $\frac{729}{256}$.

(a) 16, 12, 9, $\frac{27}{4}$....

(b) $-16, -12, -9, \frac{27}{4}$

(c) 1, 2, 9, 11...

(d) None of these

Sol. (a) Given that, 3rd term $(t_3) = 9$

7th term $(t_7) = \frac{729}{256}$

We know that, n^{th} term $(t_n) = ar^{n-1}$

Thus,

$\Rightarrow \frac{t_7}{t_3} = \frac{ar^{7-1}}{ar^{3-1}} \Rightarrow \frac{\frac{729}{256}}{9} = \frac{ar^6}{ar^2}$

$\Rightarrow r^4 = \frac{729}{256 \times 9} = \frac{81}{256} = \frac{3^4}{4^4} \Rightarrow r = \frac{3}{4}$

Then, $t_3 = ar^2 \Rightarrow 9 = a\left(\frac{3}{4}\right)^2 \Rightarrow 9 = a\left(\frac{9}{16}\right) \Rightarrow a = 16$

We know that, the series in GP are a, ar, ar^2, ar^3, ar^4, ..., ar^n

Then, GP series is $16, 16\left(\frac{3}{4}\right), 16\left(\frac{3}{4}\right)^2, 16\left(\frac{3}{4}\right)^3, 16\left(\frac{3}{4}\right)^4, \ldots = 16, 12, 9, \frac{27}{4}, \ldots$

Hence, the correct option is (a).

SUM OF GEOMETRIC PROGRESSION

Let's take the sum of first n terms of G.P. i.e. $S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$

Now, the formula of sum of G.P. has three cases:

1. If common ratio, $r > 1$, then $S_n = \frac{a(r^n - 1)}{r - 1}$
2. If common ratio, $r < 1$, then $S_n = \frac{a(1 - r^n)}{1 - r}$

3. If common ratio, r = 1, then each term of the G.P. is equal to 'a'.

Therefore, the sum of the G.P. simplifies to:

$S_n = a + a + a + a + ... + a$ (n terms)

$S_n = na$

4. Sum of infinite geometric series:

$$S = \frac{a}{1-r}, r < 1$$

Example 50. Find the sum of series 1 + 4 + 16 + … up to 7 terms.

(a) 5121 (b) 5461 (c) 5752 (d) None of these

Sol. (b) Given series: 1 + 4 + 16 + …

First term $(a_1) = 1$

Second term $(a_2) = 4$

Common ratio $(r) = \frac{4}{1} = 4$

We know that, sum of n terms $(S_n) = \frac{a(r^n - 1)}{r - 1}$

$$\Rightarrow (S_7) = \frac{1(4^7 - 1)}{4 - 1} = \frac{(16384 - 1)}{3} = 5461$$

Therefore, the sum of series 1 + 4 + 16 + … up to 7 terms is 5461.

Hence, the correct option is (b).

Example 51. The sum of terms of the G.P. 3, 6, 12, ... is 381. Find the value of n.

(a) 3 (b) 12 (c) 5 (d) 7

Sol. (d) Given G.P., 3 + 6 + 12 + ... = 381

Here, a = 3, $r = \frac{6}{3} = 2$ and $S_n = 381$

We know that,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow 381 = \frac{3[(2)^n - 1]}{2 - 1} \Rightarrow 381 = \frac{3[(2)^n - 1]}{1}$$

$$\Rightarrow \frac{381}{3} = (2)^n - 1$$

$\Rightarrow 127 = (2)^n - 1$

$\Rightarrow 128 = (2)^n$

$\Rightarrow 2^7 = (2)^n$

On comparing, we get

$\Rightarrow n = 7$

Hence, the correct option is (d).

Example 52. How many terms of the G.P. $3, \frac{3}{2}, \frac{3}{4}, \ldots$ be taken together to make $\frac{3069}{512}$?

(a) 7 (b) 8 (c) 10 (d) Cannot be determined

Sol. (c) Given, $3+\frac{3}{2}+\frac{3}{4}+\ldots=\frac{3069}{512}$

Here, a = 3, $r=\frac{\frac{3}{2}}{3}=\frac{1}{2}=0.5$ and $S_n=\frac{3069}{512}$

We know that,

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$\Rightarrow \frac{3069}{512}=\frac{3(1-(0.5)^n)}{1-0.5}$$

$$\Rightarrow \frac{3069}{512}=\frac{3(1-(0.5)^n)}{0.5}$$

$$\Rightarrow \frac{3069}{512}\times\frac{0.5}{3}=1-(0.5)^n \Rightarrow \frac{1023}{512}\times\frac{1}{2}=1-(0.5)^n$$

$$\Rightarrow \frac{1023}{1024}=1-(0.5)^n$$

$$\Rightarrow 1-\frac{1023}{1024}=\left(\frac{5}{10}\right)^n$$

$$\Rightarrow \frac{1}{1024}=\left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^{10}=\left(\frac{1}{2}\right)^n$$

$\Rightarrow n = 10$

Therefore, the number of terms is 10.

Hence, the correct option is (c).

Example 53. The sum of the infinite G. P. $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$ is

(a) 2 (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) None of these

Sol. (b) Given that,

The sum of the infinite $(S_r)=1+\left(\frac{-1}{2}\right)+\left(\frac{1}{4}\right)+\left(\frac{-1}{8}\right)+\ldots$

First term $(a_1) = 1$

Second term $(a_2)=\frac{-1}{2}$

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Common ratio $(r) = \frac{\frac{-1}{2}}{1} = \frac{-1}{2}$

We know that,

$S_\infty = \frac{a}{1-r}$

Putting the values in the above formula,

$\Rightarrow S_\infty = \frac{1}{1-\left(\frac{-1}{2}\right)} = \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

Hence, the correct option (b).

Example 54. If $y = 1 + x + x^2 + ... + \infty$, then $x =$ **(June 2019)**

(a) $\frac{y-1}{y}$ (b) $\frac{y+1}{y}$ (c) $\frac{y}{y+1}$ (d) $\frac{y}{y-1}$

Sol. (a) Given: $y = 1 + x + x^2 + ...$

Clearly, the common ratio (x) is the same, thus the series is in G.P.

Thus,

$y = \frac{1}{1-x}$ where common ratio = x

$x = 1 - \frac{1}{y}$

$x = \frac{y-1}{y}$

Hence, the correct answer is option (a) i.e. $x = \frac{y-1}{y}$.

Example 55. The common ratio of a G.P. is 3, and the last term is 486. If the sum of these terms is 728, find the first term.

(a) 2 (b) 5 (c) 6 (d) 3

Sol. (a) Given: $r = 3$, $t_n = 486$ and $S_n = 728$

We know that,

n^{th} term of G.P, $t_n = ar^{n-1}$

$486 = a(3)^{n-1}$

$486 = a(3)^n.3^{-1}$

$\Rightarrow 486 = \frac{a(3)^n}{3}$

$\Rightarrow 1458 = a(3)^n$ ...(i)

Also, Sum of terms of G.P., $S_n = \frac{a(1-r^n)}{1-r}$

$\Rightarrow 728 = \frac{a(1-(3)^n)}{1-3}$

$\Rightarrow 728 = \frac{a(1-(3)^n)}{-2}$

$\Rightarrow -1456 = a(1-(3)^n)$

$\Rightarrow -1456 = a - a(3)^n$

$\Rightarrow -1456 = a - 1458$(from (i))

$\Rightarrow 1458 - 1456 = a \Rightarrow a = 2$

Therefore, the first term is 2.

Hence, the correct option is (a).

Example 56. The series 3, 6, 9, 12, 15 are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Sol. (a) Given sequence: 3, 6, 9, 12, 15

Here, $a_1 = 3$, $a_2 = 6$, $a_3 = 9$, $a_4 = 12$, $a_5 = 15$

The sequence $a_1, a_2, a_3, a_4, ..., a_n$ is in Arithmetic Progression (A.P.),

when $a_2 - a_1 = a_2 - a_3 = ... = a_n - a_{n-1}$

Thus, $a_2 - a_1 = 6 - 3 = 3$

$a_3 - a_2 = 9 - 6 = 3$

$a_4 - a_3 = 12 - 9 = 3$

Clearly, the common difference is constant i.e. 3.

Therefore, the given sequence is in Arithmetic Progression (A.P.).

Hence, the correct option is (a).

Example 57. If the first term of a G.P. exceeds the second term by 2 and the sum to infinity is 50, the series is _____.

(a) $10, 8, \frac{32}{5}, ...$ (b) $10, 8, \frac{5}{2}, ...$ (c) $10, \frac{10}{3}, \frac{10}{9}, ...$ (d) None of these

Sol. (a) Given: $S_\infty = 50$

Let the first term be a

According to the question,

The two terms will be a, a – 2

Then, common ratio (r) = $\frac{a-2}{a}$

We know that,

Sum of infinite series is given as, $S_\infty = \frac{a}{1-r}$

$$50 = \frac{a}{1-\frac{a-2}{a}}$$

$$50 = \frac{a}{\frac{a-a+2}{a}}$$

$$50 = \frac{a^2}{2}$$

$a^2 = 100$

$a = 10$

Thus, the first two terms are $a = 10$, $a - 2 = 8$

Now, $r = \frac{10-2}{10} = \frac{4}{5}$

Third term, $ar^2 = 10 \times \left(\frac{4}{5}\right)^2$

Thus, the series is $10, 8, \frac{32}{5}, \ldots$

Hence, the correct answer is option (a) i.e., $10, 8, \frac{32}{5}, \ldots$.

Example 58. The sum of the series $\frac{1}{\sqrt{3}} + 1 + \frac{2}{\sqrt{3}} + \ldots$ to 18 terms is **(ICAI)**

(a) $\frac{9841(\sqrt{3}+1)}{\sqrt{3}}$ (b) 9841 (c) $\frac{9841}{\sqrt{3}}$ (d) None of these

Sol. (a) As we know that,

Sum of first n terms of a G.P.

$$S_n = \frac{a(r^n - 1)}{(r-1)}$$

We have,

$\frac{1}{\sqrt{3}} + 1 + \frac{1}{\sqrt{3}} + \ldots$ to 18 terms

Here, $a = \frac{1}{\sqrt{3}}, r = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}, n = 18$

So, $S_n = \frac{a(r^n - 1)}{(r-1)}$

$$\Rightarrow S_{18} = \frac{\left(\frac{1}{\sqrt{3}}\right)((\sqrt{3})^{18} - 1)}{(\sqrt{3} - 1)} \Rightarrow S_{18} = \frac{19682}{\sqrt{3}(\sqrt{3} - 1)}$$

On rationalizing,

$$\Rightarrow S_{18} = \frac{19682}{\sqrt{3}(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow S_{18} = \frac{19682(\sqrt{3}+1)}{\sqrt{3}((\sqrt{3})^2-(1)^2)}$$

$$\Rightarrow S_{18} = \frac{19682(\sqrt{3}+1)}{2\sqrt{3}}$$

$$\therefore S_{18} = \frac{9841(\sqrt{3}+1)}{\sqrt{3}}$$

Hence, the correct option is (a) i.e., $\frac{9841(\sqrt{3}+1)}{\sqrt{3}}$.

GEOMETRIC MEAN

If we take terms a, b, c which are in G.P. then, $\frac{b}{a}=\frac{c}{b}$ i.e., $b^2 = ac$ thus b is called the geometric mean of a and c.

Example 59. Insert 3 geometric means between 1 and $\frac{1}{16}$.

(a) $1,\frac{1}{2},\frac{1}{4}$ (b) $\frac{1}{2},\frac{1}{4},\frac{1}{8}$ (c) $\frac{1}{4},\frac{1}{8},\frac{1}{16}$ (d) None of these

Sol. (b) Let the three geometric means between 1 and $\frac{1}{16}$ are g_1, g_2, g_3 i.e., series is $1, g_1, g_2, g_3, \frac{1}{16}$

First term (a) = 1

We know that, n^{th} term in the sequence

$t_n = ar^{n-1}$

$$\Rightarrow \frac{1}{16} = (1)r^{5-1} \Rightarrow \frac{1}{16} = (1)r^4$$

$$\Rightarrow \frac{1}{16} = r^4 \Rightarrow r = \frac{1}{2}$$

Then,

$$\Rightarrow g_1 = T_2 = (1)\left(\frac{1}{2}\right)^{2-1} = \frac{1}{2}$$

$$\Rightarrow g_2 = T_3 = (1)\left(\frac{1}{2}\right)^{3-1} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\Rightarrow g_3 = T_4 = (1)\left(\frac{1}{2}\right)^{4-1} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Therefore, the geometric means are $\frac{1}{2},\frac{1}{4},\frac{1}{8}$

Hence, the correct option is (b).

Example 60. The A.M. of two positive numbers is 40 and their G. M. is 24. The numbers are

(a) (72, 8) (b) (70, 10) (c) (60, 20) (d) None of these **(ICAI)**

Sol. (a) **Detailed Method:**

Let the two numbers are a & b.

We know that,

A.M. of two a & b = $\frac{a+b}{2}$

G.M. of two a & b = $\sqrt{ab}$

As per question,

A.M. of two positive numbers = 40

$\Rightarrow \frac{a+b}{2} = 40 \Rightarrow a + b = 80$ ...(i)

Also, G.M. of two positive numbers = 24

$\Rightarrow \sqrt{ab} = 24$

Squaring both sides,

$\Rightarrow ab = 576 \Rightarrow a = \frac{576}{b}$

Substitute the value of a in eq (i),

$\Rightarrow \frac{576}{b} + b = 80 \Rightarrow 576 + b^2 = 80b$

$\Rightarrow b^2 - 80b + 576 = 0$

$\Rightarrow b^2 - 72b - 8b + 576 = 0$

$\Rightarrow b(b - 72) - 8(b - 72) = 0$

$\Rightarrow (b - 8)(b - 72) = 0$

$\Rightarrow b = 8$ or $b = 72$

From eq (i),

$\Rightarrow a = 80 - b$

$\Rightarrow a = 80 - 8$ or $a = 80 - 72$

$\Rightarrow a = 72$ or $a = 8$

Therefore, the two numbers are 8 and 72.

Trick: Go by choices

For option (a) only, both the conditions are satisfied.

i.e. $\frac{72+8}{2} = \frac{80}{2} = 40$ (A,M)

and $\sqrt{72 \times 8} = \sqrt{576} = 24$ (G.M)

Hence, the correct option is (a) i.e., (72, 8).

SELECTION OF TERMS OF A GEOMETRIC PROGRESSION

When the product of terms of G.P. is given

Number of Terms	Terms	Common Ratio
3	$\frac{a}{r}, a, ar$	r
4	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	r^2
5	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	r
6	$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$	r^2
7	$\frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3$	r

Example 61. The product of the first three terms of a G.P. is –1. Find the middle term.

(a) -1 (b) 0 (c) 1 (d) 3

Sol. (a) Let the three terms of G.P. be $\frac{a}{r}, a, ar$

According to the question,

Product of the first three terms of a G.P. = –1

$\Rightarrow \frac{a}{r} \times a \times ar = -1$

$\Rightarrow a^3 = -1$

$\Rightarrow a^3 = (-1)^3$

$\Rightarrow a = -1$

Therefore, the middle term is –1.

Hence, the correct option is (a).

Example 62. The sum of 3 numbers of a G.P. is 39 and their product is 729. The numbers are

(a) 2, 4, 8 (b) 3, 9, 27 (c) 3, 9, 12 (d) None of these

Sol. (b) Let the three numbers are $\frac{a}{r}, a, ar$

Given that the product of three numbers is 729.

Then,

$\Rightarrow \frac{a}{r} \times a \times ar = 729$

$\Rightarrow a^3 = 729$

$\Rightarrow a = 9$

Also, the sum of three terms is 39, then

$\Rightarrow \frac{a}{r}+a+ar=39 \Rightarrow \frac{9}{r}+9+9r=39$

$\Rightarrow \frac{9}{r}+9r=39-9=30$

$\Rightarrow 9r^2 - 30r + 9 = 0$

$\Rightarrow 9r^2 - 3r - 27r + 9 = 0$

$\Rightarrow 3r(3r - 1) - 9(3r - 1) = 0$

$\Rightarrow (3r - 9)(3r - 1) = 0$

$\Rightarrow r = \frac{1}{3}, 3$

If r = 3 then the three terms are 3, 9, 27

If $r = \frac{1}{3}$ then the three terms are 27, 9, 3

Therefore, the three terms are 3, 9, 27.

Hence, the correct option is (b).

Example 63. Three numbers are in A.P. and their sum is 21. If 1, 5, 15 are added to them respectively, they form a G.P. The numbers are **(ICAI)**

(a) 5, 7, 9 (b) 9, 5, 7 (c) 7, 5, 9 (d) None of these

Sol. (a) Let the three numbers be a – d, a, a + d where, d is the common difference

Given that, the sum of three numbers is 21

then,

$\Rightarrow a - d + a + a + d = 21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7$

Thus, the three terms are 7 – d, 7, 7 + d.

Given that, if 1, 5, 15 are added then, the series becomes G.P.

Then, the G.P. series is 7 – d + 1, 7 + 5, 7 + d + 15 = 8 – d, 12, 22 + d

$\Rightarrow 12^2 = (8 - d) \times (22 + d)$

$\Rightarrow 144 = 8d + 176 - d^2 - 22d$

$\Rightarrow d^2 + 14d + 144 - 176 = 0$

$\Rightarrow d^2 + 14d - 32 = 0$

$\Rightarrow d^2 + 16d - 2d - 32 = 0$

$\Rightarrow d(d + 16) - 2(d + 16) = 0$

$\Rightarrow (d - 2)(d + 16) = 0$

$\Rightarrow d = -16$ or $d = 2$

(i) When d = – 16

Thus, required three numbers are

First number = a – d = 7– (–16) = 23

Second number = a = 7

Third number = a + d = 7 + (–16) = –9

(ii) When d = 2

First number = a – d = 7 – 2 = 5

Second number = a = 7

Third number = a + d = 7 + 2 = 9

Therefore, the correct option is (a).

Example 64. The sum of the infinite series $1+\frac{2}{3}+\frac{4}{9}+...$ is

(a) $\frac{1}{3}$ (b) 3 (c) $\frac{2}{3}$ (d) None of these

Sol. (b) Given that,

The sum of the infinite $(S_r)=1+\left(\frac{2}{3}\right)+\left(\frac{4}{9}\right)+\ldots$

First term $(a_1) = 1$

Second term $(a_2)=\frac{2}{3}$

Common ratio $(r)=\frac{\frac{2}{3}}{1}=\frac{2}{3}$

We know that, sum of infinite terms of G.P. is given b

$$S_\infty=\frac{a}{1-r}$$

Then,

$$\Rightarrow S_\infty=\frac{a}{1-r}=\frac{1}{1-\left(\frac{2}{3}\right)}=\frac{1}{1-\frac{2}{3}}=\frac{1}{\frac{1}{3}}=3$$

Hence, the correct option is (b).

Example 65. The sum of $1.03 + (1.03)^2 + (1.03)^3 + \ldots.$ to n terms is

(a) $103\{(1.03)^n - 1\}$ (b) $\frac{103}{3}\{(1.03)^n - 1\}$

(c) $(1.03)^n - 1$ (d) None of these

Sol. (b) Given series: $1.03 + (1.03)^2 + (1.03)^3 + \ldots$. to n terms

First term $(a_1) = 1.03$

Second term $(a_2) = (1.03)^2$

Common ratio $(r) = \frac{(1.03)^2}{1.03} = 1.03$

We know that, the sum of series in a G.P. $= \frac{a(r^n - 1)}{r - 1}$

$= \frac{1.03(1.03^n - 1)}{1.03 - 1} = \frac{1.03(1.03^n - 1)}{0.03} = \frac{103}{3}(1.03^n - 1)$

Hence, the correct option is (b).

Example 66. Given x, y, z are in G.P. and $x^p = y^q = z^m$, then $\frac{1}{p}, \frac{1}{q}, \frac{1}{m}$ are in

(a) A.P. (b) G.P.

(c) Both A.P. and G.P. (d) None of these

Sol. (a) Given that,

x, y, z are in G.P.

Let us consider $x^p = y^q = z^m = k$

Thus

$k^{\frac{1}{p}} = x, k^{\frac{1}{q}} = y, k^{\frac{1}{m}} = z$

As x, y, z are in G.P. we get that $y^2 = xz$

$\Rightarrow y^2 = xy$

$\Rightarrow \left(k^{\frac{1}{q}}\right)^2 = k^{\frac{1}{p}} \times k^{\frac{1}{m}}$

$\Rightarrow \left(k^{\frac{2}{q}}\right) = k^{\frac{1}{p} + \frac{1}{m}}$

On comparing on both sides, we get

$\Rightarrow \frac{2}{q} = \frac{1}{p} + \frac{1}{m}$

Therefore, $\frac{1}{p}, \frac{1}{q}, \frac{1}{m}$ are in A.P.

Hence, the correct option is (a).

Example 67. If unity is added to the sum of any number of terms of the A.P. 3, 5, 7, 9, ... the resulting sum is

(a) 'a' perfect cube (b) 'a' perfect square

(c) 'a' number (d) None of these

Sol. (b) Given series 3, 5, 7, 9,…

First term $(a_1) = 3$

Second term $(a_2) = 5$

Common difference $(d) = a_2 - a_1 = 5 - 3 = 2$

We know that, sum of n terms of an A.P. is given by

$$(S_n)=\frac{n}{2}(2a+(n-1)d)$$

$$\Rightarrow (S_n)=\frac{n}{2}(2(3)+(n-1)2)$$

$$\Rightarrow (S_n)=\frac{n}{2}(6+2n-2)$$

$$\Rightarrow (S_n)=\frac{n}{2}(4+2n) \Rightarrow (S_n) = n\,(n+2)$$

$$\Rightarrow (S_n) = n^2 + 2n$$

If unity is added to sum of n terms (S_n), then

$$\Rightarrow (Sn) = n^2 + 2n + 1 = (n+1)^2$$

Therefore, the resulting sum is a perfect square.

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART C)

1. The 6th term of the series: 0.04, 0.2, 1, ... is

(a) 5 (b) 25 (c) 125 (d) 625

2. Find the 6th term and the common ratio of the geometric progression (G.P.):
3, 9, 27, 81, ...

(a) 255, 9 (b) 343, 3 (c) 729, 3 (d) 2187, 9

3. Which term of the series: 3, 9, 27, ... is 243?

(a) 3 (b) 4 (c) 5 (d) 6

4. The last term of the series 1, 2, 4, ... to 10 terms is **(ICAI)**

(a) 512 (b) 256 (c) 1024 (d) None of these

5. The 5th term of a G.P. is 32 and the 8th term is 256, then the common ratio of the G.P. is

(a) 2 (b) 4 (c) 6 (d) 16

6. If fifth term of a G.P. is $\sqrt[3]{3}$, then the product of first nine terms is

(a) 8 (b) 27 (c) 243 (d) 9

7. If the fifth and third terms of a G.P. are 256 and 16 respectively, then its 8th term is

(a) 16384 (b) 15560 (c) 14000 (d) None of these

8. Which term of the geometric progression (G.P.) 2, 6, 18, ... is 1458?

(a) 6th (b) 7th (c) 8th (d) 9th

9. In a geometric progression, the 3rd and 6th terms are respectively 1 and $-\frac{1}{8}$. The first term (a) and common ratio are respectively **(Jan 2021)**

(a) $4 \text{ and } \frac{1}{2}$ (b) $4 \text{ and } -\frac{1}{4}$ (c) $4 \text{ and } -\frac{1}{2}$ (d) $4 \text{ and } \frac{1}{4}$

10. The sum of first 6 terms of the G.P.: 4, 12, 36, ... is

(a) 1450 (b) 1456 (c) 728 (d) 364

11. The sum of the series 243, 81, 27, …. to 8 terms is

(a) 36 (b) $36\frac{13}{30}$ (c) $36\frac{1}{9}$ (d) None of these

12. Find the sum of the infinite geometric progression 3, 1.5, 0.75, ...

(a) 4 (b) 6 (c) 8 (d) Cannot be determined

13. If the sum of an infinite geometric progression is 10 and the common ratio is $\frac{1}{2}$, then the first term is

(a) 5 (b) 7.5 (c) 2.5 (d) 10

14. Find the sum of the infinite terms: $2, \frac{4}{y}, \frac{8}{y^2}, \frac{16}{y^3}, ...$

(a) $\frac{2y}{y-2}$ (b) $\frac{4y}{y-2}$ (c) $\frac{3y}{y-2}$ (d) None of these

15. The sum of how many terms of the sequence 256, 128, 64, ... is 511.

(a) 8 (b) 9 (c) 7 (d) None of these

16. If Arithmetic Mean and Geometric Mean between two number are 5 and 4 respectively, then these numbers are **(Dec 2022)**

(a) 2 and 3 (b) 2 and 8 (c) 4 and 6 (d) 1 and 16

Answer Key

1. (c)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (b)
11. (d)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (b)				

SPECIAL SERIES

Example 68. Sum of n terms of the series 2 + 22 + 222 + ... is

(a) $\frac{2}{9}\left(10^{n+1}-10-9n\right)$ (b) $\frac{2}{81}\left(10^{n+1}-10-9n\right)$

(c) $2(10^{n+1} - 9n)$ (d) None of these

Sol. (b) Given series: 2 + 22 + 222 + …

$\Rightarrow S_n = 2 + 22 + 222 + ...$

$$\Rightarrow S_n = \frac{2}{9}(9 + 99 + 999 + \ldots n \text{ terms})$$

$$\Rightarrow S_n = \frac{2}{9}\left((10-1) + \left(10^2 - 1\right) + \left(10^3 - 1\right) + \ldots n \text{ terms}\right)$$

$$\Rightarrow S_n = \frac{2}{9}\left(10 + 10^2 + 10^3 + \ldots 10^n - n\right)$$

$$\Rightarrow S_n = \frac{2}{9}\left(\frac{10\left(10^n - 1\right)}{10-1} - n\right)$$

$$\Rightarrow S_n = \frac{2}{9}\left(\frac{10^{n+1} - 10}{9} - n\right)$$

$$\Rightarrow S_n = \frac{2}{9}\left(\frac{10^{n+1} - 10 - 9n}{9}\right)$$

$$\Rightarrow S_n = \frac{2}{81}\left(10^{n+1} - 10 - 9n\right)$$

Hence, the correct option is (b).

Example 69. Sum of n terms of the series 0.1 + 0.11 + 0.111 + ….

(a) $2\left(9n - 1 + \left(\frac{1}{10}\right)^n\right)$ (b) $\frac{2}{81}\left(9n - 1 + \left(\frac{1}{10}\right)^n\right)$

(c) $\frac{1}{81}\left(9n - 1 + \left(\frac{1}{10}\right)^n\right)$ (d) none of these

Sol. (c) Given series: 0.1 + 0.11 + 0.111 +….

$\Rightarrow S_n$ = 0.1 + 0.11 + 0.111.....

$$\Rightarrow S_n = \frac{1}{9}\ (0.9 + 0.99 + 0.999 + \ldots..n \text{ terms})$$

$$\Rightarrow S_n = \frac{1}{9}\ [(1 - 0.1) + (1 - 0.01) + (1 - 0.001) \ldots\ldots n \text{ terms}]$$

$$\Rightarrow S_n = \frac{1}{9}\left(\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \ldots\ldots\left(1 - \frac{1}{10^n}\right)\right)$$

$$\Rightarrow S_n = \frac{1}{9}\left(n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \ldots\ldots\frac{1}{10^n}\right)\right)$$

$$\Rightarrow S_n = \frac{1}{9}\left(n - \frac{1}{10} \times \frac{1 - \frac{1}{10^n}}{1 - \frac{1}{10}}\right)$$

$$\Rightarrow S_n = \frac{1}{9}\left(n - \frac{1}{10} \times \frac{1-\frac{1}{10^n}}{\frac{9}{10}}\right)$$

$$\Rightarrow S_n = \frac{1}{9}\left(n - \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)\right)$$

$$\Rightarrow S_n = \frac{1}{81}\left(9n - 1 + \left(\frac{1}{10}\right)^n\right)$$

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART D)

1. Find the sum of the following series: 0.5 + 0.55 + 0.555 + ... to n terms

(a) $\frac{5}{81}\left[n - \frac{(1-0.1^n)}{9}\right]$ (b) $\frac{5}{9}\left[n - \frac{(1-0.1^n)}{9}\right]$

(c) $\frac{1}{9}\left[n - \frac{(1+0.1^n)}{9}\right]$ (d) None of these

2. The sum of square of first n natural numbers is

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+1)(2n+1)}{6}$

(c) $\frac{n(n-1)(n-1)}{6}$ (d) $\frac{n(n+1)(n+2)}{6}$

3. The sum to m terms of the series 1 + 11 + 111 + ……. up to m terms, is equal to:

(a) $\frac{1}{81}(10^{m+1} - 9m - 10)$ (b) $\frac{1}{27}(10^{m+1} - 9m - 1)$

(c) $\frac{1}{27}(10^{m+1} - 9m - 10)$ (d) None of these

4. Find the sum to n terms of the series: 7 + 77 + 777 + ... to n terms.

(a) $\frac{7}{9}(10^{n+1} - 10) - \frac{7n}{9}$ (b) $\frac{7}{9}(10^{n+1} - 10) + \frac{7n}{9}$

(c) $\frac{7}{9}\left(\frac{10(10^n - 1)}{9} - n\right)$ (d) $\frac{7}{81}(10^{n+1} - 10) + \frac{7n}{9}$

Answer Key

1. (b) **2.** (b) **3.** (a) **4.** (c)

FULL CHAPTER PRACTICE QUESTIONS

1. The sum of the series –8, –6, –4, …n terms is 52, then the numbers of terms n is

(a) 11 (b) 12 (c) 13 (d) 10

2. The 3rd term of a G.P is $\frac{2}{3}$ and the 6th term is $\frac{2}{81}$, then the 1st term is

(a) 6 (b) $\frac{1}{3}$ (c) 2 (d) None of these

3. Insert two arithmetic means between 68 and 260, the two terms are

(a) 132,196 (b) 130,194 (c) 70, 258 (d) None of these

4. The value of c such that a, –3, b, 5, c are in A.P is:

(a) –7 (b) 1 (c) 13 (d) 9

5. If the sum of first 'n' terms of an A.P. is $6n^2+6n$, then the fourth term of the series:

(a) 120 (b) 72 (c) 48 (d) 24

6. The sum to m terms of the series 1 + 11 + 111 + … up to n terms, is equal to:

(a) $\frac{1}{81}(10^{n+1}-9n-10)$ (b) $\frac{1}{27}(10^{n+1}-9n-10)$

(c) $(10^{n+1}-9n-10)$ (d) None of these

7. If third term and seventh term of an A.P are eighteen and thirty respectively, then sum of the first twenty terms will be:

(a) 540 (b) 610 (c) 740 (d) 810

8. If x, y, z are the terms in G.P. then the terms x^2+y^2, $xy+yz$, y^2+z^2 are in:

(a) A.P. (b) G.P. (c) H.P. (d) None of these

9. The value of $1^3+2^3+3^3+4^3+\ldots+m^3$ is equal to:

(a) $\left[\frac{m(m+1)}{2}\right]^3$ (b) $\frac{m(m+1)(2m+1)}{6}$

(c) $\left[\frac{m(m+1)}{2}\right]^2$ (d) None of these

10. The sum of the infinite G.P. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$ is equal to

(a) 1.95 (b) 1.5 (b) 1.75 (c) None of these

11. An arithmetic progression has 13 terms whose sum is 143. The third term is 5 so the first term is

(a) 4 (b) 7 (c) 9 (d) 2

12. In a G.P. the sixth term is 729 and the common difference is 3, then the first term of G.P. is

(a) 2 (b) 3 (c) 4 (d) 7

13. If the sum of n terms of an A.P be $3n^2 - n$ and its common difference is 6, then its first term is

(a) 2 (b) 3 (c) 4 (d) 5

14. If the sum of n terms of an A.P be $2n^2 + 5n$, then its nth term is

(a) $4n - 1$ (b) $3n - 4$ (c) $4n + 3$ (d) $3n + 4$

15. In an A.P., if common difference is 2, sum of n terms is 49, 7th term is 13, then n = _____

(a) 0 (b) 5 (c) 7 (d) 13

16. If 8th term of an A.P is 15, then sum of its 15 terms is

(a) 15 (b) 0 (c) 225 (d) 225/2

17. The sum of the third and ninth term of an A.P is 8. Find the sum of the first 11 terms of the progression.

(a) 44 (b) 22 (c) 19 (d) 11

18. If each month ₹100 increase in any sum then find out the total sum after 10 months, If the sum of first month is ₹2000

(a) ₹24,500 (b) ₹24,000 (c) ₹50,000 (d) ₹60,000

19. If sum of 3 arithmetic means between "a" and 22 is 42, then "a" =

(a) 14 (b) 11 (c) 10 (d) 6

20. Rohan saves ₹100 in the first week and increases his savings by ₹10 every week. If he wants to save a total of ₹500, how many weeks will it take him to achieve his goal?

(a) 10 (b) 19 (c) 20 (d) 24

21. The infinite G.P. with first term $\frac{1}{4}$ and sum $\frac{1}{3}$ is

(a) $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$ (b) $\frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, \ldots$

(c) $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ (d) None of these

22. The mth term of an A.P. is n and n^{th} term of an A.P. is m. The rth term of an A.P. is

(a) m + n + r (b) n + m – 2r (c) $m + n + \frac{r}{2}$ (d) m + n – r

23. The product of 2 numbers in G.P. is 729 and the sum of squares is 819. The numbers are **(ICAI)**

(a) 9, 3, 27 (b) 27, 3, 9 (c) 3, 9, 27 (d) None of these

24. The number of the terms of the series $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + \ldots\ldots$ will amount to 155 is

(a) 30 (b) 31 (c) 32 (d) None of these **(ICAI)**

25. The first, second and seventh term of an A.P. are in G.P. and the common difference is 2, then the second term of an A.P. is

(a) $\frac{5}{2}$ (b) 2 (c) $\frac{3}{2}$ (d) $\frac{1}{2}$

26. Find three numbers in G.P. such that their sum is 21, and the sum of their squares is 189:

(a) 5, 7, 9 (b) 3, 7, 11 (c) 3, 6, 12 (d) 4, 8, 9

27. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

(a) 1,56,375 (b) 1,56,357 (c) 1,65,375 (d) 1,65,357

28. A man employed in a company is promised a salary of ₹73,000 every month for the first year and an increment of ₹1,000 in his monthly salary every succeeding year. How much does the man earn from the company in 20 years?

(a) ₹30,00,000 (b) ₹27,50,000 (c) ₹19,10,000 (d) ₹7,90,000

29. On 1st January every year a person buys National Saving Certificates of value exceeding that of his last year's purchase by ₹100. After 10 years, he finds that the total value of the certificates purchased by him is ₹54,500. Find the value of certificates purchased by him in the first year.

(a) ₹6,000 (b) ₹4,000 (c) ₹5,000 (d) ₹5,500

Answer Key

1. (c)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (b)
11. (d)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (b)
21. (a)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (a)	**26.** (c)	**27.** (a)	**28.** (a)	**29.** (c)	

PREVIOUS YEAR QUESTION (PYQ'S)

1. The nth term of the series 3 + 7 + 13 + 21 + 31 + ... is **(Jan 2021)**

(a) 4n – 1 (b) $n^2 + 2n$ (c) $n^2 + n + 1$ (d) $n^3 + 2$

2. The number of integers 1 to 100 which are neither divisible by 3 nor 5 nor 7 is **(Jan 2021)**

(a) 67 (b) 55 (c) 45 (d) 33

3. In a geometric progression the 3rd and 6th terms are respectively 1 and $-\frac{1}{8}$. The first term (a) and common ratio are respectively **(Jan 2021)**

(a) 4 and $\frac{1}{2}$ (b) 4 and $-\frac{1}{4}$ (c) 4 and $-\frac{1}{2}$ (d) 4 and $\frac{1}{4}$

4. The number of terms of the series 5 + 7 + 9 +..... must be taken so that the sum may be 480 is **(July 2021)**

(a) 20 (b) 10 (c) 15 (d) 25

5. If the sum of 'n' terms of an AP (Arithmetic Progression) is $2n^2$, the fifth term is **(July 2021)**

(a) 20 (b) 50 (c) 18 (d) 25

6. The sum of three numbers in a geometric progression is 28. When 7, 2 and 1 are subtracted from the first, second and the third numbers respectively, then the resulting numbers are in arithmetic progression. What is the sum of squares of the original three numbers? **(July 2021)**

(a) 510 (b) 456 (c) 400 (d) 336

7. The sum of square of any real positive quantities and its reciprocal is never less than **(July 2021)**

(a) 1 (b) 2 (c) 3 (d) 4

8. If the sum and products of three numbers in G.P are 7 and 8 respectively, then 4th term of the series is **(Dec 2021)**

(a) 6 (b) 4 (c) 8 (d) 16

9. The sum of series 7 + 14 + 21 + ….. to 17th terms is: **(Dec 2021)**

(a) 1071 (b) 971 (c) 1171 (d) 1271

10. The sum of first n terms of an AP is $3n^2 + 5n$. The series is **(Dec 2021)**

(a) 8, 14, 20, 26, … (b) 8, 22, 42, 66, …

(c) 22, 68, 114, … (d) 8, 14, 28, 44, …

11. The largest value of n for which $\frac{1}{2}+\frac{1}{2^2}+....+\frac{1}{2^n}<0.998$ is _____. **(Dec 2021)**

(a) 9 (b) 6 (c) 7 (d) 8

12. If the nth term of the arithmetic progression 9, 7, 5, … is same as the nth term of the arithmetic progression 15, 12, 9, …., then n will be **(June 2022)**

(a) 7 (b) 9 (c) 15 (d) 11

13. In a geometric progression, the second term is 12 and the sixth term is 192. Find the 11th term. **(June 2022)**

(a) 3072 (b) 1536 (c) 12288 (d) 6144

14. The first and last terms of an arithmetic progression are 5 and 905. Sum of the terms is 45,955. The number of terms is **(June 2022)**

(a) 99 (b) 100 (c) 101 (d) 102

15. The sum of first eight terms of geometric progression is five times the sum of the first four terms. The common terms is **(June 2022)**

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 4 (d) 2

16. If pth term of an AP is q and its qth term is p, then what will be the value of (p + q)th terms? **(Dec 2022)**

(a) 0 (b) 1 (c) p + q - 1 (d) 2(p + q - 1)

17. In a G.P, 5th term is 27 and 8th term is 729. Find its 11th term. **(Dec 2022)**

(a) 729 (b) 6561 (c) 2187 (d) 19,683

18. How many numbers between 74 and 25,556 are divisible by 5? **(June 2023)**

(a) 5090 (b) 5097 (c) 5095 (d) 5075

19. If 9th and 19th term of an Arithmetic Progression are 35 and 75 respectively, then its 20th term is **(June 2023)**

(a) 78 (b) 79 (c) 80 (d) 81

20. If 4th, 7th and 10th terms of a Geometric Progression are p, q and r respectively, then

(a) $p^2 = q^2 + r^2$ (b) $p^2 = qr$ (b) $q^2 = pr$ (d) $pqr + pq + 1 = 0$

21. Find the value of 'x' for the following data $1 + 7 + 13 + 19 + ...+ x = 225$. **(Dec 2023)**

(a) 56 (b) 63 (c) 49 (d) 42

22. If the nth term of an A.P is $7n - 2$, then the sum of n terms is **(Dec 2023)**

(a) $0.5(7n^2 + 2n)$ (b) $0.5(7n^2 - 3n)$

(c) $0.5(7n^2 + 3n)$ (d) $0.5(7n^2 - 2n)$

23. Find the 17th term of an AP series if 15th and 21st terms are 30.5 and 39.5 respectively.

(Dec 2023)

(a) 33.5 (b) 35.5 (c) 36.0 (d) 38.0

24. Given an infinite geometric series with first term 'a' and common ratio 'r'. If its sum is 4 and the second term is $\frac{3}{4}$, then one of the correct option is: **(Dec 2023)**

(a) $a = 1, r = \frac{1}{4}$ (b) $a = 3, r = \frac{3}{4}$ (c) $a = 3, r = \frac{1}{4}$ (d) $a = 1, r = \frac{1}{2}$

25. The 3rd term of arithmetic progression is 7 and Seventh term is 2 more than thrice of third term. The common difference is **(June 2024)**

(a) 4 (b) 3 (c) 5 (d) 6

26. If the second and eighth terms of an arithmetic progression (A.P.) are equal to a constant a, then the sum of the first n terms of this A.P. is: **(June 2024)**

(a) na (b) $\frac{a}{n}$ (c) $2n + n(a-1)$ (d) $n + a(n-1)$

27. The 3rd term of arithmetic progression is 7 and Seventh term is 2 more than thrice of third term. The common difference is **(June 2024)**

(a) 4 (b) 3 (c) 5 (d) 6

28. In an AP, the seventh term is x and (x + 7)th term is zero, Then xth term is **(June 2024)**

(a) 6 (b) 7 (c) 8 (d) 9

29. A roadside tea stall merchant borrows ₹9,000 at 2.76% simple interest per annum. The principal and interest are to be paid in 10 monthly instalments, where each instalment is double the preceding one. Find the value of the last instalment. **(Sept 2024)**

(a) ₹4,608 (b) ₹1,024 (c) ₹9,207 (d) ₹4,096

30. If the fourth term of an Arithmetic Progression (A.P.) series is zero, then what is the ratio of the twenty-fifth term to the eleventh term? **(Sept 2024)**

(a) 4 (b) 5 (c) 3 (d) 2

31. If for an infinite geometric progression, first term is 'a' common ratio is 'r', the sum is 8 and the second term is '$\frac{7}{8}$', then **(Sept 2024)**

(a) $a = 4, r = \frac{7}{16}$ (b) $a = 3, r = \frac{7}{24}$ (c) $a = 7, r = \frac{1}{8}$ (d) $a = 2, r = \frac{7}{32}$

32. The numbers x, 8, y in Geometric Progression (G.P) and the numbers x, y, –8 are in Arithmetic Progression (A.P). Find the values of x and y respectively. **(Sept 2024)**

(a) 4, 16 (b) 16, 4 (c) 4, 8 (d) 8, 4

33. Find the 9th term of the A.P: 8, 5, 2, –1, –4, ... **(Jan 2025)**

(a) –24 (b) –10 (c) –16 (d) –4

34. The sum of the series 1 + 2 + 3 + ... is 55. The number of terms is: **(Jan 2025)**

(a) 30 (b) 40 (c) 20 (d) 10

35. The sum of 4th and 8th term of an AP is 10, then the sum of first eleven terms of the series is **(Jan 2025)**

(a) 22 (b) 33 (c) 44 (d) 55

36. The product of three numbers which are in GP is 512. Then the second number is: **(Jan 2025)**

(a) 3 (b) 2 (c) 6 (d) 8

37. Find the sum of series $1 + \frac{1}{2} + \frac{1}{4}$ + ...upto 6 terms. **(May 2025)**

(a) $\frac{63}{32}$ (b) $\frac{32}{63}$ (c) $\frac{26}{53}$ (d) $\frac{53}{26}$

38. The sum of first three terms of a G.P. is $\frac{21}{2}$ and their product is 27. Which of the following is not a term of the G.P. if the numbers are positive? **(May 2025)**

(a) 3 (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 6

39. Find the sum of n term of the A.P, whose n^{th} terms is 5n + 1. **(May 2025)**

(a) $\frac{n}{2}$ (b) $\frac{2n}{7}$ (c) $\frac{n(7+5n)}{2}$ (d) $\frac{n(7+4n)}{2}$

40. Insert 4 numbers between 2 and 22 such that the resulting sequence is an Arithmetic Progression (A.P). **(May 2025)**

(a) 4, 8, 12, 16 (b) 5, 9, 13, 17 (c) 4, 10, 15, 19 (d) 6, 10, 14, 18

41. The common difference of the arithmetic progression $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \ldots\ldots$ is **(Sept 2025)**

(a) –b (b) b (c) –3b (d) 3b

42. If the numbers x, 2x + 2 and 3x + 3 are in the geometric progression, then the fourth term of the progression is _____. **(Sept 2025)**

(a) 27 (b) –27 (c) 13.5 (d) –13.5

43. The sum of all natural numbers between 200 and 600, those are divisible by 13 is _____. **(Sept 2025)**

(a) 12493 (b) 14493 (c) 16493 (d) 18493

44. The sum of first two terms of a geometric progression is 14 and its infinite sum i.e., sum up to infinity is 32. What is the common ratio of the progression? **(Sept 2025)**

(a) 0.5 (b) 0.75 (c) 1.25 (d) 0.25

Answer Key

1. (c)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (a)
11. (a)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (b)	**20.** (b)
21. (c)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (a)	**28.** (b)	**29.** (a)	**30.** (c)
31. (c)	**32.** (b)	**33.** (c)	**34.** (d)	**35.** (d)	**36.** (d)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (d)
41. (a)	**42.** (d)	**43.** (a)	**44.** (b)						

SUMMARY

- An ordered collection of numbers $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ is a sequence if there is some pattern and then a_n is called the term of the sequence, corresponding to any value of the natural number n.
- An expression of the form $a_1 + a_2 + a_3 + \ldots + a_n + \ldots$ which is the sum of the elements of the sequence is called a series.
- If the series contains a finite number of elements, it is called a finite series, otherwise called an infinite series.

ARITHMETIC PROGRESSION

- A sequence $a_1, a_2, a_3, \ldots,$ is called an Arithmetic Progression (A.P.) when
  $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$
- This constant 'd' is called the common difference of the A.P.
  (a) If 3 numbers a, b, c are in A.P., then b – a = c – b or a + c = 2b; b is called the arithmetic mean between a and c.

  'or' A.M. of a and b = $\frac{a+b}{2}$

  (b) nth term of a_n A.P., a_n = a + (n – 1)d where a = first term, d = common difference = $a_n - a_{n-1}$

- Sum of n terms of AP: $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a + l]$

  where, l is the last term of A.P.
- Sum of 1st n natural or counting numbers: $S = \frac{n(n+1)}{2}$
- Sum of 1st n odd numbers : $S = n^2$
- Sum of 1st n even numbers : $S = n(n + 1)$
- Sum of the squares of the first n natural numbers : $\frac{n(n+1)(2n+1)}{6}$
- Sum of the cubes of the first n natural numbers is $\left\{\frac{n(n+1)}{2}\right\}^2$

GEOMETRIC PROGRESSION

- If in a sequence of terms, each term is a constant multiple of the preceding term, then the sequence is called a Geometric Progression (G.P.).
- n^{th} term of a GP = ar^{n-1}
- The constant multiplier is called the common ratio $= \frac{\text{any term}}{\text{preceding term}} = \frac{a_n}{a_{n-1}} = r$
- Sum of first n terms of a GP:

  $S_n = a\frac{(1-r^n)}{1-r}$ when $r < 1$

  $S_n = a\frac{(r^n-1)}{r-1}$ when $r > 1$
- Sum of infinite geometric series $S_\infty = \frac{a}{1-r}, r < 1$
- If a, b, c are in G.P. we get $\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$, b is called the geometric mean between a and c

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CHAPTER

07

Sets, Relations and Functions, Basics of Limits and Continuity Functions

Unit-1: Sets, Relations and Functions

SETS

DEFINITION

A set is a collection of well-defined objects or elements. It is a fundamental concept in mathematics that allows us to organize and categorize objects based on common characteristics or properties.

- Objects of sets can be called as elements or members of a set.
- Sets are generally denoted by Capital letters like – A, B, C, D, etc.
- The elements of sets are generally denoted by small letters like a, b, c, d, p, q, r, s, etc.

E.g.: A = {a, e, i, o, u}

B = {2, 3, 5, 7, 11, 13, 17, 19}

where, A is the set of vowels in the alphabet and a,e,i,o,u are its elements. Similarly, B is the set of prime numbers less than 20 where 2, 3, 5, 7, 11, 13, 17, 19 are its elements.

REPRESENTATION OF SETS

Roster Method (Tabular form) or Bracket form: This method involves listing all the elements of a set, separating them by commas, and enclosing them in curly braces {}. The order of elements in a set does not matter. Each element is typically distinct and appears only once in the set.

E.g.: Set A = {1, 2, 3, 4, 5} represents a set with elements 1, 2, 3, 4 and 5.

Set B = {apple, banana, orange} represents a set with elements apple, banana, and orange.

Set Builder or Algebraic Form or Rule Method: This method involves defining a common property or rule that all elements in the set possess, and no element outside the set possesses. The set is then represented by expressing the property or rule in a concise form.

E.g.: Set C = {x: x is an even number} represents a set of all even numbers. Here, the property or rule is that the elements must be even numbers.

Set D = {y: y > 0} represents a set of all positive numbers. Here, the property or rule is that the elements must be greater than zero.

FEW IMPORTANT AND WIDELY USED SETS

- N: The set of all natural numbers: 1, 2, 3, 4, 5, 6,…..
- Z: The set of all integers: …. –3, –2, –1, 0, 1, 2, 3,…..

- Q: The set of all rational numbers: $\frac{2}{5}, \frac{-3}{8}, \frac{-19}{5}, \ldots..$
- R: The set of real numbers: 0.2 ,1, 2, –3, $\frac{2}{5}, \frac{-3}{8}, \ldots..$
- Z^+: The set of positive integers: 1, 2, 3, 4,……
- Q^+: The set of positive rational numbers: $\frac{2}{5}, \frac{3}{8}, \frac{19}{5}, \ldots\ldots\ldots$
- R^+: The set of positive real numbers: 0.2, 1, 2, $\frac{2}{5}, \ldots..$

Example 1. Write the below sets in roster form?

A = {x : x is an odd natural number and x < 5}

B = {x : x is an integer, $-3.4 < x < 2.5$}

C = {x : x is an integer, $x^2 \leq 9$}

D = {x : x is a letter in the word "SCHOOL"}

Sol. According to the question,

(I) A = {x : x is an odd natural number and x < 5}

As we know,

Odd natural numbers less than 5 are 1 and 3.

Thus, the roster form will be given as, A = {1, 3}.

Therefore, A = {1, 3}.

(II) B = {x : x is an integer, $-3.4 < x < 2.5$}

Since, B is the set of integers between –3.4 and 2.5 (excluding –3.4 and 2.5). So, the elements in set B are –3, –2, –1, 0, 1 and 2.

Therefore, B = {–3, –2, –1, 0, 1, 2}

(III) C = {x : x is an integer, $x^2 \leq 9$}

Here, C is the set of integers whose square is less than or equal to 9. So, the elements in set C are: –3, –2, –1, 0, 1, 2 and 3.

Therefore, C = {–3, –2, –1, 0, 1, 2, 3}

(IV) D = {x : x is a letter in the word "SCHOOL"}

Here, D is the set of letters in the word "SCHOOL". So, the elements in set D are S, C, H, O and L.

Therefore, D = {S, C, H, O, L}

Example 2. Write the following sets in the set-builder form:

A = {2, 4, 6, ……, 100}

B = {P, R, I, N, C, A, L}

C = {a, e, i, o, u}

D = {–10, –8, –6, –4, –2}

Sol. (I) A = {2, 4, 6, ……, 100}

Here, A is the set of even natural numbers from 2 to 100. In set-builder form, it can be represented as:

A = {x : x is an even natural number, $2 \leq x \leq 100$}

(II) B = {P, R, I, N, C, A, L}

Here, B is the set of letters in the word "PRINCIPAL". In set-builder form, it can be represented as:

B = {x : x is a letter in the word "PRINCIPAL"}

(III) C = {a, e, i, o, u}

C is the set of vowels in the English alphabet. In set-builder form, it can be represented as:

C = {x : x is a vowel in the English alphabet}

(IV) D = {–10, –8, –6, –4, –2}

D is the set of even negative integers between –10 and –2 (inclusive). In set-builder form, it can be represented as:

D = {x : x is an even negative integer, $-10 \leq x \leq -2$}

SOME SPECIAL SETS

- **Finite sets:** A finite set is a set that contains a specific number of elements, which can be counted.

  E.g.: The set {1, 2, 3, 4} is a finite set with four elements.
- **Infinite sets:** An infinite set is a set that has an unlimited number of elements and cannot be counted.

  E.g.: The set of all natural numbers {1, 2, 3, 4, ...} is an infinite set.
- **Null (Empty) set:** The null set, also known as the empty set, is a set that does not contain any elements. It is denoted by the symbol φ or {}.

  E.g.: The set of all prime numbers less than 1 is an empty set.
- **Singleton set:** A singleton set is a set that contains only one element. It is a finite set with a cardinality of one.

  E.g.: {5} is a singleton set that contains the element 5.
- **Equal Set and Equivalent set:** Two sets are said to be equal if they have exactly the same elements.

  E.g.: If A = {1, 2, 3} and B = {3, 2, 1}, then A = B since they contain the same elements, even though the order may differ.

  On the other hand, two sets are said to be equivalent if they have the same cardinality, i.e., they contain the same number of elements. The elements in the equivalent sets may or may not be the same.

  E.g.: {1, 2, 3} and {4, 5, 6} are equivalent sets since both sets have three elements.

Note: All equal sets are equivalent sets but equivalent sets may not be equal sets.
E.g.: {1, 2, 3, 4} and {a, b, c, d} are equivalent sets but not equal.

- **Disjoint Sets:** Disjoint sets are sets that have no common elements. In other words, there is no element that belongs to both sets.
  E.g.: {1, 2, 3} and {4, 5, 6} are disjoint sets since they do not share any common elements.

Example 3. Which of the following sets is an infinite set?

(a) {1, 2, 3, 4, ….. 50}

(b) {-5, -4, -3, -2, -1, 0, 1, 2}

(c) Set of months of a year

(d) Set of positive integers greater than 1000

Sol. (d) Here,

- {1, 2, 3, 4, ….. 50} is a finite set.
- {-5, -4, -3, -2, -1, 0, 1, 2} is a finite set.
- Set of months of a year is a finite set.
- Set of positive integers greater than 1000 = {1001, 1002, 1003, …..} is an infinite set.

Hence, the correct option is (d).

Example 4. Which of the following is an example of a null set?

(a) Set of even prime numbers.

(b) Set of odd natural numbers divisible by 2.

(c) Set of natural numbers greater than 5 but less than 7.

(d) All of them

Sol. (b) We know that,

- Set of even prime numbers = {2}
- Set of odd natural numbers divisible by 2 is a null set.
- Set of natural numbers greater than 5 but less than 7 = {6}

Therefore, the set of odd natural numbers divisible by 2 is an example of a null set.

Hence, the correct option is (b).

Example 5. Which of the following pairs of sets is disjoint?

(a) {1, 2, 3, 4} and {x : x is a natural numbers such that $4 \le x \le 6$}

(b) {a, e, i, o, u} and {c, d, e, f}

(c) {x : x is an even integer} and {x : x is an odd integer}

(d) None of these

Sol. (c) For option (a):

{1, 2, 3, 4} and {x : x is a natural numbers such that x = 4, 5, 6}

i.e. {1, 2, 3, 4} and {4, 5, 6}

Clearly, 4 belongs to both the sets, thus it is not disjoint sets.

For option (b):

{a, e, i, o, u} and {c, d, e, f}

Here, e belongs to both the sets, thus these are not disjoint sets.

For option (c):

Clearly, {x : x is an even integer} ∩ {x : x is an odd integer} = φ

Thus, these are disjoint sets.

Hence, the correct option is (c).

Example 6. Which of the following pair of sets is equal?

(a) A = {1, 2, 3, 4} and B = {d, c, a, b}

(b) A = {1, 4, 9, 16} and B = {16, 1, 9, 4}

(c) A = {x : x is multiple of 10} and B = {10, 20, 25, 30, 40,...}

(d) None of these

Sol. (b) For option (a):

A = {1, 2, 3, 4} and B = {d, c, a, b}

Clearly, the elements in both the sets are not same but the number of elements are equal.

Thus, it is an equivalent set but not an equal set.

For option (b):

A = {1, 4, 9, 16} and B = {16, 1, 9, 4}

Since, the order in which the elements are listed does not matter and all the elements in A and B are the same, thus it is an equal set.

For option (c):

A = {x : x is multiple of 10} and B = {10, 20, 25, 30, 40,...}

Here, 25 ∈ B but 25 ∉ A

Thus, these are not equal sets.

Therefore, the sets A = {1, 4, 9, 16} and B = {16, 1, 9, 4} are equal.

Hence, the correct option is (b).

Example 7. Check if the below given sets are equal sets or equivalent sets:

(I) {3, 5, 7} and {1, 3, 7}

(II) {8, 6, 10, 12} and {3, 2, 4, 6}

(III) {1, 4, 9, 16, 25} and {12, 22, 32, 42, 52}

Sol. We know that,

If two (finite) sets contain the same number of elements then the sets are equivalent sets.

Two sets are equal if they have exactly the same elements.

Here,

(I) {3, 5, 7} and {1, 3, 7}

Both the sets contain the same number of elements, thus the sets are equivalent sets.

(II) {8, 6, 10, 12} and {3, 2, 4, 6}

Here, the given sets are not equal but they are equivalent as number of elements is same.

(III) {1, 4, 9, 16, 25} and {1, 16, 9, 25, 4}

The given sets are equal as well as equivalent sets.

OTHER FREQUENTLY USED SYMBOLS ARE

- $\in$ **(belongs to):** The symbol "$\in$" is used to indicate that an element belongs to a set. For example, if x is an element of set A, we can write it as $x \in A$.
- $\notin$ **(Does not belong to):** The symbol "$\notin$" is used to indicate that an element does not belong to a set. For example, if y does not belong to set B, we can write it as $y \notin B$.

Example 8. If set, A = {2, 4, 7}, B = {1, 3, 5, 8, 9} and C = {11, 13, 15, 19, 22}. Fill in the blanks.

$11 \in$ ___

$8 \in$ ___

Sol. Given: A = {2, 4, 7}, B = {1, 3, 5, 8, 9} and C = {11, 13, 15, 19, 22}

Clearly,

$11 \in C$ and $8 \in B$.

Example 9. Show that φ, {0} and 0 are all different.

Sol. We know that, φ is set containing no element at all.

Also, {0} is a set containing one element, mainly 0.

Additionally, 0 is a number, not a set.

Hence, φ, {0} and 0 are all different.

Hence, proved.

SUBSETS

A set A is said to be a subset of a set B if every element of A is also an element of B. In other words, if all the elements of set A are contained within set B, we represent it as $A \subseteq B$.

E.g.: A = {1, 2, 3} and B = {1, 2, 3, 4, 5}

Here, A is a subset of B, because every element of A (1, 2 and 3) is also present in B.

Properties:

- If $A \subseteq B$ and $B \subseteq A$, then it implies that A = B. This means that A and B are equal sets.
- Every set is a subset of itself, denoted as $A \subseteq A$.
- The empty set (φ) is a subset of every set. In other words, the empty set has no elements, and thus, it is contained within every set.

PROPER SUBSET AND SUPER SET

Proper set: If A is a subset of B, but A is not equal to B, then A is called a proper subset of B, denoted as $A \subset B$. It means that A is a subset of B, because B contains element/elements that are not present in A.

E.g.: A = {1, 2} and B = {1, 2, 3}

Here, A is a proper subset of B because A is a subset of B, but B contains an additional element (3) that is not in A.

Super set: If A is a subset of B, then B is a superset of A.

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POWER SET

The power set of a set is the set of all possible subsets of the given set.

For example, if we have a set A = {1, 2}, then the power set of A, denoted as P(A), will contain all the subsets of A, including the empty set and the set itself.

E.g.: A = {1, 2}

P(A) = {φ, {1}, {2}, {1, 2}}

Here, P(A) contains all the possible subsets of A, which are the empty set (∅), the individual elements {1} and {2}, and the set itself {1, 2}.

Thus, the number of power sets in a set of cardinality 'n' = 2^n

Also, if n is the number of elements in a set,

- Total number of subsets = 2^n
- Total number of proper subsets = $2^n - 1$

SUBSETS OF SET OF REAL NUMBERS

We know that,

- N: Set of all natural numbers = {1, 2, 3, 4, 5, …… }
- Z: Set of all integers = { ….., -4, -3, -2, -1, 0, 1, 2, 3, ……}
- Q: Set of all rational numbers = $\left\{\frac{p}{q}, \text{ where p and q are integers and } q \neq 0\right\}$
- R: Set of real numbers = Sets of all numbers on a number line

Then, $N \subset Z \subset Q \subset R$

Example 10. The number of subsets of the set {2, 3, 5} is

(a) 3 (b) 8 (c) 6 (d) None of these

Sol. (b) Given set: {2,3,5}

Number of the subsets of a set = 2^n

where n is the number of elements of a set

Here, n = 3

Number of subsets for given set {2, 3, 5} = $2^3 = 8$

Hence, the correct option is (b).

Example 11. If A = {x : x is a positive integer and x < 6} then the number of elements in the power set of A is

(a) 4 (b) 16 (c) 32 (d) 64

Sol. (c) Given set: A = {x : x is a positive integer and x < 6}

A = {1, 2, 3, 4, 5}

Here, number of elements (n) = 5

Thus, number of elements in the power set of A = $2^n = 2^5 = 32$

Hence, the number of elements in the power set of A is 32.

Example 12. Which of the following is a subset of the set A = {2, 4, 6, 8}?

(a) {1, 2, 3, 4} (b) {4, 8, 12} (c) {2, 4, 6} (d) {2, 4, 6, 8, 10}

Sol. (c) Here,

$\{1, 2, 3, 4\} \not\subset \{2, 4, 6, 8\}$ since $1 \notin \{2, 4, 6, 8\}$

$\{4, 8, 12\} \not\subset \{2, 4, 6, 8\}$ since $12 \notin \{2, 4, 6, 8\}$

$\{2, 4, 6\} \subset \{2, 4, 6, 8\}$ since $8 \notin \{2, 4, 6, 8\}$

$\{2, 4, 6, 8, 10\} \not\subset \{2, 4, 6, 8\}$ since $10 \notin \{2, 4, 6, 8\}$

Hence, the correct option is (c) i.e., {2, 4, 6}.

Example 13. Which of the following is a universal set for the set A = {x : x is a prime number less than 10}?

(a) {1, 3, 4, 5, 6, 7, 8, 9}

(b) {2, 3, 5, 7, 11}

(c) {x: x is a positive integer less than 10}

(d) {x : x is a multiple of 2}

Sol. (b) Given, A = {x : x is a prime number less than 10} i.e., A = {2, 3, 5, 7}

Universal set of the given set is {2, 3, 5, 7, 11}.

Hence, the correct option is (b).

Universal set: Universal set is a set which consists of all sets in a given situation. Generally denoted by the letter 'U'.

E.g.: Suppose we are working with the following sets of numbers:

- N: The set of natural numbers {1, 2, 3, 4, ...}
- Z: The set of integers {..., -3, -2, -1, 0, 1, 2, 3, ...}
- Q: The set of rational numbers $\{\frac{1}{2}, -\frac{3}{4}, 0, \frac{5}{2}, ...\}$
- R: The set of real numbers (includes all rational and irrational numbers)

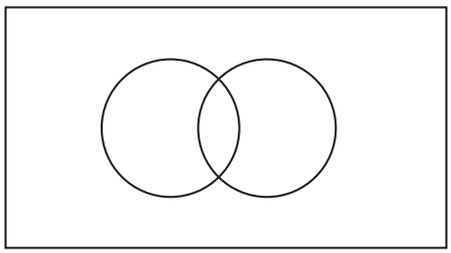
  In this case, the universal set U would be the set of all real numbers, encompassing all the other sets:

  In the above case, U = R

  Thus, the universal set U in this example consists of all possible real numbers, including both rational and irrational numbers.

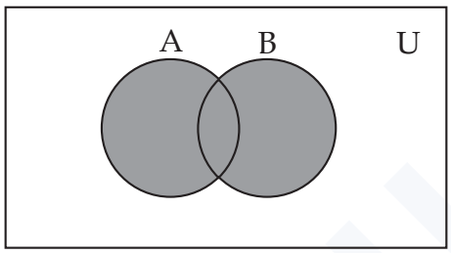
VENN DIAGRAM

Venn diagrams are named after the English logician, John Venn (1834 – 1883). These diagrams consist of rectangles and circles inside it. Universal set is a rectangle and other sets (which will eventually be subsets of the universal set.) are circles inside rectangles.



OPERATION ON SETS

Union of sets: Let's take sets A and B. Union of sets A and B is the set of all the elements either in A or B or in both, the common elements being taken only once. To denote union, we use the symbols '$\cup$'. thus A union B will be written as $A \cup B$.

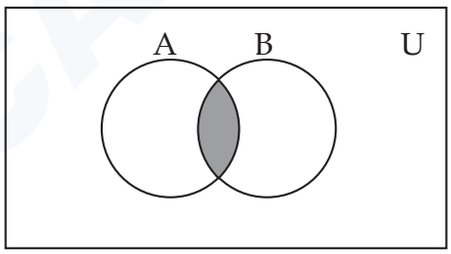


In set builder form, definition of $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

E.g.: If $A = \{2, 4, 6, 8\}$ and $B = \{5, 6, 8, 10\}$, then

$A \cup B = \{2, 4, 5, 6, 8, 10\}$

Intersection of sets: Intersection of sets A and B is the set of all elements which are in A and in B. The symbol '$\cap$' is used to denote the intersection. Thus, the intersection of A and B will be elements which are in both sets.



In set builder form definition of $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

If $A \cap B = \varphi$, then set is called Disjoint sets.

E.g.: (1) If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{3, 5, 7\}$, then

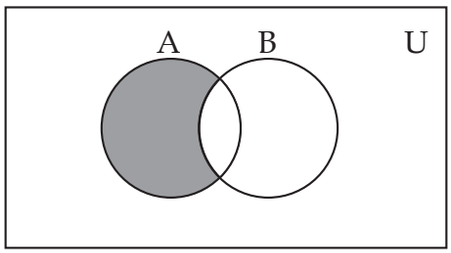
$A \cap B = \{3, 5, 7\}$

(2) If $C = \{2, 4, 6, 8, 10\}$ and $D = \{1, 3, 5, 7, 9\}$, then

$C \cap D = \varphi$ i.e., C and D are disjoint sets

Difference of two sets: The difference of two sets A and set B, if taken in order A difference B, will consist of all elements in A leaving elements which are in intersection with B. It is denoted as A - B.

In set builder from, definition of $A - B = \{x : x \in A \text{ and } x \notin B\}$.



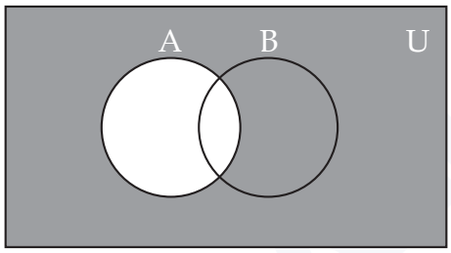
Note: $A - B \neq B - A$

E.g.: If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$

then $A - B = \{1, 3, 5\}$ and $B - A = \{8, 10\}$

Clearly, $A - B \neq B - A$

Complement of a set: Complement of a set is denoted by A^C or A' and it includes all elements in the universal set except for elements in A i.e, $A^C = U - A$



In set builder form, definition of $A' = \{x : x \in U \text{ and } x \notin A\}$.

E.g.: If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 3, 5, 7, 9\}$ then

$A' = \{2, 4, 6, 8\}$

Properties: Complement laws

1. $A \cup A' = U$
2. $A \cap A' = \varnothing$

E.g.: If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 3, 5, 7, 9\}$ then $A' = \{2, 4, 6, 8\}$

Thus, $A \cup A' = \{1, 3, 5, 7, 9\} \cup \{2, 4, 6, 8\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} = U$

Also, $A \cap A' = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8\} = \varnothing$

DE MORGAN'S LAW

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$
   - Law of double complementation: $(A')' = A$
   - Laws of empty set and universal set: $\varnothing' = U$ and $U' = \varnothing$

Example 14. Find the union and intersection of below given pairs of sets.

(I) $A = \{7, 9, 12, 21\}$ and $B = \{5, 9, 12, 18\}$

(II) $C = \{a, b, c\}$ and $D = \{d, e, f\}$

(III) $E = \{x : n \in Z, x = 3n, -2 < n < 2\}$ and $F = \{x : x \in W, x < 5\}$

Sol. We know that,

Union is the set of all elements that are in A or B or in both.

Intersection is the set of all elements which are in A and in B.

According to the question,

(I) $A = \{7, 9, 12, 21\}$ and $B = \{5, 9, 12, 18\}$

$\Rightarrow A \cup B = \{5, 7, 9, 12, 18, 21\}$

$\Rightarrow A \cap B = \{9, 12\}$

(II) $C = \{a, b, c\}$ and $D = \{d, e, f\}$

$\Rightarrow C \cup D = \{a, b, c, d, e, f\}$

$\Rightarrow C \cap D = \phi$

(III) $E = \{x : n \in Z, x = 3n, -2 < n < 2\}$ and $F = \{x : x \in W \text{ and } x < 5\}$

i.e., for set E:

$n = -1, 0, 1$

$\Rightarrow x = -3, 0, 3$

Thus, $E = \{-3, 0, 3\}$ and $F = \{0, 1, 2, 3, 4\}$

Therefore, $E \cup F = \{-3, 0, 1, 2, 3, 4\}$ and $E \cap F = \{0, 3\}$

Example 15. If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $C = \{2, 4, 6\}$ then find:

(I) $A \cup B$ (II) $C \cup B$ (III) $A \cup B \cup C$ (IV) $A \cap C$

(V) $A - B$ (VI) $A \cap B \cap C$ (VII) $C - B$ (VIII) $B - C$

Sol. Given, $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ and $C = \{2, 4, 6\}$

(I) $A \cup B = \{1, 2, 3, 4, 5\}$

(II) $C \cup B = \{2, 3, 4, 5, 6\}$

(III) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6\}$

(IV) $A \cap C = \{2\}$

(V) $A - B = \{1, 2\}$

(VI) $A \cap B \cap C = \phi$

(VII) $C - B = \{2, 6\}$

(VIII) $B - C = \{3, 5\}$

Example 16. Let $U = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{a, c, e, g, i\}$ and $B = \{b, c, d, e, f, g\}$

(I) A' (II) B' (III) $A' - B'$ (IV) $A' \cup B'$ (V) $A' \cap B'$

Sol. Given, $U = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{a, c, e, g, i\}$ and $B = \{b, c, d, e, f, g\}$

(I) $A' = U - A$

$\Rightarrow A' = \{a, b, c, d, e, f, g, h, i, j\} - \{a, c, e, g, i\}$

$\Rightarrow A' = \{b, d, f, h, j\}$

(II) $B' = U - B$

$\Rightarrow B' = \{a, b, c, d, e, f, g, h, i, j\} - \{b, c, d, e, f, g\}$

$\Rightarrow B' = \{a, h, i, j\}$

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(III) $A' - B' = \{b, d, f, h, j\} - \{a, h, i, j\}$

$\Rightarrow A' - B' = \{b, d, f\}$

(IV) $A' \cup B' = \{b, d, f, h, j\} \cup \{a, h, i, j\}$

$\Rightarrow A' \cup B' = \{a, b, d, f, h, i, j\}$

(V) $A' \cap B' = \{b, d, f, h, j\} \cap \{a, h, i, j\}$

$\Rightarrow A' \cap B' = \{h, j\}$

Example 17. The union of two sets $A = \{1, 2, 7, 9\}$ and $B = \{x : x^2 - 10x + 16 = 0\}$ is

(a) $\{2\}$ (b) $\{1, 2, 7, 9\}$ (c) $\{1, 2, 7, 8, 9\}$ (d) $\{1, 2, 7, 8, 9, 10\}$

Sol. (c) Given, $A = \{1, 2, 7, 9\}$ and $B = \{x : x^2 - 10x + 16 = 0\}$

Here, $x^2 - 10x + 16 = 0$

$\Rightarrow x^2 - 8x - 2x + 16 = 0$

$\Rightarrow x(x - 8) - 2(x - 8) = 0$

$\Rightarrow (x - 8)(x - 2) = 0$

$\Rightarrow x - 8 = 0$ or $x - 2 = 0$

$\Rightarrow x = 8, x = 2$

Thus, $B = \{2, 8\}$

Therefore, $A \cup B = \{1, 2, 7, 8, 9\}$

Hence, the correct option is (c).

Example 18. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$. What is $(A - B) \cup (B - A)$?

(a) $\{1, 2, 3, 4, 5, 6\}$ (b) $\{3, 4, 5, 6\}$

(c) $\{1, 2, 3, 4\}$ (d) $\{1, 2, 5, 6\}$

Sol. (d) Given, $A = \{1, 2, 3, 4\}$ and $B \{3, 4, 5, 6\}$

$(A - B) = \{1, 2, 3, 4\} - \{3, 4, 5, 6\} = \{1, 2\}$

Now, $(B - A) = \{3, 4, 5, 6\} - \{1, 2, 3, 4\} = \{5, 6\}$

Thus, $(A - B) \cup (B - A) = \{1, 2, 5, 6\}$

Hence, the correct option is (d).

Example 19. If $A \Delta B = (A - B) \cup (B - A)$ and $A = \{1, 2, 3, 4\}$, $B = \{3, 5, 7\}$ than $A \Delta B$ is

(a) $\{1, 2, 4, 5, 7\}$ (b) $\{3\}$

(c) $\{1, 2, 3, 4, 5, 7\}$ (d) None of these

Sol. (a) Given, $A \Delta B = (A - B) \cup (B - A)$

$A = \{1, 2, 3, 4\}$,

$B = \{3, 5, 7\}$

We know that, $A - B = \{x : x \in A, x \notin B\}$

Then,

$A - B = \{1, 2, 3, 4\} - \{3, 5, 7\} = \{1, 2, 4\}$

$B - A = \{3, 5, 7\} - \{1, 2, 3, 4\} = \{5, 7\}$

Now, $A \Delta B = (A - B) \cup (B - A)$

$\Rightarrow A \Delta B = \{1, 2, 4\} \cup \{5, 7\}$

$\Rightarrow A \Delta B = \{1, 2, 4, 5, 7\}$

Hence, the correct answer is option (a).

CARDINAL NUMBER

Let A and B be finite sets and if $A \cap B = \varnothing$, then $n(A \cup B) = n(A) + n(B)$

Let A and B be finite sets and if $A \cap B \neq \phi$, then

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(B - A) = n(B) - n(A \cap B)$

Directions (Q. 20-22): By using the data

If $P = \{1, 2, 3, 5, 7\}$, $Q = \{1, 3, 6, 10, 15\}$,

Universal Set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

Example 20. The cardinal number of $P \cap Q$ is

(a) 3 (b) 2 (c) 0 (d) None of these

Sol. (b) Given, $P = \{1, 2, 3, 5, 7\}$

$Q = \{1, 3, 6, 10, 15\}$

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

The cardinality of the set is the measure of the number of the elements of the set.

$P \cap Q = \{1, 3\}$

Therefore, the cardinality of $P \cap Q$ is 2.

Hence, the correct option is (b).

Example 21. The cardinal number of $P \cup Q$ is

(a) 10 (b) 9 (c) 8 (d) None of these

Sol. (c) Given, $P = \{1, 2, 3, 5, 7\}$

$Q = \{1, 3, 6, 10, 15\}$

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

The cardinality of the set is the measure of the number of the elements of the set.

$P \cup Q = \{1, 2, 3, 5, 6, 7, 10, 15\}$

Therefore, the cardinality of $P \cup Q$ is 8.

Hence, the correct option is (c).

Example 22. $n(P')$ is **(ICAI)**

(a) 10 (b) 5 (c) 6 (d) None of these

Sol. (a) Given: $P = \{1, 2, 3, 5, 7\}$ and Universal Set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

To find: $n(P')$

We know that,

$n(P') = n(S) - n(P)$

$n(S) = 15$

$n(P) = 5$

Thus, $n(P') = 15 - 5 = 10$

Hence, the correct answer is option (a).

Example 23. If $A = \{1, 2, 3, 5, 7\}$ and $B = \{x^2 : x \in A\}$

(a) $n(B) = n(A)$ (b) $n(B) > n(A)$ (c) $n(A) < n(B)$ (d) None of these

Sol. (a) Given, $A = \{1, 2, 3, 5, 7\}$

$B = \{x^2 : x \in A\}$

Here, $n(A) = 5$

And, $B = \{1^2, 2^2, 3^2, 5^2, 7^2\} = \{1, 4, 9, 25, 49\}$

Thus, $n(B) = 5$

Therefore, $n(A) = n(B)$

Hence, the correct option is (a).

Example 24. $A \cap E$ is equal to (E is a superset of A) **(ICAI)**

(a) A (b) E (c) ϕ (d) None of these

Sol. (a) Given, $A \cap E$

As we know, if $A \subseteq B$, then B is called the super set of A.

Here, $A \subseteq E$, thus the union of A and E will be set A as E is the bigger set and all elements of A are in E.

Hence, the correct option is (a).

Example 25. If $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the subset of E satisfying $5 + x > 10$ is **(ICAI)**

(a) $\{5, 6, 7, 8, 9\}$ (b) $\{6, 7, 8, 9\}$

(c) $\{7, 8, 9\}$ (d) None of these

Sol. (b) Given, $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the subset of E satisfying $5 + x > 10$ is

The set which must be the subset of E with a given condition that is $5 + x > 10$

For the values of x:

$x = 1, 6 > 10$, not possible thus discarded

$x = 2, 7 > 10$, not possible thus discarded

$x = 3, 8 > 10$, not possible thus discarded

$x = 4, 9 > 10$, not possible thus discarded

$x = 5, 10 > 10$, not possible thus discarded

$x = 6, 11 > 10$, possible thus included in the set that is subset of E

$x = 7, 12 > 10$, possible thus included in the set that is subset of E

$x = 8, 13 > 10$, possible thus included in the set that is subset of E

$x = 9, 14 > 10$, possible thus included in the set that is subset of E

Thus, the required set is: B = {6, 7, 8, 9}

Hence, the correct option is (b).

Example 26. If A and B are two sets such that A has 50 elements. $A \cup B$ has 80 elements and $A \cap B$ has 15 elements, how many elements does B have?

(a) 25 (b) 50 (c) 75 (d) 45

Sol. (d) Let the set B has x elements.

Then, $n(A) = 50, n(B) = x, n(A \cup B) = 80, n(A \cap B) = 15$

As we know,

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

Put the values and compute,

$\Rightarrow 15 = 50 + x - 80$

$\Rightarrow x = 15 - 50 + 80$

$\Rightarrow x = 45$

Hence, the correct option is (d).

Example 27. Out of 20 members in a family, 11 like to take tea and 14 like coffee. Assume that each one likes at least one of the two drinks. Find how many like both tea and coffee.

(a) 2 (b) 3 (c) 4 (d) 5

Sol. (d) Let T represents the members who like tea and C represents the members who like coffee.

According to the question,

$n(T \cup C) = 20, n(T) = 11$ and $n(C) = 14$

We know that,

$n(T \cap C) = n(T) + n(C) - n(T \cup C)$

$n(T \cap C) = 11 + 14 - 20$

$n(T \cap C) = 5$

Therefore, 5 members like both tea and coffee.

Hence, the correct option is (d) i.e., 5.

Example 28. In a survey of 500 people, it was found that 350 people enjoy reading books and 250 people enjoy watching movies. How many people enjoy watching movies only?

(a) 150 (b) 120 (c) 100 (d) None of these

Sol. (a) Given, People who enjoy reading books n(B) = 350

People who enjoy watching movies n(M) = 250

Total people in a survey = $n(B \cup M) = 500$

We know that,

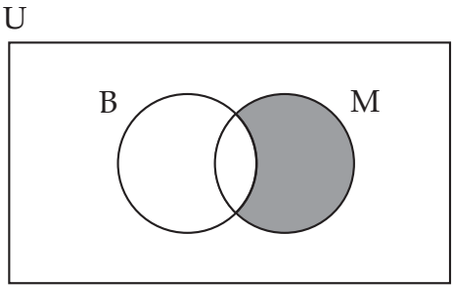
$n(B \cap M) = n(B) + n(M) - n(B \cup M)$

$\Rightarrow n(B \cap M) = 350 + 250 - 500$

$\Rightarrow n(B \cap M) = 600 - 500$

$\Rightarrow n(B \cap M) = 100$

Now, people enjoy watching movies only can be calculated by



$= n(M) - n(B \cap M) = 250 - 100 = 150$

$\therefore$ 150 people enjoy watching movies only.

Hence, the correct option is (a).

Example 29. In a group of 100 tourists, 55 can speak English and 85 can speak German. How many people can speak both English and German?

(a) 40 (b) 50 (c) 60 (d) 70

Sol. (a) Let the number of tourists who can speak English be, $n(A) = 55$

Let the number of tourists who can speak German be, $n(B) = 85$

Now, number of total tourists $(A \cup B) = 100$

Now, number of people who can speak both English and German will be given by $(A \cap B)$

As we know, $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

Put the values and compute,

$\Rightarrow n(A \cap B) = 55 + 85 - 100$

$\Rightarrow n(A \cap B) = 140 - 100$

$\Rightarrow n(A \cap B) = 40$

Hence, the correct option is (a).

Example 30. In a conference with 400 attendees, 300 attendees are from the finance industry and 200 attendees are from the marketing industry. How many attendees are from the finance industry only?

(a) 70 (b) 150 (c) 200 (d) None of these

Sol. (c) Given, Attendees from the finance industry $n(F) = 300$

Attendees from the marketing industry $n(M) = 200$

Total attendees $= n(F \cup M) = 400$

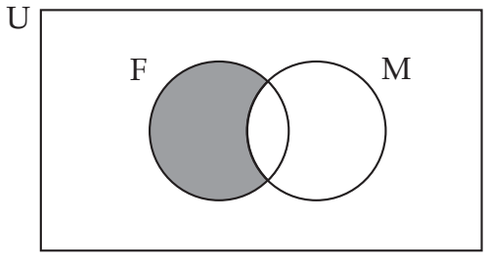
We know that,

$n(F \cup M) = n(F) + n(M) - n(F \cup M)$

$\Rightarrow n(F \cap M) = 300 + 200 - 400$

$\Rightarrow n(F \cap M) = 500 - 400$

$\Rightarrow n(F \cap M) = 100$



Now, attendees are from the finance industry only can be calculated by:

$= n(F) - n(F \cap M)$

$= 300 - 100 = 200$

$\therefore$ 200 attendees are from the finance industry only.

Hence, the correct option is (c).

Example 31. The number of items in the set A is 40 in the Set B is 32; in the Set C is 50; in both A and B is 4; in both A and C is 5; in both B and C is 7; in all the set is 2. How many are in only one set? **(Dec 2020)**

(a) 65 (b) 110 (c) 96 (d) 84

Sol. (c) Given, $n(A) = 40$

$n(B) = 32$

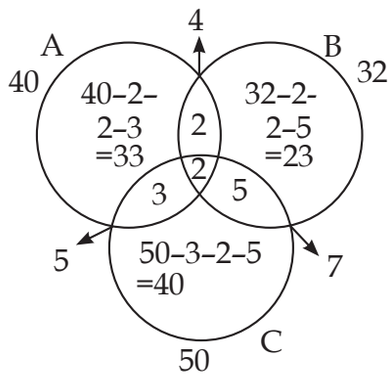
$n(C) = 50$

$n(A \cap B) = 4$

$n(A \cap C) = 5$

$n(B \cap C) = 7$

and $n(A \cap B \cap C) = 2$



No. of items in only one set.

= only A + only B + only C

$= (40 - 2 - 2 - 3) + (32 - 2 - 2 - 5) + (50 - 3 - 2 - 5) = 33 + 23 + 40 = 96$

Hence, the correct option is (c).

Example 32. In a group of 100 students, 60 students play football, 50 students play basketball, and 40 students play cricket. Out of these, 15 students play both football and basketball, 10 students play both basketball and cricket, 8 students play both football and cricket and 5 students play all three sports. How many students play cricket only?

(a) 14 (b) 29 (c) 34 (d) None of these

Sol. (b) Let F be the set of students who play football, B be the set of students who play basketball, and C be the set of students who play cricket.

Given, $n(F) = 60$, $n(B) = 50$ and $n(C) = 40$

$n(F \cap B) = 15$, $n(B \cap C) = 10$, $n(F \cap C) = 8$ and $n(F \cap B \cap C) = 5$

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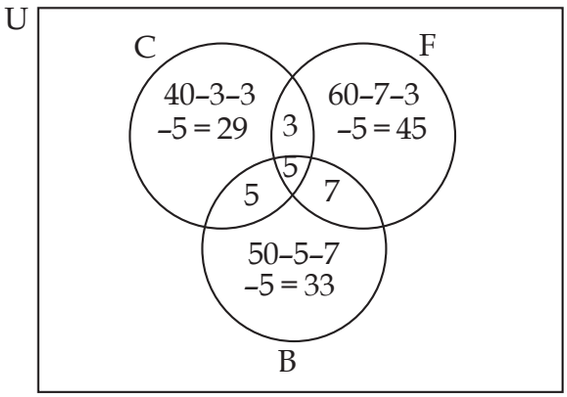
From the Venn diagram,



Therefore, number of students who play cricket only = 29.

Hence, the correct option is (b).

Example 33. X and Y are two sets such that n(X) = 17, n(Y) = 23 and n(X ∪ Y) = 38, find n(X ∩ Y).

(a) 1 (b) 2 (c) 3 (d) 4

Sol. (b) Given, n(X) = 17, n(Y) = 23, n(X ∪ Y) = 38, n(X ∩ Y) = A

As we know, n(X ∩ Y) = n(X) + n(Y) – n(X ∪ Y)

Put the values and compute,

⇒ A= 17 + 23 – 38

⇒ A = 40 – 38

⇒ A = 2

Hence, the correct option is (b).

Example 34. In a class of 40 students, 30 like cricket, 10 like both cricket and football. How many like football only and not cricket? How many like football?

(a) 20, 2 (b) 5, 10 (c) 20, 7 (d) 20, 10

Sol. (d) Let C denote the set of people who like cricket, F denote the set of people who like football.

∴ n(C ∪ F) = 40, n(C) = 30, n(C ∩ F) = 10

We know that,

n(C ∪ F) = n(C) + n(F) – n(C ∩ F)

∴ 40 = 30 + n(F) – 10

⇒ 40 = 20 + n(F)

⇒ n(F) = 40 – 20

⇒ n(F) = 20

Therefore, 20 people like football.

Now, (F – C) ∪ (F ∩ C) = F

Also, (F – C) ∩ (F ∩ C) = ϕ

∴ n(F) = n(F – C) + n(F ∩ C)

$\Rightarrow 20 = n(T - C) + 10$

$\Rightarrow n(T - C) = 20 - 10$

$\Rightarrow n(T - C) = 10$

Thus, 10 people only like football.

Hence, the correct option is (d).

PRACTICE QUESTIONS (PART A)

1. Write the following set in Roster form.

A = {x : x is an integer and $-4 \leq x < 2$}

(a) {–3, –2, –1, 0, 1}
(b) {–4, –3, –2, –1, 0, 1}
(c) {0, 1}
(d) None of these

2. Write the following set in set-builder form.

{1, 4, 9, ..., 121}

(a) $\{x : x = 2n, n \in N \text{ and } 1 \leq x \leq 11\}$
(b) $\{x : x = n^2, n \in N \text{ and } 1 < x < 11\}$
(c) $\{x : x = n^2, n \in N \text{ and } 1 \leq x \leq 11\}$
(d) None of these

3. Write the set B = {6, 9, 12, 15, ...} in set-builder form.

(a) $\{x : x = 3n, n \in W\}$
(b) $\{x : x = 3n, n \in N\}$
(c) $\{x : x = 6n - 3, n \in W\}$
(d) None of these

4. Write the set {8, 27, 64, 125, 216} in set-builder form.

(a) $\{x : x = n^2, n \in N\}$
(b) $\{x : x = n^3, n \in N\}$
(c) $\{x : x = n^3, n \in N \text{ and } 2 < n < 6\}$
(d) $\{x : x = n^3, n \in N \text{ and } 2 \leq n \leq 6\}$

5. Write the set $\{x : x \in Z, -5 \leq x \leq 1\}$ in roster form.

(a) {–5, –4, –3, –2, –1, 0, 1}
(b) {–4, –3, –2, –1, 0}
(c) {–5, –4, –3, –2, –1, 0}
(d) {–5, –4, –3, –2, –1}

6. The set of squares of positive integers is

(a) A finite set
(b) An infinite set
(c) A null set
(d) None of these

7. The set of cubes of natural numbers is **(Dec 2020)**

(a) Null set
(b) Finite set
(c) Infinite set
(d) A finite set of three numbers

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8. Which of the following is an example of a singleton set?

(a) {a, b, c}
(b) {x : x is a negative even number}
(c) {x : x is a vowel}
(d) {π}

9. Which of the following sets is finite?

(a) The set of months of a year.
(b) {1, 2, 3, ... }
(c) The set of positive integers greater than 100.
(d) None of these

10. The null set is represented by

(a) (φ)
(b) (O)
(c) φ
(d) None of these

11. Write down all the subsets of the following set: {1, 2, 3}

(a) {1}, {2}, {3}
(b) {1}, {2}, {3}, {1, 2}, {2, 3}, {2, 3}, {1, 3}
(c) ϕ, {1}, {2}, {3}, {1, 2}, {2, 3}, {2, 3}, {1, 3}
(d) ϕ, {1}, {2}, {3}, {1, 2}, {2, 3}, {2, 3}, {1, 3}, {1, 2, 3}

12. The number of proper subsets of the set {3, 4, 5, 6, 7} is

(a) 32
(b) 31
(c) 30
(d) 25

13. The number of the subsets of the set {0, 1, 2, 3} is **(Dec 2022)**

(a) 2
(b) 4
(c) 8
(d) 16

14. If A = {0, 1} state which of the following statements are true:

(I) {1} ⊂ A
(II) {1} ∈ A
(III) ϕ ∈ A
(IV) 0 ∈ A
(V) 1 ⊂ A
(VI) {0} ∈ A
(VII) ϕ ⊂ A

(a) (I), (IV) and (VII) only are true
(b) (I), (IV) and (VI) only are true
(c) (II), (III) and (VI) only are true
(d) None of these

15. The number of proper subsets of A ∩ B, if A = {1, 2, 3, 4, 5, 7, 8, 9, 10} and B = {2, 4, 6, 7, 9} is

(a) 8
(b) 15
(c) 16
(d) 64

16. A = {1, 2, 3}; B = {2, 3, 4}; C = {2, 3, 5}; D = {1, 3, 5, 7} then (A ∪ B) ∩ (C ∪ D) is equal to

(a) {1, 2, 3}
(b) {1, 2, 3, 5, 7}
(c) {1, 2, 3, 4}
(d) None of these

17. If U = {1, 2, 3, 4, 5} and A = {2, 4}, then A' =

(a) {1, 2, 3, 4, 5}
(b) {1, 3, 5}
(c) {1, 3, 4, 5}
(d) {1, 2, 3}

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18. If A = {1, 2, 3, 4, 5}; B = {2, 4, 6} and C = {3, 5, 7}, then A – (B ∪ C) =

(a) {1} (b) {1, 2, 3} (c) {1, 2, 4} (d) None of these

19. If A = {x : x is a prime number less than 10}, B = {x : x is an odd natural number less than 10} then (A ∩ B) is

(a) {1, 3, 5, 7, 9} (b) {2, 3, 5, 7} (c) {3, 5, 7} (d) None of these

20. If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}; B {1, 3, 4, 5, 7, 8}; C = {2, 6, 8}, then find (A – B) ∪ C **(June 2019)**

(a) {2, 6} (b) {2, 6, 8} (c) {2, 6, 8, 9} (d) None of these

21. If U = {1, 2, 3, 4, 5, 6, 7, 8}, A = {1, 2, 3, 8}, B = {3, 4, 5, 7} and C = {2, 3, 7, 8}. Find (A – B) ∪ C'.

(a) {1, 2, 3,5} (b) {1, 2, 4, 6, 8}

(c) {3} (d) {1, 2, 3, 4, 5, 6, 7, 8}

22. If P = {1, 2, 3, 5, 7}, Q = {1, 3, 6, 10, 15}, Universal set S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} then n(Q') is **(ICAI)**

(a) 4 (b) 10 (c) 4 (d) None of these

23. If A = {1, 2, 3, 4, 5, 6, 7} and B = {2, 4, 6, 8}. Cardinal number of A - B is **(Nov 2018)**

(a) 4 (b) 3 (c) 9 (d) 7

24. If P and Q are two sets such that P has 50 elements, P ∪ Q has 70 elements, and P ∩ Q has 20 elements, how many elements does Q have?

(a) 20 (b) 40 (c) 60 (d) None of these

25. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket and how much like tennis?

(1) 35, 25 (2) 25, 35 (3) 15, 25 (4) None of these

26. In a meeting of 50 employees, 27 employees like coffee, 22 like tea and 9 like both coffee and tea. How many employees like at least one of the two drinks?

(a) 47 (b) 40 (c) 50 (d) None of these

27. In a survey of 100 students, 70 enjoy playing football, 60 enjoy playing basketball, and at least 50 students enjoy both sports. How many students enjoy both football and basketball?

(a) 70 (b) 30 (c) 50 (d) 80

28. In a classroom, there are 35 students who study French or Spanish. Of these, 20 study French, and 5 study both French and Spanish. How many students study Spanish?

(a) 20 (b) 40 (c) 50 (d) 65

Answer Key

1. (b)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (c)
11. (d)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (c)
21. (b)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (d)	**28.** (a)		

ORDERED PAIR

An ordered pair is a simple way of keeping track of two elements by writing them in a specific order. It is denoted by (a, b).

CARTESIAN PRODUCT OF SETS

A set of all ordered pairs constructed by different sets. Let A and P be two sets with elements {a, b} and {p, q} respectively. The Cartesian product of A and P is denoted by A × P and is the set of all ordered pairs.

If we have two sets, A and P, with elements A = {a, b} and P = {p, q} respectively, the Cartesian product of A and P, denoted by A × P, is the set of all possible ordered pairs that can be formed by taking one element from A and one element from P.

So, the Cartesian product A × P would be:

A × P = {(a, p); (a, q); (b, p); (b, q)}

The Cartesian product P × A, on the other hand, would be:

P × A = {(p, a); (p, b); (q, a); (q, b)}

PROPERTIES

- $A \times P \neq P \times A$
- $A \times \phi = \phi$
- $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (Distributive law)
- $(A \cup B) \times C = (A \times C) \cup (B \times C)$ (Associative law)

Example 35. If A = {3, 5} and B = {4, 6} then A × B?

(a) {3, 5} (b) {(3, 4), (3, 6), (5, 4), (5, 6)}

(c) {4, 6} (d) None of these

Sol. (b) Given, A= {3, 5} and B = {4, 6}

We know that,

If A and B are two non-empty sets then the cartesian product of A and B is the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$.

So, A × B = {(3, 4), (3, 6), (5, 4), (5, 6)}

Hence, the correct option is (b).

Example 36. If A = {1, 3, 5} and B = {2, 3}, then find A × B.

(a) {(1, 2), (1, 3), (3, 2)}

(b) {(1, 2), (1, 3), (3, 3), (5, 5)}

(c) {(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)}

(d) None of these

Sol. (c) Given, A = {1, 3, 5} and B = {2, 3}

Thus, A × B = {1, 3, 5} × {2, 3}

= {(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)}

Hence, the correct option is (c).

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- We can represent the product set of ordered pairs by points in the XY plane.
- If X = Y = the set of all natural numbers, the product set XY represents an infinite lattice of points in the first quadrant of the XY plane.

RELATION

A relation is any subset of the product of sets A and B. It is denoted by aRb, where 'a' belongs to set A and 'b' belongs to set B.

Suppose we define a relation R where set B (women) represents the mothers of set A (men):

Geeta has two sons, Ramesh and Brijesh; Babita has one son, Kamlesh; and Sunita has two sons, Suresh and Rajesh.

We can represent this relation as follows:

R = {(Ramesh, Geeta), (Brijesh, Geeta), (Kamlesh, Babita), (Suresh, Sunita), (Rajesh, Sunita)}

Now, let's find A × B, which represents the Cartesian product of sets A and B:

A × B = {(Ramesh, Geeta), (Ramesh, Babita), (Ramesh, Sunita), (Suresh, Geeta), (Suresh, Babita), (Suresh, Sunita), (Brijesh, Geeta), (Brijesh, Babita), (Brijesh, Sunita), (Rajesh, Geeta), (Rajesh, Babita), (Rajesh, Sunita), (Kamlesh, Geeta), (Kamlesh, Babita), (Kamlesh, Sunita)}

By comparing the relation R and the set A × B, we can see that they contain the same set of ordered pairs. The only difference is the representation and the context in which they are used.

Example 37. If A = {1, 2, 3, ..., 13, 14}. Find relation R in the set A defined by R = {(x, y) : 3x – y = 0}

Sol. Given relation, R = {(x, y) : 3x – y = 0}

and A = {1, 2, 3, ..., 13, 14}

Therefore, R = {(1, 3), (2, 6), (3, 9), (4, 12)}

KEY CONCEPTS

- If there are no elements that satisfy a particular relation, the relation is considered a void relation (R = φ).
- If R is a relation defined from A to A, it is called a relation defined on A.
- In this case, R_1, R_2, and R_3 represent different relations between the elements of sets A and B.
- R = φ represents a void relation, which means there are no elements in the relation. This occurs when there are no elements in the Cartesian product A × B.
- If R is a relation defined from a set A to a set B, and A × B contains all possible ordered pairs between elements of A and B, then R is called a universal relation. In other words, R includes all possible combinations of elements from A and B.
- If R is a relation defined from a set A to itself (A × A), it is called a relation defined on A. In this case, the elements of the relation are ordered pairs of elements from A.

  E.g.: Let A = {1, 2, 3} and B = {2, 5, 7}, and the relation is "the difference is odd":

  R = A × B = {(1, 2), (2, 5), (2, 7), (3, 2)}

  In this case, R is not a void relation because there are elements in the relation.

Let A = {2, –2} and B = {4}, and the relation is "is square root of":

R = A × B = {(2, 4), (–2, 4)}

In this case, R is a universal relation because it includes all possible combinations of elements from A and B.

If R is a relation defined from a set A to itself, such as A = {2, -2}, then the relation is defined on A.

DOMAIN AND RANGE OF A RELATION

For a relation from set A to set B i.e., aRb, all the elements of set A are called the domain of the relation R and the set of all second elements in a relation R from a set A to a set B is called the range of the relation R.

So, Domain (R) = {a : (a, b) ∈ R} and Range (R) = {b : (a, b) ∈ R}

E.g.: Consider the relation R defined on the set A = {1, 2, 3} as R = {(1, 2), (2, 3)}, then

Domain (R) = {1, 2}

Range (R) = {2, 3}

Example 38. Write the domain and range of the following relations.

R_1 = {(1, 5), (2, 4), (2, 5), (1, 4), (1, 6), (1, 7)}

Sol. According to the question,

R_1 = {(1, 5), (2, 4), (2, 5), (1, 4), (1, 6), (1, 7)}

As we know, the domain of a function is all x-values or inputs of a function, and the range of a function is all y-values or outputs of a function.

So, Domain: {1, 2} and Range: {4, 5, 6, 7}

Hence, Domain is {1, 2} and Range is {4, 5, 6, 7}.

Example 39. Find the domain of the relation R defined by {(x, y): x, y ∈ N, xy = 8}

(a) {1, 2} (b) {1, 2, 4, 8} (c) {1, 2, 4, 8, 16} (d) None of these

Sol. (b) Given, {(x, y): x, y ∈ N, xy = 8}

Since, x, y ∈ N and xy = 8

Thus, the possible ordered pairs are:

(1, 8), (2, 4), (4, 2), (8, 1).

Therefore, the domain of the given relation = {1, 2, 4, 8}

Hence, the correct option is (b).

IDENTITY RELATION

The identity relation I = {(a, a) : a ∈ A} is called the identity relation on A. In the identity relation, every element is related to itself.

For example, let A = {1, 2, 3} and the relation "is equal to":

I = {(1, 1), (2, 2), (3, 3)}

INVERSE RELATION

If R is a relation on A, then the relation R^{-1} on A, defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$, is called the inverse relation on A. In the inverse relation, the ordered pairs are reversed

For example, let A = {1, 2, 3} and R = {(1, 2), (2, 2), (3, 1), (3, 2)}:

Now, let's determine the values of the following:

1. Domain (R): The domain of relation R is the set of all first elements in the ordered pairs of R.
   Dom (R) = {1, 2, 3}
2. Range (R): The range of relation R is the set of all second elements in the ordered pairs of R.
   Range (R) = {1, 2}
3. R^{-1}: The inverse relation of R is obtained by reversing the ordered pairs of R.
   R^{-1} = {(2, 1), (2, 2), (1, 3), (2, 3)}
4. Dom (R^{-1}): The domain of the inverse relation R^{-1} is the set of all first elements in the ordered pairs of R^{-1}.
   Dom (R^{-1}) = {1, 2}
5. Range (R^{-1}): The range of the inverse relation R–1 is the set of all second elements in the ordered pairs of R^{-1}.
   Range (R^{-1}) = {1, 2, 3}

SOME IMPORTANT RELATIONS WHEN A RELATION IS DEFINED ON ITSELF

Let S = {a, b, c, ...} be any set, and the relation R is a subset of the product set S × S.

1. If R contains all ordered pairs of the form (a, a) in S × S, then R is called Reflexive. In a reflexive relation, every element is related to itself.
   For example, the relation "is equal to" is reflexive because every element is equal to itself.
2. If (a, b) ∈ R implies (b, a) ∈ R for every a, b ∈ S, then R is called Symmetric. In a symmetric relation, if element a is related to element b, then element b is also related to element a.
   For example, the relation "is equal to" is symmetric because if a = b, then b = a.
3. If (a, b) ∈ R and (b, c) ∈ R implies (a, c) ∈ R for every a, b, c ∈ S, then R is called Transitive. In a transitive relation, if element a is related to element b and element b is related to element c, then element a is also related to element c.
   For example, the relation "is equal to" is transitive because if a = b and b = c, then a = c.
   A relation that is reflexive, symmetric, and transitive is called an **Equivalence relation**. The relation "is equal to" is an equivalence relation.

Example 40. If a relation R is represented by {(8, 2), (6, 4), (1, 9), (3, 7)}, then the domain of R^{-1} is

(a) (1, 3, 6, 8)
(b) (1, 2, 3, 4, 6, 7, 8, 9)
(c) (2, 4, 7, 9)
(d) None of these

Sol. (c) Given, R = {(8, 2), (6, 4), (1, 9), (3, 7)}

Thus, R^{-1} = {(2, 8), (4, 6), (9, 1), (7, 3)}

Therefore, Domain of R^{-1} = (2, 4, 7, 9)

Hence, the correct option is (c) i.e. (2, 4, 7, 9).

Example 41. A relation R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)} is

(a) Symmetric, Transitive but not Reflexive

(b) Reflexive, Symmetric but not Transitive

(c) Reflexive, Transitive but not Symmetric

(d) An Equivalence relation

Sol. (b) We know that,

Relation is reflexive if (a, a) $\in$ R for every a $\in$ R

Clearly, (1, 1), (2, 2), (3, 3) $\in$ R for 1, 2, 3 $\in$ R

Thus, relation is Reflexive.

Now, Relation is symmetric if (a, b) $\in$ R then (b, a) $\in$ R for every (a, b) $\in$ R

We have, (1, 2) $\in$ R and (2, 1) $\in$ R

Also, (2, 3) $\in$ R and (3, 2) $\in$ R

Thus, Relation is symmetric.

Now, for a relation to be transitive

If (a, b) $\in$ R and (b, c) $\in$ R then (a, c) $\in$ R

Here, (1, 2) $\in$ R and (2, 3) $\in$ R but (1, 3) $\notin$ R

Thus, relation is not Transitive.

Hence, the correct option is (b).

Example 42. If R be a relation on set A = {1, 2, 3, 4} such that R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} then R is

(a) Not Symmetric (b) Transitive (c) Reflexive (d) Equivalence

Sol. (a) Given, R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)}

Clearly, (1, 1), (2, 2), (3, 3), (4, 4) $\notin$ R for 1, 2, 3, 4 $\in$ A

Thus, the given relation is not Reflexive.

Now, we have, (1, 3) $\in$ R and (3, 1) $\in$ R

Also, (2, 4) $\in$ R and (4, 2) $\in$ R

But (2, 3) $\in$ R and (3, 2) $\notin$ R

Thus, Relation is not symmetric.

Now, (2, 3) $\in$ R and (3, 1) $\in$ R but (2, 1) $\notin$ R

Thus, the given relation is not transitive.

Therefore, the relation R is not Symmetric.

Hence, the correct option is (a).

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Example 43. Which of the following is an example of an equivalence relation?

(a) $\{(x, y) : x < y\}$ (b) $\{(x, y) : x > y\}$

(c) $\{(x, y) : x - y \text{ is an integer}\}$ (d) None of these

Sol. (c) Let's check each option:

For option (a): $\{(x, y) : x < y\}$

It does not satisfy reflexivity because an element x cannot be less than itself, i.e., (x, x) cannot belong to the relation. For example, if we take x = 5, then (5, 5) is not in the relation because 5 is not less than 5. Therefore, this relation does not satisfy reflexivity.

For option (b): $\{(x, y) : x > y\}$

It does not satisfy reflexivity because an element x cannot be greater than itself, i.e., (x, x) cannot belong to the relation.

For option (c): $\{(x, y) : x - y \text{ is an integer}\}$

Reflexivity: For any x in the set, (x, x) is in the relation. This is true because x = x for any x.

Symmetry: For any x and y in the set, if (x, y) is in the relation, then (y, x) is also in the relation. This is true because if x = y, then y = x.

Transitivity: For any x, y, and z in the set, if (x, y) and (y, z) are in the relation, then (x, z) is also in the relation. This is true because if x = y and y = z, then x = z.

Since the relation satisfies all three properties, it is an equivalence relation.

Hence, the correct option is (c).

Example 44. "is perpendicular to" over the set of straight lines in a given plane is **(ICAI)**

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

Sol. (b) We know that,

For some set A,

Reflexive relation: If $(a, a) \in R$ for all $a \in A$

Symmetric relation: If $a, b \in A$ such that $(a, b) \in R$ then $(b, a) \in R$.

Transitive relation: If $a, b, c \in A$ such that $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$.

Equivalence relation: A relation which is a reflexive, symmetric as well as transitive.

Here,

We know, a line cannot be perpendicular to itself, i.e. it is not reflexive.

If a line 'a' is perpendicular to line 'b', then line 'b' will also be perpendicular to line 'a', thus it is symmetric.

Also, if a line 'a' is perpendicular to line 'b', and line 'b' is perpendicular to line 'c', then line 'a' will be parallel to line 'c' but not perpendicular, thus it is not transitive.

Therefore, it is a symmetric relation.

Hence, the correct option is (b).

Example 45. Is equal to" over the set of all rational numbers is **(ICAI)**

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

Sol. (d) We know that,

Every number is equal to itself, thus the given set is reflexive.

Now, for two numbers a and b, if a = b then b = a, thus it is a symmetric relation.
Now, for three numbers a, b and c,
If a = b and b = c then we can say a = c, thus it is a transitive relation as well.
Since, the given set is reflexive, symmetric and transitive.
Therefore, it is an equivalence relation.
Hence, the correct option is (d) i.e., (E).

Example 46. "Is smaller than" over the set of eggs in a box is **(ICAI)**

(a) Transitive (T) (b) Symmetric (S)
(c) Reflexive (R) (d) Equivalence (E)

Sol. (a) Let A be the required relation.
The relation R is defined on A by x < y
i.e., egg x is smaller than egg y

For Reflexive:
x is never less than x for any x > A.
Thus, x and x are not connected.
Therefore, R does not reflex.

For Symmetric:
If x < y
It is not implied that egg y is smaller than egg x.
Thus, R is not symmetric.

For Transitive:
Suppose x < y and y < z.
Then obviously x < z
Therefore, R is transitive relation.
Hence, the correct option is (a).

PRACTICE QUESTIONS (PART B)

1. If P = {1, 5, 9} and Q = {2, 4) then P × Q?
(a) {(1, 2), (1, 4), (5, 2), (5, 4), (9, 2), (9, 4)}
(b) {(2, 1), (4, 1), (2, 5), (4, 5), (2, 9), (4, 9)}
(c) {(1, 2), (5, 2), (9, 2), (9, 4)}
(d) {(2, 1), (1, 4), (5, 2), (5, 4), (9, 2), (9, 4)}

2. If A × B = {(1, 2), (1, 3), (4, 2), (4, 4)}, then find B × A.
(a) {(2, 1), (3, 1), (2, 4), (4, 4)} (b) {(1, 2), (1, 3), (4, 2), (4, 4)}
(c) {(1, 2), ((1, 3), (4, 2)} (d) None of these

3. If A × B = {(1, 4), (1, 3), (2, 3), (2, 4)}, then what is A and B?
(a) A = {2, 1} and B = {3, 4} (b) A = {1, 2} and B = {4, 3}
(c) A = {1, 3} and B = {2, 4} (d) A = {1, 2} and B = {3, 4}

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4. If A = {2, 3}, B = {4, 5} and C = {5, 6}, then find A × (B ∩ C).

(a) {(5, 2), (5, 3)} (b) {(2, 5), (3, 5)}

(c) {(2, 4), (5, 3)} (d) {(3, 5), (2, 6)}

5. The range of the relation {(1, 0), (2, 0), (3, 0), (4, 0), (0, 0)} is

(a) {1, 2, 3, 4, 0} (b) {0}

(c) {1, 2, 3, 4} (d) None of these

6. The domain and range of $\{x, y : y = x^2\}$ where $x, y \in R$ is **(ICAI)**

(a) (reals, natural numbers) (b) (reals, non-negative reals)

(c) (reals, reals) (d) None of these

7. Let R be a relation in the set {1, 2, 3, 4} given by R = {(1, 2); (2, 2); (1, 1); (4, 4); (1, 3); (3, 3); (3, 2)}. Then

(a) R is reflexive and symmetric but not transitive.

(b) R is reflexive and transitive but not symmetric.

(c) R is symmetric and transitive but not reflexive.

(d) R is an equivalence relation.

8. If R be the relation in the set A = {1, 2, 3, 4} given by R = {(2, 2); (1, 1); (4, 4); (1, 3); (3, 3); (3, 2)}; then the relation is

(a) Reflexive

(b) Reflexive and Symmetric

(c) Reflexive, Symmetric and Transitive

(d) None of these

9. {(x, y) : x + y = 2x, where x and y are positive integers} is **(ICAI)**

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

10. If R be the relation in the set {1, 2, 3} given by R = {(1, 3), (1, 2), (3, 2)}, then the relation is

(a) Reflexive (b) Reflexive and Symmetric

(c) Transitive (d) Symmetric and Transitive

11. "Is the square of" over n set of real numbers is **(ICAI)**

(a) Reflexive (b) Symmetric (c) Transitive (d) None of these

12. "Is greater than" over the set of mangoes in a box is which type of relation?

(a) Symmetric (b) Transitive (c) Reflexive (d) Equivalence

13. The relation "a number is the square of another number" over n set of 'real numbers' is

(a) Reflexive (b) Symmetric (c) Transitive (d) None of these

Answer Key

1. (a) **2.** (a) **3.** (d) **4.** (b) **5.** (b) **6.** (b) **7.** (b) **8.** (a) **9.** (d) **10.** (c)
11. (d) **12.** (b) **13.** (d)

FUNCTION

If we have two sets A and B, the relation 'f' from set A to B is a function if and only if every element of A has a unique image in B. In other words, in a function, each element in the domain maps to a unique element in the codomain.

Example 47. Identify the function from the following: **(Nov 2018)**

(a) {(1, 1); (1, 2); (1, 3)}

(b) {(1, 1); (2, 1); (2, 3)}

(c) {(1, 2); (2, 2); (3, 2); (4, 2)}

(d) None of these

Sol. (c) In a function f: (x, y), on one value of x, there shall be only one value of y, however there can be same value of y associated with different values of x.

This case is only with option (c) i.e. {(1, 2); (2, 2); (3, 2); (4, 2)}

Hence, the correct answer is option (c) i.e. {(1, 2), (2, 2), (3, 2), (4, 2)}.

Example 48. If A = {2, 4, 6, 8} and B = {4, 64, 36, 16}, and the relation f is defined as "is square of" between elements of set A and set B. Which of the following represent the function?

(a) f = {(2, 4), (4, 16), (6, 36), (8, 64)}

(b) f = {(4, 2), (16, 4), (36, 6), (64, 8)}

(c) f = {(2, 4), (4, 8), (6, 12), (8, 16)}

(d) None of these

Sol. (a) Given, A = {2, 4, 6, 8} and B = {4, 64, 36, 16}, and the relation f is defined as "is square of" between elements of set A and set B.

Then we can represent the function as follows:

f = {(2, 4), (4, 16), (6, 36), (8, 64)}

In this function, each element in set A has a unique image in set B. For instance, 2 is mapped to 4, 4 is mapped to 16, 6 is mapped to 36, and 8 is mapped to 64.

Hence, the correct option is (a).

It can be framed also in a way, if we take a function f from set A to set B such that $(x, y) \in f$.

- Then $f(x) = y$ is true only when for every element 'x' there is only one element 'y' or void.
- The function f from A to B is denoted by $f : A \rightarrow B$ and this is called mapping.

DOMAIN, CO-DOMAIN AND RANGE OF FUNCTION

For a function f from set A to set B ($f : A \rightarrow B$), where $f(a) = b$, where $a \in A$ and $b \in B$

The set A is called the domain of the function f.

The set B is called the codomain of the function f.

The set of images or the set f(A) = {f(x): x ∈ A} is called the range of the function f.

The function f is denoted as f: A → B, and this is called a mapping.

- For a function from set A to set B (f : A → B) i.e. f(a) = b where a ∈ A and b ∈ B
- All the elements of set A are called as the *domain* of the function f.
- While all the elements of set B are called as the co-domain of the function f and the set of image or set f(A) = {f(x): x ∈ A} is called the range of f.

Note: Range is always a subset of Co-domain, i.e. Range ⊆ Co-domain.

Example 49. If A = {2, 4, 6, 8}, B = {4, 64, 36, 16}

And let's say relation f "is square of" between elements of set A and set B, then domain, co-domain and range will be

Sol. Given, A = {2, 4, 6, 8} and B = {4, 64, 36, 16}

Here, the function f = {(2, 4), (4, 16), (6, 36), (8, 64)}:

Domain (f) = {2, 4, 6, 8}

Codomain (f) = {4, 64, 36, 16}

Range (f) = {4, 64, 36, 16}

Example 50. If A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 8, 10, 12} and a function is defined from set A to B where f(x) – 2x, then range of f is:

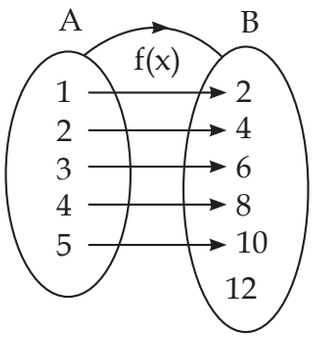
(a) {1, 2, 3, 4, 5} (b) {2, 4, 6, 8, 10}

(c) {1, 4, 9, 16, 25} (d) {2, 4, 6, 8, 10, 12}

Sol. (b) Given, A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 8, 10, 12}

Since, f(x) = 2x

Thus,



Therefore, Range = {2, 4, 6, 8, 10}

Hence, the correct option is (b) i.e., {2, 4, 6, 8, 10}.

Example 51. The range of the function f defined by $f(x)=\sqrt{16-x^2}$ is **(July 2021)**

(a) [–4, 0] (b) [–4, 4] (c) [0, 4] (d) (–4, 4)

Sol. (c) Given, $f(x) = \sqrt{16 - x^2}$

We know that,

For domain, $16 - x^2$ should not be negative

$\Rightarrow 16 - x^2 \geq 0$

$\Rightarrow x^2 \leq 16$

$\Rightarrow x \leq 4$ or $x \geq -4$

$\Rightarrow -4 \leq x \leq 4$

$\Rightarrow$ Domain $= [-4, 4]$

Now, for Range:

f(x) will be maximum value if x = 0

$\Rightarrow f(0) = \sqrt{16} = 4$

f(x) will be minimum value if $x = \pm 4$

$\Rightarrow f(\pm 4) = \sqrt{16 - 16} = 0$

Thus, $0 \leq f(x) \leq 4$

Therefore, Range of f(x) = [0, 4]

Hence, the correct answer is option (c) i.e. [0, 4].

VARIOUS TYPES OF FUNCTIONS

One to One Function

A function is said to be one-to-one if each element in set A is uniquely mapped to a different element in set B. In other words, there are no repetitions in the mapping.

E.g.: Let A = {1, 2, 3} and B = {4, 5, 6}. The function f: A → B defined as f(x) = x + 3 is a one-to-one function because each element in set A is mapped to a unique element in set B without any repetitions.

Many to One Function

A function is called many-to-one if multiple elements in set A are mapped to the same element in set B. In this case, the mapping is referred to as a many-to-one mapping.

E.g.: Let A = {1, 2, 3, 4} and B = {1, 2}. The function f: A → B defined as f(x) = 1 for all $x \in A$ is a many-to-one function because all elements in set A are mapped to the same element 1 in set B.

Note:

- If there is a greater-than sign (>) from set A to set B, it indicates a many-to-one function.
- If there is a less-than sign (<) from set A to set B, it implies that the relation is not a function.
- If the mapping is a straight line, it represents a one-to-one function.

Example 52. Let N be the set of natural numbers and the relation R be defined on N such that R = {(x, y): y = 2x, x, y ∈ N}. What is the domain, co domain and range of R? Is this relation a function?

Sol. According to the question, R = {(x, y): y = 2x, x, y ∈ N}

Domain of R = Set of Natural Numbers

Co-Domain of R = Set of natural numbers

Range of R = Set of even natural numbers

Thus, Clearly, every natural number is related to a unique image, so this relation is a function.

Example 53. Let A = {–3, –1, 1, 3, 5, 7} and B = {–1, 1, 3, 5, 9, 25}

Consider the rule

(a) $f(x) = x + 2$, where $x \in A$ (b) $f(x) = x^2$

Tell whether it is one to one or many to one function. Also, find the domain and range of the function.

Sol. (a) $f(x) = x + 2$, where $x \in A$:

To determine if it is one-to-one, we need to ensure that each element in set A is uniquely mapped to a different element in set B.

Let's apply the function and check the mapping:

For x = –3, f(–3) = –3 + 2 = –1

For x = –1, f(–1) = –1 + 2 = 1

For x = 1, f(1) = 1 + 2 = 3

For x = 3, f(3) = 3 + 2 = 5

For x = 5, f(5) = 5 + 2 = 7

For x = 7, f(7) = 7 + 2 = 9

From the above mapping, we can see that each element in set A is uniquely mapped to a different element in set B. Hence, the function f(x) = x + 2 is a one-to-one function.

Domain = {–3, –1, 1, 3, 5, 7}

Range = {–1, 1, 3, 5, 7, 9}

(b) $f(x) = x^2$:

To determine if it is one-to-one, we need to ensure that each element in set A is uniquely mapped to a different element in set B.

Let's apply the function and check the mapping:

For x = –3, $f(-3) = (-3)^2 = 9$

For x = –1, $f(-1) = (-1)^2 = 1$

For x = 1, $f(1) = (1)^2 = 1$

For x = 3, $f(3) = (3)^2 = 9$

For x = 5, $f(5) = (5)^2 = 25$

For x = 7, $f(7) = (7)^2 = 49$

From the above mapping, we can see that different elements in set A are mapped to the same elements in set B (e.g., -3 and 3 both map to 9).

Therefore, the function $f(x) = x^2$ is a many-to-one function.

Domain = {–3, –1, 1, 3, 5, 7}

Range = {1, 9, 25, 49}

ONTO OR SUBJECTIVE FUNCTIONS

Let f: A → B. If every element in B has at least one pre-image in A, then f is said to be an onto function. In other words, for every $y \in B$, there exists an $x \in A$ such that $y = f(x)$. A function f is onto if and only if the range of f is equal to the set B.

Example 54. Let N be the set of natural numbers and the relation R be defined on N such that $R = \{(x, y) : y = x, x, y \in N\}$. Tell whether it is onto function or not?

Sol. Given, the set of natural numbers N and the relation R defined on N such that

$R = \{(x, y) : y = x, x, y \in N\}$

In this case, every natural number y has at least one pre-image x such that $y = x$.

Therefore, the function defined by R is onto.

BIJECTIVE FUNCTION

A function that is both one-to-one (injective) and onto (subjective) is called a bijective function. A bijective function establishes a one-to-one correspondence between the elements of the domain and the elements of the co-domain.

E.g.: Let's consider the function $f : A \to B$ defined as $f(x) = 2x$, where A and B are sets of real numbers. This function is both one-to-one and onto. Each element in the domain A has a unique image in the co-domain B, and every element in B has a pre-image in A. Therefore, f is a bijective function.

IDENTITY FUNCTION

The identity function on a non-empty set A, denoted by $I : A \to A$, is defined as $I(x) = x$ for all $x \in A$. The identity function simply maps each element to it-self.

E.g.: Consider the set A = {1, 2, 3}. The identity function on A is defined as $I(x) = x$. So, $I(1) = 1$, $I(2) = 2$, and $I(3) = 3$. The identity function preserves the elements of the set.

Let A be a non-empty set. Then, the function I defined by $I : A \to A : I(x) = x$ for all $x \in A$ is called an identity function on A.

INTO FUNCTIONS

If there exists at least one element in the co-domain B that has no pre-image in the domain A, then the function f: A → B is said to be an into function.

E.g.: Let's consider the function $f : A \to B$ defined as $f(x) = x^2$, where A and B are sets of real numbers. In this case, the function is not into because there are negative numbers in the co-domain B (e.g., -1) that do not have a pre-image in the domain A.

CONSTANT FUNCTION

A constant function is a function where all the elements in the domain have the same image in the co-domain.

E.g.: Let f: A → B be a constant function, where A = {1, 2, 3} and B = {5}. In this case, f(x) = 5 for all x ∈ A. Regardless of the input, the function always produces the same output, which is 5.

EQUAL FUNCTION

Two functions f and g are considered equal, denoted as f = g, if they have the same domain and satisfy the condition f(x) = g(x) for all x in their common domain.

E.g.: Let's consider two functions f: A → B and g: A → B, where A = {1, 2} and B = {3, 6}. If f(x) = x + 2 and g(x) =3x, then f = g because both functions produce the same output for each input in their common domain.

INVERSE FUNCTION

If f is a one-to-one and onto function from set A to set B, then the inverse function f^{-1} is defined as f^{-1}: B → A, where f^{-1} (y) = x if and only if f(x) = y. The inverse function undoes the action of the original function.

A FUNCTION IS INVERTIBLE IF AND ONLY IF F IS ONE-ONE AND ONTO

Example 55. Let A = {1, 2, 3} and B = {4, 5, 6, 7}. Consider the relation R from set A to set B, defined as R = {(1, 4), (2, 5), (3, 6)}. Then the relation is

(a) One-one function (b) Many-one function

(c) One-one and onto function (d) Many-one and onto function

Sol. (a) Given, R: A → B is defined as R = {(1, 4), (2, 5), (3, 6)}

Thus, f(1) = 4, f(2) = 5 and f(3) = 6

Clearly, different elements of A have different and unique images in B.

Thus, it is a one-one function.

Now, since there exist an element 7 which belongs to B, has no pre-image in A.

Thus, it is not an onto function.

Hence, the correct option is (a).

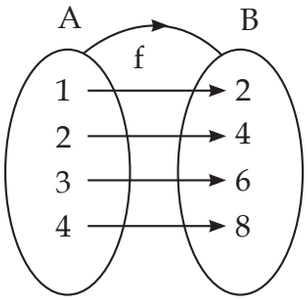
Example 56. If A = {1, 2, 3, 4}, B = {2, 4, 6, 8}, f(1) = 2, f(2) = 4, f(3) = 6 and f(4) = 8 and f : A → B then f^{-1} is

(a) {(2, 1), (4, 2), (6, 3), (8, 4)} (b) {(1, 2), (2, 4), (3, 6), (4, 8)}

(c) {(1, 4), (2, 2), (3, 6), (4, 8)} (d) None of these

Sol. (a) Given, A = {1, 2, 3, 4}, B = {2, 4, 6, 8}

According to the question,



Thus, $f^{-1} = \{(2, 1), (4, 2), (6, 3), (8, 4)\}$

Hence, the correct option is (a).

Example 57. The Inverse function f^{-1} of $f(x) = 2x$ is **(ICAI)**

(a) $\frac{1}{2x}$ (b) $\frac{x}{2}$ (c) $\frac{1}{x}$ (d) None of these

Sol. (b) Given, $f(x) = 2x$

Let $y = f(x) = 2x$

$\Rightarrow y = 2x$

$\Rightarrow x = \frac{y}{2}$

$\Rightarrow f^{-1}(y) = \frac{y}{2}$ (since, $y = f(x)$ thus $x = f^{-1}(y)$)

$\Rightarrow f^{-1}(y) = \frac{y}{2}$

$\Rightarrow f^{-1}(x) = \frac{x}{2}$

Hence, the answer is option (b).

Example 58. If f is an invertible function defined by $f(x) = \frac{3x-4}{6}$, then $f^{-1}(x) = ?$

(a) $2x + 3$ (b) $2x$ (c) $2x + \frac{4}{3}$ (d) None of these

Sol. (c) Given, $f(x) = \frac{3x-4}{6}$

Let, $y = \frac{3x-4}{6}$

$\Rightarrow 6y = 3x - 4$

$\Rightarrow 3x = 6y + 4$

$\Rightarrow x = \frac{6}{3}y + \frac{4}{3}$

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$\Rightarrow x = 2y + \frac{4}{3}$

$\Rightarrow f^{-1}(y) = 2y + \frac{4}{3}$

$\Rightarrow f^{-1}(x) = 2x + \frac{4}{3}$

Hence, the correct option is (c).

COMPOSITE FUNCTION

A composite function is created when one function is substituted into another function. The composite function is formed by applying one function to the output of another function.

E.g.: Let's consider two functions f : A → B and g : B → C. The composite function (gof): A → C is defined as (gof) (x) = g(f(x)). The output of f(x) becomes the input for g(x)

Example 59. Given $f(x) = x^2 + 6$ and $g(x) = 2x - 1$, find (fog) (x).

(a) $x^2 + x - 1$ (b) $4x^2 - 4x + 5$ (c) $x + 1$ (d) None of these

Sol. (b) Given, $f(x) = x^2 + 6$ and $g(x) = 2x - 1$

Now, (fog) (x)

$\Rightarrow f(g(x)) = f(2x - 1)$

$\Rightarrow (2x - 1)^2 + 6$

$\Rightarrow 4x^2 + 1 - 4x + 6$

$\Rightarrow 4x^2 - 4x + 5$

Hence, the correct option is (b).

Example 60. If $f(x) = x^2$ and $g(x) = \sqrt{x}$, then **(June 2019)**

(a) gof(3) = 3 (b) gof(–3) = 9 (c) gof(9) = 3 (d) gof(–9) = 3

Sol. (a) Given: $f(x) = x^2$ and $g(x) = \sqrt{x}$

Thus, $gof(x) = g(x^2) = \sqrt{x^2} = x$

So, gof(3) = 3

Hence, the correct option is option (a) i.e. gof(3) = 3.

Example 61. If $f(x) = \frac{x}{\sqrt{1+x^2}}$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$. Find fog(x).

(a) x (b) $\frac{1}{x}$ (c) $\frac{x}{\sqrt{1-x^2}}$ (d) $x\sqrt{1-x^2}$

Sol. (a) Given, $f(x) = \frac{x}{\sqrt{1+x^2}}$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$

Thus, $fog = f(g(x)) = f\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{1+\left(\frac{x}{\sqrt{1-x^2}}\right)^2}} = \frac{x}{\sqrt{1-x^2}\sqrt{1+\frac{x^2}{1-x^2}}} = \frac{x}{\sqrt{1-x^2}\sqrt{\frac{1-x^2+x^2}{1-x^2}}}$$

$$= \frac{x}{\sqrt{1-x^2}\sqrt{\frac{1}{1-x^2}}} = \frac{x}{\sqrt{1-x^2}\times\frac{1}{\sqrt{1-x^2}}} = x$$

Therefore, the correct option is (a) i.e., x.

Example 62. If $f(x) = x^2 - 1$ and $g(x) = |2x + 3|$, then $fog(3) - gof(-3) =$ **(July 2021)**

(a) 71 (b) 61 (c) 41 (d) 51

Sol. (b) Given, $f(x) = x^2 - 1$ and $g(x) = |2x + 3|$

Thus,

$g(3) = |2(3) + 3| = |6 + 3| = 9$

$f(-3) = (-3)^2 - 1 = 9 - 1 = 8$

Now, $fog(3) - gof(-3)$

$= f(g(3)) - g(f(-3)) = f(9) - g(8)$

$= (9^2 - 1) - |16 + 3| = 80 - 19 = 61$

Hence, the correct answer is option (b) i.e. 61.

Example 63. Let the domain of x be the set {1}. Which of the following functions are equal to 1?

(a) $f(x) = x^2, g(x) = x$ (b) $f(x) = x, g(x) = 1 - x$

(c) $f(x) = x^2 + x + 2, g(x) = (x + 1)^2$ (d) None of these

Sol. (a) Go by options:

Option (a): Given $f(x) = x^2, g(x) = x$

In function f(x, y), x represents the domain.

If $x = 1$, then $f(x) = (1)^2 = 1, g(x) = (1) = 1$

Option (b): Given $f(x) = x, g(x) = 1 - x$

In function f(x, y), x represents the domain.

If $x = 1$, then $f(x) = (1), g(x) = 1 - x = 1 - 1 = 0$

Option (c): Given $f(x) = x^2 + x + 2, g(x) = (x + 1)^2$

In function f(x, y), x represents the domain.

If $x = 1$ then $f(x) = (1)^2 + (1) + 2 = 1 + 1 + 2 = 4, g(x) = (x + 1)^2 = (1 + 1)^2 = 4$

Hence, the correct answer is option (a).

Example 64. If $f(x) = \frac{1}{1-x}$, f(–1) is

(a) 0 (b) $\frac{1}{2}$ (c) 0 (d) None of these

Sol. (b) Given, $f(x)=\frac{1}{1-x}$

Let us consider x = –1, then

$f(-1)=\frac{1}{1-(-1)}=\frac{1}{1+1}=\frac{1}{2}$

Hence, the correct option is (b).

Example 65. If $f(x)=\frac{1}{1-x}$ and $g(x)=\frac{x-1}{x}$, then gof(x) is

(a) x – 1 (b) x (c) $\frac{1}{x}$ (d) None of these

Sol. (b) Given, $f(x)=\frac{1}{1-x}$ and $g(x)=\frac{x-1}{x}$

We know that, gof(x) = g(f(x))

$=g\left(\frac{1}{1-x}\right) \quad \left(\because f(x)=\frac{1}{1-x}\right)$

$=\frac{\frac{1}{1-x}-1}{\frac{1}{1-x}}$

Take L.C.M and solve,

$=\frac{\frac{1-(1-x)}{1-x}}{\frac{1}{1-x}}=\frac{\frac{x}{1-x}}{\frac{1}{1-x}}=x$

Hence, the correct answer is option (b).

Example 66. The range of the function $f(x) = \log_{10}(1 + x)$ for the domain of real values of x when $0 \le x \le 9$ is **(ICAI)**

(a) [0, 1] (b) [0, 1, 2] (c) [0, 2] (d) None of these

Sol. (a) Given, $f(x) = \log_{10}(1 + x)$

Domain of real values of x, i.e. $x \in 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

For lowest value i.e.

$f(0) = \log_{10}(1 + 0)$

$= \log_{10}(1) = 0$

For highest value i.e. $f(9) = \log_{10}(1 + 9)$

$= \log_{10}(10) = 1$ $(\because \log_{10} 10 = 1)$

Alternate solution:

Given, $0 \le x \le 9$

$\Rightarrow 1 + 0 \le 1 + x \le 1 + 9$

$\Rightarrow 1 \le (1 + x) \le 10$

$\Rightarrow \log_{10} 1 \leq \log_{10} (1 + x) \leq \log_{10} 10$

$\Rightarrow 0 \leq \log_{10} (1 + x) \leq 1$

Therefore, the range of $f(x) = \log_{10} (1 + x)$ is $[0, 1]$.

Hence, the answer is option (a).

Example 67. Let A = R – {3} and B = R – {1}. Let f: A → B defined by $f(x) = \frac{x-2}{x-3}$. What is the value of $f^{-1}\left(\frac{1}{2}\right)$? **(July 2021)**

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) 1 (d) –1

Sol. (c) Given, $f(x) = \frac{x-2}{x-3}$

Let $y = f(x) = \frac{x-2}{x-3}$

then, $y = \frac{x-2}{x-3}$

$xy - 3y = x - 2$

$xy - x = 3y - 2$

$x = \frac{3y-2}{y-1}$

$f^{-1}(x) = \frac{3x-2}{x-1}$

Thus, $f^{-1}(x) = \frac{3x-2}{x-1}$

$$f^{-1}\left(\frac{1}{2}\right) = \frac{3\left(\frac{1}{2}\right)-2}{\frac{1}{2}-1} = \frac{\frac{-1}{2}}{\frac{-1}{2}} = 1$$

Hence, the correct answer is option (c) i.e. 1.

Example 68. The h^{-1} when $h(x) = \log_{10} x$ is **(ICAI)**

(a) $\log_{10} x$ (b) 10^x (c) $\log_{10}\left(\frac{1}{x}\right)$ (d) None of these

Sol. (b) Given, $h(x) = \log_{10} x$

Replace h(x) by y

$\Rightarrow y = \log_{10} x$

Taking the inverse of the function,

$\Rightarrow x = e^y$

Replace the original function, so e = 10

Then, $h^{-1}(x) = 10^x$

Hence, the correct answer is option (b).

PRACTICE QUESTIONS (PART C)

1. If A = {x, y, z}, B = {p, q, r, s}, which of the relations on A to B is a function?

(a) {(n, p), (x, q), (y, r), (z, s)} (b) {(s, x), (y, s), (z, s)}

(c) {(y, p), (y, q), (y, r), (z, s)} (d) {(x, p), (y, r), (z, s)}

2. If $f(x)=\frac{3x+1}{2x+5}$ for $x \neq -\frac{5}{2}$, then $f^{-1}(2)$ =

(a) 7 (b) -2 (c) 10 (d) -9

3. {(x, y) | x – y = 3}, where x, y ∈ R is a

(a) not a function (b) a function

(c) one-one mapping (d) None of these

4. If $f(x) = 8x^3$ and $g(x)=x^{\frac{1}{3}}$, then find gof(x)

(a) 8x (b) 2x (c) $2x^3$ (d) None of these

5. The number of elements in range of constant function is:

(a) One (b) Zero (c) Infinite (d) Undetermined

6. If f: R→R is a function, defined by $f(x) = 10x^{-7}$, if $g(x)=f^{-1}(x)$, then the value of g(x) is equal to

(a) $\frac{1}{10x-7}$ (b) $\frac{1}{10x+7}$ (c) $\frac{x+7}{10}$ (d) $\frac{x-7}{10}$

7. The function $g(x) = 2^x$ is

(a) one-many mapping (b) one-one mapping

(c) many-one mapping (d) none of these

8. The range of the function f: N → N; $f(x) = (-1)^{x-1}$ is

(a) {-1,0} (b) {-1,1} (c) {0,1} (d) {1,0,-1}

9. X = {x, y, w, z}; Y = {1, 2, 3, 4}; H = {(x, 1), (y, 2), (y, 3), (z, 4), (x, 4)}

(a) H is a function from x to y (b) H is not a function from x to y

(c) H is a relation from y to x (d) None of these

10. If A = {1, 2, 3, 4}, B = {2, 4, 6, 8}, f(1) = 2,f(2) = 4, f(3) = 6 and f(4) = 8; and f : A → B then f^{-1} is

(a) {(2, 1), (4, 2), (6, 3), (8, 4)} (b) {(1, 2), (2, 4), (3, 6), (4, 8)}

(c) {(1, 4), (2, 2), (3, 6), (4, 8)} (d) None of these

Answer Key

1. (d) **2.** (d) **3.** (c) **4.** (b) **5.** (a) **6.** (c) **7.** (b) **8.** (b) **9.** (b) **10.** (a)

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FULL CHAPTER PRACTICE QUESTIONS

1. If $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$ then (gof) x is

(a) $2x^2 + 6x + 1$ (b) $x^2 - 3$ (c) $4x^2 - 6x + 1$ (d) None of these

2. The no. of subsets of the set {3, 4, 5} is

(a) 4 (b) 8 (c) 16 (d) 32

3. A = {1, 2, 3, 4, ..., 10} a relation on A, R = {(x, y) | x + y = 10, x ∈ A, y ∈ A, x ≥ y}, then domain of R^{-1} is

(a) {1, 2, 3, 4, 5} (b) {0, 3, 5, 7, 9} (c) {1, 2, 4, 5, 6, 7} (d) None of these

4. The numbers of proper subset of the set {3, 4, 5, 6, 7} is

(a) 32 (b) 31 (c) 30 (d) 25

5. A is {1, 2, 3, 4} and B is {1, 4, 9, 16, 25} if a function f is defined from set A to B where $f(x) = x^2$ then the range of f is

(a) {1, 2, 3, 4} (b) {1, 4, 9, 16} (c) {1,4,9,16,25} (d) None of these

6. If A = {1, 2} and B = {3, 4}. Determine the number of relations from A to B

(a) 3 (b) 16 (c) 5 (d) 6

7. Let N be the set of all natural numbers, E be the set of all even natural numbers then the function; F: N → E defined as f(x) = 2x, x ∈ N is

(a) One – one-into (b) Many – one-into

(c) One – one-onto (d) Many –one-onto

8. In a class of 35 students, 24 like to play cricket and 16 like to play football. Also each student likes to play at least one of the two games. How many students like to play both cricket and football?

(a) 5 (b) 11 (c) 19 (d) 8

9. The domain (D) and range (R) of the function $f(x) = 2 - |x + 1|$ is

(a) D = Real numbers, R = (2, ∞) (b) D = Integers, R = (0, 2)

(c) D = Integers, R = (–∞ ,∞) (d) D = Real numbers, R = (–∞, 2)

10. If R is the set of all real numbers, then the function f: R → R defined by f(x) = 2x

(a) One-one-onto (b) One-one-into

(c) Many-one-into (d) Many-one-onto

11. The inverse function f^{-1} of f(x) = 100x is **(June 2016)**

(a) $\frac{x}{100}$ (b) $\frac{1}{100x}$ (c) $\frac{1}{x}$ (d) None of these

12. The number of subsets of the formed by the word 'ALLAHABAD' is **(June 2016)**

(a) 128 (b) 16 (c) 32 (d) 64

13. The range of function f defined by $f(x)=\frac{x}{x^2+1}$ is

(a) $\left\{x:\frac{-1}{2}<x<\frac{1}{2}\right\}$ (b) $\left\{x:\frac{-1}{2}\le x<\frac{1}{2}\right\}$

(c) $\left\{x:\frac{-1}{2}\le x\le\frac{1}{2}\right\}$ (d) $\left\{x:x>\frac{1}{2}\text{ or }x<\frac{-1}{2}\right\}$

14. In a group of students 80 can speaks Hindi, 60 can speak English and 40 can speak English and Hindi both, then number of students is

(a) 100 (b) 140 (c) 180 (d) 60

15. The number of non-empty subsets of the set {1, 2, 3, 4} is

(a) 15 (b) 14 (c) 16 (d) 17

16. If R be a relation on set A = {a, b, c} such that R = {(a, b), (a, c), (b, c)} then the range of the relation is

(a) {b, c} (b) {c} (c) {a, b, c} (d) Not defined

17. The domain of the function f = {(1, 3), (3, 5), (2, 6)} is

(a) {1, 2, 3} (b) {1, 2} (c) {3, 5, 6} (d) {5, 6}

18. If two finite sets A and B are such that n(A) = 2 and n(B) = 3, then the total number of relations from A to B is

(a) 8 (b) 16 (c) 32 (d) 64

19. If $f(x)=\frac{x-1}{x}$ and $g(x)=\frac{1}{1-x}$ then (fog)(x) is equal to **(June 2017)**

(a) x – 1 (b) x (c) 1 – x (d) –x

20. If A = {1, 2, 3}, then which of the following given set is not an equivalence relation on A?

(a) {(1, 1), (2, 2), (3, 3)} (b) {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

(c) {(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)} (d) {(1, 2), (2, 3)}

21. If $f(x)=\frac{-x}{x+1}$ then (fof)x is

(a) x^2 (b) x (c) x^3 (d) None of these

22. If N be the set of all natural numbers and E be the set of all even natural numbers then the function f : N → E, such that f(x) = 2x for all x ∈ N is

(a) One- one onto (b) One-one-into

(c) Many–one onto (d) None of these

23. If f(x) = x + 7 and g(x) = x – 7, then (fog)(7) =

(a) –7 (b) 7 (c) 0 (d) 14

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24. In a class of 100 students, 55 students have passed in Business Mathematics and 67 students have passed in Accountancy, then the number of students who have passed in Accountancy only is

(a) 25 (b) 35 (c) 15 (d) 45

25. In a town 20000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C if 2% families buy all the three newspaper, then the number of families which buy A only is

(a) 3300 (b) 6300 (c) 5600 (d) 600

Answer Key

1. (a)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (d)	**10.** (b)
11. (a)	**12.** (c)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (d)
21. (b)	**22.** (a)	**23.** (b)	**24.** (d)	**25.** (a)					

PREVIOUS YEAR QUESTION (PYQ'S)

1. The set of cubes of natural numbers is **(Jan 2021)**

(a) Null set (b) A finite set (c) An infinite set (d) Singleton set

2. In the set of all straight lines on a plane which of the following is not 'TRUE'? **(Jan. 2021)**

(a) Parallel to an equivalence relation

(b) Perpendicular to is a symmetric relation

(c) Perpendicular to is an equivalence relation

(d) Parallel to a reflexive relation

3. Let F: R → R be defined by

$f(x)=\{2x \text{ for } x > 3\} = \{x^2 \text{ for } 1 < x \leq 3 = \{3x \text{ for } x \leq 1\}$

The value of f (-1) + f(2) + f(4) is **(Jan 2021)**

(a) 9 (b) 14 (c) 5 (d) 6

4. Let U be the universal set, A and B be the subsets of U. If n(U) = 650, n(A) = 310, $n(A \cap B) = 95$ and n(B) = 190, then $n(\underline{A} \cap \underline{B})$ is equal to ($\underline{A}$ and $\underline{B}$ are the complement of A and B, respectively) **(July 2021)**

(a) 400 (b) 200 (c) 300 (d) 245

5. The range of the function f defined by $f(x)=\sqrt{(16-x^2)}$ is **(July 2021)**

(a) [-4,0] (b) [-4,4] (c) [0,4] (d) (-4,4)

6. Let A = R – {3} and B = R – {1}. Let f : A → B defined by $f(x) = \frac{x-2}{x-3}$. What is the value of $f^{-1}\left(\frac{1}{2}\right)$? **(July 2021)**

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) 1 (d) -1

7. If $f(x) = x^2-1$ and $g(x)=|2x-3|$, then $fog(3)-gof(-3)=$ **(July 2021)**

(a) 71 (b) 61 (c) 41 (d) 51

8. Out of a group of 20 teachers in school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry. How many teach Chemistry and Physics? How many teach only Physics?

(a) 2, 3 (b) 3, 2 (c) 4, 6 (d) 6, 4

9. If a is related to b if and only if the difference in a and b is an even integer. This relation is **(Dec 2021)**

(a) symmetric, reflexive and not transitive

(b) symmetric, transitive and not reflexive

(c) transitive, reflexive and not symmetric

(d) Equivalence relation

10. If $u(x) = \frac{1}{1-x}$, then $u^{-1}(x)$ is **(Dec 2021)**

(a) $\frac{1}{1-x}$ (b) 1-x (c) $1-\frac{1}{x}$ (d) $\frac{1}{x}-1$

11. If $f(p) = \frac{1}{1-p}$, then f^{-1} is **(June 2022)**

(a) 1- p (b) $\frac{p-1}{p}$, (c) $\frac{p}{p-1}$ (d) $\frac{1}{p}$

12. Let R={(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. Then the relation is **(June 2022)**

(a) an equivalence relation (b) reflexive and transitive only

(c) reflexive only (d) reflexive and symmetric only

13. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are **(June 2022)**

(a) 6, 3 (b) 7, 6 (c) 5, 1 (d) 8, 7

14. If A = {1, 2, 3, 4, 5, 7, 8, 9} and B = {2, 4, 6, 7, 9} then how many proper subset of $A \cap B$ can be created? **(June 2022)**

(a) 16 (b) 15 (c) 32 (d) 31

15. Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}, then R is **(Dec 2022)**

(a) Symmetric and transitive (b) Reflexive but not transitive

(c) Reflexive but not symmetric (d) Neither symmetric nor transitive

16. The number of subjects of the set {0, 1, 2, 3} is **(Dec 2022)**

(a) 2 (b) 4 (c) 8 (d) 16

17. Given the relation R = {(1, 2), (2, 3)} on the set A = {1, 2, 3}, the minimum number of ordered pairs which when added to R make it equivalence relation is **(June 2023)**

(a) 5 (b) 7 (c) 6 (d) 8

18. A survey shows that 74% of Canadians like grapes whereas 68% like bananas. What percentage of Canadians like both grapes and bananas, if everybody likes either of two? **(June 2023)**

(a) 32% (b) 26% (c) 6% (d) 42%

19. If R be a relation defined, on the set of Natural numbers as 'xRy $\Leftrightarrow$ (x – y) is divisible by 5" $\forall$ x, y $\in$ N, then the relation is **(June 2023)**

(a) Equivalence (b) Anti-symmetric

(c) Symmetric but not transitive (d) Symmetric but not reflexive

20. If A = {a, b, c), B = (b, c, d) and C = (a, d, c), then (A – B) × (B ∩ C) is equal to **(June 2023)**

(a) {(a, d), (c, d)} (b) {(a, c), (a, d)}

(c) {(c, a), (d, a)} (d) {(a, c), (a, d), (b, d)}

21. If f(x)=N→R is a function defined as f(x)=4x + 3, $\forall$ x $\in$ N, then f^{-1} (x) is **(June 2023)**

(a) $4+\frac{x+3}{4}$ (b) $\frac{x+3}{4}$ (c) $\frac{x-3}{4}$ (d) $\frac{3x+4}{4}$

22. If B = {1, 2, 3, 4, 5} then the number of proper subsets of B is **(Dec 2023)**

(a) 120 (b) 30 (c) 31 (d) 32

23. In a survey of 100 boys, it was found that 50 used white shirts, 40 red shirts and 30 blue shirts. 20 were habituated in using both white & red shirts. 15 were using both red & blue and 10 were using blue & white shirts. Find the number of boys who are using all colours. **(Dec 2023)**

(a) 20 (b) 25 (c) 30 (d) 35

24. If A = {2, 4} and B = {1, 2, 3} then (A ∪ B) × (A ∩ B) is equal to **(Dec 2023)**

(a) {(1, 2), (2, 2), (3, 2)} (b) {(1, 2), (2, 2), (2, 3), (2, 4)}

(c) {(2, 1), (2, 2), (2, 4)} (d) {(1, 2), (2, 2), (3, 2), (4, 2)}

25. If A = {1, 2}, B = {3, 4} and C = {5, 6} then the value of A × (B ∪ C)= **(Dec 2023)**

(a) {(1, 2), (3, 4), (5, 6)}

(b) {(1, 3), (2, 3), (1, 4), (2, 4), (2, 5), (1, 5), (1, 6), (2, 6)}

(c) {(1, 3), (2, 3), (1, 4), (2, 4), (2, 5), (1, 5)}

(d) {(3, 1), (2, 3), (4, 1), (2, 4), (2, 5), (1, 5), (1, 6), (2, 6)}

26. Consider the following relations on A = {1, 2, 3}, R = {(1, 1), (1, 2), (1, 3), (3, 3)}, S = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)}, T = {(1, 1), (1, 2), (2, 2), (2, 3)}, and Ø = empty set. Which of these forms an equivalence relation? **(June 2024)**

(a) R (b) S (c) T (d) Ø

27. Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} then R is **(Sept 2024)**

(a) Reflexive but not symmetric (b) Reflexive but not transitive
(c) Symmetric and Transitive (d) Neither symmetric nor transitive

28. A town has a total population of 50,000. Out of it 28,000 read the newspaper X and 23,000 read Y while 4,000 read both the papers. The number of persons not reading X and Y both is - **(Sept 2024)**

(a) 2,000 (b) 3,000 (c) 2,500 (d) none of these

29. If A = {a, b, c, d, e} then the number of proper subsets is **(Jan 2025)**

(a) 32 (b) 31 (c) 30 (d) 29

30. A = {a, b, p}, B = {2, 3} and C = {p, q, r, s} then n[(A ∪ C) × B] is; **(Jan 2025)**

(a) 20 (b) 8 (c) 12 (d) 16

31. Which of the following relations is transitive but not reflexive for the set S = {3, 4, 6}? **(May 2025)**

(a) R = {(3, 4), (4, 6), (3, 6)} (b) R = {(1, 2), (1, 3), (1, 4)}
(c) R = {(3, 3), (4, 4), (6, 6)} (d) R = {(3, 4), (4, 3)}

32. If A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6} then value of A – (B ∪ C) is **(May 2025)**

(a) {1, 2, 3} (b) {2, 3, 4, 5} (c) {1} (d) {0}

33. The range of the function f(x) = 3x – 2 is **(May 2025)**

(a) (–∞, ∞) (b) R – {3} (c) (–∞, 0) (d) (0, – ∞)

34. Let S = {a, b, c, d, e}. The number of non-empty proper subsets of S is _____. **(Sept 2025)**

(a) 30 (b) 31 (c) 32 (d) 28

35. For a relation R, aRb represents that a is related to b. If for all a, b, c, aRb and bRc gives that aRc, then the relation is _____. **(Sept 2025)**

(a) Reflexive (b) Symmetric (c) Transitive (d) Asymmetric

36. If $f(y) = \frac{1}{1+y}$ and $g(y) = \frac{y+1}{y}$, then fog(y) = _____. **(Sept 2025)**

(a) $\frac{y}{2y+1}$ (b) $\frac{2y}{1+y}$ (c) $\frac{2y+1}{y}$ (d) $\frac{y+1}{y}$

37. The inverse of the function $f(x) = \frac{2+3x}{x+5}$, by taking f(x) as y is _____. **(Sept 2025)**

(a) $\frac{2+5y}{y+3}$ (b) $\frac{2-5y}{y+3}$ (c) $\frac{2-5y}{y-3}$ (d) $\frac{2+5y}{y-3}$

Answer Key

1. (c)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (c)
11. (b)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (b)
21. (c)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (b)	**30.** (c)
31. (a)	**32.** (c)	**33.** (a)	**34.** (a)	**35.** (c)	**36.** (a)	**37.** (c)			

SUMMARY

- A set is defined to be a collection of well-defined distinct objects. This collection may be listed or described.
- Each object is called an element of the set. We usually denote sets by capital letters and their elements by small letters.
- Representation of Sets: Roster form & Set Builder Form
- **Subset -** Subset of a set is a collection of a few elements of a set.
- No. of sets for n number of elements = 2^n subsets.
- **Power Set -** The collection of all possible subsets of a given set A is called the power set of A, to be denoted by P(A).
- **Proper Subset and Super Set -** A set containing n elements has $2^n - 1$ proper subsets.
- **Equivalent Set -** Two finite sets A & B are said to be equivalent if n(A) = n(B).
- **Null Set -** A set containing zero element is called null set.
- **Singleton Set -** A set containing one element is called a Singleton set.
- **Equal Set -** Two sets A & B are said to be equal, written as A = B if every element of A is in B and every element of B is in A.
- **Universal Set -** The set which contains all the elements under consideration in a particular problem is called the universal set denoted by S. Suppose that P is a subset of S.
- **Venn Diagram -** Venn diagrams are the diagrams that are used to represent the sets, relation between the sets and operation performed on them, in a pictorial way.
- **Union of Sets -** The union of two sets X and Y is equal to the set of elements that are present in set X, in set Y, or in both the sets X and Y. This operation can be represented as $X \cup Y = \{a : a \in X \text{ or } a \in Y\}$.
- **Intersection of sets -** Intersection of two sets A and B is the set containing elements that are common to both A and B. This can be expressed as: $A \cap B = \{x : x \in A \text{ and } x \in B\}$.
- **Complement of a set -** If U is a universal set and A be any subset of U then the complement of A is the set of all members of the universal set U which are not the elements of A.
- **The difference of two sets -** A and B is defined as the lists of all the elements that are in set A but that are not present in set B. The set notation used to represent the difference between the two sets A and B is A – B or A, B
- $A - B = \{x \mid x \in A \text{ and } x \notin B\}$
- If A and B are two sets then, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

- If A and B are disjoint sets, then $n(A \cup B) = n(A) + n(B)$ as $n(A \cap B) = 0$
- For three sets P, Q and R
- $n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - R(Q \cap R) - n(P \cap R) + n(P \cap Q \cap R)$
- When P, Q and R are disjoint sets $n(P \cup Q \cup R) = n(P) + n(Q) + n(R)$

RELATIONS

- **Ordered Pair:** Two elements a and b, listed in a specific order, form an ordered pair, denoted by (a, b).
- **Cartesian Product of sets:** If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B, is called the Cartesian product of A and B, to be denoted by $A \times B$.
- If $A = \Phi$ or $B = \Phi$, we define $A \times B = \Phi$
- **Relation:** For x and y are two sets of ordered pairs. And set x has relation with set y, then the values of set x are called domain whereas the values of set y are called range.
- **Domain & Range of a relation:** If R is a relation from A to B, then the set of all first coordinates of elements of R is called the domain of R, while the set of all second coordinates of elements of R is called the range of R.
- So, $Dom(R) = \{a : (a, b) \in R\}$ & $Range(R) = \{b : (a, b) \in R\}$

FUNCTION

- Any relation from X to Y in which no two different ordered pairs have the same first element is called a function.
- Let A and B be two non-empty sets. Then, a rule or a correspondence f which associates to each element x of A, a unique element, denoted by f(x) of B, is called a function or mapping from A to B and we write $f : A \rightarrow B$
- The element f(x) of B is called the image of x, while x is called the pre-image of f(x).
- Let $f : A \rightarrow B$, then A is called the domain of f, while B is called the co-domain of f.
- The set $f(A) = \{f(x): x \in A\}$ is called the range of f.

RELATION AND FUNCTION

- **One-one Function:** Let $f : A \rightarrow B$, If different elements in A have different images in B, then f is said to be a one-one or an injective function or mapping.
- **Onto or Surjective Functions:** Let $f : A \rightarrow B$, If every element in B has at least one preimage in A, then f is said to be an onto function. If f is onto if and only if Range of f = B.
- **Bijection Function:** A one-one and onto function is said to be bijective. A bijective function is also known as a one-to-one correspondence.
- **Inverse Function** A function is invertible if and only if f is one-one onto.
- Let f be a one-one onto function from A to B. Then there exists an element x in A such that $f(x) = y$ then $f^{-1}(y) = x$.

- **Identity function:** For $x \in A$ if $f(x) = x$, then f is identity function.
- **Constant function:** For $x \in A$ if $f(x) = c$, then f is constant function.
- **Equal Functions:** Two functions f and g are said to be equal, written as $f = g$ if they have the same domain and they satisfy the condition - $f(x) = g(x)$, for all x.
- Let $S = \{a, b, c, \ldots.\}$ be any set then the relation R is a subset of the product set $S \times S$
  (i) If R contains all ordered pairs of the form (a, a) in $S \times S$, then R is called reflexive. In a reflexive relation 'a' is related to itself.
  (ii) If $(a, b) \in R \Rightarrow (b, a) \in R$ for every $a, b \in S$, then R is called symmetric.
  (iii) If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \Rightarrow R$ for every $a, b, c, \in S$, then R is called transitive.

A relation which is reflexive, symmetric and transitive is called an equivalence relation or simply an equivalence.

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Chapter 7 - Unit-2: Limits and Continuity

LIMIT

In calculus, the concept of a limit is crucial for understanding how a function behaves as the input variable (commonly denoted as 'x') approaches a specific value ('a').

Let's explore the idea of limits with a new example.

Consider the function: g(x) = 2x + 3 and we want to determine its limit as x approaches 1.

The notation for the limit of a function is represented as:

$$\lim_{x \to a} g(x) = L$$

In the context of the function g(x) = 2x + 3, and as x approaches 1, we are interested in:

$$\lim_{x \to 1} (2x + 3)$$

This signifies our interest in understanding how the function 2x + 3 behaves as x gets closer and closer to 1 while going from 0 to 1 and while going from 2 to 1.

In this limit, when we approach g(x) from left hand side of 1:

When x = 0.97	$\lim_{x \to 0.97} (2x+3) = 2(0.97) + 3 = 1.94 + 3 = 4.94$
When x = 0.98	$\lim_{x \to 0.98} (2x+3) = 2(0.98) + 3 = 1.96 + 3 = 4.96$
When x = 0.99	$\lim_{x \to 0.99} (2x+3) = 2(0.99) + 3 = 1.98 + 3 = 4.98$
When x = 0.9999	$\lim_{x \to 0.9999} (2x+3) = 2(0.9999) + 3 = 1.9998 + 3 = 4.9998$
When x =1.0	$\lim_{x \to 1.0} (2x+3) = 2(1.0) + 3 = 2.0 + 3 = 5$

Now, when we approach g(x) from right hand side of 1:

When x = 1.2	$\lim_{x \to 1.2} (2x+3) = 2(1.2) + 3 = 2.4 + 3 = 5.4$
When x = 1.1	$\lim_{x \to 1.1} (2x+3) = 2(1.1) + 3 = 2.2 + 3 = 5.2$
When x = 1.01	$\lim_{x \to 1.01} (2x+3) = 2(1.01) + 3 = 2.02 + 3 = 5.02$
When x = 1.001	$\lim_{x \to 1.001} (2x+3) = 2(1.001) + 3 = 2.002 + 3 = 5.002$
When x = 1.0	$\lim_{x \to 1.0} (2x+3) = 2(1.0) + 3 = 2.0 + 3 = 5.0$

From the above table, we can conclude that:

When x gets closer and closer to 1, regardless of whether x approaches from left (x < 1) or x approaches from right (x > 1), the corresponding value of the function is approaching 5.

Thus, we conclude that the limit of the function g(x) = 2x + 3 as x approaches 1 is equal to 5.

Therefore, we can write as:

$$\lim_{x\to 1^-}(2x+3) = \lim_{x\to 1^+}(2x+3) = 5$$

$$\Rightarrow \lim_{x\to 1^-}(2x+3) = \lim_{x\to 1^+}(2x+3) = \lim_{x\to 1}(2x+3) = 5$$

We can conclude that $\lim_{x\to a} f(x)$ is said to exist when:

- Both left-hand limit and right-hand limit exist and are equal.
- $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a} f(x)$

Let's consider the function, f(x) = {2x + 1, if x ≠ 2}

= {4, if x = 2

Here, when x approaches from left-hand limit of 2 and right-hand limit of 2 then

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 2(2) + 1 = 5$$

But $\lim_{x\to 2} f(x) = 2$

In this case, both left-hand and right-hand limit exist and are equal but

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \neq \lim_{x\to 2} f(x)$$

Thus, we can conclude $\lim_{x\to a} f(x)$ does not exist

PROPERTIES OF LIMITS

Let $\lim_{x\to a} f(x) = I$ and $\lim_{x\to a} g(x) = m$ where l and m are finite quantities

1. $\lim_{x\to a} f\{f(x)+g(x)\} = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = l + m$ that is limit of the sum of the difference of two functions is equal to the sum of their limits
2. $\lim_{x\to a}\{f(x)-g(x)\} = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = I - m$ that is limit of the difference of two functions is equal to difference of their limits.
3. $\lim_{x\to a}\{f(x)g(x)\} = \lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x) = lm$ that is limit of the product of two functions is equal to the product of their limits
4. $\lim_{x\to a}\left\{\frac{f(x)}{g(x)}\right\} = \frac{\left\{\lim_{x\to a} f(x)\right\}}{\left\{\lim_{x\to a} g(x)\right\}} = l/m$ if m ≠ 0 that is limit of the quotient of two functions is equal to the quotient of their limits.
5. $\lim_{x\to a} c = c$ where is constant that is limit of a constant is the constant
6. $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$

7. $\lim_{x\to a} F\{f(x)\} = F\left\{\lim_{x\to a} F(x)\right\} = F(a)$

8. $\lim_{x\to 0^+} \frac{1}{x} = \lim_{h\to 0} \frac{1}{h} \to +\infty \ (h > 0)$

$\lim_{x\to 0^-} \frac{1}{x} = \lim_{h\to 0} \frac{1}{-h} \to -\infty \ (h > 0)$

∞ is a very-very large number called infinity

Thus $\lim_{x\to 0^+} \frac{1}{x}$ does not exist.

Example 1. Evaluate $\lim_{x\to 2} 9$

(a) 2 (b) 9 (c) 18 (d) Cannot be determined

Sol. (b) We know

$\lim_{x\to a} b = b$

Thus, $\lim_{x\to 2} 9 = 9$

Hence, the correct option is (b).

Example 2. Evaluate $\lim_{x\to -2} (3x + 10)$

(a) 1 (b) –2 (c) 4 (d) 10

Sol. (c) We know that,

$\lim_{x\to a} (f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

Thus, $\lim_{x\to -2} (3x + 10) = \lim_{x\to -2} 3x + \lim_{x\to -2} 10$

$= 3(-2) + 10 = -6 + 10 = 4$

Therefore, $\lim_{x\to -2} (3x + 10) = 4$

Hence, the correct option is (c).

Example 3. Evaluate $\lim_{x\to 1} (3x^2 - 2x)$.

(a) 1 (b) 2 (c) 3 (d) 4

Sol. (a) We know that,

$\lim_{x\to a} (f(x) - g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

Thus, $\lim_{x\to 1} (3x^2 - 2x) = \lim_{x\to 1} 3x^2 - \lim_{x\to 1} 2x$

$= 3(1)^2 - 2(1) = 3 - 2 = 1$

Therefore, $\lim_{x\to 1} (3x^2 - 2x) = 1$

Hence, the correct option is (a).

Example 4. Evaluate $\lim_{x\to 2} 5x^2$.

(a) 2 (b) 5 (c) 10 (d) 20

Sol. (d) $\lim_{x\to 2} 5x^2$

$=5\,(2)^2 = 5 \times 4 = 20$

Therefore, $\lim_{x\to 2} 5x^2 = 20$

Hence, the correct option is (d).

Example 5. Evaluate $\lim_{x\to 3} \sqrt[4]{x^2+7}$.

(a) 2 (b) 3 (c) 4 (d) None of these

Sol. (a) $\lim_{x\to 3} \sqrt[4]{x^2+7}$

$= \sqrt[4]{(3)^2+7} = \sqrt[4]{16} = 2$

Therefore, $\lim_{x\to 3} \sqrt[4]{x^2+7} = 2$

Hence, the correct option is (a).

Example 6. Evaluate $\lim_{x\to 1} \frac{x^2-1}{x-1}$.

(a) 3 (b) 2 (c) 4 (d) None of these

Sol. (b) We know that,

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1}$$

$$= \lim_{x\to 1} (x+1) = 1+1 = 2$$

Therefore, $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$

Hence, the correct option is (b).

Example 7. Evaluate $\lim_{x\to 5} \frac{x^2-3x-10}{x^2-6x+5}$.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{7}{4}$ (d) Cannot be determined

Sol. (c) Here, $\lim_{x\to 5} \frac{x^2-3x-10}{x^2-6x+5} = \frac{25-15-10}{25-30+5} = \frac{0}{0}$ which is indeterminate form.

Now, on factorizing the numerator and denominator, we get

$$\lim_{x\to 5}\frac{x^2-3x-10}{x^2-6x+5}$$

$$=\lim_{x\to 5}\frac{x^2-5x+2x-10}{x^2-5x-x+5}$$

$$=\lim_{x\to 5}\frac{x(x-5)+2(x-5)}{x(x-5)-1(x-5)}$$

$$=\lim_{x\to 5}\frac{(x+2)(x-5)}{(x-1)(x-5)}$$

$$=\lim_{x\to 5}\frac{(x+2)}{(x-1)}=\frac{7}{4}$$

Hence, the correct option is (c).

Example 8. Evaluate $\lim_{x\to 1}\{(x^2+4)(3x+2)\}$.

(a) 5 (b) 25 (c) 10 (d) Does not exist

Sol. (b) $\lim_{x\to 1}\{(x^2+4)(3x+2)\}$

$$=\lim_{x\to 1}\{(x^2+4)\}\lim_{x\to 1}\{(3x+2)\}$$

$= (1 + 4)\ (3 + 2) = 5(5) = 25$

Therefore, $\lim_{x\to 1}\{(x^2+4)(3x+2)\}=25.$

Hence, the correct option is (b).

Example 9. Evaluate $\lim_{x\to 4}\frac{1}{x-4}$.

(a) 4 (b) 2 (c) 0 (d) Does not exist

Sol. (d) We know,

$\lim_{x\to a}\frac{1}{x-a}=\frac{1}{0}$, which does not exist

Therefore, $\lim_{x\to 4}\frac{1}{x-4}$ does not exist.

Hence, the correct option is (d).

Example 10. Evaluate $\lim_{x\to 2}\sqrt{\frac{x^2+7x-6}{x+1}}$.

(a) 2 (b) 4 (c) 12 (d) Does not exist

Sol. (a) $\lim_{x\to 2}\sqrt{\frac{x^2+7x-6}{x+1}}$

$$= \sqrt{\frac{\lim\limits_{x\to 2}(x^2+7x-6)}{\lim\limits_{x\to 2}(x+1)}} = \sqrt{\frac{(2^2+7(2)-6)}{(2+1)}}$$

$$= \sqrt{\frac{(2^2+7(2)-6)}{(2+1)}} = \sqrt{\frac{12}{4}} = \sqrt{4} = 2$$

Therefore, $\lim\limits_{x\to 2}\sqrt{\frac{x^2+7x-6}{x+1}} = 2$.

Hence, the correct option is (a).

SOME IMPORTANT LIMITS

We now state some important limits

1. $\lim\limits_{x\to 0}\frac{(e^x-1)}{x} = 1$
2. $\lim\limits_{x\to 0}\frac{a^x-1}{x} = \log_e a \ (a>0)$
3. $\lim\limits_{x\to 0}\frac{\log(1+x)}{x} = 1$
4. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$
5. $\lim\limits_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$
6. $\lim\limits_{x\to 0}\frac{(1+x)^n-1}{x} = n$

LIMITS AT INFINITY

Consider the function, $f(x)=\frac{1}{x}$.

Now,

At x = 1, $f(x)=\frac{1}{1}=1$

At x = 10, $f(x)=\frac{1}{10}=0.1$

At x = 100, $f(x)=\frac{1}{100}=0.01$

At x = 1000, $f(x)=\frac{1}{1000}=0.001$

Thus, when x gets larger and larger, the values of f(x) approaches to 0.

It can be represented as $\lim\limits_{x\to\infty}\frac{1}{x}=0$

Similarly, as x gets smaller and smaller, the values of f(x) approaches to 0.

It can be represented as $\lim_{x\to-\infty}\frac{1}{x}=0$

In general,

$\lim_{x\to\infty}\frac{1}{x^p}=0$, where p > 0.

Example 11. Evaluate $\lim_{x\to2}\frac{\sqrt{x+2}-2}{x-2}$.

(a) $\frac{1}{5}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) Does not exist

Sol. (c) Here, $\lim_{x\to2}\frac{\sqrt{x+2}-2}{x-2}=\frac{\sqrt{2+2}-2}{2-2}=\frac{0}{0}$ which is an indeterminate form.

Now, on factorizing the numerator, we get

$$\lim_{x\to2}\frac{\sqrt{x+2}-2}{x-2}=\lim_{x\to2}\frac{\sqrt{x+2}-2}{x-2}\times\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}$$

$$=\lim_{x\to2}\frac{(\sqrt{x+2})^2-2^2}{(x-2)(\sqrt{x+2}+2)}=\lim_{x\to2}\frac{x+2-4}{(x-2)(\sqrt{x+2}+2)}=\lim_{x\to2}\frac{x-2}{(x-2)(\sqrt{x+2}+2)}$$

$$=\lim_{x\to2}\frac{1}{(\sqrt{x+2}+2)}=\frac{1}{(\sqrt{2+2}+2)}=\frac{1}{(2+2)}=\frac{1}{4}$$

Therefore, $\lim_{x\to2}\frac{\sqrt{x+2}-2}{x-2}=\frac{1}{4}$

Hence, the correct option is (c).

Example 12. Evaluate $\lim_{x\to0}\frac{e^{3x}-1}{x}$.

(a) 1 (b) 3 (c) 0 (d) Does not exist

Sol. (b) We know,

$$\lim_{x\to0}\frac{e^x-1}{x}=1 \text{ or } \lim_{x\to0}\frac{e^{ax}-1}{ax}=1$$

$$\Rightarrow\lim_{x\to0}\frac{e^{3x}-1}{x}=\lim_{x\to0}\frac{e^{3x}-1}{3x}\times3=(1)(3)=3$$

Hence, the correct option is (b).

Example 13. Find $\lim_{x\to0}\frac{3x+|x|}{7x-5|x|}$.

(a) 2 (b) $\frac{1}{6}$ (c) 2 or $\frac{1}{6}$ (d) Does not exist

Sol. (d) To evaluate: $\lim_{x\to 0}\frac{3x+|x|}{7x-5|x|}$

LHL: $\lim_{x\to 0^-}\frac{3x+|x|}{7x-5|x|}=\lim_{x\to 0}\frac{3x-x}{7x+5x}=\frac{2}{12}=\frac{1}{6}$

RHL: $\lim_{x\to 0^+}\frac{3x+|x|}{7x-5|x|}=\lim_{x\to 0}\frac{3x+x}{7x-5x}=\frac{4}{2}=2$

Thus, LHL ≠ RHL.

Therefore, the limit of the given function does not exist.

Example 14. Find $\lim_{x\to 0}\frac{e^x-e^{-x}}{x}$.

(a) 0 (b) 1 (c) 2 (d) Does not exist

Sol. (c) To evaluate: $\lim_{x\to 0}\frac{e^x-e^{-x}}{x}=\lim_{x\to 0}\frac{(e^x-1)-(e^{-x}-1)}{x}$

$=\lim_{x\to 0}\frac{(e^x-1)}{x}-\lim_{x\to 0}\frac{(e^{-x}-1)}{x}=\lim_{x\to 0}\frac{(e^x-1)}{x}+\lim_{x\to 0}\frac{(e^{-x}-1)}{-x}=1+1=2$

Therefore, $\lim_{x\to 0}\frac{e^x-e^{-x}}{x}=2$.

Example 15. Find $\lim_{x\to\infty}\frac{2x^2+5x+1}{3x^3-2x+4}$.

(a) 2 (b) 1 (c) 0 (d) Does not exist

Sol. (c) $\lim_{x\to\infty}\frac{2x^2+5x+1}{3x^3-2x+4}=\frac{\lim_{x\to\infty}(2x^2+5x+1)}{\lim_{x\to\infty}(3x^3-2x+4)}=\frac{\infty}{\infty}$, which is the indeterminate form.

Thus, divide both numerator and denominator by the highest power of x that occurs in the denominator. In given function, highest power of x is x^3.

Thus, dividing numerator and denominator by x^3, we get

$$\lim_{x\to\infty}\frac{\frac{2}{x}+\frac{5}{x^2}+\frac{1}{x^3}}{3-\frac{2}{x^2}+\frac{4}{x^3}}=\frac{0+0+0}{3-0+0}=0$$

Therefore, $\lim_{x\to\infty}\frac{2x^2+5x+1}{3x^3-2x+4}=0$

Example 16. Find $\lim_{x\to\infty}\frac{2x^3+5x^2+x+1}{3x^2+4x+5}$.

(a) ∞ (b) 1 (c) 0 (d) Does not exist

Sol. (a) $\lim\limits_{x\to\infty}\dfrac{2x^3+5x^2+x+1}{3x^2+4x+5}=\dfrac{\lim\limits_{x\to\infty}(2x^3+5x^2+x+1)}{\lim\limits_{x\to\infty}(3x^2+4x+5)}=\dfrac{\infty}{\infty}$, which is the indeterminate form.

Thus, divide both numerator and denominator by the highest power of x that occurs in the denominator. In given function, highest power of x is x^2. Thus, dividing numerator and denominator by x^2, we get

$$\lim_{x\to\infty}\frac{2x+5+\frac{1}{x}+\frac{1}{x^2}}{3+\frac{2}{x}+\frac{5}{x^2}}=\frac{2(\infty)+5+0+0}{3+0+0}=\infty$$

Therefore, $\lim\limits_{x\to\infty}\dfrac{2x^3+5x^2+x+1}{3x^2+4x+5}=\infty$.

PRACTICE QUESTIONS (PART A)

1. Find $\lim\limits_{x\to3}7x$.

(a) 7 (b) 21 (c) 15 (d) Does not exist

2. Find $\lim\limits_{x\to1}x^3-5x^2-3x+2$.

(a) –5 (b) 2 (c) 11 (d) Cannot be determined

3. Evaluate $\lim\limits_{x\to2}\dfrac{5x+2}{x-1}$.

(a) 8 (b) 7 (c) 12 (d) Does not exist

4. Evaluate $\lim\limits_{x\to4}\dfrac{x^2-16}{x-4}$.

(a) 8 (b) 0 (c) 4 (d) Does not exist

5. Evaluate $\lim\limits_{x\to2}\dfrac{x^2+2x+1}{\sqrt{x^2+2}}$

(a) $\dfrac{7}{6}$ (b) $\dfrac{7}{\sqrt{6}}$ (c) 1 (d) Does not exist

6. Evaluate $\lim\limits_{x\to3}\sqrt[5]{x^2+7x+2}$.

(a) 5 (b) 8 (c) 10 (d) 2

7. Evaluate $\lim\limits_{x\to4}\dfrac{x^2-9x+20}{x^2-3x-4}$.

(a) $\dfrac{1}{5}$ (b) $\dfrac{1}{8}$ (c) $-\dfrac{1}{5}$ (d) Does not exist

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8. Find $\lim\limits_{x\to4}\dfrac{2x^3-3x^2+1}{9x^2+8x+7}$

(a) 0 (b) 1 (c) 2 (d) Does not exist

9. Find $\lim\limits_{x\to0}\dfrac{x}{|x|}$.

(a) 1 (b) –2 (c) 0 (d) Does not exist

10. Find $\lim\limits_{x\to2}\dfrac{x^2-4}{4x}$.

(a) 0 (b) 1 (c) 2 (d) Does not exist

11. Find $\lim\limits_{x\to0}\dfrac{e^{6x}-1}{x}$.

(a) 0 (b) 1 (c) 6 (d) Does not exist

12. Find $\lim\limits_{x\to\infty}\dfrac{2x^3+3x^2+5}{-5x^3+8x-17}$.

(a) ∞ (b) 1 (c) 0 (d) $-\dfrac{2}{5}$

13. Find $\lim\limits_{x\to\infty}\dfrac{3x^2+5x+8}{2x^3+x^2-5x+7}$.

(a) ∞ (b) 3 (c) 0 (d) None of these

14. Evaluate $\lim\limits_{h\to0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$ if $h > 0$

(a) $\dfrac{1}{\sqrt{x}}$ (b) $\dfrac{1}{2\sqrt{x}}$ (c) $\dfrac{1}{x}$ (d) Does not exist

15. Evaluate: $\lim\limits_{x\to0}\dfrac{4x+2x^2}{2x-3x^2}$.

(a) 1 (b) 2 (c) 3 (d) None of these

16. Evaluate the limit: $\lim\limits_{x\to0}\left(x-\dfrac{|x|}{x}\right)$.

(a) 0 (b) 1 (c) 2 (d) Does not exist

17. If the function $y=f(x)=\dfrac{x^2-x-12}{x-4}$, then evaluate the limit of y as $x\to4$.

(a) ∞ (b) 3
(c) 7 (d) Cannot be determined

Answer Key

1. (b) **2.** (a) **3.** (c) **4.** (b) **5.** (b) **6.** (d) **7.** (c) **8.** (a) **9.** (d) **10.** (a)
11. (c) **12.** (d) **13.** (c) **14.** (b) **15.** (b) **16.** (d) **17.** (c)

CONTINUITY

In mathematics, the concept of continuity is a fundamental idea, particularly in calculus and real analysis. When we say a function is continuous, we mean that it can be smoothly drawn without any abrupt breaks or interruptions, much like the notion of sketching a curve without having to lift your pencil.

DEFINITION

A function y = f(x) is continuous at x =a if and only if

1. f(a) exists i.e., f is defined at x =a
2. $\lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a^-} f(x) = f(a)$
3. $\lim\limits_{x\to a} f(x) = f(a)$

Thus, a function is continuous over an interval if a function is continuous at each point of the interval. A function is not continuous at a point 'a' is said to be discontinuous at point 'a'.

PROPERTIES

1. A polynomial function is always continuous.
2. The sum, difference and product of two continuous functions is a continuous function.
3. The quotient of two continuous functions is a continuous function, provided that the denominator is not zero.

Example 17. Check if the function $f(x) = \begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$ is continuous at x = 0.

Sol. Given function: $f(x) = \begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} 2x+3 = 2(0)+3 = 3$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} 3(x+1) = 3(0+1) = 3(1) = 3$

f (0) = 2(0) + 3 = 3

Therefore, $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = f(0)$

Hence, the given function is continuous at x = 0.

Example 18. Is the function $f(x) = \dfrac{x^2-9}{x-3}$ is continuous at x = 3?

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Sol. Given function: $f(x)=\dfrac{x^2-9}{x-3}$

Here, $f(x)=\dfrac{x^2-9}{x-3}=\dfrac{(x+3)(x-3)}{(x-3)}=x+3$

As we know, linear function is always continuous.

Therefore, the given function is continuous at x = 3.

Example 19. Find the points of discontinuity, if, any, of the function $f(x)=\dfrac{x^2-7}{x^3-x^2+x-1}$.

Sol. Given, $f(x)=\dfrac{x^2-7}{x^3-x^2+x-1}$

The given function will be discontinuous at points where the denominator is zero.

Here, denominator = $x^3 - x^2 + x - 1$

To check the points at which denominator is zero, put $x^3 - x^2 + x - 1 = 0$

$x^2(x - 1)+1 (x - 1) = 0$

$\Rightarrow (x^2 + 1) (x - 1) = 0$

$\Rightarrow (x - 1) = 0 \Rightarrow x = 1$

Therefore, the given function is discontinuous at x = 1.

Example 20. A function f is defined as $f(x)=\{x+1, \text{ if } -1\le x\le 0$
$\{x, \text{ if } 0<x\le 1$
$\{2-x, \text{ if } 1<x\le 2$

(a) x = 1 (b) x = 0 (c) Both (a) and (b) (d) None of these

Sol. (a) Given function,

$f(x)=\{x+1, \text{ if } -1\le x\le 0$
$\{x, \text{ if } 0<x\le 1$
$\{2-x, \text{ if } 1<x\le 2$

At x = 0

$\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}(x)=0$

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}(x+1)=0+1=1$

Thus, $\lim\limits_{x\to 0^+} f(x)\ne \lim\limits_{x\to 0^-} f(x)$

Therefore, the given function is discontinuous at x = 0.

At x = 1

$\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}(2-x)=(2-1)=1$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1$$

$f(1) = 1$

Thus, $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = f(1)$

Therefore, the function is continuous at x = 1.

Hence, the correct option is (a).

Example 21. The value of k such that the function is continuous at x = 2 is

$$f(x) = \begin{cases} kx^2, \text{if } x \le 2 \\ 3, \text{if } x > 2 \end{cases}$$

(a) 1 (b) 3 (c) $\frac{3}{4}$ (d) None of these

Sol. (c) Since, the function is continuous at x = 2

$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{x\to 2^-} kx^2 = \lim_{x\to 2^+} 3 = f(2)$

$\Rightarrow k(2)^2 = 3 = 4k$

$\Rightarrow 4k = 3$

$\Rightarrow k = \frac{3}{4}$

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART B)

1. A function f is defined as

$$f(x) = \begin{cases} x+1, \text{ if } -1 \le x \le 0 \\ x, \text{ if } 0 < x \le 1 \\ 2-x, \text{ if } 1 < x \le 2 \end{cases}$$

then, the function is discontinuous at

(a) x = 2 (b) x = 1 (c) x = 0 (d) Everywhere

2. The value of k such that the function is continuous at x = 5 is

$$f(x) = \begin{cases} Kx+1, \text{if } x \le 5 \\ 3x-5, \text{if } x > 5 \end{cases}$$

(a) 5 (b) $\frac{1}{5}$ (c) $\frac{9}{5}$ (d) None of these

3. Is the function f(x) = |x| continuous at x = 0?

(a) Yes (b) No

(c) May or may not (d) Cannot be determined

4. Find the points of discontinuity, if, any, of the function $f(x)=\frac{x^2+2x+5}{x^2-3x+2}$.

(a) x = 1 (b) x = 2 (c) x = 1 or x = 2 (d) None of these

5. The function f(x) is defined by

$f(x)=\{5x-4,\ \text{if } 0<x\le 1$

$=\{4x^3-3x, \text{if} 1<x<2$ then the function is continuous at

(a) x = 0 (b) x = 1 (c) x = 2 (d) None of these

6. The function $f(x)=\frac{x^2-9}{x-3}$ is to be continuous at x = 3. What value must be assigned to f(3), if f(x) is to be continuous at x = 3.

(a) 1 (b) 3 (c) 6 (d) Cannot be determined

7. Find $\lim_{x\to 1} f(x)$ where $f(x)=\begin{cases}x^2-1, & x\le 1\\ -x^2-1, & x>1\end{cases}$

(a) 1 (b) 0 (c) –2 (d) Does not exist

8. A function f(x) is defined as

$f(x)=\{3+2x, \text{if } -\frac{3}{2}\le x\le 0$

$\{3-2x, \text{if } 0<x\le\frac{3}{2}$ then, the function is continuous at

$\{-3-2x, \text{if } x>\frac{3}{2}$

(a) x = 0 (b) $x=\frac{3}{2}$ (c) Both (a) and (b) (d) None of these

9. A function f is defined as

$f(x)=\{-x^2, \text{ if } x\le 0$

$\{5x-4, \text{ if } 0<x\le 1$

$\{4x^2-3x, \text{if } 1<x\le 2$ then, the function is discontinuous at

$\{3x+4, \text{ if } x\ge 2$

(a) x = 2 (b) x = 1 (c) x = 0 (d) None of these

10. Find the value of a so that the given function is continuous at x = 2

f(x) = {ax + 5, if x ≤ 2

{x – 1, if x > 2

(a) 2 (b) –2 (c) 1 (d) None of these

Answer Key

1. (c) **2.** (c) **3.** (a) **4.** (c) **5.** (b) **6.** (c) **7.** (c) **8.** (a) **9.** (c) **10.** (b)

PREVIOUS YEAR QUESTION (PYQ'S)

1. The $\lim_{x\to 2}\frac{x^2-4x+4}{x-2}=$ _____ **(June 2024)**

(a) 0 (b) 1 (c) 2 (d) 3

2. Evaluate, $\lim_{x\to 3}\frac{x^2+4x+3}{x^2+6x+9}$ **(Jan 2025)**

(a) $\frac{2}{8}$ (b) $\frac{2}{3}$ (c) 2 (d) $\frac{1}{3}$

3. Find the value of $\lim_{x\to 4}\frac{x^2-2x-8}{(x-4)}$. **(May 2025)**

(a) 0 (b) 2 (c) 8 (d) 6

Answer Key

1. (a) **2.** (b) **3.** (d)

SUMMARY

LIMIT

The limit of a function represents the value that the function approaches as the input (or variable) gets arbitrarily close to a certain point. This point may be a specific number or infinity.

Formally, we say that the limit of a function f(x) as x approaches a certain value c is L, denoted by:

$$\lim_{x\to c} f(x) = L$$

This notation indicates that as x gets arbitrarily close to c, the values of f(x) get arbitrarily close to L. If f(x) approaches a specific number as x approaches c, then that number is the limit. If f(x) becomes arbitrarily large or small without bound, we say the limit is infinity or negative infinity, respectively.

PROPERTIES

1. $\lim_{x\to a}[f(x)\pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
2. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
3. $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
4. $\lim_{x\to a}[c\,f(x)] = c\lim_{x\to a} f(x)$

5. $\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$

6. $\lim_{x\to 0^+}\frac{1}{x} = \lim_{h\to 0}\frac{1}{h} \to +\infty (h > 0)$

   $\lim_{x\to 0^+}\frac{1}{x} = \lim_{h\to 0}\frac{1}{-h} \to -\infty (h > 0)$

   ∞ is a very-very large number called infinity

   Thus $\lim_{x\to 0^+}\frac{1}{x}$ does not exist.

CONTINUITY

Continuity is a fundamental concept that describes the smooth and unbroken nature of a mathematical object, particularly a function. A function is considered continuous at a point if, intuitively, the values of the function don't exhibit abrupt jumps or breaks as the input approaches that point.

Formally, a function f(x) is continuous at a point c in its domain if three conditions are met:

1. f(c) is defined: The function must be defined at the point c
2. The limit of f(x) as x approaches c exists: The $\lim_{x\to c} f(x)$ must exist.
3. The limit and the function value are equal: The value of the function at c i.e. f(c) must be equal to the limit of the function as x approaches c i.e. $\lim_{x\to c} f(x)$

In mathematical notation, this can be expressed as:

$$\lim_{x\to c} f(x) = f(c)$$

If these three conditions are satisfied, then the function is said to be continuous at the point c. If a function is continuous at every point in its domain, it is simply referred to as a continuous function.

PROPERTIES

1. A polynomial function is always continuous.
2. The sum, difference and product of two continuous functions is a continuous function.
3. The quotient of two continuous functions is a continuous function, provided that the denominator is not zero.

❑❑❑

CHAPTER

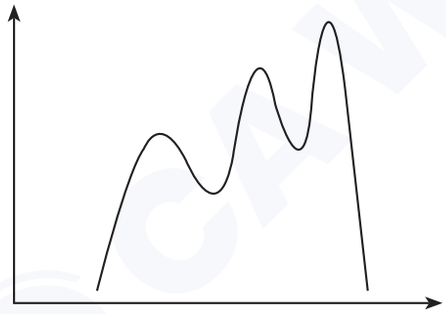
08

Basic Applications of Differential and Integral Calculus in Business and Economics

Unit-1: Differential Calculus

INTRODUCTION

Differential calculus is a branch of calculus focused on studying the rates of change and derivatives of functions. It involves finding a function's derivative, representing its instantaneous rate of change at any point. The derivative measures how a function changes as the input variable changes, providing information about slope, direction, and concavity. In differentiation, "increment" signifies a small change in the independent variable, while "change" is the corresponding function change. As this change approaches zero, the derivative gives the exact rate of change. It's widely used in math, physics, engineering, economics, and more for function analysis, optimization, differential equations, rate understanding, and real-world modeling. Differential calculus is fundamental for understanding dynamic systems and solving various scientific problems.



To express the rate of change in any function we introduce concept of derivative

DERIVATIVE OR DIFFERENTIAL COEFFICIENT

The derivative or differential coefficient of a function f(x) is defined as the limit of the ratio of the change (increment) in the function corresponding to a small change (increment) in the independent variable x, as that change approaches zero. It represents the rate at which the function value changes with respect to the independent variable.

Let y = f(x) be a function. If h (or Δx) be the small increment in x and the corresponding increment in y or f(x) be $\Delta y = f(x + h) - f(x)$, then the derivative of f(x) is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This is denoted as $f'(x), \frac{dy}{dx}$ or $\frac{df(x)}{dx}$.

The derivative of f(x) is also known as the differential coefficient of f(x) with respect to x. This process of differentiation is called the first principle (or abinitio) (Ab-initio).

A function f(x) is said to be differentiable at a specific point x = c if the derivative f '(c) exists. This means that the function has a well-defined rate of change at that point.

i.e., $\lim_{h\to c}\frac{f(x)-f(c)}{x-c}$ exists which is denoted by f (c) or $\left[\frac{dy}{dx}\right]_{x=c}$.

FEW IMPORTANT FUNCTIONS AND THEIR DERIVATIVES

Function	Derivative of the Function
$f(x)$	$f'(x)$
x^n	nx^{n-1}
e^x	e^x
$\log_e x$	$\frac{1}{x}$
a^x	$a^x \cdot (\log a)$
c (a constant)	0
cf (x)	cf '(x)

BASIC LAWS FOR DIFFERENTIATION

- h(x) = cf(x) where c is any real constant **(Scalar multiple of a function)**

  Derivative of the function: h' (x) = cf '(x)

  E.g.: If $h(x) = 3x^2$

  then $h'(x) = \frac{d}{dx}(3x^2) = 3\frac{d}{dx}(x^2) = 3(2x^{2-1}) = 6x$

- h(x) = f(x) ± g(x) **(Sum/Difference of function)**

  Derivative of the function: h '(x) = f '(x) ± g '(x)

  E.g.: $h(x) = 2x + 5x^3$

  then $h'(x) = 2 + 5(3x^{3-1}) = 2 + 15x^2$

- Derivative of Product of functions: **(Product Rule)**: h(x) = f(x) g(x)

  If two functions f and g are differentiable then

  $$\frac{d}{dx}(f(x).g(x)) = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

  $\Rightarrow h'(x) = f(x)g'(x) + f'(x)g(x)$

  That is, the derivative of the product of two functions is the sum of the first function multiplied by the derivative of the second and the second function multiplied by the derivative of the first function.

E.g.: $h(x) = e^x x^2$

then $h'(x) = e^x \frac{d(x^2)}{dx} + x^2 \frac{d(e^x)}{dx}$

$h'(x) = e^x (2x) + e^x x^2$

$h'(x) = 2xe^x + e^x x^2$

$h'(x) = xe^x (2+x)$

- **Quotient Rule:** $h(x) = \frac{f(x)}{g(x)}$

If f(x) and g(x) are two differentiable functions then

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$\Rightarrow h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

E.g.: $h(x) = \frac{x^2}{e^x}$

then $h'(x) = \frac{e^x(2x) - x^2e^x}{(e^x)^2} = \frac{xe^x(2-x)}{(e^x)^2} = \frac{x(2-x)}{e^x}$

Example 1. Differentiate $x^{\frac{3}{2}}$ with respect to x.

(a) $x^{\frac{3}{2}}$ (b) $\frac{3}{2}\sqrt{x}$ (c) $\frac{3}{2}x$ (d) None of these

Sol. (b) Given, $x^{\frac{3}{2}}$

We know, $\frac{d}{dx}x^n = nx^{n-1}$

$$\Rightarrow \frac{d}{dx}\left(x^{\frac{3}{2}}\right) = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$

Hence, the correct option is (b).

Example 2. If $y = 4x^3 - 7x^4$, then $\frac{dy}{dx}$ is

(a) $2x(-14x^2 + 6x)$ (b) $2x(14x^2 - 6x)$

(c) $2x(14x^2 + 6x)$ (d) None of these

Sol. (a) Given that, $y = 4x^3 - 7x^4$

Now, differentiate y with respect to x using the formula $\frac{d}{dx}x^n = nx^{n-1}$

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$$\Rightarrow \frac{dy}{dx} = 4\frac{d}{dx}(x^3) - 7\frac{d}{dx}(x^4)$$

$$\Rightarrow \frac{dy}{dx} = 4(3x^2) - 7(4x^3)$$

$$\Rightarrow \frac{dy}{dx} = 12x^2 - 28x^3$$

$$\Rightarrow \frac{dy}{dx} = 2x(6x - 14x^2)$$

Hence, option (a) is the correct answer.

Example 3. Differentiate each of the following function with respect to x and choose the correct option:

I. $5x^2 + 2x$

(a) $5x + 2$ (b) $10x + 2$ (c) $10x$ (d) $5x$

II. $a^x + x^a + a^a$

(a) $a^x \log a + ax^{a-1}$ (b) $a^x + ax^{a-1}$

(c) $a^x \log a + x^{a-1}$ (d) $a^x + x^{a-1}$

III. $\frac{2}{3}x^3 - 7x^2 + 3x - 5\log x + 3$

(a) $2x^2 - 14x + 3 - \frac{1}{x}$ (b) $6x^2 - 7x + 3 - \frac{5}{x}$

(c) $2x^2 - 14x + 3 - \frac{5}{x}$ (d) $6x^2 - 14x + 3 - \frac{5}{x}$

IV. $e^x \log x$

(a) $e^x(1 + x \log x)$ (b) $\frac{e^x}{x}(1 + \log x)$

(c) $e^x(1 + \log x)$ (d) $\frac{e^x}{x}(1 + x \log x)$

V. $y = \frac{e^x + 1}{e^x - 1}$

(a) $\frac{-2e^x}{(e^x - 1)^2}$ (b) $\frac{2e^x}{(e^x - 1)^2}$ (c) $\frac{-2}{(e^x - 1)^2}$ (d) None of these

Sol. I. (b) We know, $\frac{d}{dx}x^n = nx^{n-1}$,

So, $\frac{d}{dx}(5x^2 + 2x) = 5(2x) + 2$

$\Rightarrow 10x + 2$

Hence, option (b) is correct.

II. (a) Given function: $a^x + x^a + a^a$

We know that,

$\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(a^x) = a^x \log a$

Also, a^a = constant $\Rightarrow \frac{d}{dx}(a^a) = 0$

Thus, $\frac{d}{dx}(a^x + x^a + a^a)$

$\Rightarrow a^x \log a + ax^{a-1} + 0 = a^x . \log a + ax^{a-1}$

Therefore, $\frac{d}{dx}(a^x + x^a + a^a) = a^x \log a + ax^{a-1}$

Hence, option (a) is correct.

III. (c) $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(\log x) = \frac{1}{x}$

Thus, $\frac{d}{dx}\left(\frac{2}{3}x^3 - 7x^2 + 3x - 5\log x + 3\right)$

$\Rightarrow \frac{2}{3}(3x^2) - 14x + 3 - \frac{5}{x}$

$\Rightarrow 2x^2 - 14x + 3 - \frac{5}{x}$

Hence, option (c) is correct.

IV. (d) Product rule of derivative is:

$\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v\frac{du}{dx}$

Using this, we get:

$\frac{d}{dx}(e^x \log x) = e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(e^x)$

$\Rightarrow \frac{e^x}{x} + e^x \log x$

$\Rightarrow \frac{e^x}{x}(1 + x\log x)$

Hence, option (d) is correct.

V. (a) Given: $y = \frac{e^x + 1}{e^x - 1}$

Differentiating y wrt x,

$\Rightarrow \frac{dy}{dx} = \frac{(e^x - 1)\frac{d}{dx}(e^x + 1) - (e^x + 1)\frac{d}{dx}(e^x - 1)}{(e^x - 1)^2}$

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$$\Rightarrow \frac{dy}{dx} = \frac{(e^x - 1)e^x - (e^x + 1)e^x}{(e^x - 1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(e^{2x} - e^x - e^{2x} - e^x)}{(e^x - 1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2e^x}{(e^x - 1)^2}$$

Hence, option (a) is correct.

Example 4. If $f(x) = \frac{x^2}{e^x}$, then $f'(1)$ is equal to **(ICAI)**

(a) -1/e (b) $\frac{1}{e}$ (c) e (d) None of these

Sol. (b) Given, $f(x) = \frac{x^2}{e^x}$

On differentiating using quotient rule, we get

$$f'(x) = \frac{d}{dx}\left(\frac{x^2}{e^x}\right) = \frac{e^x(2x) - x^2e^x}{e^{2x}} = \frac{2xe^x - x^2e^x}{e^{2x}}$$

Now, $f'(1) = \frac{2(1)e^1 - (1)^2e^1}{e^{2(1)}} = \frac{2e - e}{e^2} = \frac{e}{e^2} = \frac{1}{e}$

Hence, option (b) is correct.

DERIVATIVE OF A FUNCTION OF FUNCTION

If y = f [h(x)], where u = h(x), then $\frac{dy}{dx} = ?$

If we have a function y = f [h(x)], where u = h(x), then the derivative $\frac{dy}{dx}$ can be calculated using the chain rule of differentiation.

The chain rule states that if y = f(u), where u = h(x), then the derivative $\frac{dy}{dx}$ is given by:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

In this case, u = h(x), so we can rewrite the equation as:

y = f(u) = f[h(x)]

Now, let's differentiate both sides of the equation with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}[f(h(x)]$$

Using the chain rule, we have:

$$\frac{dy}{dx}=\frac{df}{du}.\frac{du}{dx}$$

Since u = h(x), the derivative $\frac{du}{dx}$ is simply the derivative of h(x) with respect to x.

Now, the derivative $\frac{df}{du}$ represents the derivative of the function f with respect to its argument u.

Thus, $\frac{dy}{dx}=f'(u)\times h'(x)$

Example 5. Differentiate log $(2x + 3x^2)$ w.r.t. x.

(a) $\frac{1}{2x+3x^2}$ (b) $\frac{2+6x}{2x+3x^2}$ (c) $\frac{6x}{2x+3x^2}$ (d) $\frac{2+3x}{2x+3x^2}$

Sol. (b) $\frac{d}{dx}(\log x)=\frac{1}{x}$

Using this rule derivative in $y = \log (2x + 3x^2)$

Let $(2x + 3x^2) = t$

So, function becomes log t

$$\Rightarrow \frac{d}{dt}(\log t)=\frac{1}{t}$$

Also, $\frac{dt}{dx}=2+6x$

So, $\frac{dy}{dx}=\frac{1}{2x+3x^2}(2+6x)$

$$\Rightarrow \frac{dy}{dx}=\frac{2+6x}{2x+3x^2}$$

Hence, option (b) is correct.

Example 6. Differentiate e^{3x+8} w.r.t. x

(a) $3e^{3x+8}$ (b) e^{3x+8} (c) $(3x + 8)\, e^{3x+8}$ (d) $8e^{3x+8}$

Sol. (a) Derivative of y = f [h(x)] is given by $\frac{dy}{dx}=\frac{df}{dh}.\frac{dh}{dx}$

Using this, we get: $y = e^{3x+8}$

Let u = 3x + 8

$$\frac{dy}{dx}=e^u\times(3)$$

$$\Rightarrow \frac{dy}{dx}=3e^{3x+8}$$

Hence, option (a) is correct.

Example 7. If $y = e^{\sqrt{2x}}$, then $\frac{dy}{dx}$ is equal to _____ **(ICAI)**

(a) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (b) $e^{\sqrt{2x}}$ (c) $\frac{e^{\sqrt{2x}}}{2\sqrt{x}}$ (d) None of these

Sol. (a) Given, $y = e^{\sqrt{2x}}$

Differentiating both sides:

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(e^{\sqrt{2x}})$$

$$\Rightarrow \frac{dy}{dx} = e^{\sqrt{2x}} \frac{d}{dx}(\sqrt{2x})$$

$$\Rightarrow \frac{dy}{dx} = e^{\sqrt{2x}} \times \sqrt{2} \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{dy}{dx} = e^{\sqrt{2x}} \left(\sqrt{2} \times \frac{1}{2\sqrt{x}}\right)$$

$$\Rightarrow \frac{dy}{dx} = e^{\sqrt{2x}} \left(\frac{1}{\sqrt{2}\sqrt{x}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

Hence, the correct option is (a) i.e. $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$.

Example 8. If $y = \sqrt{x^2 + m^2}$, then $y \times y_1 \left(\text{where } y_1 = \frac{dy}{dx}\right)$ is equal to **(ICAI)**

(a) -x (b) x (c) $\frac{1}{x}$ (d) None of these

Sol. (b) Given, $y = \sqrt{x^2 + m^2}$

Squaring both sides,

$$\Rightarrow y^2 = \left(\sqrt{x^2 + m^2}\right)^2$$

$$\Rightarrow y^2 = x^2 + m^2$$

Differentiating both sides,

$$\Rightarrow 2y\frac{dy}{dx} = 2x + 0 \qquad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$$\Rightarrow y\frac{dy}{dx} = \frac{2x}{2}$$

$$\Rightarrow y \times y_1 = x$$

Hence, the correct option is (b) i.e., x.

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Example 9. If $y=\left(x+\sqrt{x^2+m^2}\right)^n$, then $\frac{dy}{dx}$ is equal to **(ICAI)**

(a) ny (b) $\frac{ny}{\sqrt{x^2+m^2}}$ (c) $\frac{-ny}{\sqrt{x^2+m^2}}$ (d) None of these

Sol. (b) Here we have: $y=\left(x+\sqrt{x^2+m^2}\right)^n$

Differentiating both sides

$$\frac{dy}{dx}=\left\{n\left(x+\sqrt{x^2+m^2}\right)^{n-1}\right\}\cdot\left\{1+\left(\frac{1}{2\sqrt{x^2+m^2}}\right)\cdot 2x\right\}$$

$$\Rightarrow \frac{dy}{dx}=\left(n\cdot\left(x+\sqrt{x^2+m^2}\right)^{n-1}\right)\cdot\left(\frac{\sqrt{x^2+m^2}+x}{\sqrt{x^2+m^2}}\right)$$

$$\Rightarrow \frac{dy}{dx}=n\cdot\frac{\left(\left(x+\sqrt{x^2+m^2}\right)^{n-1}\right)\left(\sqrt{x^2+m^2}+x\right)}{\sqrt{x^2+m^2}}$$

$$\Rightarrow \frac{dy}{dx}=n\cdot\frac{\left(\left(x+\sqrt{x^2+m^2}\right)^{n}\right)}{\sqrt{x^2+m^2}}$$

$$\Rightarrow \frac{dy}{dx}=n\cdot\left(\frac{y}{\sqrt{x^2+m^2}}\right)$$

$$\Rightarrow \frac{dy}{dx}=\left(\frac{ny}{\sqrt{x^2+m^2}}\right)$$

Hence, the correct answer is option (b).

IMPLICIT FUNCTIONS

An implicit function is a function that is defined implicitly by an equation in the form f(x, y) = 0, where y cannot be directly expressed as a function of x.

For example: $x^2y^2 + 7xy + x + y$ where y cannot be directly defined as a function of x is called an implicit function of x.

Example 10. Find $\frac{dy}{dx}$ when $x^3 + y^3 = xy$.

(a) $\frac{xy-3x^2}{3y^2-x}$ (b) $\frac{y-3x^2}{3y^2-x}$ (c) $\frac{y-3x^2}{3y^2-x^2}$ (d) None of these

Sol. (b) Given, $x^3 + y^3 = xy$

Differentiating above function with respect to x, we get

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(xy)$$

$$\Rightarrow 3x^2 + 3y^2 \frac{dy}{dx} = x\frac{dy}{dx} + y\frac{dx}{dx}$$

$$\Rightarrow 3x^2 + 3y^2 \frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\Rightarrow 3y^2 \frac{dy}{dx} - x\frac{dy}{dx} = y - 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$$

Hence, the correct option is (b).

Example 11. If $x^5 + y^5 - 5xy = 0$, then $\frac{dy}{dx}$ is **(Dec 2022)**

(a) $\frac{y + x^4}{x + y}$ (b) $\frac{y - x^4}{y^4 - x}$ (c) $\frac{x - y^4}{x - y}$ (d) $\frac{x + y^4}{x^4 + y}$

Sol. (b) Given, $x^5 + y^5 - 5xy = 0$

Differentiating above function with respect to x, we get

$$\frac{d}{dx}(x^5 + y^5 - 5xy) = \frac{d}{dx}(0)$$

$$\Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} - 5\left(x\frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow x^4 + y^4 \frac{dy}{dx} - x\frac{dy}{dx} - y = 0$$

$$\Rightarrow \frac{dy}{dx}(y^4 - x) = y - x^4$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x^4}{y^4 - x}$$

Hence, the correct option is (b).

Example 12. If $xy = 1$, then $y^2 + \frac{dy}{dx}$ is equal to **(ICAI)**

(a) 1 (b) 0 (c) –1 (d) None of these

Sol. (b) Given, $xy = 1$

Differentiate above equation with respect to x using product rule i.e., $\frac{d}{dx}(pq) = p\frac{dq}{dx} + q\frac{dp}{dx}$

$$\Rightarrow x\frac{dy}{dx}+y=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{-y}{x}$$

Therefore, $y^2+\frac{dy}{dx}=y^2-\frac{y}{x}$

$$=\frac{y^2x-y}{x}=\frac{y^2\left(\frac{1}{y}\right)-y}{\frac{1}{y}}=y(y-y)=0$$

Hence, option (b) is the correct answer i.e., 0.

PARAMETRIC EQUATION

Parametric equations are a way of representing a curve or a set of points in terms of a parameter, usually denoted as "t" Instead of expressing the variables x and y directly in terms of each other, they are expressed individually as functions of the parameter.

For parametric equations x = f(t) and y = h(t), where f(t)and h(t) are functions of the parameter t, the derivative $\frac{dy}{dx}$ can be obtained using the chain rule of differentiation.

Applying the chain rule to the parametric equations x = f(t) and y = h(t), we can find the derivative $\frac{dy}{dx}$ as follows:

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{dy}{dt}.\frac{dt}{dx}$$

Example 13. If x = ct and $y=\frac{c}{t}$, then $\frac{dy}{dx}$ is equal to **(June 2012)**

(a) $\frac{1}{t}$ (b) $t.e^t$ (c) $-\frac{1}{t^2}$ (d) None of these

Sol. (c) Given, x = ct and $y=\frac{c}{t}$

$$\Rightarrow \frac{dx}{dt}=c \text{ and } \frac{dy}{dt}=-\frac{c}{t^2}$$

$$\Rightarrow \frac{dy}{dx}=-\frac{c}{c.t^2}=-\frac{1}{t^2}$$

Therefore, $\frac{dy}{dx}=-\frac{1}{t^2}$

Hence, the correct option is (c).

Example 14. Given $x = 2t + 5$, $y = t^2 - 2$, $\frac{dy}{dx}$ is calculated as **(ICAI)**

(a) t (b) $\frac{-1}{t}$ (c) $\frac{1}{t}$ (d) None of these

Sol. (a) Given: $x = 2t + 5$, $y = t^2 - 2$

Now, differentiate x and y with respect to t, we get

$$\frac{dx}{dt} = 2 \quad ...(i)$$

and $$\frac{dy}{dt} = 2t \quad ...(ii)$$

$$\left[\because \frac{d}{dx}x^n = nx^{n-1}\right]$$

Divide equation (ii) by equation (i), we get

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{2} = t$$

Hence, option (a) is the correct answer.

Example 15. If $x = at^2$, $y = 2at$ then $\left[\frac{dy}{dx}\right]_{t=2}$ is equal to **(ICAI)**

(a) $\frac{1}{2}$ (b) -2 (c) $-\frac{1}{2}$ (d) None of these

Sol. (a) Given, $x = at^2$, $y = 2at$

$$\frac{dx}{dt} = 2at \ \& \ \frac{dy}{dt} = 2a$$

Now, to find $\frac{dy}{dx}$ we will find $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$$\Rightarrow \frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$$

At t = 2,

$$\frac{dy}{dx} = \frac{1}{2}$$

Hence, option (a) is correct i.e. $\frac{1}{2}$.

Example 16. Given $x = t + t^{-1}$ and $y = t - t^{-1}$ then the value of $\frac{dy}{dx}$ at t = 2 is **(ICAI)**

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{5}{3}$ (d) None of these

Sol. (c) Given, $x = t + t^{-1}$ and $y = t - t^{-1}$

Differentiate x and y with respect to t, we get

$$\frac{dy}{dx} = 1 - t^{-2} \quad \text{...(i)}$$

and $$\frac{dy}{dx} = 1 + t^{-2} \quad \text{...(ii)}$$

Now, divide equation (ii) by equation (i),

$$\Rightarrow \frac{dy}{dx} = \frac{1 + t^{-2}}{1 - t^{-2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}$$

Now, at t = 2

$$\frac{dy}{dx} = \frac{4 + 1}{4 - 1} = \frac{5}{3}$$

Hence, option (c) is the correct answer i.e., $\frac{5}{3}$.

LOGARITHMIC DIFFERENTIATION

The process of finding a derivative by taking logarithm in the first instance is called logarithmic differentiation. The procedure is convenient to adopt when the function to be differentiated involves a function in its power or when the function is the product of the number of functions.

Example 17. Differentiate x^x w.r.t. x.

(a) $x^x (1 + \log x)$ (b) $x^x \log x$

(c) $x^x (1 - \log x)$ (d) None of these

Sol. (a) Let $y = x^x$

Taking logarithm on both sides, we get $\log y = x \log x$

Differentiating with respect to x,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot (1)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dy}{dx} = y(1+\log x)$$

$$\Rightarrow \frac{dy}{dx} = x^x(1+\log x)$$

Hence, the correct option is (a).

Example 18. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is

(a) $\frac{\log x}{(1-\log x)^2}$ (b) $\frac{\log x}{(1+\log x)^2}$ (c) $\frac{\log x}{(1-\log x)}$ (d) $\frac{\log x}{(1+\log x)}$

Sol. (b) Given, $x^y = e^{x-y}$

Now taking log both sides,

$\Rightarrow \log x^y = \log e^{x-y}$

$\Rightarrow y \log x = (x - y)\log e$

$\Rightarrow y \log x = (x - y)$

$\Rightarrow y \log x + y = x$

$\Rightarrow y(1 + \log x) = x$

$$\Rightarrow y = \frac{x}{1+\log x}$$

Now differentiating both sides w.r.t x,

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\log x)\times 1 - x\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$$

$$= \frac{\log x}{(1+\log x)^2}$$

Hence, option (b) is correct

Example 19. If $x^y \cdot y^x = M$, where M is constant then $\frac{dy}{dx}$ is equal to **(ICAI)**

(a) $\frac{-y}{x}$ (b) $\frac{-y(y+x\log y)}{x(y\log x+x)}$

(c) $\frac{y+x\log y}{y\log x+x}$ (d) None of these

Sol. (b) Given that,

$x^y \cdot y^x = M$ where M is a constant

Taking log both the sides, we get

$\log(x^y \cdot y^x) = \log M$

$\Rightarrow \log x^y + \log y^x = \log M$ $(\log pq = \log p + \log q)$

$\Rightarrow y \log x + x \log y = \log M$ $(\log p^q = q \log p)$

Differentiate above equation with respect to x using the product rule,

$$\Rightarrow y\left(\frac{1}{x}\right)+\log x\left(\frac{dy}{dx}\right)+x\left(\frac{1}{y}\right)\frac{dy}{dx}+\log y(1)=0$$

$$\Rightarrow\left(\log x+\frac{x}{y}\right)\frac{dy}{dx}=-\log y-\frac{y}{x}$$

$$\Rightarrow\frac{dy}{dx}=\frac{-\log y-\frac{y}{x}}{\log x+\frac{x}{y}}$$

$$\Rightarrow\frac{dy}{dx}=\frac{-\left(\frac{x\log y+y}{x}\right)}{\left(\frac{y\log x+x}{y}\right)}$$

$$\Rightarrow\frac{dy}{dx}=-\frac{y}{x}\left(\frac{x\log y+y}{y\log x+x}\right)$$

Hence, option (b) is the correct answer.

Example 20. Differentiate the following w.r.t. x: **(ICAI)**

I. $\log\left[e^x\left\{\frac{x-2}{x+2}\right\}^{3/4}\right]$

(a) $\frac{x^2+1}{x^2+4}$ (b) $\frac{x^2-1}{x^2-4}$ (c) $\frac{1}{x^2-4}$ (d) None of these

II. If $\log\left(\frac{x}{y}\right)=x+y,\frac{dy}{dx}$ is

(a) $\frac{y(1-x)}{x(1+y)}$ (b) $\frac{y}{x}$ (c) $\frac{1-x}{1+y}$ (d) None of these

Sol. I. (b) We know that, $\frac{d}{dx}(\log x)=\frac{1}{x},\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$

To find derivative of $\log\left[e^x\left\{\frac{x-2}{x+2}\right\}^{3/4}\right]$

We will apply chain rule here,

$$\Rightarrow\left[e^x\left\{\frac{x-2}{x+2}\right\}^{3/4}\right]^{-1}\left[e^x\left(\frac{3}{4}\left(\frac{x-2}{x+2}\right)^{-1/4}\right)\left\{\frac{x+2-(x-2)}{(x+2)^2}\right\}+\left\{\frac{x-2}{x+2}\right\}^{3/4}e^x\right]$$

$$\Rightarrow\left[e^x\left\{\frac{x-2}{x+2}\right\}^{3/4}\right]^{-1}\left[e^x\left(\frac{3}{4}\left(\frac{x-2}{x+2}\right)^{-1/4}\right)\left\{\frac{4}{(x+2)^2}\right\}+\left\{\frac{x-2}{x+2}\right\}^{3/4}e^x\right]$$

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Taking common $\left[e^x\left\{\frac{x-2}{x+2}\right\}^{3/4}\right]$ from numerator, we get

$$\Rightarrow\left[3\left\{\frac{1}{(x+2)(x-2)}\right\}+1\right]=\frac{3}{(x^2-4)}+1$$

$$\Rightarrow\frac{3+x^2-4}{x^2-4}=\frac{x^2-1}{x^2-4}$$

Hence, option (b) is correct.

II. (a) Given: $\log\left(\frac{x}{y}\right)=x+y$

Differentiate with respect to x, we get

$$\frac{d}{dx}\left(\log\left(\frac{x}{y}\right)\right)\times\frac{d}{dx}\left(\frac{x}{y}\right)=1+\frac{dy}{dx} \qquad \left[\because\frac{d}{dx}\log x=\frac{1}{x}\right]$$

$$\Rightarrow\frac{1}{\left(\frac{x}{y}\right)}\times\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)=1+\frac{dy}{dx} \qquad \left[\because\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{d}{dx}(u)-u\frac{d}{dx}(v)}{v^2}\right]$$

$$\Rightarrow\frac{y}{x}\times\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)=1+\frac{dy}{dx}$$

$$\Rightarrow\frac{1}{x}\times\left(\frac{y-x\frac{dy}{dx}}{y}\right)=1+\frac{dy}{dx}$$

$$\Rightarrow y-x\frac{dy}{dx}=xy+xy\frac{dy}{dx}$$

$$\Rightarrow xy\frac{dy}{dx}+x\frac{dy}{dx}=y-xy$$

$$\Rightarrow\frac{dy}{dx}(xy+x)=y-xy$$

$$\Rightarrow\frac{dy}{dx}=\frac{y-xy}{xy+x}$$

$$\Rightarrow\frac{dy}{dx}=\frac{y(1-x)}{x(1+y)}$$

Hence, option (a) is the correct answer.

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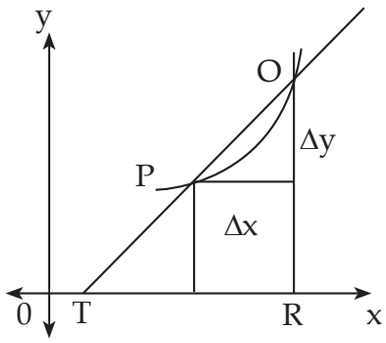
GEOMETRICAL INTERPRETATION OF THE DERIVATIVE



Geometrically, the derivative of a function can be interpreted as the slope of the tangent line to the graph of the function at a given point. The tangent line touches the graph at that point and represents the best linear approximation to the function's behavior near that point.

Consider a function f(x) and two points on its graph, P (x, f(x)) and Q (x + Δx, f(x + Δx)). The secant line passing through these two points represents the average rate of change of the function over the interval Δx.

The slope of the secant line is given by the formula:

$$m = \frac{\Delta y}{\Delta x} = \frac{(f(x+\Delta x) - f(x))}{\Delta x}$$

As Δx approaches zero, the secant line becomes closer and closer to the tangent line to the graph at the point (x, f(x)). The derivative of the function at that point is defined as the limit of the secant line slope as Δx approaches zero.

Mathematically, the derivative is given by:

$$f'(x) = \lim_{\Delta x \to 0} \frac{(f(x+\Delta x) - f(x))}{\Delta x}$$

The derivative represents the instantaneous rate of change of the function at the point (x, f(x)). It describes the slope of the tangent line at that point, which provides information about the function's behavior and how it is changing locally.

Example 21. The gradient of the curve $y = 2x^3 - 5x^2 - 3x$ at $x = 0$ is **(ICAI)**

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) None of these

Sol. (b) Given equation: $y = 2x^3 - 5x^2 - 3x$

The gradient of the curve at the given point is the value of the slope of the tangent at a given point.

The slope of the tangent is the value of $\frac{dy}{dx}$ (derivative of y with respect to x) at a given point.

$y = 2x^3 - 5x^2 - 3x$

Differentiate with respect to x, we get

$$\frac{dy}{dx} = 2(3x^2) - 5(2x) - 3$$

$$\frac{dy}{dx} = 6x^2 - 10x - 3$$

Given, x = 0

$$\left(\frac{dy}{dx}\right)_{x=0} = 6(0)^2 - 10(0) - 3$$

$$\left(\frac{dy}{dx}\right)_{x=0} = -3$$

Hence, the correct answer is option (b) i.e., -3.

Example 22. The gradient of the curve y - xy + 2px + 3qy = 0 at the point (3, 2) is $\frac{-2}{3}$. The value of p and q are **(ICAI)**

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) (2, 2) (c) $\left(\frac{-1}{2}, \frac{-1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{1}{6}\right)$

Sol. (d) Given that,

The equation of the curve y - xy + 2px + 3qy = 0 where $\frac{dy}{dx} = \frac{-2}{3}$ at the point (3, 2)

i.e., x = 3, y = 2 ...(i)

It implies that, the point (3, 2) satisfies the equation of the curve. Substitute x = 3, y = 2 in y - xy + 2px + 3qy = 0.

$\Rightarrow 2 - 3 \times 2 + 2p(3) + 3q(2) = 0$

$\Rightarrow 2 - 6 + 6p + 6q = 0$

$\Rightarrow 6p + 6q = 4$

$$\Rightarrow p + q = \frac{2}{3} \quad ...(ii)$$

Now, differentiate the equation of the curve with respect to x,we get

$$\frac{dy}{dx} - \left(x\frac{dy}{dx} + y\right) + 2p + 3q\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} - x\frac{dy}{dx} - y + 2p + 3q\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{-2}{3} - 3\left(\frac{-2}{3}\right) - 2 + 2p + 3q\left(\frac{-2}{3}\right) = 0 \text{ from equation (i)}$$

$$\Rightarrow \frac{-2}{3} + 2 - 2 + 2p - 2q = 0$$

$$\Rightarrow 2p - 2q = \frac{2}{3}$$

$$\Rightarrow p - q = \frac{1}{3} \quad ...(iii)$$

On solving equation (ii) and (iii), we get

$p = \frac{1}{2}$ and $q = \frac{1}{6}$

Hence, option (d) is the correct answer.

Example 23. The gradient of the curve $y = 2x^3 - 3x^2 - 12x + 8$ at $x = 0$ is **(ICAI)**

(a) -12 (b) 12 (c) 0 (d) None of these

Sol. (a) We know that,

The gradient of the curve at the given point is the value of the slope of the tangent at a given point.

The slope of the tangent is the value of $\frac{dy}{dx}$ (derivative of y with respect to x) at a given point.

Since, $y = 2x^3 - 3x^2 - 12x + 8$

Differentiate with respect to x, we get

$$\frac{dy}{dx} = 2(3x^2) - 3(2x) - 12$$

$$\frac{dy}{dx} = 6x^2 - 6x - 12$$

Given, x= 0

$$\left(\frac{dy}{dx}\right)_{x=0} = 6(0)^2 - 6(0) - 12$$

$$\left(\frac{dy}{dx}\right)_{x=0} = -12$$

Hence, the correct option is (a) i.e., -12.

Example 24. The slope of the tangent to the curve $y = \sqrt{4 - x^2}$ at the point, where the ordinate and the abscissa are equal is **(ICAI)**

(a) -1 (b) 1 (c) 0 (d) None of these

Sol. (a) Given: Equation of the curve is $y = \sqrt{4 - x^2}$

$\Rightarrow y^2 = 4 - x^2$ (Squaring both the sides)

$\Rightarrow 2y^2 = 4$

$\Rightarrow y = \sqrt{2}$

Since, the ordinate and the abscissa are equal i.e., x = y

$\Rightarrow y = x = \sqrt{2}$

Now, differentiate $y = \sqrt{4 - x^2}$ with respect to x using chain rule,

$$\left(\text{Chain rule}\ \frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x))) \times \frac{d}{dx}g(x)\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times \frac{d}{dx}(4-x^2) \qquad \left(\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (0-2x) \qquad \left(\frac{d}{dx}x^n = nx^{n-1}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{4-x^2}}$$

Substitute $x = y = \sqrt{2}$, we get $\frac{dy}{dx} = \frac{-\sqrt{2}}{\sqrt{4-2}}$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{2}}{\sqrt{2}} = -1$$

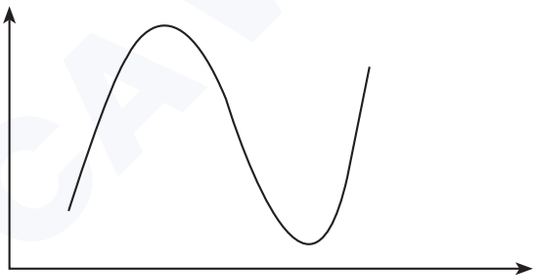
Thus, the slope of tangent is -1.

Hence, option (a) is the correct answer.

CONCEPT OF MAXIMUM AND MINIMUM IN DIFFERENTIATION

How to Find Maximum and Minimum Points Using Differentiation?

The value of the function at a maximum point is called the maximum value of the function and the value of the function at a minimum point is called the minimum value of the function



How to find the maximum or minimum of a curve?

Steps for finding maximum or minimum:

1. Differentiate the given function f(x) with respect to x to find its derivative f′(x).
2. Set the derivative f′(x) equal to zero and solve for x to find the critical points. These are the values of x where the slope of the curve is zero or undefined.
3. Find the second derivative f′′(x) by differentiating the derivative f′(x) obtained in step 1.
4. Evaluate the second derivative f′′(x) at each critical point found in step 2.
   - If f′′(x) < 0 at a critical point, then the function has a local maximum at that point.
   - If f′′(x) > 0 at a critical point, then the function has a local minimum at that point.

- If $f''(x) = 0$ at a critical point, further investigation is needed to determine the nature of the point (it could be a point of inflection or a saddle point).

5. To find the maximum and minimum values of the function, substitute the x-values of the critical points (including any endpoints of the domain) into the original function f(x) and compare the corresponding function values.

- The largest function value obtained is the maximum value of the function.
- The smallest function value obtained is the minimum value of the function.

Example 25. The maxima and minima of the function $y = 2x^3 - 15x^2 + 36x + 10$ occurs respectively at **(Dec 2022)**

(a) $x = 2$ and $x = 3$ (b) $x = 1$ and $x = 3$

(c) $x = 3$ and $x = 2$ (d) $x = 3$ and $x = 1$

Sol. (a) Given, $y = 2x^3 - 15x^2 + 36x + 10$

On differentiating y wrt x, we get

$$\frac{dy}{dx} = 6x^2 - 30x + 36 \quad ...(i)$$

and $$\frac{d^2y}{dx^2} = 12x - 30 \quad ...(ii)$$

Now, substitute $\frac{dy}{dx} = 0$

$\Rightarrow 6x^2 - 30x + 36 = 0$

$\Rightarrow 6(x^2 - 5x + 6) = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x - 3) - 2(x - 3) = 0$

$\Rightarrow (x - 3)(x - 2) = 0$

$\Rightarrow x = 3$ and $x = 2$

Now, put $x = 3$ in (ii), we get

$\frac{d^2y}{dx^2} = 36 - 30 = 6 > 0$, attaining minima

Now, put $x = 2$ in (ii), we get

$\frac{d^2y}{dx^2} = 24 - 30 = -6 < 0$, attaining maxima

Therefore, the given function attains maxima at $x = 2$ and minima at $x = 3$.

Hence, the correct option is (a).

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APPLICATIONS OF DIFFERENTIAL CALCULUS

Differentiation is a powerful mathematical tool that is applied in various scenarios to analyze and calculate important quantities related to rates of change and cost functions.

For example:

- Rate of change in price
- Change in demand with respect to change in output
- Change in revenue obtained with respect to change in price
- Change in demand with respect to change in income

Few things to be taken care of:

- Rate of Change in Quantities: Let there two quantities x and y such that rate of change in y is dependent of x then $\frac{dy}{dx}$ represents the rate of change of y with respect to x.
- Always remember that, "rate of Change of a function" means the derivative of the function.
- We can also write f(x) in place of y and similarly f'(x) in place of $\frac{dy}{dx}$.

Cost Function: Total cost consists of two parts (i) Variable Cost (ii) Fixed Cost. If C(X) denotes the cost producing x units of a product then C(x) = V(x) + F(x) , where V(x) denotes the variable cost and F(x) is the fixed cost. Variable cost depends upon the number of units produced (i.e value of x) whereas fixed cost is independent of the level of output

The formulas for average cost (AC or C), average variable cost (AVC), and average fixed cost (AFC) are as follows:

- Average cost (AC or C) $= \frac{C(x)}{x}$
- Average variable cost (AVC) $= \frac{V(x)}{x}$
- Average Fixed Cost (AFC) $= \frac{F(x)}{x}$

Marginal Cost: If C(x) the total cost producing x units then the increase in cost in producing one more unit is called marginal cost at an output level of x units and is given as Marginal Cost (MC).

- Marginal Cost (MC) = Rate of change in cost C per unit change in output at an output level of x units $= \frac{dC}{dx}$.
- To increase profits, a company may decide to increase its production. The question that concerns the management is how will the cost be affected by an increase in production. Economists use the marginal cost to answer the question.

REVENUE FUNCTION

The revenue function, R(x) represents the total amount of money obtained (total turnover) by selling x units of a product. If x units are sold at a price of 'P' per unit, then the revenue function is given by: R(x) = P.x

MARGINAL REVENUE

The marginal revenue (MR) is the rate of change of revenue with respect to a unit change in output. If R is the revenue and x is the output, then the marginal revenue is given by:

$$MR = \frac{dR}{dx}$$

PROFIT FUNCTION

The profit function, P(x), represents the difference between the total revenue, R(x), and the total cost, C(x). It is expressed as:

P(x) = R(x) - C(x)

MARGINAL PROFIT

The marginal profit is the rate of change of profit with respect to a unit change in output. It is given by:

$$\text{Marginal Profit} = \frac{dP}{dx}$$

Example 26. Find the marginal cost function if the total cost function to produce x units is defined by:

$$C(x) = \frac{3x+2}{x+4}$$

(a) $\frac{2x}{(x+4)^2}$ (b) $\frac{10}{(x+4)^2}$ (c) $\frac{10}{(x+4)}$ (d) None of these

Sol. (b) We know that,

Marginal cost = Rate of change in cost C per unit change in output which is given by $\frac{dC}{dx}$

Given, $C(x) = \frac{3x+2}{x+4}$

Thus, differentiating it w.r.t x, we get

$$C'(x) = \frac{(x+4)\frac{d}{dx}(3x+2) - (3x+2)\frac{d}{dx}(x+4)}{(x+4)^2}$$

$$= \frac{(x+4)(3)-(3x+2)(1)}{(x+4)^2} = \frac{3x+12-3x-2}{(x+4)^2} = \frac{10}{(x+4)^2}$$

Hence, option (b) is correct.

Example 27. The cost function for the production of x units of a commodity is given by $C(x) = x^2 - 24x + 600$. The cost will be minimum when x will be

(a) 12 (b) 14 (c) 15 (d) 20

Sol. (a) Given, $C(x) = x^2 - 24x - 600$

Differentiating w.r.t x, we get

$C'(x) = 2x - 24$

Again differentiating, we get

$C''(x) = 2 > 0$

For maxima or minima, put $C'(x) = 0$

$\Rightarrow 2x - 24 = 0 \Rightarrow 2x = 24 \Rightarrow x = 12$

Thus, the function is minimum at $x = 12$.

Hence, the correct option is (a) i.e., 12.

Example 28. If the cost function is given by $C(x) = 15x + 0.055x^2$ and the demand function is given by $p(x) = 100 - 0.055x$, where x is the number of units produced, then the marginal profit function when $x = 50$ is

(a) 85 (b) 11 (c) 74 (d) 25

Sol. (c) We know that,

Revenue function = p(x). x

$= (100 - 0.055x)x$

$= 100x - 0.055x^2$

Thus, Profit function $P(x) = R(x) - C(x)$

$= 100x - 0.055x^2 - (15x + 0.055x^2)$

$= 85x - 0.11x^2$

Now, marginal profit function is given by

$P'(x) = 85 - 0.22x$

Therefore, $P'(50) = 85 - 0.22(50)$

$= 85 - 11 = 74$

Hence, the correct option is (c) i.e., 74.

Example 29. The cost function of a company $C(x) = 100x - 2x^2 + 120$, $0 \le x \le 100$ and the revenue function $R(x) = 500 - x$, then the profit function is

(a) $2x^2 - 101x + 380$ (b) $2x^2 - 101x + 620$

(c) $x^2 - 101x + 380$ (d) None of these

Sol. (a) Given, $C(x) = 100x - 2x^2 + 120$

$R(x) = 500 - x$

Thus, the profit function = $P(x) = R(x) - C(x)$

$= 500 - x - (100x - 2x^2 + 120) = 500 - x - 100x + 2x^2 - 120 = 2x^2 - 101x + 380$

Hence, option (a) is correct.

Example 30. A computer software company wishes to start the production of floppy disks. It was observed that the company had to spend ₹2 lakhs for the technical informations. The cost of setting up the machine is ₹88,000 and the cost of producing each unit is ₹30, while each floppy could be sold at ₹45. Find:

(I) the total cost function for producing x floppies; and

(II) the break-even point.

(a) 30x and 19,200 (b) 30x + 88,000 and 19,200

(c) 30x + 2,88,000 and 19,200 (d) None of these

Sol. (c) Given, Amount spend on technical information = ₹2,00,000

Amount spend on setting up the machine = ₹88,000

Cost of production of each unit = ₹30

(I) Thus, total cost function, C(x) for producing 'x' floppies

$= 30x + 2,00,000 + 88,000 = 30x + 2,88,000$

(II) Since, each floppy is sold for ₹45

Thus, Revenue function, R(x) on selling 'x' floppies = 45x

For break-even point, R(x) =C(x)

$\Rightarrow 45x = 30x + 2,88,000 \Rightarrow 15x = 2,88,000$

$\Rightarrow x = 19,200$, which is the break-even point.

Hence, the correct option is (c).

Example 31. If the demand function of a product is given by p = 200 - 5x, where p is the price per unit and x is the quantity demanded, then the marginal revenue function is given by

(a) $200x - 5x^2$ (b) 200 - 10x (c) -10x (d) None of these

Sol. (b) Given, Demand function (p) = 200 - 5x

We know,

Revenue function $R = p.x = (200 - 5x)\, x$

$= 200x - 5x^2$

Therefore, Marginal revenue function $= \frac{dR}{dx} = \frac{d}{dx}(200x - 5x^2) = 200 - 10x$

Hence, the correct option is (b).

Example 32. If the cost function is given by $C(x) = (x^2 + 3)^3$, where x denotes the number of items produced, then the marginal average function at x = 10 is

(a) 51626 (b) 52726 (c) 62726 (d) 52420

Sol. (b) Given, $C(x) = (x^2 + 3)^3$

Since, Average cost $(AC) = \frac{C(x)}{x} = \frac{(x^2+3)^3}{x}$

We know that,

Marginal average cost function is given by

$$\frac{d}{dx}(AC) = \frac{x \times 3(x^2+3)^2(2x) - (x^2+3)^3 \cdot 1}{x^2}$$

$$= \frac{6x^2\ (x^2+3)^2 - (x^2+3)^3}{x^2} = \frac{(x^2+3)^2(6x^2 - x^2 - 3)}{x^2}$$

$$= \frac{(x^2+3)^2(5x^2-3)}{x^2}$$

Now, at x = 10

$$AC' = \frac{(10^2+3)^2\left(5(10)^2-3\right)}{(10)^2} = \frac{(103)^2(497)}{100} = 52726$$

Hence, the correct option is (b) i.e., 52726.

Example 33. The cost function for the production of x units of a commodity is given by $C(x) = 2x^3 - 15x^2 + 36 + 15$. The cost will be minimum when 'x' equal to

(a) 3 (b) 2 (c) 1 (d) 4

Sol. (a) Given, $(x) = 2x^3 - 15x^2 + 36x + 15$

Differentiating w.r.t x, we get

$C'(x) = 6x^2 - 30x + 36$

Again on differentiating, we get

$C''(x) = 12x - 30$

For minima or maxima, put $C'(x) = 0$ i.e.

$\Rightarrow 6x^2 - 30x + 36 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow (x - 2)(x - 3) = 0$

$\Rightarrow x = 2, x = 3$

Now, at x = 2

$C''(x) = 12(2) - 30 = -6 < 0$

Thus, C(x) is maximum at x = 2.

Now, at x = 3

$C''(x) = 12(3) - 30 = +6 > 0$

Thus, C(x) is minimum at x = 3

Therefore, the cost will be minimum at x = 3.

Hence, the correct option is (a) i.e., 3.

Example 34. The cost function of production is given by $C(x) = 100x - 8x^2 + \frac{x^3}{3}$ where x denotes the number of items produced. The level of output for which marginal cost is minimum and level of output for which the average cost is minimum are given by, respectively.

(a) 12 and 14 (b) 8 and 12 (c) 10 and 15 (d) None of these

Sol. (b) Given, Cost function $C(x) = 100x - 8x^2 + \frac{x^3}{3}$

Thus,

$$M(x) = \text{Marginal Cost} = C(x) = \frac{d}{dx}\left(100x - 8x^2 + \frac{x^3}{3}\right) = 100 - 16x + x^2$$

$$A(x) = \text{Average Cost} = \frac{C(x)}{x} = 100 - 8x + \frac{x^2}{3}.$$

(i) M(x) is maximum or minimum when

$M'(x) = -16 + 2x = 0$

$\text{or}, x = 8.$

$[M''(8) = M''(x)]_{x=8} = [2]_{x=8} = 2 > 0$

Hence, marginal cost is minimum at x = 8.

(ii) A(x) is maximum or minimum when

$$A'(x) = -8 + \frac{2x}{3} = 0$$

$\text{or}, x = 12.$

$$A''(12) = A''(x)]_{x=12} = \frac{2}{3}\Big]_{x=12} = \frac{2}{3} > 0$$

Hence, marginal cost is minimum at x = 12.

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART A)

1. Differentiate each of the following function with respect to x:

(i) $5x^2$ (ii) $x^5 + x^2$ (iii) $3^x x^4$ (iv) $\frac{x^2}{\log x}$

(v) $\frac{e^x}{x}$ (vi) $5^x e^x$ (vii) $\frac{4x}{3x^2+5}$ (viii) $\frac{x^3+1}{x^3}$

(ix) $(x^2 - 3x + 7)(2x^2 + 5x + 9)$ (x) $3^x \cdot x^3$

2. Differentiate $(5x + 4)^5$ with respect to x.

(a) $25(5x + 4)^5$ (b) $25(5x + 4)^4$ (c) $5(5x + 4)^4$ (d) None of these

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3. If $y = \log(5x^2 + 7)$, find $\frac{dy}{dx}$.

(a) $\frac{1}{5x^2+7}$ (b) $\frac{5}{5x^2+7}$ (c) $\frac{10x+7}{5x^2+7}$ (d) $\frac{10x}{5x^2+7}$

4. Differentiate e^{2x^2+3x+1} with respect to x.

(a) $(4x+3)e^{2x^2+3x+1}$ (b) $(4x)e^{2x^2+3x+1}$

(c) $(2x^2+3x)e^{2x^2+3x+1}$ (d) None of these

5. Find the derivative of $\log(\log x)$.

(a) $\frac{1}{x\log x}$ (b) $\frac{1}{x}$ (c) $\frac{1}{\log x}$ (d) None of these

6. Find the derivative of $(\log x)^3$.

(a) $3 \log x^2$ (b) $\frac{3(\log x)^2}{x}$ (c) $\frac{1}{x^3}$ (d) None of these

7. Find $\frac{dy}{dx}$ when $x^5 + y^5 = 5ax^2y^2$

(a) $\frac{x(2ay^2-x^3)}{y(y^3-2ax^2)}$ (b) $\frac{y-3x^2}{3y^2-x}$ (c) $\frac{y-3x^2}{3y^2-x^2}$ (d) None of these

8. Find the derivative of the function: $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

(a) $\frac{y^{\frac{2}{3}}}{y^{\frac{2}{3}}}$ (b) $\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$ (c) $-\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$ (d) None of these

9. Let $xy = 1$, then $y^2 + \frac{dy}{dx} = ?$ **(Nov 2018)**

(a) 1 (b) 0 (c) 2 (d) None of these

10. If $x^3 - 2x^2y^2 + 5x + y = 5$, then $\frac{dy}{dx}$ at $x = 1$ and $y = 1$ is

(a) $\frac{4}{3}$ (b) $-\frac{5}{4}$ (c) $\frac{4}{5}$ (d) $-\frac{4}{3}$

11. Find $\frac{dy}{dx}$ for $x^2y^2 + 3xy + y = 0$

(a) $-\frac{2xy^2 + 3y}{2x^2y + 3x + 1}$ (b) $\frac{2xy - 3y}{2x^2y + 3x + 1}$ (c) $\frac{2yx^2+3y}{2y^2x-3x+1}$ (d) None of these

12. Let $x = at^3, y = \frac{a}{t^2}$, then $\frac{dy}{dx}$ is **(Nov 2018)**

(a) $\frac{-3a}{t^6}$ (b) $\frac{-1}{t^6}$ (c) $\frac{1}{3at^2}$ (d) None of these

13. If $x = t^3 - t$ and $y = 4 - t^2$, then $\frac{dy}{dx}$ is

(a) $\frac{-2t}{3t^2-1}$ (b) $\frac{2t}{3t^2-1}$ (c) $\frac{1}{3t^2-1}$ (d) None of these

14. If $x = t^2$ and $y = t^3$, then $\frac{dy}{dx} = ?$

(a) $\frac{3t}{2}$ (b) $\frac{2}{3t}$ (c) t (d) None of these

15. If $x = 2t + 5$ and $y = t^2 - 5$, then $\frac{dy}{dx} = ?$

(a) t (b) $-\frac{1}{t}$ (c) $\frac{1}{t}$ (d) 0

16. If $x = \frac{3at}{1+t^2}$ and $y = \frac{3at^2}{1+t^2}$ then $\frac{dy}{dx} = ?$

(a) $\frac{2at}{1+t^2}$ (b) $\frac{2t}{1+t^2}$ (c) $\frac{t}{1+t}$ (d) $\frac{3at}{1+t}$

17. If $x^y = e^{x+y}$, then the value of $\frac{dy}{dx}$ is

(a) $\frac{y(x-1)}{x(1-y)}$ (b) $\frac{x(x-1)}{y(1-y)}$ (c) $\frac{y(y-1)}{x(1-x)}$ (d) None of these

18. If $x^m \cdot y^n = (x + y)^{m+n}$, then the value of $\frac{dy}{dx}$ is

(a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) $\frac{1}{xy}$ (d) None of these

19. If $y = \left(\frac{1}{x}\right)^x$, then $\frac{dy}{dx}$ is

(a) $-y(1 + \log x)$ (b) $y(1 + \log x)$ (c) $x(1 + \log y)$ (d) None of these

20. If $y = x^x$, then $\frac{dy}{dx}$ at $x = 1$ is equal to **(Dec 2022)**

(a) 0 (b) 1 (c) -1 (d) 2

21. If $x^y = y^x$ then $\frac{dy}{dx}$ will be equal to

(a) $\frac{x(x-y\log x)}{y(y-x\log x)}$ (b) $\frac{y(y-x\log y)}{x(x-y\log x)}$ (c) $\frac{y-x\log y}{x-y\log x}$ (d) None of these

22. The minima of the function $y = x^5 - 5x^4 + 5x^3 - 1$ occur at

(a) $x = 1$ (b) $x = 3$ (c) $x = 5$ (d) None of these

23. The maxima of the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 1$ is at

(a) $x = 1$ (b) $x = 3$ (c) $x = 2$ (d) All of these

24. The maxima and minima of the function $f(x) = 2x^3 - 15x^2 + 36x + 18$ occur respectively at

(a) $x = 2$ and $x = 3$ (b) $x = 2$ and $x = -3$

(c) $x = -2$ and $= -3$ (d) None of these

25. If the cost of producing x units is given by the function $C(x) = x^3 + 10x^2 - 30x + 50$, then the average cost function is

(a) $x^3 + 10x^2 - 30x + 50$ (b) $x^3 + 10x^2 - 30x$

(c) $3x^2 + 20x^2 - 30$ (d) $x^2 + 10x - 30 + \frac{50}{x}$

26. The cost function of a company $C(x) = 100x - 2x^2 + 120$, $0 \le x \le 100$ and the revenue function $R(x) = 500 - x$, then the profit function is

(a) $2x^2 - 101x + 380$ (b) $2x^2 - 100x + 620$

(c) $x^2 + 101x + 380$ (d) None of these

27. If the cost function of a firm is given by $C(x) = 3x^2 + 5x + 2$, where x is the output. Find the average cost and the marginal cost.

(a) $3x + \frac{2}{x}$ and $6x + 5$ (b) $3x + 5 + \frac{2}{x}$ and $6x + 5$

(c) $3x^3 + 5x^2 + 2x$ and $6x^2 + 5$ (d) None of these

28. If the demand function is $p(x) = 50 - 2x$, then the marginal revenue function is given by

(a) $50 - 4x$ (b) $50 - 2x$

(c) $50x - 4x^2$ (d) None of these

29. The total cost function of a firm is: $C(x) = 0.0005x^3 - 0.03x^2 + 6x + 2000$, where x is the output. The marginal average cost will be

(a) $0.001x - 0.03 - \frac{2000}{x^2}$ (b) $0.0015x^2 - 0.03x$

(c) $0.001x - 0.03 + 6 - \frac{2000}{x^2}$ (d) None of these

30. The cost function of a good is $2Q + 6 + \frac{13}{Q}$, where Q is the quantity produced. The approx. cost at $Q = 15$ is

(a) 42 (b) 36 (c) 66 (d) 130

31. The profit gained by selling x units of an item is given by $P(x) = \frac{x^2}{x-1}$. Find the marginal profit if 10 units are sold.

(a) 0.987 (b) 20.87 (c) 0.5 (d) 10.5

32. The cost function is given by $C(x) = 0.00002x^3 - 0.03x^2 + 300x + 78000$ and the demand function is $p(x) = 500 - 0.05x$. Then the marginal profit at $x = 2500$ is

(a) -275 (b) -375 (c) 575 (d) None of these

33. The total revenue received from the sale of x units of a product is given by $R(x) = 20x - \frac{x^2}{2}$. The marginal revenue when $x = 5$ is

(a) 5 (b) 10 (c) 15 (d) None of these

34. If $C = 0.05q^2 + 16 + \frac{100}{q}$ is the manufacturer's average cost function, then the marginal cost when 50 units are produced is

(a) 16 (b) 165 (c) 281 (d) 391

35. If the demand function of the monopolist is given by $p = 200 - 0.5q$ and its cost function is given by $C = 100 + 5q + 7q^2$, then the value of q at which the profit is maximum is

(a) 8 (b) 13 (c) 20 (d) None

Answer Key

1. (i) $10x$ (ii) $5x^4 + 2x$ (iii) $x^3 3^x (4 + x\log 3)$ (iv) $\frac{2x}{\log x} - \frac{x}{(\log x)^2}$

(v) $\frac{e^x(x-1)}{x^2}$ (vi) $5^x e^x (1 + \log 5)$ (vii) $\frac{20 - 12x^2}{(3x^2+5)^2}$

(viii) $2x - \frac{1}{x^2}$ (ix) $8x^3 - 3x^2 + 16x + 8$ (x) $(3 + x \log 3)\, 3^x x^2$

2. (b)	**3.** (d)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (a)	**11.** (a)
12. (d)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (a)	**20.** (b)	**21.** (b)
22. (b)	**23.** (c)	**24.** (a)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (b)	**31.** (a)
32. (a)	**33.** (c)	**34** (d)	**35.** (b)						

PREVIOUS YEAR QUESTION (PYQ'S)

1. If the given cost function of commodity is given by $C(x) = 150x - 5x^2 + \frac{x^3}{6}$, where C stands for cost and x stands for output, if the average cost is equal to the marginal cost then the output x= **(June 2019)**

(a) 5 (b) 10 (c) 15 (d) 20

2. If $2^x - 2^y = 2^{x-y}$, then $\frac{dy}{dx}$ at $x = y = 2$ **(June 2019)**

(a) 1 (b) 2 (c) 4 (d) 5

3. $f(x) = \begin{cases} x, x < 0 \\ 0, x = 0 \\ x^2, x > 0 \end{cases}$ then find $\lim_{x \to 0} f(x)$ **(Dec 2019)**

(a) 1 (b) 0 (c) –2 (d) 2

4. Find the value of $\frac{dy}{dx}$ if $y = x^x$. **(Dec 2019)**

(a) $x^x \log x$ (b) $1 + \log x$ (c) $y \log x$ (d) None of these

5. If $f(x) = a(x^2 + x + 1)^2$ and $f^1(-1) = -6$, then the value of a = **(Dec 2019)**

(a) 1 (b) 2 (c) 3 (d) 4

6. If $y = x(x-1)(x-2)$ then $\frac{dy}{dx}$ is **(Dec 2020)**

(a) -6x (b) $3x^2-6x+2$ (c) $6x + 4$ (d) $3x^2 - 6x$

7. The cost function of a good is $2Q+6+\frac{13}{Q}$, where Q is the quantity produced. The approx cost at Q = 15 is **(Dec 2020)**

(a) 42 (b) 36 (c) 66 (d) 130

8. The cost function of production is given by $C(x) = \frac{x^3}{2} - 15x^2 + 36x$ where x denotes the number of items produced. The level of output for which marginal cost is minimum and the level of output for which the average cost is minimum are given by, respectively **(Jan 2021)**

(a) 10 and 15 (b) 10 and 12 (c) 12 and 15 (d) 15 and 10

9. The cost function $C(x) = 125 + 500x - x^2 + \frac{x^3}{3}, 0 \le x \le 100$ and the demand function for the items is given by, p(x)=1500-x, then the marginal profit when 18 items are sold is **(July 2021)**

(a) 751 (b) 571 (c) 676 (d) 875

10 If $f(x) = 3e^{x^4}$, then $f'(x)-4x^3 f(x) + \left(-\frac{1}{3}\right) f(0) - f'(0)$ is equal to **(July 2021)**

(a) 0 (b) e^{x^2} (c) 1 (d) -1

11. The cost of producing x units is $500 - 20x^2 + \frac{x^3}{3}$. The material cost is minimum at x = _____. **(Dec 2021)**

(a) 5 (b) 10 (c) 20 (d) 50

12. If $y = \frac{x^4}{e^x}$, then $\frac{dy}{dx}$ is equal to; **(Dec 2021)**

(a) $\frac{x^3(4-x)}{(e^x)^2}$ (b) $\frac{x^3(4-x)}{e^x}$ (c) $\frac{x^2(4-x)}{e^x}$ (d) $\frac{x^2(4-x)}{e^x}$

13. The speed of a train at a distance x (from the starting point) is given by $3x^2 - 5x + 4$. What is the rate of change (of distance) at x = 1? **(Dec 2021)**

(a) -1 (b) 0 (c) 1 (d) 2

14. Which of the following is the differentiation of $e^t (\log_e t)$ with respect to 't'? **(June 2022)**

(a) $e^t (t.\log_e t)$ (b) $\frac{e^t (1 + t.\log_e t)}{t}$

(c) $\frac{e^t}{t}$ (d) $e^t (1 - \log_e t)$

15. The maxima and minima of the function $y = 2x^3 - 15x^2 + 36x + 10$ occurs respectively at **(Dec 2022)**

(a) x = 2 and x = 3 (b) x = 1 and x = 3

(c) x = 3 and x = 2 (d) x = 3 and x = 1

16. If $y = x^2$, then $\frac{dy}{dx}$ at x=1 is equal to **(Dec 2022)**

(a) 0 (b) 1 (c) -1 (d) 2

17. If $x^5 + y^5 - 5xy = 0$, then $\frac{dy}{dx}$ is **(Dec 2022)**

(a) $\frac{y + x^4}{x + y^4}$ (b) $\frac{y - x^4}{y^4 - x}$ (c) $\frac{x - y^4}{x^4 + y}$ (d) $\frac{x + y^4}{x^4 + y}$

18. For a given curve $y = 2 - x^2$, when 'x' increases at the rate of 3 units/s, then the slope of curve will be: **(June 2023)**

(a) increase at 6 units/s (b) increase at 3 units/s

(c) decrease at 6 units/s (d) decrease at 3 units/s

19. If $y = \frac{x}{x+5}$, then $\frac{dx}{dy}$ is equal to **(June 2023)**

(a) $\frac{5}{(1-y)^2}$ (b) $\frac{5}{(1+y)^2}$ (c) $\frac{3}{(1-y)^2}$ (d) $\frac{3}{(1+y)^2}$

20. If x y = 1, then $y^2 + \frac{dy}{dx}$ is equal to **(June 2023)**

(a) 1 (b) 0 (c) -1 (d) $\frac{1}{2}$

21. If $x = 5t^2 + 3$ and $y = 2t^2 + 1$, then $\frac{dy}{dx}$ can be calculated as **(Dec 2023)**

(a) 2 (b) 1 (c) 2t (d) $\frac{2}{5}$

22. If $x^3 + y^3 - 3axy = 0$, then $\frac{dy}{dx}$ is equal to **(Dec 2023)**

(a) $\frac{ay - x^2}{y^2 - ax}$ (b) $\frac{x^2 - ay}{y^2 - ax}$ (c) $\frac{ay - x^2}{ax - y^2}$ (d) $\frac{x^2 - ay}{ax - y^2}$

23. If $f(x) = (x - 1)(x)(x + 1)$ then $\frac{dy}{dx} = ?$ **(June 2024)**

(a) $3x^2 - 1$ (b) $3x^2 + 1$ (c) $x^2 - 3$ (d) $x^2 + 3$

24. If $f(x) = (x + 1)^{x+1}$, then find $f'(0)$ **(June 2024)**

(a) 0 (b) 1 (c) 2 (d) –1

25. If a function is given by $f(x) = e^{3x}$, what is the derivative of the function? **(June 2024)**

(a) $3e^{3x}$ (b) e^{3x} (c) $3xe^{3x}$ (d) $3e^{3x} + 3$

26. Find $\frac{dy}{dx}$ for $x^2y^2 + y = 0$ **(June 2024)**

(a) $\frac{-2y^2x}{2y^2x^2 + 1}$ (b) $\frac{-2y^2x}{2yx^2 + 1}$ (c) $\frac{-2y^2x}{2y^2x^2}$ (d) $\frac{2y^2x}{2y^2x^2}$

27. If $xy \times yx = 16$ then the value of $\frac{dy}{dx}$ at (2, 2) is: **(Sept 2024)**

(a) –1 (b) 0 (c) 2 (d) –2

28. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to: **(Sept 2024)**

(a) $\frac{3t}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3}{2}$

29. If $x = at^2$ and $y = a(t^3 - t)$ then $\frac{dy}{dx} =$ **(Jan 2025)**

(a) $\frac{3t^2 - t}{2t}$ (b) $\frac{3t^2 - 1}{2t}$ (c) $\frac{3t^2 - t}{t}$ (d) $\frac{3t^2 + t}{2t}$

30. The marginal revenue function for a product $MR = 5 - 4x + 3x^2$. Then the total revenue function is **(Jan 2025)**

(a) $5x - 2x^2 + x^3$ (b) $5x + 2x^2 + x^3$

(c) $5x + 2x^2 + x^3 + 3$ (d) $5x - 2x^2 - x^3$

31. Determine f(x) given that $f'(x) = 12x^2 - 4x$ and $f(-3) = 17$ **(May 2025)**

(a) $f(x) = 4x^3 - 2x^2 + 143$ (b) $f(x) = 6x^2 - x^4 + 137$

(c) $f(x) = 3x^4 - x^3 - 137$ (d) $f(x) = 4x^3 - 2x^2 - 143$

32. What is the differential function of $\sqrt{x^2+2}$? **(May 2025)**

(a) $x\sqrt{x^2+2}dx$ (b) $\left[\frac{x}{\sqrt{x^2+2}}\right]dx$ (c) $\left[\frac{x^2}{\sqrt{x^2-2}}\right]dx$ (d) $\left[\frac{-x}{x^2+2}\right]dx$

33. Find $\frac{dy}{dx}$ where $x = \frac{e^t + e^{-t}}{2}$ and $y = \frac{e^t - e^{-t}}{2}$ **(May 2025)**

(a) $\frac{y}{x}$ (b) $\frac{x}{y}$ (c) $\frac{e^t}{e^{-t}}$ (d) $\frac{1}{et}$

34. Find $\frac{dy}{dx}$ for $x^2y^2 + y = 0$. **(Sept 2025)**

(a) $\frac{2y^2x}{2y^2x^2+1}$ (b) $\frac{-2y^2x}{2yx^2+1}$ (c) $\frac{-2y^2x+1}{2y^2x^2}$ (d) $\frac{2y^2x-1}{2y^2x^2}$

35. The cost function of an organization is given by $C(x) = 500 - 5x^2 + \frac{x^2}{3}$, where x denotes the output. Find the level of output at which the marginal cost is the minimum. **(Sept 2025)**

(a) 5 (b) 4 (c) 10 (d) 6

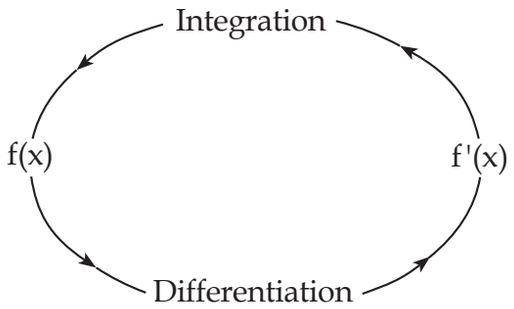
Answer Key

1.	(c)	**2.**	(a)	**3.**	(b)	**4.**	(a)	**5.**	(c)	**6.**	(b)	**7.**	(b)	**8.**	(a)	**9.**	(c)	**10.**	(c)
11.	(c)	**12.**	(b)	**13.**	(d)	**14.**	(b)	**15.**	(a)	**16.**	(b)	**17.**	(b)	**18.**	(c)	**19.**	(a)	**20.**	(b)
21.	(d)	**22.**	(d)	**23.**	(a)	**24.**	(b)	**25.**	(a)	**26.**	(b)	**27.**	(a)	**28.**	(b)	**29.**	(b)	**30.**	(a)
31.	(a)	**32.**	(b)	**33.**	(b)	**34.**	(b)	**35.**	(a)										

Chapter 8 - Unit-2: Integral Calculus

What is Integration and why do we need to study?

The below figure represents the relation between Integration and Differentiation.



We can say that integration is the inverse process of differentiation which is denoted with the symbol $\int$.

So, if we have to find integration of x^n

Then, integral of x^n with respect to x is equal to $\frac{x^{n+1}}{n+1}$

Thus, if we differentiate $\frac{x^{n+1}}{n+1}$ we will get back x^n.

Again if we differentiate $\frac{x^{n+1}}{n+1}+C$, where C being a constant then we will get back the same x^n

i.e., $\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}+C\right]=x^n$

Hence, $\int x^n dx=\frac{x^{n+1}}{n+1}+C$, where C is constant of integration.

Remember; always add constant c in an integration result.

Integral calculus was primarily invented to determine the area bounded by the curves dividing the entire area into an infinite number of infinitesimal small areas and taking the sum of all these small areas.

BASIC FORMULAE FOR INTEGRATION

1. $\int x^n dx=\frac{x^{n+1}}{n+1}+c, n\neq -1$
2. $\int dx=x, \text{since} \int 1\,dx=\int x^0 dx=\frac{x^1}{1}=x$
3. $\int e^x dx=e^x+c$
4. $\int e^{ax} dx=\frac{e^{ax}}{a}+c$
5. $\int \frac{dx}{x}=\log_e x+c$
6. $\int a^x dx=\frac{a^x}{\log_e a}+c$

ELEMENTARY RULES

$\int c f(x)dx = c\int f(x)dx$, where c is constant

$\int \{f(x)dx \pm g(x)\}dx = \int f(x)dx \pm \int g(x)dx$

Example 1. Evaluate $\int 5x^2dx$

(a) $\frac{5}{3x^3}+k$ (b) $\frac{5x^3}{3}+k$ (c) $5x^3$ (d) None of these

Sol. (b) To evaluate the integral $\int 5x^2dx$

We know that, $\int x^n\,dx = \frac{x^{n+1}}{n+1}+k$, where k is the constant of integration

Thus, $\int 5x^2dx = \frac{5x^3}{3}+k$

Hence, option (b) is correct.

Example 2. Evaluate $\int (x^2-1)dx$.

(a) $\frac{x^5}{5}-\frac{2}{3}x^3+x+k$ (b) $\frac{x^3}{3}-x+k$

(c) 2x (d) None of these

Sol. (b) Given, $\int (x^2-1)dx$

As we know, if $f(x) = x^n$

$\Rightarrow \int f(x)dx = \frac{x^{n+1}}{n+1}+C$ [C → Arbitrary Constant]

$\Rightarrow \int (x^2-1)dx = \int x^2\,dx - 1\int dx$

$= \frac{x^{2+1}}{2+1}-x+k$

$= \frac{x^3}{3}-x+k$ [k → Arbitrary Constant]

Hence, the correct option is (b) i.e., $\frac{x^3}{3}-x+k$.

Example 3. Find the following integrals **(ICAI)**

I. $\int \sqrt{x}\,dx$

(a) $\frac{2}{3}x^{3/2}+k$ (b) $\frac{3}{2}x^{3/2}+k$ (c) $x^{3/2}+k$ (d) None of these

II. $\int \frac{x\sqrt{x}}{2}$

(a) $\frac{2}{5}x^{\frac{5}{2}}+k$ (b) $\sqrt{2x}+k$ (c) $\frac{1}{3\sqrt{x}}+k$ (d) $\frac{1}{5}x^{\frac{5}{2}}+k$

III. $\int\left[\sqrt{x}-\frac{1}{\sqrt{x}}\right]dx$ **(ICAI)**

(a) $\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+k$ (b) $\frac{2}{3}\sqrt{x}-2\sqrt{x}+k$

(c) $\frac{1}{2\sqrt{x}}+\frac{1}{2x\sqrt{x}}+k$ (d) None of these

Sol. I. (a) We know that,

$\int x^n\,dx=\frac{x^{n+1}}{n+1}+k$, where k is the constant of integration.

Thus, $\int x^{1/2}dx=\frac{2}{3}x^{3/2}+k$

Hence, option (a) is correct.

II. (d) $\int \frac{x\sqrt{x}}{2}=\int \frac{x^{1+\frac{1}{2}}}{2}dx=\frac{1}{2}\int x^{\frac{3}{2}}dx$

$=\frac{1}{2}.\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+k \;=\frac{1}{2}.\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+k \;=\frac{1}{5}x^{\frac{5}{2}}+k$

Therefore, the correct option is (d).

III. (a) Given, $\int\left[\sqrt{x}-\frac{1}{\sqrt{x}}\right]dx$

$\Rightarrow \int\sqrt{x}\,dx-\int\frac{1}{\sqrt{x}}dx$

As we know, if $f(x) = x^n$

$\Rightarrow \int f(x)dx=\frac{x^{n+1}}{n+1}+C$ [C → Arbitrary Constant]

$\Rightarrow \int x^{\frac{1}{2}}dx-\int x^{\frac{-1}{2}}dx$

$\Rightarrow \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+k$ [k → Arbitrary Constant]

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$$\Rightarrow \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + k$$

$$\Rightarrow \frac{2x^{\frac{3}{2}}}{3} - 2x^{\frac{1}{2}} + k$$

Hence, the correct answer is option (a).

Example 4. $\int (1-3x)(1+x)dx$ is equal to **(ICAI)**

(a) $x - x^2 - x^3$ (b) $x^3 - x^2 + x$ (c) $x - x^2 - x^3 + k$ (d) None of these

Sol. (c) As we know, if $f(x) = x^n$, then

$$\int f(x)dx = \frac{x^{n+1}}{n+1} + C$$ [C is arbitrary constant]

Given, $\int (1-3x)(1+x)dx$

Multiply and expanding the brackets,

$$= \int (1+x-3x-3x^2)dx$$

$$= \int (1-2x-3x^2)dx$$

$$= 1\int dx - 2\int xdx - 3\int x^2dx$$

$$= x - \frac{2x^2}{2} - \frac{3x^3}{3} + k$$ [K → Arbitrary Constant]

$= x - x^2 - x^3 + k$

Hence, the correct option is (c).

METHOD OF SUBSTITUTION (CHANGE OF VARIABLE)

It is sometimes possible by a change of independent variables to transform a function into another which can be reality integrated.

Example 5. Find the value of $\int (4x+5)^6 dx$ is equal to **(Dec 2019)**

(a) $\frac{1}{7}(4x+5)^7 + c$ (b) $\frac{1}{28}(4x+5)^7 + c$

(c) $\frac{1}{4}(4x+5)^7 + C$ (d) None of these

Sol. (b) Given, $f(x) = (4x + 5)^6$

Here, $I = (4x + 5)^6$

Let $4x + 5 = t$

$\Rightarrow 4dx = dt$

$\Rightarrow dx = \frac{1}{4}dt$

As we know,

If $f(x) = x^n$

$$\Rightarrow \int f(x)dx = \frac{x^{n+1}}{n+1} + C, \text{ where C is arbitrary constant}$$

On Integrating, we get

$$\Rightarrow I = \frac{1}{4}\int t^6 dt = \frac{1}{4}\left(\frac{t^{6+1}}{6+1}\right)$$

$$= \frac{1}{4}\left(\frac{t^7}{7}\right) + k, \text{ where k is arbitrary constant}$$

$$= \frac{t^7}{28} + k = \left(\frac{(4x+5)^7}{28}\right) + k$$

Hence, the correct answer is option (b).

Example 6. $\int (2x-3)^5 dx$ **(Dec 2022)**

(a) $\frac{(2x - 3)^6}{6}$ (b) $\frac{(2x - 3)^6}{2}$

(c) $\frac{(2x - 3)^6}{12}$ (d) $\frac{(2x - 3)^6}{3}$

Sol. (c) To evaluate : $\int (2x-3)^5 dx$

$$\text{Using}, \int x^n = \frac{x^{n+1}}{n+1} + C$$

Thus,

$$\int (2x-3)^5 = \frac{(2x-3)^{5+1}}{(5+1)\cdot\frac{d}{dx}(2x)} + c = \frac{(2x-3)^6}{6\times 2} + c = \frac{(2x-3)^6}{12} + c$$

where, c is constant of integration

Hence, the correct option is (c).

Example 7. Integrate w.r.t x, $(x^3 + 2)^2\, 3x^2$.

(a) $\frac{1}{3}(x^3+2)^3 + k$ (b) $3(x^3 + 2)^3 + k$

(c) $3x^2(x^3 + 2)^3 + k$ (d) $9x^2(x^3 + 2)^3 + k$

Sol. (a) To find: $\int (x^3+2)^2 3x^2 dx$

Substitute $x^3 + 2 = t$

$\Rightarrow 3x^2\, dx = dt$

Thus, $\int (x^3+2)^2 3x^2 dx$

$= \int t^2 dt$

$= \frac{t^{2+1}}{2+1} + k$ $\quad \because \int x^n dx = \frac{x^{n+1}}{n+1} + k, n \neq -1$

$= \frac{1}{3}t^3 + k = \frac{1}{3}(x^3+2)^3 + k$

Thus, the value of $\int (x^3+2)^2 3x^2 dx$ is $\frac{1}{3}(x^3+2)^3 + k$.

Hence, option (a) is correct i.e., $\frac{1}{3}(x^3+2)^3 + k$.

Example 8. Evaluate $\int \frac{(\log x)^2}{x} dx$.

(a) $\frac{(\log x)^3}{x^2} + c$ (b) $\frac{(\log x)^3}{3} + c$ (c) $\frac{\log x}{x} + c$ (d) None of these

Sol. (b) To evaluate: $\int \frac{(\log x)^2}{x} dx$

Put log x = t

then $\frac{1}{x} dx = dt$

Thus, $\int t^2 \, dt = \frac{t^3}{3} + c = \frac{(\log x)^3}{3} + c$, where c is the constant of integration

Hence, the correct option is (b).

Example 9. Find the following integral: $\int \frac{8x^2}{(x^3+2)^3} dx$ **(ICAI)**

(a) $\frac{-4}{3}(x^3+2)^2 + k$ (b) $\frac{-4}{3(x^3+2)^2} + k$

(c) $\frac{4}{3(x^3+2)^2} + k$ (d) None of these

Sol. (b) To find, $\int \frac{8x^2}{(x^3+2)^3} dx$

Let, $x^3 + 2 = t$

On differentiating both the sides w.r.t x,

$\Rightarrow 3x^2 = \frac{dt}{dx}$

$\Rightarrow x^2 \, dx = \frac{dt}{3}$ …(i)

Thus, $\int \frac{8x^2}{(x^3+2)^3}dx$

$\int \frac{8}{3t^3}dt$ (from i)

$= \frac{8}{3}\int \frac{dt}{t^3} = \frac{8}{3}\int t^{-3}dt$

$= \frac{8}{3}\left(\frac{t^{-3+1}}{-3+1}\right)+k$ [k → Arbitrary Constant]

$= \frac{8}{3}\left(\frac{t^{-2}}{-2}\right)+k = \frac{8}{3\times -2}\left(\frac{1}{t^2}\right)+k$

$= \frac{-4}{3}\left(\frac{1}{(x^3+3)^2}\right)+k$ [$\because x^3 + 2 = t$]

Hence, the correct option is (b).

Example 10. Solve $\int x^x(1+\log x)dx$ **(ICAI)**

(a) $x^x \log x + k$ (b) $e^{x^2}+k$ (c) $\frac{x^2}{2}+k$ (d) $x^x + k$

Sol. (d) Given, $f(x) = \int x^x(1+\log x)dx$

Let $x^x = t$

Taking log both sides,

$\Rightarrow \log x^x = \log t$ [$\because \log a^x = x \log a$]

$\Rightarrow x \log x = \log t$

Differentiation by product rule,

$\Rightarrow \left(x\times\frac{1}{x}+\log x\right) = \frac{1}{t}dt$

$\Rightarrow (1 + \log x)\, dx = \frac{1}{t}dt$

$\Rightarrow dx = \frac{dt}{t\,(1+\log x)}$

As we know, $f(x) = x^n$

$\int f(x)dx = \frac{x^{n+1}}{n+1}+C$ [C → Arbitrary Constant]

Using dx in the function,

$\Rightarrow \int t(1+\log x)\times\frac{dt}{t(1+\log x)}$

$\Rightarrow \int 1dt$

$\Rightarrow t + k$ [k → Arbitrary Constant]

Put the value of $x^x = t$

$\Rightarrow x^x + k$

Hence, the correct answer is option (d).

INTEGRATION BY PARTS

Integral of product of two functions:

= First function × Integral of second function $-\int$ (differentiation of first function × Integral of second function) dx

$$\int f(x).g(x)dx = f(x)\int g(x) - \int\left[\frac{df(x)}{dx} \times \int g(x)dx\right]dx$$

LAE method:

'L' stands for Logarithm function (log x, log x^3 etc.)

'A' stands for Algebraic function $(x^3, \sqrt{x}, x^5$ etc.)

'E' stands for Exponential function $(e^x, e^{\sqrt{x}}$ etc.)

Example 11.

I. $\int xe^x dx$

(a) $e^x + 1 + k$ (b) $xe^x + e^x + k$ (c) $xe^x - e^x + k$ (d) None of these

II. $\int x\log x\,dx$ **(ICAI)**

(a) $\frac{x^2}{2} + \log x + k$ (b) $\frac{x^2}{2}\log x - \frac{x^2}{4} + k$

(c) $\frac{x^2}{2}\log x - \frac{x}{4} + k$ (d) None of these

III. $\int x^2 e^{ax}$ **(ICAI)**

(a) $\frac{xe^{ax}}{a} - \frac{2xe^{ax}}{a} + \frac{2}{a^3}e^{ax} + k$ (b) $\frac{x^2e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + e^{ax} + k$

(c) $x^2e^{ax} - 2xe^{ax} + 2e^{ax} + k$ (d) $\frac{x^2e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2}{a^3}e^{ax} + k$

IV. $\int (\log x)^2 dx$

(a) $x (\log x)^2 - 2x \log x + 2x + k$ (b) $x (\log x)^2 - 2x + k$

(c) $2x \log x - 2x + k$ (d) None of these

Sol. I. (c) Here, according to 'LAE' rule

First function = x and second function $=e^x$

Thus, on Integrating by parts, we get

$$\int xe^x\,dx = x\int e^x - \int\left\{\frac{d}{dx}(x)\int e^x dx\right\}dx$$

$$\Rightarrow xe^x - \int 1.e^x dx = xe^x - e^x + k$$

Hence, option (c) is correct.

II. (b) Here, according to 'LAE' rule

First function = log x and second function = x

Integrating by parts we get :

$$\int x\log\ x\ dx = \log x\int x\,dx - \int\left\{\frac{d}{dx}(\log x)\int x\,dx\right\}dx$$

$$\Rightarrow \frac{x^2}{2}\log x - \int\frac{1}{x}.\frac{x^2}{2}dx$$

$$\Rightarrow \frac{x^2}{2}\log x - \frac{x^2}{4} + k$$

Hence, option (b) is correct.

III. (d) Integrating by parts we get :

$$\int x^2e^{ax}\,dx = x^2\int e^{ax} - \int\left\{\frac{d}{dx}(x^2)\int e^{a\,x}dx\right\}dx$$

$$\Rightarrow \frac{x^2}{a}e^{ax} - \int 2x\frac{e^{ax}}{a}dx = \frac{x^2}{a}e^{ax} - \frac{2}{a}\left\{\frac{xe^{a\,x}}{a} - \int 1.\frac{e^{a\,x}}{a}dx\right\}$$

$$\Rightarrow \frac{x^2e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2}{a^3}e^{ax} + k$$

Hence, option (d) is correct.

IV. (a) Given, $\int(\log x)^2\,dx = \int(\log x)^2\ .1\,dx$

Here, according to 'LAE' rule

First function = $(\log x)^2$ and second function = 1

Integration by parts is given as,

$$\int u\,v\,dx = u\int v\,dx - \int u'\left(\int v\,dx\right)dx$$

According to 'LAE' rule,

$u = (\log x)^2$, $v = 1$

As we know, if $f(x) = x^n$,

$$\int f(x)dx = \frac{x^{n+1}}{n+1} + C \qquad [C \rightarrow \text{Arbitrary Constant}]$$

$$\int (\log x)^2 \cdot 1 = (\log x)^2 \int 1dx - \int \frac{d}{dx}(\log x)^2 \left(\int 1\,dx\right)dx$$

$$= x(\log x)^2 - \int 2\log x \cdot \frac{1}{x} \cdot x\,dx \qquad \left[\because \frac{d}{dx}\log x = \frac{1}{x}\right]$$

$$= x(\log x)^2 - 2\int \log x \cdot 1\,dx$$

Again, Integrating by parts,

$$\Rightarrow x(\log x)^2 - 2\left[\log x \int 1\,dx - \int \frac{d}{dx}\log x (1\,dx)dx\right]$$

$$\Rightarrow x(\log x)^2 - 2\left[x \cdot \log x - \int \frac{1}{x} \times x\,dx\right]$$

$$\Rightarrow x(\log x)^2 - 2[x\log x - dx]$$

$\Rightarrow x\,(\log x)^2 - 2(x \log x - x) + k$

$\Rightarrow x\,(\log x)^2 - 2x \log x + 2x + k$

Hence, the correct answer is option (a).

SOME MORE IMPORTANT FORMULAE

1. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$
2. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$
3. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$
4. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c$
5. $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$
6. $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\,(x + \sqrt{x^2 + a^2}) + c$
7. $\sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log(x + \sqrt{x^2 - a^2}) + c$
8. $\int \frac{f'(x)}{f(x)}dx = \log\,[f(x)] + c$

Example 12. Evaluate $\int \frac{1}{x^2-9}dx$

(a) $\frac{1}{9}\log\left|\frac{x-3}{x+3}\right|+c$ (b) $\frac{1}{3}\log\left|\frac{x+3}{x-3}\right|+c$

(c) $\frac{1}{6}\log\left|\frac{x-3}{x+3}\right|+c$ (d) None of these

Sol. (c) To evaluate: $\int \frac{1}{x^2-9}dx$

We know that,

$$\int \frac{dx}{x^2-a^2}=\frac{1}{2a}\log\frac{x-a}{x+a}+c$$

Thus, $\int \frac{1}{x^2-9}dx=\int \frac{1}{x^2-3^2}dx$

$$=\frac{1}{2(3)}\log\left|\frac{x-3}{x+3}\right|+c=\frac{1}{6}\log\left|\frac{x-3}{x+3}\right|+c$$

Hence, the correct option is (c).

Example 13. Evaluate $\int \frac{1}{\sqrt{x^2-25}}dx$

(a) $\log\left|x+\sqrt{x^2-25}\right|+c$ (b) $\frac{1}{5}\log\left|x+\sqrt{x^2-25}\right|+c$

(c) $\frac{1}{25}\log\left|x+\sqrt{x^2-25}\right|+c$ (d) None of these

Sol. (a) To evaluate: $\int \frac{1}{\sqrt{x^2-25}}dx$

We know that,

$$\int \frac{dx}{\sqrt{x^2-a^2}}=\log\left|x+\sqrt{x^2-a^2}\right|+c$$

Thus, $\int \frac{1}{\sqrt{x^2-25}}dx=\int \frac{1}{\sqrt{x^2-5^2}}dx$

$$=\log\left|x+\sqrt{x^2-25}\right|+c$$

Hence, the correct option is (a).

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Example 14. Evaluate $\int \frac{e^x}{e^{2x}-4}dx$

(a) $\frac{1}{2}\log\left|\frac{e^x+2}{e^x-2}\right|+c$ (b) $\frac{1}{2}\log\left|\frac{e^x-4}{e^x+4}\right|+c$

(c) $\frac{1}{4}\log\left|\frac{e^x-4}{e^x+4}\right|+c$ (d) $\frac{1}{4}\log\left|\frac{e^x-2}{e^x+2}\right|+c$

Sol. (d) $e^x = t$

$e^x = \frac{dt}{dx}$

$\Rightarrow e^x dx = dt$

$\int \frac{e^x dx}{(e^x)^2-2^2}dx = \int \frac{dt}{t^2-2^2}$

$= \frac{1}{2(2)}\log\left|\frac{t-2}{t+2}\right|+c = \frac{1}{4}\log\left|\frac{e-2}{e^x+2}\right|+c$

Hence, the correct option is (d).

Example 15. Evaluate $\int \frac{(2-x)e^x}{(1-x)^2}dx$ and the value is

(a) $\frac{e^x}{1-x}+k$ (b) $e^x + k$ (c) $\frac{1}{1-x}+k$ (d) None of these

Sol. (a) To evaluate: $\int \frac{(2-x)e^x}{(1-x)^2}dx$

$\therefore \int \frac{(2-x)e^x}{(1-x)^2}dx = \int \frac{(1+1-x)e^x}{(1-x)^2}dx$

$= \int\left[\frac{1}{(1-x)^2}+\frac{(1-x)}{(1-x)^2}\right]e^x dx = \int\left[\frac{1}{(1-x)^2}+\frac{1}{(1-x)}\right]e^x dx$

Let $f(x) = \frac{1}{1-x}$, then $f'(x) = \frac{-1}{(1-x)^2}(-1) = \frac{1}{(1-x)^2}$

$\therefore \int \frac{(2-x)e^x}{(1-x)^2}dx = \int\left[\frac{1}{(1-x)^2}+\frac{1}{(1-x)}\right]e^x dx$

Using $\int e^x[f(x)+f'(x)]\,dx = e^x f(x)+k$

Thus, $\int\left[\frac{1}{(1-x)^2}+\frac{1}{(1-x)}\right]e^x dx = e^x\left(\frac{1}{1-x}\right)+k = \frac{e^x}{1-x}+k$

where k is the constant of integration.

Hence, the correct answer is option (a) i.e., $\frac{e^x}{1-x}+k$.

Example 16. Evaluate $\int e^x(x^3+3x^2)dx$

(a) $xe^x + c$ (b) $x^2e^x + c$ (c) $x^3e^x + c$ (d) None of these

Sol. (c) Clearly, it is of the form $\int e^x[f(x)+f'(x)]dx = e^x f(x)+c$

where, $f(x) = x^3$

$\Rightarrow f'(x) = 3x^2$

Thus, $\int e^x(x^3+3x^2)dx = e^x x^3 + c$

Hence, the correct option is (c).

METHOD OF PARTIAL FRACTION

Form of function	Form of partial fraction
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}$
$\frac{px+q}{(x-a)^2(x-b)}, a \neq b$	$\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-b)}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{(x-a)}+\frac{Bx+C}{x^2+bx+c}$

Example 17. Evaluate $\int \frac{1}{(x+1)(x+2)}dx$

(a) $\log\frac{1}{(x+1)(x+2)}+c$ (b) $\log[(x+1)(x+2)] + c$

(c) $\log\left|\frac{x+1}{x+2}\right|+c$ (d) None of these

Sol. (c) Let $\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)}+\frac{B}{(x+2)}$

Multiplying both sides by (x + 1) (x +2), we get

$\Rightarrow 1 = A(x + 2) + B(x + 1)$

Putting x = –2, we get

$1 = A(-2 + 2) + B(-2 + 1)$

$\Rightarrow 1 = B(-1) \Rightarrow B = -1$

Now, putting x = –1, we get

$1 = A(-1 + 2) + B(-1 + 1)$

$\Rightarrow 1 = A(1)$

$\Rightarrow A = 1$

Thus, $\frac{1}{(x+1)(x+2)} = \frac{1}{(x+1)}-\frac{1}{(x+2)}$

$$\Rightarrow \int \frac{1}{(x+1)(x+2)}dx = \int \left[\frac{1}{(x+1)} - \frac{1}{(x+2)}\right]dx$$

$= \log(x + 1) - \log(x + 2) + c$

$$= \log\left|\frac{x+1}{x+2}\right| + c$$

Hence, the correct option is (c).

Example 18. Integrate the following and choose the correct option: $\int \frac{(x+5)}{(x+1)(x+2)^2}dx$ **(ICAI)**

(a) $4\log(x+1) - 4\log(x+2) + \frac{3}{x} + 2 + k$

(b) $4\log(x+2) - \frac{3}{x} + 2 + k$

(c) $4\log(x + 1) - 4\log(x + 2)$

(d) None of these

Sol. (d) Given, $f(x) = \int \frac{(x+5)}{(x+1)(x+2)^2}dx$

Integrating by partial fraction,

$$\Rightarrow \frac{(x+5)}{(x+1)(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x+1}$$

$$\Rightarrow \frac{(x+5)}{(x+1)(x+2)^2} = \frac{A(x+1)(x+2) + B(x+1) + C(x+2)^2}{(x+1)(x+2)^2}$$

$\Rightarrow x + 5 = A(x + 1)(x + 2) + B(x + 1) + C(x + 2)^2$

Put x = -1,

$\Rightarrow (-1 + 5) = A(-1 + 1)(-1 + 2) + B(-1 + 1) + C(-1 + 2)^2$

$\Rightarrow C = 4$

Put x = -2,

$\Rightarrow (-2 + 5) = A(-2 + 1)(-2 + 2) + B(-2 + 1) + C(-2 + 2)^2$

$\Rightarrow B = -3$

Put x = 0

$\Rightarrow (0 + 5) = A(0 + 1)(0 + 2) + B(0 + 1) + C(0 + 2)^2$

$\Rightarrow 5 = 2A + B + 4C$

Put the values of B and C in the above equation,

$\Rightarrow 2A - 3 + 4(4) = 5$

$\Rightarrow 2A - 3 + 16 = 5$

$\Rightarrow 2A = -8$

$\Rightarrow 2A = -4$

Put the values of A, B and C in the partial fraction,

$$\Rightarrow \frac{(x+5)}{(x+1)(x+2)^2} = \frac{-4}{x+2} + \frac{-3}{(x+2)^2} + \frac{4}{x+1}$$

Integrating with respect to x,

$$\Rightarrow \int \frac{(x+5)}{(x+1)(x+2)^2} = -4\int \frac{dx}{x+2} - 3\int \frac{dx}{(x+2)^2} + 4\int \frac{dx}{x+1}$$

Now, $-4\int \frac{1}{x+2}dx = -4\log\,(x+2) + c_1$

Now, $-3\int \frac{1}{(x+2)^2}dx$

Let $(x + 2) = t$

$\Rightarrow dx = dt$

$$\Rightarrow -3\int \frac{dx}{(x+2)^2} = -3\left(\int \frac{dt}{t^2}\right)$$

$$= -3\int t^{-2}\,dt \qquad \left[\because \int f(x)dx = \frac{x^{n+1}}{n+1} + C\right]$$

$$= -3\left(\frac{t^{-2+1}}{-2+1}\right) + C_2 \qquad [C \rightarrow \text{Arbitrary Constant}]$$

$$= 3\left(\frac{1}{t}\right) + C_2 = \frac{3}{x+2} + C_2$$

Now, $4\int \frac{dx}{x+1} = 4\log|x+1| + C_3$

Combining all three integrals, C_1, C_2, C_3 are arbitrary constants.

$$\Rightarrow -4\log|x+2| + \frac{3}{x+2} + 4\log|x+1| + k \qquad [k \rightarrow \text{Arbitrary Constant}]$$

Hence, the correct answer is option (d).

Example 19. Evaluate the integral: $\int \frac{3x^2 - 2x + 5}{(x-1)(x^2+5)}dx$

(a) $\log|(x-1)(x^2+5)| + c$

(b) $\log|(x^3-5)| + c$

(c) $\log\left|\frac{x-1}{x^2+5}\right| + c$

(d) None of these

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Sol. (a) To integrate: $\int \frac{3x^2-2x+5}{(x-1)(x^2+5)}dx$

$$\Rightarrow \frac{3x^2-2x+5}{(x-1)(x^2+5)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+5)}$$

$$\frac{3x^2-2x+5}{\cancel{(x-1)(x^2+5)}} = \frac{A(x^2+5)+(Bx+C)(x-1)}{\cancel{(x-1)(x^2+5)}}$$

$3x^2 - 2x + 5 = A(x^2) + 5A + Bx^2 - Bx + Cx - x$

coefficient of $x^2 \Rightarrow 3 = A + B$

constant $\Rightarrow 5 = 5A - C$

coefficient of $x \Rightarrow -2 = -B + C$

$\Rightarrow A + B = 3$

Also, $-B + C = -2$

On adding both, we get

$A + C = 1$ ...(i)

Also we have, $5A - C = 5$ ...(ii)

Now, adding (i) and (ii), we get

$A + 5A = 6$

$\Rightarrow 6A = 6$

$\Rightarrow A = 1$

$\Rightarrow B = 2$ and $C = 0$

$$= \int \frac{1}{x-1}dx + \int \frac{2x}{x^2+5}dx$$

$x^2 + 5 = t$

$2x\,dx = dt$

$$= \log|x-1| + \int \frac{dt}{t}$$

$= \log|x - 1| + \log|x^2 + 5| + C$

$= \log|(x - 1)(x^2 + 5)| + C$

Hence, the correct option is (a).

Example 20. Using method of partial fraction the integration of f(x) when $f(x)=\frac{1}{x^2-a^2}$ and the answer is

(a) $\log x - \frac{a}{x-a} + k$

(b) $\log(x - a) - \log(x + a) + k$

(c) $\frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+k$

(d) None of these

Sol. (c) Given, $f(x)=\frac{1}{x^2-a^2}$

Integrating by Partial Fraction,

$\Rightarrow I=\frac{1}{x^2-a^2}$ [$\because a^2 - b^2 = (a - b)(a + b)$]

This can be written as,

$\Rightarrow \frac{1}{x^2-a^2}=\frac{A}{x+a}+\frac{B}{x-a}$

$\Rightarrow A = A(x - a) + B(x + a)$

Let, $x = a$

$\Rightarrow 1 = (2a) \times B$

$\Rightarrow B=\frac{1}{2a}$

Let, $x = -a$

$\Rightarrow 1 = (-2a) \times A$

$\Rightarrow A=\frac{1}{-2a}$

So we can rewrite the equation and Integrate with respect to x,

$$\int\left(\frac{1}{x^2-a^2}\right)=\int\left(\frac{\frac{-1}{2a}}{x+a}\right)dx+\int\left(\frac{\frac{1}{2a}}{x-a}\right)dx$$

$$=\frac{-1}{2a}\int\frac{1}{x+a}dx+\frac{1}{2a}\int\frac{1}{x-a}dx\left[\because\int\frac{1}{x}dx=\log|x|+C\right]$$ [$C \rightarrow$ Arbitrary Constant]

$$=\frac{-1}{2a}\log|x+a|+\frac{1}{2a}\log|x-a|+k$$ [$k \rightarrow$ Arbitrary Constant]

$$=\frac{1}{2a}\log|x-a|-\frac{1}{2a}\log|x+a|+k \qquad \left[\because \log a-\log b=\log\left(\frac{a}{b}\right)\right]$$

$$=\frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+k$$

Hence, the correct answer is option (c).

DEFINITE INTEGRATION

Let's say f(x) is a function with x that varies from a to b.

Then the function limit will be between f(a) to f(b).

Thus, if we say $\int F(x)dx=f(x)+c$

By indefinite integral

$\int_a^b F(x)dx=f(b)-f(a)$

where 'b' is the upper limit of integration and 'a' is the lower limit of integration

To find the definite integral, we have to find the indefinite integral and then put values of limits.

In the process of evaluating definite integrals, the constant of integration (C) does not affect the final result because it cancels out when we subtract the values at the limits. Therefore, there is no need to consider the constant when finding definite integrals.

Example 21. $\int_0^2 3x^2\,dx$ is

(a) 7 (b) -8 (c) 8 (d) None of these

Sol. (c) To evaluate: $\int_0^2 3x^2\,dx$

$$\therefore \int_0^2 3x^2\,dx = 3\int_0^2 x^2\,dx$$

$$= 3\left[\frac{x^3}{3}\right]_0^2 = \left[x^3\right]_0^2$$

$= [2^3 - 0^3] = 8$

Hence, the correct option is (c) i.e., 8.

Example 22. $\int_0^1 (2x^2 - x^3)dx$

(a) $\frac{4}{3}+k$ (b) $\frac{5}{12}$ (c) $-\frac{4}{3}$ (d) None of these

Sol. (b) Given, $f(x) = \int_0^1 (2x^2 - x^3)dx$

$$\Rightarrow \int_0^1 (2x^2 - x^3)dx = 2\int_0^1 x^2\,dx - \int_0^1 x^3\,dx$$

$$= 2\left(\frac{x^{2+1}}{2+1}\right)_0^1 - \left(\frac{x^{3+1}}{3+1}\right)_0^1 = \frac{2}{3}\left(x^3\right)_0^1 - \frac{1}{4}\left(x^4\right)_0^1 = \frac{2}{3}\left((1)^3 - (0)^3\right) - \frac{1}{4}\left((1)^4 - (0)^4\right)$$

$$= \frac{2}{3} - \frac{1}{4} = \frac{4\times 2 - 3\times 1}{12} = \frac{8-3}{12} = \frac{5}{12}$$

Hence, the correct option is (b).

Example 23. $\int_0^1 (x^2 - x)dx$

(a) $\frac{4}{3}$ (b) $-\frac{1}{6}$ (c) $-\frac{4}{3}$ (d) None of these

Sol. (b) Given, $f(x) = \int_0^1 (x^2 - x)dx$

$$\Rightarrow \int_0^1 (x^2 - x)dx = \int_0^1 x^2 dx - \int_0^1 x\,dx$$

$$=\left(\frac{x^{2+1}}{2+1}\right)_0^1-\left(\frac{x^{1+1}}{1+1}\right)_0^1=\frac{1}{3}\left(x^3\right)_0^1-\frac{1}{2}\left(x^2\right)_0^1$$

$$=\frac{1}{3}\left((1)^3-(0)^3\right)-\frac{1}{2}\left((1)^2-(0)^2\right)=\frac{1}{3}-\frac{1}{2}=\frac{3-2}{6}=-\frac{1}{6}$$

Hence, the correct option is (b).

Example 24. $\int_1^2 x\log x\,dx$ **(ICAI)**

(a) $2\log 2$ (b) $\frac{-3}{4}$ (c) $2\log 2-\frac{3}{4}$ (d) None of these

Sol. (c) To evaluate: $\int_1^2 x\log x\,dx$

$$\int_1^2 x\log x\,dx=\left[\log x\int x\,dx\right]_1^2-\int_1^2\left(\frac{d(\log x)}{dx}.\int x\,dx\right)dx$$

$$=\left[\log x\,.\frac{x^2}{2}\right]_1^2-\int_1^2\left(\frac{1}{x}\left(\frac{x^2}{2}\right)\right)dx=\left[\frac{x^2}{2}\log x\right]_1^2-\int_1^2\frac{x}{2}dx=\left[\frac{x^2}{2}\log x\right]_1^2-\frac{1}{2}\int_1^2 x\,dx$$

$$=\left[\frac{x^2}{2}\log x\right]_1^2-\frac{1}{2}\left[\frac{x^2}{2}\right]_1^2=\left[\frac{2^2}{2}\log 2-\frac{1}{2}\log 1\right]-\frac{1}{2}\left(\frac{2^2}{2}-\frac{1^2}{2}\right)=2\log 2-\frac{3}{4}$$

Hence, the correct option is (c).

Example 25. Evaluate $\int_1^4(2x+5)\,dx$ and the value is **(ICAI)**

(a) 3 (b) 10 (c) 30 (d) None of these

Sol. (c) To evaluate: $\int_1^4(2x+5)\,dx$

$$\therefore\int_1^4(2x+5)\,dx=\int_1^4(2x)\,dx+\int_1^4 5\,dx=2\int_1^4 x\,dx+5\int_1^4 dx$$

We know that,

$$\int x^n\,dx=\frac{x^{n+1}}{n+1},\ \int dx=x+K$$

Thus, $\int_1^4(2x+5)\,dx=2\left[\frac{x^2}{2}\right]_1^4+5[x]_1^4$

$= (4^2 - 1^2) + 5\,(4 - 1) = 15 + 15 = 30$

Hence, the correct option is (c) i.e., 30.

Example 26. $\int_1^2 \frac{2x}{1+x^2}dx$ is equal to

(a) $\log_e\left(\frac{5}{2}\right)$ (b) $\log_e 5 - \log_e 2 + k$

(c) $\log_e\left(\frac{2}{5}\right)$ (d) None of these

Sol. (a) To evaluate: $\int_1^2 \frac{2x}{1+x^2}dx$

Let, $1 + x^2 = t$, then $2x\,dx = dt$

When $x = 1$, then $t = 1 + 1 = 2$

When $x = 2$, then $t = 1 + (2^2) = 1 + 4 = 5$

By substituting above values in the given integral, we get

$$\therefore \int_1^2 \frac{2}{1+x^2}dx = \int_2^5 \frac{1}{t}dt$$

$$= [\log_e t]_2^5 = \log_e 5 - \log_e 2 = \log_e\left(\frac{5}{2}\right)$$

Hence, the correct option is (a).

Example 27. $\int_0^4 \sqrt{(3x-4)}dx$ is equal to **(ICAI)**

(a) $\frac{9}{112}$ (b) $\frac{112}{9}$ (c) $\frac{11}{9}$ (d) None of these

Sol. (b) Given: $\int_0^4 \sqrt{(3x-4)}dx$

We know that,

$$\int (ax+k)^n dx = \frac{(ax+k)^{n+1}}{a(n+1)}$$

$$\therefore \int_0^4 \sqrt{(3x+4)}dx = \left[\frac{(3x+4)^{1+\frac{1}{2}}}{3\left(1+\frac{1}{2}\right)}\right]_0^4$$

$$= \left[\frac{(3x+4)^{\frac{3}{2}}}{\frac{3}{2}(3)}\right]_0^4 = \frac{2}{9}\left[(16)^{\frac{3}{2}} - (4)^{\frac{3}{2}}\right] = \frac{2}{9}[64-8] = \frac{2}{9}[56] = \frac{112}{9}$$

Hence, the correct answer is option (b).

Example 28. Evaluate $\int_0^2 \frac{x+2}{x+1}dx$

(a) $2 + \log_e 2$ (b) $2 + \log_e 3$ (c) $\log_e 3$ (d) None of these

Sol. (b) Given, $\int_0^2 \frac{x+2}{x+1}dx$

$$\therefore \int_0^2 \frac{x+2}{x+1}dx = \int_0^2 \frac{x+1+1}{x+1}dx$$

$$= \int_0^2 \frac{(x+1)+1}{x+1}dx = \int_0^2 \left[\frac{x+1}{x+1} + \frac{1}{x+1}\right]dx$$

$$= \int_0^2 \frac{x+1}{x+1}dx + \int_0^2 \frac{1}{x+1}dx = \int_0^2 1\,dx + \int_0^2 \frac{1}{x+1}dx$$

We know that,

$$\int dx = x \text{ and } \int \frac{1}{x}dx = \log|x|$$

$$\therefore \int_0^2 \frac{x+2}{x+1}dx = [x]_0^2 + [\log_e(1+x)]_0^2$$

$= [2 - 0] + [\log_e (1 + 2) - \log_e (1 + 0)]$

$= 2 + [\log_e (3) - \log_e (1)]$

$= 2 + \log_e (3)$

Hence, the correct answer is option (b).

Example 29. Evaluate $\int_1^{e^2} \frac{1}{x(1+\log x)^2}dx$ and the value is **(ICAI)**

(a) $\frac{3}{2}$ (b) $\frac{1}{3}$ (c) $\frac{26}{3}$ (d) $\frac{2}{3}$

Sol. (d) To evaluate: $\int_1^{e^2} \frac{1}{x(1+\log x)^2}dx$

Let, $1 + \log x = t$, then $\frac{1}{x}dx = dt$

$\Rightarrow dx = x\,dt$

When $x = 1$, then $t = 1 + \log 1 = 1 + 0 = 1$

When $x = e^2$, then $t = 1 + \log e^2 = 1 + 2 \log e = 1 + 2 = 3$

$$\therefore \int_1^3 \frac{1}{x(1+\log x)^2}dx = \int_1^3 \frac{x\,dt}{x(t)^2}$$

$$= \int_1^3 \frac{dt}{t^2} = \left[\frac{-1}{t}\right]_1^3 = \left[\frac{-1}{3} - \left(\frac{-1}{1}\right)\right] = \frac{2}{3}$$

Hence, the correct option is (d).

Example 30. $\int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}}dx$ is equal to

(a) $51\frac{1}{5}$ (b) $\frac{48}{5}$ (c) 48 (d) $55\frac{7}{15}$

Sol. (d) To evaluate: $\int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}}dx$

$$\therefore \int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}}dx = \int_0^4 \frac{x^2+5x+4}{\sqrt{x}}dx$$

$$= \int_0^4 \frac{x^2}{\sqrt{x}}dx + \int_0^4 \frac{5x}{\sqrt{x}}dx + \int_0^4 \frac{4}{\sqrt{x}}dx$$

$$= \int_0^4 x^{\frac{3}{2}}dx + 5\int_0^4 x^{\frac{1}{2}}dx + 4\int_0^4 x^{\frac{-1}{2}}dx$$

We know that, $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$\therefore \int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}}dx = \left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^4 + 5\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 + 4\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^4$$

$$= \frac{2}{5}\left[x^{\frac{5}{2}}\right]_0^4 + \frac{10}{3}\left[x^{\frac{2}{2}}\right]_0^4 + 8\left[x^{\frac{1}{2}}\right]_0^4 \text{ p1} = \frac{2}{5}(32) + \frac{10}{3}(8) + 8(2)$$

$$= \frac{64}{5} + \frac{80}{3} + 16 = \frac{192+400+240}{15} = \frac{832}{15} = 55\frac{7}{15}$$

Hence, the correct answer is option (d).

PRACTICE QUESTIONS (PART B)

1. Evaluate the following integral:

I. $\int\left(x+\frac{1}{x}\right)^2 dx$

II. $\int \frac{1}{\sqrt{x}}dx$

III. $\int e^{3x} + e^{-3x}dx$

IV. $\int 3^x dx$

2. Evaluate $x^2\left(1-\frac{1}{x^2}\right)$.

(a) $\frac{x^3}{3}\left(1+\frac{1}{x^3}\right)$

(b) $\frac{x^3}{3} - x + c$

(c) $\frac{x^3}{3} + \frac{x^2}{2} + c$

(d) None of these

3. Evaluate $\int x^2 e^x dx$

(a) $\left(\frac{x^3}{3}\right)e^x + c$

(b) $(x^2 + 2)\, e^x + c$

(c) $(x^2 - 2x + 2)e^x + c$

(d) None of these

4. Evaluate $\int x^3 \log x \, dx$

(a) $x^4\left(\frac{1}{4}\log x - \frac{1}{16}\right) + c$ (b) $x^4\left(\frac{1}{4}\log x + \frac{1}{4}\right) + c$

(c) $x^2\left(\frac{1}{4}\log x\right) + c$ (d) None of these

5. Evaluate $\int (1+5x)e^{2x} \, dx$

(a) $\frac{1}{2}e^{2x} + \frac{1}{4}e^{2x} + c$ (b) $\frac{5}{2}x^2e^{2x} + c$

(c) $\frac{5}{2}xe^{2x} - \frac{3}{4}e^{2x} + c$ (d) None of these

6. Evaluate $\int \frac{1}{x^2 - 16} dx$

(a) $\frac{1}{8}\log\left|\frac{x-4}{x+4}\right| + c$ (b) $\frac{1}{4}\log\left|\frac{x+4}{x-4}\right| + c$

(c) $\frac{1}{8}\log\left|x^2 - 16\right| + c$ (d) None of these

7. Evaluate $\int \frac{1}{\sqrt{x^2 - 1}} dx$

(a) $\log\left|x + \sqrt{x^2 - 1}\right|$ (b) $\log\left|x + \sqrt{x^2 + 1}\right| + c$

(c) $\log\left|x - \sqrt{x^2 + 1}\right| + c$ (d) None of these

8. Evaluate $\int \frac{1}{\sqrt{9 + x^2}} dx$

(a) $\log\left|\sqrt{x^2 + 9}\right| + c$ (b) $\log\left|x + \sqrt{x^2 + 9}\right| + c$

(c) $\log\left|x - \sqrt{x^2 + 9}\right| + c$ (d) None of these

9. Evaluate $\int \frac{\sqrt{16 + (\log x)^2}}{x} dx$ (**Hint:** Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$)

(a) $\frac{\log x}{2}\sqrt{16 + (\log x)^2} + 8\left|(\log x) + \sqrt{16 + (\log x)^2}\right| + c$

(b) $\frac{\log x}{2}\sqrt{16 - (\log x)^2} + 8\left|(\log x) - \sqrt{16 + (\log x)^2}\right| + c$

(c) $\frac{\log x}{2}\sqrt{16 + (\log x)^2} + c$

(d) None of these

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10. $\int \frac{(x-1)e^x}{x^2}dx$ is equal to **(ICAI)**

(a) $\frac{e^x}{x}+k$ (b) $\frac{e^{-x}}{x}+k$ (c) $\frac{-e^x}{x}+k$ (d) None of these

11. Evaluate the following integrals:

(i) $\int_2^3 x^2\,dx$ (ii) $\int_0^2 (2x^2+3x+1)dx$

(iii) $\int_2^3 \frac{1}{x}dx$ (iv) $\int_0^1 \frac{2x}{1+x^2}dx$

(v) $\int_1^3 \frac{x}{(x+1)(x+2)}dx$ (vi) $\int_0^1 xe^x\,dx$

(vii) $\int_1^2 x\log x\,dx$ (viii) $\int_0^1 (e^x+e^{-x})dx$

Answer Key

1. I. $\frac{x^3}{3}+2x-\frac{1}{x}+c$ II. $2\sqrt{x}+c$

III. $\frac{e^{3x}}{3}-\frac{e^{-3x}}{3}+c$ IV. $\frac{3^x}{\log_e 3}+c$

2. (b) **3.** (c) **4.** (a) **5.** (c) **6.** (a) **7.** (a) **8.** (b) **9.** (a) **10.** (a)

11. (i) $\frac{19}{3}$ (ii) $\frac{40}{3}$ (iii) $\log\frac{3}{2}$ (iv) $\log 2$ (v) $\log\frac{25}{18}$ (vi) 1

(vii) $2\log 2-\frac{3}{4}$ (viii) $e - e^{-1}$

PROPERTIES OF DEFINITE INTEGRATION

1. $\int_a^b f(x)dx=\int_a^b f(t)dt$

2. $\int_a^b f(x)dx=-\int_b^a f(x)dx$

3. $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx, a<c<b$

4. $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

5. $\int_0^a f(x)dx=\int_0^a f(a+0-x)dx=\int_0^a f(a-x)dx$

6. $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ if f(x) is an even function i.e., f(-x) = f(x)

= 0 if f(x) is an odd function i.e., f(-x) = -f(x)

Example 31. Evaluate $\int_{-1}^{1} \log\left(\frac{2+x}{2-x}\right)dx$

(a) 0 (b) 1 (c) $\log\frac{3}{2}$ (d) None of these

Sol. (a) We know,

$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ if f(x) is an even function i.e., f(-x) = f(x)

= 0 if f(x) is an odd function i.e., f(-x) = -f (x)

Here, $f(x) = \log\left(\frac{2+x}{2-x}\right)$

$\Rightarrow f(-x) = \log\left(\frac{2-x}{2+x}\right) = -\log\left(\frac{2+x}{2-x}\right) = -f(x)$

$\Rightarrow$ f(x) is an odd function

Thus, $\int_{-1}^{1} \log\left(\frac{2+x}{2-x}\right)dx = 0$

Hence, the correct option is (a).

Example 32. Evaluate $\int_{1}^{3} \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

Sol. (a) Let $I = \int_{1}^{3} \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$ …(i)

We know, $\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx$

$\Rightarrow I = \int_{1}^{3} \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx = \int_{1}^{3} \frac{\sqrt{4-(1+3-x)}}{\sqrt{1+3-x}+\sqrt{4-(1+3-x)}}dx$

$= \int_{1}^{3} \frac{\sqrt{4-(4-x)}}{\sqrt{4-x}+\sqrt{4-(4-x)}}dx$

$= \int_{1}^{3} \frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}dx$ …(ii)

Adding (i) and (ii), we get

$$I + I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx + \int_1^3 \frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}dx$$

$$\Rightarrow 2I = \int_1^3 \frac{\sqrt{4-x}+\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}dx = \int_1^3 1.dx = (x)_1^3 = (3-1) = 2$$

$\Rightarrow 2I = 2$

$\Rightarrow I = 1$

Therefore, $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx = 1$

Hence, the correct option is (a).

Example 33. $\int_{-1}^{1}(e^x - e^{-x})dx =$ **(Dec 2020)**

(a) 0 (b) 1 (c) 12 (d) None of these

Sol. (b) Here, $f(x) = e^x - e^{-x}$

then, $f(-x) = e^{-x} - e^x = -(e^x - e^{-x}) = -f(x)$

Thus, f(x) is an odd function.

We know,

$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if f(x) is an even function i.e., f(-x) = f(x)

= 0 if f(x) is an odd function i.e., f(-x) = -f(x)

Therefore, $\int_{-1}^{1}(e^x - e^{-x})dx = 0$

Hence, the correct option is (b).

Example 34. Evaluate: $\int_0^2 x\sqrt{2-x}\,dx$

(a) $\frac{8\sqrt{2}}{15}$ (b) $\frac{16\sqrt{2}}{3} - \frac{8}{5}$ (c) $\frac{16\sqrt{2}}{15}$ (d) None of these

Sol. (c) To evaluate: $\int_0^2 x\sqrt{2-x}\,dx$

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Thus, $\int_0^2 x\sqrt{2-x}\,dx = \int_0^2 (2-x)\sqrt{2-(2-x)}\,dx$

$= \int_0^2 (2-x)\sqrt{x}\,dx = \int_0^2 \left(2\sqrt{x} - x^{\frac{3}{2}}\right)dx$

$$= \left[\frac{4x^{\frac{3}{2}}}{3} - \frac{2x^{\frac{5}{2}}}{5} \right]^2 = \frac{4(2)^{\frac{3}{2}}}{3} - \frac{2(2)^{\frac{5}{2}}}{5} = \frac{16\sqrt{2}}{15}$$

Example 35. $\int_{-1}^{1} \frac{|x|}{x} dx =$

(a) -1 (b) 0 (c) 1 (d) 2

Sol. (b) Here, $f(x) = \frac{|x|}{x}$

then, $f(-x) = \frac{|-x|}{-x} = \frac{|x|}{-x} = -\frac{|x|}{x} = -f(x)$

Thus, f(x) is an odd function.

We know,

$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ if f(x) is an even function i.e., f(-x) = f(x)

= 0 if f(x) is an odd function i.e., f(-x) = -f(x)

Therefore, $\int_{-1}^{1} \frac{|x|}{x} dx = 0$

Hence, the correct option is (b).

Example 36. The value of $\int_{-2}^{2} f(x)dx$, where f(x) = 1 + x, $x \le 0$ and f(x) = 1 - 2x when $x \ge 0$ is **(July 2021)**

(a) 20 (b) -2 (c) -4 (d) 0

Sol. (b) Given, f(x) = 1 + x, $x \le 0$ and f(x) = 1 - 2x when $x \ge 0$

Thus, $\int_{-2}^{2} f(x)dx = \int_{-2}^{0} f(x)dx + \int_{0}^{2} f(x)dx$

$$= \int_{-2}^{0} (1+x)dx + \int_{0}^{2} (1-2x)dx = \left[x - \frac{x^2}{2} \right]_{-2}^{0} + \left[x - \frac{2x^2}{2} \right]_{0}^{2}$$

$$= -\left[-2 - \frac{(-2)^2}{2} \right] + \left[2 - \frac{2(2)^2}{2} \right]_{0}^{2} = +4 - 2 = -2$$

Hence, the correct option is (b).

Example 37. The equation of the curve which passes through the point (1, 3) and has the slope 4x - 3 at any point (1, 3) is **(ICAI)**

(a) $y = 2x^3 - 3x + 4$

(b) $y = 2x^2 - 3x + 4$

(c) $x = 2y^2 - 3y + 4$

(d) None of these

Sol. (b) Given, Slope = f'(x) = 4x - 3

We know that, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int dx = x + k$

Thus, the equation of the curve is given by:

$y = \int f'(x)dx$

$\Rightarrow y = \int (4x - 3)dx$

$\Rightarrow y = 4\int x\,dx - 3\int dx$

$\Rightarrow y = 4\left(\frac{x^2}{2}\right) - 3x + k$

$\Rightarrow y = 2x^2 - 3x + k$, where k is the constant of integration

Since, the curve passes through the point (1, 3)

$y = 2x^2 - 3x + k$

$\Rightarrow 3 = 2(1)^2 - 3(1) + k$

$\Rightarrow k = 4$

By substituting the value of k, we get

$y = 2x^2 - 3x + 4$

Hence, the correct answer is option (b).

Example 38. The value of $\int_2^3 f(5-x)dx - \int_2^3 f(x)dx$ is **(ICAI)**

(a) 1 (b) 0 (c) -1 (d) None of these

Sol. (b) Let $A = \int_2^3 f(5-x)dx - \int_2^3 f(x)dx$

By properties of integration, we have

$\int_a^b f(x)dx = \int_a^b f(b+a-x)dx$

$\Rightarrow \int_2^3 f(x)dx = \int_2^3 f(3+2-x)dx = \int_2^3 f(5-x)dx$

Since, $A = \int_2^3 f(5-x)dx - \int_2^3 f(x)dx$

$\Rightarrow A = \int_2^3 f(5-x)dx - \int_2^3 f(5-x)dx$

$\Rightarrow A = 0$

Hence, the correct answer is option (b)

PREVIOUS YEAR QUESTION (PYQ'S)

1. $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ **(June 2019)**

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

2. $\int e^x\left(x^2+2x\right)dx$ **(June 2019)**

(a) $x^2.e^x + c$ (b) $e^x.x+c$

(c) $-x^2.e^x + c$ (d) $-x.e^x + c$

3. Find the value of $\int xe^x\,dx$ **(Dec 2019)**

(a) $e^x\,(x-1) + c$ (b) $e^x\,(2x - 1) + c$

(c) $e^x\,(x-1)$ (d) None of these

4. Find the value of $\int (4x+5)^6\,dx$ is equal to **(Dec 2019)**

(a) $\frac{1}{7}(4x+5)^7+c$ (b) $\frac{1}{28}(4x+5)^7+c$

(c) $\frac{1}{4}(4x+5)^7+c$ (d) None of these

5. $\int_{-1}^1\left(2x^2-x^3\right)dx =$ **(Dec 2019)**

(a) 14 (b) 104 (c) $\frac{2x^3}{3}-\frac{x^4}{4}$ (d) 4/3

6. $\int_{-1}^1\left(e^x-e^{-x}\right)dx = ?$ **(Dec 2020)**

(a) 0 (b) 1 (c) 12 (d) None

7. $\int_1^2\frac{1-x}{1+x}dx =$ **(Dec 2020)**

(a) $2\log\frac{3}{2}-1$ (b) 2 log 3-1 (c) $\frac{1}{2}\log\frac{3}{2}-1$ (d) 2 log 2-1+k

8. The value of $\int_0^2\frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx$ is **(Dec 2020)**

(a) 0 (b) 3 (c) 2 (d) 1

9. $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx =$ **(Jan 2021)**

(a) $e\left(\frac{e}{2}-1\right)$ (b) e(e-1) (c) a (d) $e^2\,(e - i)$

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10. The value of $\int_{-2}^{2} f(x)dx$, where f(x) = 1 + x, x < 0; f(x) = 1 - 2x, x ≥ 0 is **(July 2021)**

(a) 20 (b) -2 (c) -4 (d) 0

11. Integrate with respect to x, $\frac{1}{[x(\log x)^2]}$ **(Dec 2021)**

(a) $-\frac{1}{\log x}+k$ (b) $\frac{1}{\log x}+k$ (c) $\log x$ (d) x

12. $\int_0^1 x.e^x dx$ **(June 2022)**

(a) -1 (b) 1 (c) e^1 (d) $-e^1$

13. Determine f(x), given that f′ (x)=$12x^2$ - 4x and f(-3) = 17 **(June 2022)**

(a) f(x) = $4x^3 - 2x^2 + 143$ (b) f(x) = $6x^2 - x^4 + 137$

(c) f(x) = $3x^4 - x^3 - 137$ (d) f(x) = $4x^3 - 2x^2 - 143$

14. Find the area under the curve f(x)=x^2 + 5x + 2 with limits 0 to 1. **(Dec 2022)**

(a) 3.833 (b) 4.388 (c) 4.833 (d) 3.338

15. $\int (2x-3)^6 dx$ is **(Dec 2022)**

(a) $\frac{(2x-3)^6}{6}$ (b) $\frac{(2x-3)^6}{2}$ (c) $\frac{(2x-3)^6}{12}$ (d) $\frac{(2x-3)^6}{3}$

16. $\int_2^4 \frac{x}{x^2+1}dx$ is **(Dec 2022)**

(a) $\frac{1}{2}\log\left(\frac{17}{5}\right)$ (b) $2\log\left(\frac{17}{5}\right)$ (c) $\frac{1}{2}\log\left(\frac{5}{17}\right)$ (d) $2\log\left(\frac{5}{17}\right)$

17. Evaluate the integral $\int \frac{1}{(x-1)(x-2)}dx$ **(June 2023)**

(a) $\log\left(\frac{x-2}{x-1}\right)+c$ (b) log [(x - 2)(x - 1)] + c

(c) $\log\left(\frac{x-1}{x-2}\right)+c$ (d) log [(x - 2)(x + 1)] + c

18. $\int_0^1 u(1-u)^{10} dx$ **(June 2024)**

(a) $\frac{1}{10\times11}$ (b) $\frac{1}{12\times11}$ (c) $\frac{1}{10\times9}$ (d) $\frac{1}{12\times13}$

19. $\int \log_e x dx$ is equal to: **(Sept 2024)**

(a) $x\log_e(ex) + c$

(b) $x\log_e\left(\frac{x}{e}\right) + c$

(c) $x\log_e\left(\frac{e}{x}\right) + c$

(d) $\log_e\left(\frac{x}{e}\right) + c$

20. Evaluate the following integral: **(Sept 2024)**

$$\int \frac{1}{x\left(x^5+1\right)}dx$$

(a) $\log\left(\frac{x^5}{x^5+1}\right)+C$

(b) $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right)+C$

(c) $\frac{1}{3}\log\left(\frac{x^5}{x^5+1}\right)+C$

(d) $\frac{1}{3}\log\left(\frac{x^5+1}{x^5}\right)+C$

21. $\int(2x+5)^7\,dx$ is equal to; **(Jan 2025)**

(a) $\frac{(2x+5)^7}{7}$ (b) $\frac{(2x+5)^8}{16}$ (c) $\frac{\left(2x^2+5x\right)^7}{2}$ (d) $\frac{\left(2x^2+5x\right)^7}{5}$

22. $\int_2^4(3x-2)^2\,dx$ **(May 2025)**

(a) 104 (b) 100 (c) 10 (d) 52

23. The value of $\int_0^4 \frac{x+4}{x+2}dx$ is **(Sept 2025)**

(a) $4 - \log_e 6 - \log_e 2$

(b) $4 + \log_e 6 - \log_e 2$

(c) $4 + \log_e 6$

(d) $4 + \log_{10} 6 - \log_{10} 2$

24. The value of $\int_3^4 \frac{2x}{1+x^2}dx$ is _____. **(Sept 2025)**

(a) $\log\frac{16}{10}$ (b) $\log\frac{17}{10}$ (c) $\log\frac{16}{9}$ (d) $\log\frac{17}{9}$

Answer Key

1.	(b)	**2.**	(a)	**3.**	(a)	**4.**	(b)	**5.**	(d)	**6.**	(a)	**7.**	(a)	**8.**	(d)	**9.**	(a)	**10.**	(b)
11.	(a)	**12.**	(b)	**13.**	(a)	**14.**	(c)	**15.**	(c)	**16.**	(c)	**17.**	(a)	**18.**	(b)	**19.**	(b)	**20.**	(b)
21.	(b)	**22.**	(a)	**23.**	(b)	**24**	(b)												

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FULL CHAPTER PRACTICE QUESTIONS

1. The gradient of the curve $y = 2x^3 - 5x^2 - 3x$ at $x = 0$ is **(ICAI)**

   (a) 3 (b) -3 (c) 1/3 (d) None of these

2. If $f(x) = e^{ax^2 + bx + c}$ then $f'(x)$ is

   (a) $e^{ax^2 + bx + c}$ (b) $e^{ax^2 + bx + c}(2ax + b)$

   (c) $2ax + b$ (d) None of these

3. If $y = x(x-1)(x-2)$ then $\frac{dy}{dx} =$

   (a) $3x^2 - 6x + 2$ (b) $-6x + 2$

   (c) $3x^2 + 2$ (d) None of these

4. The gradient of the curve at (1, 1) is 1/2. The values of p and q are

   (a) (–1, 1) (b) (2, –1)

   (c) (1, 2) (d) (0, -1)

5. The derivative of the function $\sqrt{x^2 + \sqrt{x}}$ is

   (a) $\frac{2x + \frac{1}{2}\sqrt{x}}{2\sqrt{x^2 + \sqrt{x}}}$ (b) $1 + \frac{1}{2\sqrt{x}}$

   (c) $\frac{1}{2\sqrt{x + \sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)$ (d) None of these

6. If $\log(x) = 2(x + y)$, $\frac{dy}{dx}$ may be found to be

   (a) $\frac{1}{2x}$ (b) $\frac{1}{2x} - 1$ (c) $\frac{1-x}{1+y}$ (d) None of these

7. For the curve $x^2 + y^2 + 2gx + 2hy = 0$ the value of $\frac{dy}{dx}$ at (0, 0) is

   (a) $-\frac{g}{h}$ (b) $\frac{g}{h}$ (c) $\frac{h}{g}$ (d) None of these

8. If $x = \frac{a}{t}$ and $y = 2at$, then $\left[\frac{dy}{dx}\right]_{t=2}$ is equal to

   (a) –8 (b) 16 (c) –16 (d) None of these

9. If $y = \left(x + \sqrt{x^2 + m^2}\right)$ then $\frac{dy}{dx}$ is equal to

   (a) $1 + \frac{x}{\sqrt{x^2 + m^2}}$ (b) $\frac{x}{\sqrt{x^2 + m^2}}$ (c) $1 + \frac{2x}{\sqrt{x^2 + m^2}}$ (d) None of these

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10. If $y = 4e^{\sqrt{2x}}$ then $\frac{dy}{dx}$ is equal to _____ .

(a) $\frac{4e^{\sqrt{2x}}}{\sqrt{2x}}$ (b) $e^{\sqrt{2x}}$ (c) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (d) None of these

11. If $f(x) = \frac{4-2x}{2+3x+3x^2}$ then the value of x for which f′ (x) = 0 is

(a) $2\left(1 \pm \sqrt{\frac{5}{3}}\right)$ (b) $\left(1 \pm \sqrt{3}\right)$ (c) 2 (d) None of these

12. $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx =$

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

13. If the given cost function of commodity is given by $C(x) = 150x - 5x^2 + \frac{x^3}{6}$, where C stands for cost, x stands for output, if the average cost is equal to the marginal cost, then the output x =

(a) 5 (b) 10 (c) 15 (d) 20

14. $\int_1^3 \left(1 + 3x - x^3\right) dx$ is equal to

(a) -4 (b) 4 (c) 3 (d) -3

15. $\int x\left(x^5 + 4\right)^5 dx$ is equal to

(a) $(x^2+4)^6 + c$ (b) $\frac{1}{12}\left(x^2+4\right)^6 + c$

(c) $\frac{1}{6}\left(x^2+4\right)^6 + c$ (d) None of the above

16. $\int \frac{8x^2}{\left(x^3+2\right)^3} dx$ is equal to

(a) $\frac{-4}{3}\left(x^3+2\right)^2 + k$ (b) $-\frac{4}{3\left(x^3+2\right)^2} + k$

(c) $-\frac{4}{3\left(x^3+2\right)^2} + k$ (d) None of these

17. If $y = \log x^x$, then $\frac{dy}{dx}$ is equal to:

(a) log ex (b) $\log \frac{e}{x}$ (c) $\log \frac{x}{e}$ (d) 1

18. The value of $\int_1^2 \frac{1-x}{1+x}dx$ dx is equal to: **(May 2018)**

(a) $\log\frac{3}{2}-1$ (b) $2\log\frac{3}{2}-1$ (c) $\frac{1}{2}\log\frac{3}{2}-x$ (d) $\frac{1}{2}\log\frac{3}{2}-1$

19. $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}dx$ is equal to **(May 2018)**

(a) $\frac{2\sqrt{2}}{\log_e 3}$ (b) 0 (c) $\frac{2\left(3\sqrt{2}-1\right)}{\log_e 3}$ (d) $\frac{3\sqrt{2}}{\sqrt{2}}$

20. The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx$ is: **(May 2018)**

(a) 0 (b) 3 (c) 2 (d) 1

21. $\int x\cdot e^{x^2}dx$ is equal to: **(Dec 2017)**

(a) $2e^{x^2}$ (b) $\frac{1}{3}e^{x^2}+c$ (c) $\frac{1}{2}e^{x^2}+c$ (d) None of these

22. $\int_0^1 \log\left(\frac{1}{x}-1\right)$ is equal to **(June 2018)**

(a) 1 (b) 0 (c) 2 (d) - 1

23. The values of $\int_1^2 \frac{x}{x^2+1}dx$ is equal to :

(a) $\log_e\left(\frac{5}{2}\right)$ (b) $\frac{1}{2}\log_e\left(\frac{5}{2}\right)$

(c) $\log_e(5) - \log_e 2 + 0$ (d) None of these

24. $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}dx$ is equal to **(May 2018)**

(a) $\frac{2\sqrt{2}}{\log_e x}$ (b) 0 (c) $\frac{2\left(3\sqrt{2}-1\right)}{\log_e x}$ (d) $\frac{3\sqrt{2}}{\sqrt{2}}$

25. Evaluate $\int\sqrt{x^2-1}$ .2x dx is equal to **(May 2018)**

(a) $\frac{1}{2}(x^2-1)^{\frac{1}{2}}+c$ (b) $\frac{2}{3}(x^2-1)^{\frac{3}{2}}+c$

(c) $(x^2-1)^{\frac{3}{2}}+c$ (d) None

Answer Key

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (a)
11. (a)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (c)	**20.** (d)
21. (c)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (b)					

SUMMARY

DIFFERENTIATION RULES

$$\frac{d}{dx}(f(x)) = f'(x)$$

- Product rule: $\{f(x)g(x)\}' = f(x)g'(x) + f'(x)g(x)$
- Quotient rule: $\left\{\frac{f(x)}{g(x)}\right\}' = \frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$
- Chain rule: $f(g(x))' = f'(g(x)) \times g'(x)$

DERIVATIVE OF VARIOUS FUNCTIONS

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\text{constant}) = 0$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \log(a)$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

Parametric equations: If $x = f(t)$ and $y = h(t)$, then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

APPLICATIONS OF DIFFERENTIAL CALCULUS

- $C(x) = V(x) + F(x)$, where $V(x)$ denotes the variable cost and $F(x)$ is the fixed cost
- Average cost (AC or C) = $\frac{C(x)}{x}$
- Average variable cost (AVC) = $\frac{V(x)}{x}$

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- Average Fixed Cost (AFC) = $\frac{F(x)}{x}$
- Marginal Cost (MC) = $\frac{dC}{dx}$
- Revenue function, R(x) = P . x
- Marginal revenue, (MR) = $\frac{dR}{dx}$
- Profit function, P(x) = R(x) - C(x)
- Marginal Profit = $\frac{dP}{dx}$

INTEGRATION RULES

- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$
- $\int x^{-1}\,dx = \int \frac{1}{x}dx = \ln|x| + C$
- $\int e^x\,dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\log a} + c$

INTEGRATION BY PARTS: USING 'LAE' RULE

$$\int f(x).g(x)dx = f(x)\int g(x) - \int\left[\frac{d\,f(x)}{dx} \times \int g(x)dx\right]dx$$

SOME MORE IMPORTANT FORMULAE

1. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$
2. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$
3. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$
4. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c$
5. $\int e^x\{f(x) + f'(x)\}\,dx = e^x\,f(x) + c$
6. $\int \sqrt{x^2 + a^2}dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log(x + \sqrt{x^2 + a^2}) + c$
7. $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log(x - \sqrt{x^2 - a^2}) + c$

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8. $\int \frac{f'(x)}{f(x)} dx = \log[f(x)] + c$

METHOD OF PARTIAL FRACTION

Form of function	Form of partial fraction
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
$\frac{px+q}{(x-a)^2(x-b)}, a \neq b$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{(x-a)} + \frac{Bx+C}{x^2+bx+c}$

DEFINITE INTEGRAL

$\int_a^b F(x)\,dx = f(b) - f(a)$

PROPERTIES

- $\int_a^b f(x)dx = \int_a^b f(t)dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)dx = \int_0^a f(a+0-x)dx = \int_0^a f(a-x)dx$
- $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ when f(-x) = f(x)

  = 0 when f(-x) = -f(x)

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PART

B

Logical Reasoning

CHAPTER

09

Number Series, Coding-Decoding and Odd Man Out Series

THIS CHAPTER CONSISTS OF 3 PARTS

- SERIES
- CODING DECODING
- ODD MAN OUT SERIES

SERIES:

- Number Series
- Alphabet Series
- Letter Series

NUMBER SERIES

Number series refers to a sequence of numbers that follows a specific pattern. In this type of series, one term is usually missing, and the task is to identify the missing term based on the given pattern. These questions test your ability to recognize numerical patterns and apply them to solve problems.

Here are a few examples of number series:

E.g.: 1, 2, 3, 4, __ , 6, 7, 8

In this series, the numbers increase by 1 in each step, except for the missing term. The missing term is 5, as it follows the pattern of incrementing by 1.

E.g.: __ , 9, 16, 25, 36, 49

Here, __ , 3^2, 4^2, 5^2, 6^2, 7^2

In this series, the numbers are squares of consecutive numbers.

The missing term is 4, as it is the square of 2 i.e., $2^2 = 4$

Example 1. What comes next in the sequence: 7, 10, 14, 19, 25, ?

(a) 30 (b) 32 (c) 36 (d) 42

Sol. (b) Given sequence: 7, 10, 14, 19, 25, ?

The sequence follows a pattern:

i.e., 7 + 3 = 10

10 + 4 = 14

14 + 5 = 19

19 + 6 = 25

Thus, the missing number is 25 + 7 = 32

Hence, the correct option is (b) i.e., 32.

Example 2. What number should come next: 58, 52, 46, 40, 34, ...?

(a) 40 (b) 28 (c) 30 (d) 26

Sol. (b) Given: 58, 52, 46, 40, 34, ...

The given sequence decreases by 6 each time i.e.,

58 − 6 = 52

52 − 6 = 46

46 − 6 = 40

40 − 6 = 34

Following this pattern, the next number = 34 − 6 = 28

Therefore, the next number in the sequence should be 28.

Hence, the correct option is (b) i.e., 28.

Example 3. What number should come next in the following sequence?

8, 16, 32, 64,...

(a) 80 (b) 96 (c) 128 (d) 256

Sol. (c) Given: 8, 16, 32, 64,...

In this sequence, each term is obtained by multiplying the previous term by 2.

Starting with 8, we multiply it by 2 to get 16. Then, we multiply 16 by 2 to get 32, and so on.

Continuing the sequence, we get

The missing number = 64 × 2 = 128

Therefore, the missing number is 128.

Hence, the correct option is (c).

Example 4. What comes next in the sequence: 16, 8, 4, 2, 1, ...

(a) 0 (b) –2 (c) 0.2 (d) 0.5

Sol. (d) Given sequence: 16, 8 , 4, 2, 1, ...

In this sequence, each term is obtained by dividing the previous term by 2 i.e.,

16 ÷ 2 = 8

8 ÷ 2 = 4

4 ÷ 2 = 2

2 ÷ 2 = 1

Thus, the missing term $= 1 \div 2 = \frac{1}{2} = 0.5$

Hence, the correct option is (d) i.e, 0.5.

Example 5. What comes next in the sequence?

7, 10, 8, 11, 9, 12, ...

(a) 10 (b) 15 (c) 9 (d) None of these

Sol. (a) Given sequence: 7, 10, 8, 11, 9, 12, ...

On observing the pattern:

$7 + 3 = 10$

$10 - 2 = 8$

$8 + 3 = 11$

$11 - 2 = 9$

$9 + 3 = 12$

Thus, the pattern is: +3, −2, +3, −2, ...

Therefore, the next number is $12 - 2 = 10$

Hence, the correct option is (a) i.e., 10.

Example 6. In the sequence, 8, 6, 9, 23, 87, ... What number should come next?

(a) 174 (b) 226 (c) 324 (d) 429

Sol. (d) Given: 8, 6, 9, 23, 87, ...

On observing the pattern, we get

$8 \times 1 - 2 = 6$

$6 \times 2 - 3 = 9$

$9 \times 3 - 4 = 23$

$23 \times 4 - 5 = 87$

$87 \times 5 - 6 = 429$

Therefore, the missing term is 429.

Hence, the correct option is (d) i.e., 429.

Example 7. The next two terms of the series: 9, 12, 11, 14, 13, 16, 15, ...

(a) 14, 13 (b) 18, 21 (c) 14, 17 (d) 18, 17

Sol. (d) This is a simple alternating addition and subtraction series i.e.,+3, −1, +3, −1, ...

Thus,

$9 + 3 = 12$

$12 - 1 = 11$

$11 + 3 = 14$

$14 - 1 = 13$

$13 + 3 = 16$

$16 - 1 = 15$

$15 + 3 = 18$

$18 - 1 = 17$

Therefore, the next two numbers are 18 and 17.

Hence, option (d) is the correct answer.

PRACTICE QUESTIONS (PART A)

1. What number should come next?
544, 509, 474, 439, ...
(a) 484 (b) 445 (c) 404 (d) 474

2. What number should come next: 3, 8, 27, 112, ... ?
(a) 256 (b) 408 (c) 565 (d) None of the above

3. 5, 2, 7, 9, 16, 25, 41, ? **(ICAI)**
(a) 65 (b) 66 (c) 67 (d) 68

4. The number that comes next in the sequence 5.2, 4.8, 4.4, 4, ... is
(a) 3.3 (b) 3.6 (c) 3.8 (d) 4.2

5. Find the missing number in the sequence, 165, 195, 255, 285, ?, 375
(a) 345 (b) 390 (c) 335 (d) 395

6. Look at the series: 14, 28, 20, 40, 32, 64, … What number should come next?
(a) 52 (b) 56 (c) 96 (d) 128

7. Look at the series: 1.5, 2.3, 3.1, 3.9, … What number should come next?
(a) 4.2 (b) 4.4 (c) 4.7 (d) 5.1

8. In the sequence, 80, 10, 70, 15, 60, ... What number should come next?
(a) 25 (b) 30 (c) 40 (d) 55

9. 120, 99, ?, 63, 48, 35 **(ICAI)**
(a) 80 (b) 36 (c) 45 (d) 40

10. What is the next term of the series: 6, 16, 44, 126, 370, ?
(a) 1100 (b) 1250 (c) 1055 (d) None of the above

Answer Key

1. (c) **2.** (c) **3.** (b) **4.** (b) **5.** (a) **6.** (b) **7.** (c) **8.** (a) **9.** (a) **10.** (a)

ALPHABET SERIES

Alphabet series involves a sequence of alphabets arranged in a specific pattern. The goal is to identify the pattern and find the missing term or predict the next term in the series.

The alphabetical position of each letter is as follow:

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

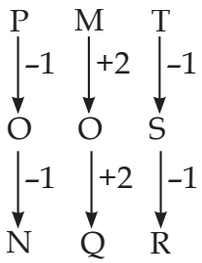
E.g.: . B, D, F, H, __

To find the next term in this series, we need to consider the pattern. Here, each letter is two positions ahead of the previous one in the English alphabet.

Applying this pattern, the next term is H + 2 = J.

Example 8. Complete this series PMT, OOS, NQR, ?

We have,



By examining the series, we can observe the following pattern:

The first letter in each term is moving in reverse alphabetical order.

The second letter in each term is increasing by 2 positions in the English alphabet.

The third letter in each term is also moving in reverse alphabetical order.

Applying this pattern, the next term in the series would be MSQ.

Therefore, the complete series would be: PMT, OOS, NQR, MSQ.

LETTER SERIES

Letter series involves a sequence of lowercase letters following a consistent pattern. The task is to identify the pattern and determine the missing term.

E.g.: __ __ a b a __ __ b a __ a b

Here, the given series is following the pattern:

ab / ab / ab / ab / ab / ab

i.e., the pattern 'ab' is repeated.

Therefore, the pattern will be a b a b a b a b a b a b

Thus, the missing letters are: a b b a b

Example 9. What is the missing term?

RST, WXY, BCD, _____ , KLM

(a) CDE (b) DEF (c) GIH (d) GHI

Sol. (d) Given: RST, WXY, BCD, _____ , KLM

On observing the pattern, we see that

The letters in each term are in order and the first letter of the next term is three positions ahead of the last letter of preceding term i.e.,

First term = RST

First letter of next term = T + 3 = W

Thus, second term = WXY

First letter of next term= Y + 3 = B

Thus, third term = BCD

Now, first letter of next term = D + 3 = G

Thus, the missing term = GHI

Hence, the correct option is (d) i.e., GHI.

PRACTICE QUESTIONS (PART B)

1. What is the missing term?

QPO, NML , KJI, _____ , EDC

(a) HGF (b) CAB (c) JKL (d) GHI

2. The missing letters in __ __ c d c __ __ d c __ c d are

(a) c d c c d (b) c d d c d (c) c c d c d (d) d d d c d

3. What comes at last place in R, U, X, A, D ?

(a) E (b) F (c) G (d) H

4. Find missing term of the letter series A, CD, GHI, ?, UVWXY.

(a) LMNO (b) MNO (c) MNOP (d) NOPQ

5. Find the next alphabet series in the given sequence: ALN, DNP, GPR, ?

(a) KLN (b) JRT (c) RNU (d) RNV

Answer Key

1. (a) **2.** (b) **3.** (c) **4.** (c) **5.** (b)

CODING AND DECODING

When we talk about Coding and Decoding, there are 2 types:

- Letter coding
- Number coding

LETTER CODING

In letter coding, the real alphabets in a word are replaced by certain other alphabets based on a specific rule. The task for the candidate is to identify the common rule and apply it to answer the given questions. This type of coding requires a keen understanding of the patterns and rules governing the replacement of alphabets.

E.g.: If GOLD is written as IQNF, WIND can be written in the same code as _____

Here, the pattern followed is:

Adding 2 to the place values of the letters.

G + 2 = I, O + 2 = Q, L + 2 = N, D + 2 = F

Therefore, WIND can be written as:

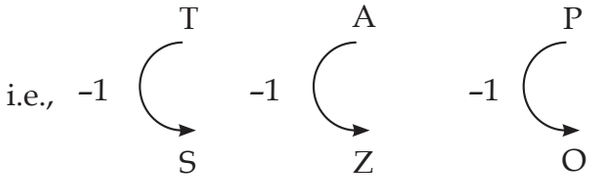
W + 2 = Y, I + 2 = K, N + 2 = P, D + 2 = F

Thus, WIND can be coded as YKPF.

Example 10. If TAP is coded as SZO, then how is FRIEND coded?

(a) EQJDNC (b) EQHDMC (c) GSIEND (d) None of above

Sol. (b) Given: TAP is coded as SZO



Similarly, FRIEND will be coded as EQHDMC.

Hence, the correct option is (b) i.e., EQHDMC.

Example 11. In certain code 'BILLION' is written as 'I B L L O I N'. How is 'HILTON' written in that code?

(a) I H T L O N (b) I H T L N O (c) O H T L I N (d) H I T L N O

Sol. (b) Given: 'BILLION' is coded as 'I B L L O I N'

We observe that the letters are exchanged in pairs

i.e., B I becomes I B, L L becomes L L, I O becomes O I and N remains same since it does not have any pair.

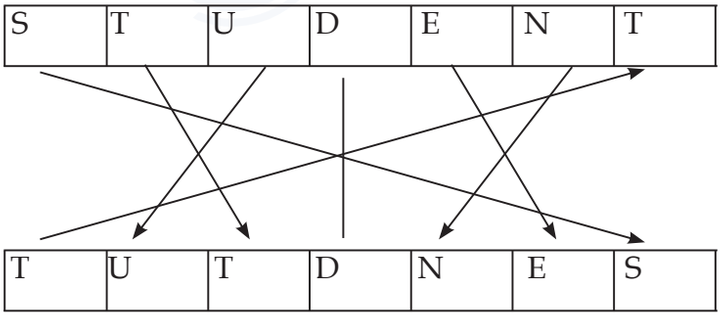
Similarly 'HILTON' can be coded as 'I H T L N O'

Hence, the correct option is (b) i.e., I H T L N O.

Example 12. In a certain code STUDENT is written as TUTDNES. How will SOURCES be written in that code ?

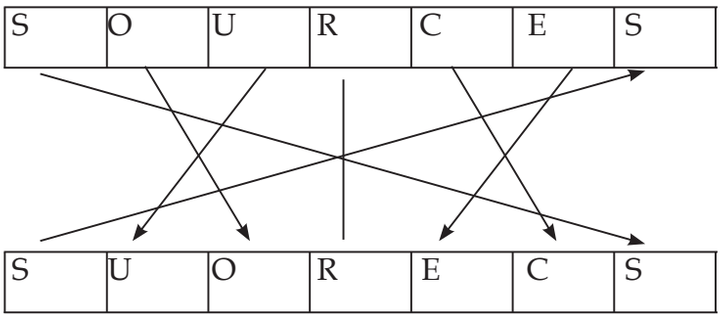
(a) SOURECS (b) SOCRSEU (c) SUORECS (d) SORCESU

Sol. (c) STUDENT is written as TUTDNES,



Similarly,

SOURCES be written in that code as:



Therefore, SOURCES can be written in that code as SUORECS.

Hence, the correct option is (c).

Example 13. In a certain code language, 'PICTURE' is written as 'QHDSVQF'. How would 'BROWSER' be written in that same code language?

(a) CQVVTDS (b) CQPVTDS (c) CQPUTDS (d) CQVPPDS

Sol. (b) 'PICTURE' is written as 'QHDSVQF'

It can be observed that:

P + 1 = Q , I - 1 = H, C + 1 = D, T - 1 = 5, U + 1 = V, R - 1 = Q, E + 1 = F

Similarly, for 'BROWSER'

B + 1 = C, R - 1 = Q, 0 + 1 = P, W - 1 = V , S + 1 = T, E – 1 = D, R + 1 = S

Therefore, BROWSER is written as CQPVTDS in the same code.

Hence, the correct option is (b).

NUMBER CODING:

Number coding involves replacing alphabets with corresponding numerical values based on a specific rule. Each alphabet is assigned a unique numerical value, and the task is to decipher the code by understanding the assigned values and applying the given rule.

There are two types of number coding:

- **When letter is given in a particular number code:**

Example 14. If 'GLOSSORY' is coded as '97533562' and 'GEOGRAPHY' is coded as '915968402', then 'GEOLOGY' can be coded as

(a) 915692 (b) 9157592 (c) 9057592 (d) 9157591

Sol. (b) G → 9, L → 7, O → 5, S → 3, S → 3, O → 5, R → 6, Y → 2

Similarly, 'GEOGRAPHY' is coded as '915968402'

Therefore, GEOLOGY can be coded as: -

G→9, E → 1, 0 → 5, L → 7, 0 → 5, G → 9, Y → 2

i.e., 9157592

Hence, option (b) is the correct answer

Example 15. In a certain code ATE is written as 145 and CHAIR is written as 09173, then how TEACHER can be written in the code?

(a) 4501953 (b) 4510953 (c) 4510934 (d) 4530943

Sol. (b) ATE is written as 145

A→1, T→4, E→5

Similarly, CHAIR is written as 09173

C→0, H→9, A→1, 1→7, R→3

Therefore, TEACHER can be written as: -

T→4, E→5, A→1, C→0, H→9, E→5, R→3

i.e., 4510953

Hence, the correct option is (b).

❑ **When number is given in a particular letter code:**

Example 16. In a certain code, a number 18462 is written as BETKO and 7935 is written as RAHU. How is 43857 written in that code ?

(a) THOEB (b) THROB (c) THKOB (d) THEUR

Sol. (d) 18462 is written as BETKO

1→B, 8→E, 4→T, 6→K, 2→O

Similarly,

7935 is written as RAHU

7→R, 9→A, 3→H, 5→U

Therefore, 43857 can be written as:

4→T, 3→H, 8→E, 5→U, 7→R

i.e., THEUR

Hence, option (d) is the correct answer.

Example 17. If 'RAJU' is coded as 11-12-13-14 and 'JUNK' is coded as 13-14-10-9, then how will you code 'RANK'?

(a) 9-10- 11- 12 (b) 10-11-12-9 (c) 11-12-10-9 (d) 12-11-10-9

Sol. (c) 'RAJU' is coded as 11 − 12 − 13 − 14

R - 11

A - 12

J - 13

U - 14

Similarly, 'JUNK' is coded as 13 − 14 − 10 − 9

J − 13

U − 14

N − 10

K − 9

Therefore, 'RANK' can be coded as: -

R – 11

A – 12

N – 10

K – 9

i.e. 11 – 12 – 10 – 9

Hence, option (c) is the correct answer.

PRACTICE QUESTIONS (PART C)

1. If EARTH is written as FCUXM in a certain code. How is DELHI written in that code?

(a) EFMIJ (b) EGOLN (c) EFNJL (d) FGNKM

2. In a certain code, NEWYORK is written as 111, how is NEWZEALAND written in that code?

(a) 115 (b) 111 (c) 110 (d) 105

3. In a certain code, "TIGER" is written as "72159" and "LION" is written as "6247". How is "GOAT" written in that code?

(a) 1673 (b) 1467 (c) 4178 (d) 1437

4. **LETTER:** L E A D I N G

CODE DIGIT: 3 6 1 4 2 5 7

Find out the correctly coded alternative for NGADLIA from amongst the given four alternatives.

(a) 5114312 (b) 4716321 (c) 5714321 (d) None of the above

5. If CLOCK is coded 34235 and TIME is 8679. What will be the code of MOTEL?

(a) 72894 (b) 77684 (c) 72964 (d) 27894

6. If BROTHER is coded 2456784, SISTER coded as 919684, what is coded for BORBERS?

(a) 2542849 (b) 2542898 (c) 2454889 (d) 2524889

7. If HONEY is coded as JQPGA, which word is coded as VCTIGVU?

(a) CARPETS (b) TRAPETS (c) TARGETS (d) UMBRELU

8. If HEALTH is coded as GSKZDG, then how NORTH be written in that code?

(a) OPSUI (b) GSQNM (c) FRPML (d) IUSPO

9. In a certain code, TEACHER is written as VGCEJGT, how is CHILDREN written in that code?

(a) EJKNEGTP (b) EJKNEITP (c) EJKNFGTO (d) EJKNFTGP

10. If SUMMER is coded as RUNNER, the code for WINTER will be

(a) SUITER (b) VIOUER (c) WALKER (d) SUFFER

Answer Key

1. (b) **2.** (d) **3.** (d) **4.** (c) **5.** (a) **6.** (a) **7.** (c) **8.** (b) **9.** (d) **10.** (b)

ODD MAN OUT

Odd Man Out is a problem-solving concept where a series of elements is presented, and the task is to identify the element that does not fit the given pattern or follow the same rule as the other elements.

To identify the Odd Man Out, you need to carefully analyze the given elements and look for any distinguishing features, characteristics, or patterns that set the odd element apart from the others. By identifying the unique element, you can determine the odd one out.

E.g.: 2, 4, 6,8 , 10, 12, 15

In this series of numbers, all the elements are even except for "15." The odd number "15" stands out and does not follow the same pattern as the other even numbers. Therefore, "15" is the Odd Man Out in this series.

Example 18. Find the odd man out of the following given numbers.

(a) 12 (b) 13 (c) 19 (d) 17

Sol. (a) Prime numbers are the numbers which have only two factors,i.e.1 and the number itself.

Here, factors of 12 are:

1, 2, 3, 4, 6, 12 (i.e. more than 2 factors)

whereas all the other numbers except 12 have only two factors (1 and the number itself).

Hence, except for 12, all other numbers are prime numbers.

Therefore, option (a) is the correct answer.

Example 19. Find odd man out of the following:6,9 , 15, 21, 24, 28, 30

(a) 28 (b) 21 (c) 24 (d) 30

Sol. (a) On observing the pattern, we see that

$3 \times 2 = 6$, $3 \times 3 = 9$, $3 \times 5 = 15$, $3 \times 7 = 21$, $3 \times 8 = 24$, $3 \times 10 = 30$

All the numbers except 28 are the multiples of 3.

Hence, option (a) is the correct answer.

Example 20. Select the odd man out:

(a) Cricket (b) Volleyball (c) Fencing (d) Baseball

Sol. (c) Cricket, Volleyball and Baseball are games that are played with the help of a ball while Fencing is a combat sport that features sword fighting.

Hence, Fencing is the odd man out.

Therefore, option (c) is the correct answer.

Example 21. Find odd one out of the following:

41, 43, 47, 53, 61, 71, 75, 83

(a) 75 (b) 43 (c) 71 (d) 53

Sol. (a) We have, 41, 43, 47, 53, 61, 71, 75, 83

All the numbers except the number 75 are prime numbers.

Hence, the correct option is (a).

Example 22. There are four groups of letters in each. Three of these groups are alike in some way while one is different. Find the one which is different.

TRP, YWU, SQO, TVX

(a) TRP (b) YWU (c) SQO (d) TVX

Sol. (d) On observing the pattern, we see that

TRP → T − 2 = R, R − 2 = P

YWU → Y − 2 = W, W − 2 = U

SQO → S − 2 = Q, Q − 2 = O

All the words follow a certain pattern except TVX.

Hence, the correct answer is option (d) i.e. TVX.

Example 23. Find odd man out of the following:

2, 5, 10, 17, 26, 37, 50, 64

(a) 50 (b) 26 (c) 37 (d) 64

Sol. (d) On observing the pattern, we see that

$1^2 + 1 = 2$,

$2^2 + 1 = 5$,

$3^2 + 1 = 10$,

$4^2 + 1 = 17$,

$5^2 + 1 = 26$,

$6^2 + 1 = 37$,

$7^2 + 1 = 50$,

$8^2 + 1 = 65 \neq 64$

Hence, the correct answer is option (d) i.e. 64.

PRACTICE QUESTIONS (PART D)

1. Find the odd one out of the following:

1, 5, 7, 11, 14, 17, 21

(a) 5 (b) 11 (c) 14 (d) 21

2. Find the odd one out of the following:

10, 15, 20, 25, 45, 54, 60, 75, 80

(a) 80 (b) 75 (c) 45 (d) 54

3. Find the odd one out of the following:

8, 27, 64, 81, 125, 216, 343

(a) 27 (b) 81 (c) 216 (d) 343

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4. Find odd man out of the following:
4, 9, 256, 529, 573
(a) 529 (b) 9 (c) 573 (d) 256

5. Find odd man out of the following:
January, May, July, November
(a) January (b) May (c) July (d) November

6. Find odd man out of the following:
6, 9, 15, 21, 24, 28, 30
(a) 28 (b) 21 (c) 24 (d) 30

7. Find odd man out of the following:
1, 5, 14, 30, 51, 55, 91
(a) 5 (b) 55 (c) 51 (d) 91

8. Find the odd man out of the following: 13, 14, 18, 27, 32, 43, 68
(a) 27 (b) 43 (c) 32 (d) 68

9. Find the odd one out of the following: 4, 9, 16, 25, 36, 48, 64
(a) 25 (b) 16 (c) 48 (d) 9

10. Which of the following is odd one?
(a) CEHL (b) KMPT (c) OQTX (d) NPSV

11. Find wrong number of the series; 1, 5, 5, 9, 7, 11, 11, 15, 12, 17.
(a) 11 (b) 12 (c) 17 (d) 15

12. Find the odd man out in the following series; 190, 145, 136, 352, 460, 324, 631, 244.
(a) 136 (b) 244 (c) 460 (d) 324

Answer Key

1. (c) **2.** (d) **3.** (b) **4.** (c) **5.** (d) **6.** (a) **7.** (c) **8.** (c) **9.** (c) **10.** (d)
11. (b) **12.** (d)

FULL CHAPTER SOLVED QUESTIONS

Example 24. In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code? **(ICAI, May 2018)**

(a) 318826 (b) 318286 (c) 618826 (d) 338816

Sol. (a) Word RIPPLE is coded as 613382

R I P P L E
6 1 3 3 8 2

Similarly,

Word LIFE is coded as 8192

L I F E

8 1 9 2

On observing the above patterns, we can see that the numbers represent the code for the alphabets.

Thus, the word PILLER can be coded as 318826.

Hence, option (a) is the correct answer.

Example 25. If LOSE is coded as 1357 and GAIN is coded as 2468 what does figure 82146 stands for? **(ICAI, May 2018)**

(a) NGLAI (b) NGLIA (c) GNLIA (d) GNIA

Sol. (a) LOSE is coded as 1357

L - 1, O – 3, S-5, E – 7

Similarly,

GAIN is coded as 2468

G – 2, A – 4, I – 6, N – 8

Therefore, the figure 82146 is for

8 – N, 2 – G, 1 – L, 4 – A, 6 – I

i.e., NGLAI

Hence, option (a) is the correct answer.

Example 26. If PLAY is coded as 8123 and RHYME is coded as 49367. What will be the code of MALE? **(Nov, 2018)**

(a) 6217 (b) 6198 (c) 6395 (d) 6285

Sol. (a) PLAY is coded as 8123

P – 8, L – 1, A – 2, Y – 3

RHYME is coded as 49367

R – 4, H – 9, Y – 3, M – 6, E – 7

On observing the above patterns, we can see that the numbers represent the code for the alphabets.

Then, the code of MALE is:

M – 6

A – 2

L – 1

E – 7

i.e., 6217

Hence, option (a) is the correct answer.

Example 27. Find out the next number in the following series 7, 11, 13, 17, 19, 23, 25, 29 ?

(a) 30 (b) 31 (c) 32 (d) 33 **(Nov, 2018)**

Sol. (b) 7 + 4 = 11

11 + 2 = 13

13 + 4 = 17

17 + 2 = 19

19 + 4 = 23

23 + 2 = 25

25 + 4 = 29

29 + 2 = 31

The next term in the series is 31.

Hence, option (b) is the correct answer.

Example 28. If HONEY is coded as JQPGA.What is the code for VCTIGVU? **(Nov, 2018)**

(a) XEVKIXW (b) TRAPETS (c) TARGETS (d) UMBRELU

Sol. (a) HONEY is coded as JQPGA

H + 2 = J

O + 2 = Q

N + 2 = P

E + 2 = G

Y + 2 = A

Similarly,

VCTIGVU is coded as

V + 2 = X

C + 2 = E

T + 2 = V

I + 2 = K

G + 2 = I

V + 2 = X

U + 2 = W

i.e. XEVKIXW

Hence, option (a) is the correct answer.

Example 29. Find odd out of the following series: 15, 21, 63 ,81, 69. **(Nov, 2018)**

(a) 15 (b) 21 (c) 63 (d) 81

Sol. (d) 15, 21, 63, 81, 69

From the above series 81 is odd one out because only 81 is a perfect square of a number.

Hence, option (d) is the correct answer.

Example 30. Find the odd man out of the following series 7, 9, 13, 17, 19. **(Nov, 2018)**

(a) 7 (b) 9 (c) 19 (d) 13

Sol. (b) Given series: 7, 9, 13, 17, 19

From the above series 9 is odd one out because only 9 is a perfect square of a number.

i.e., $3^2 = 9$

Hence, option (b) is the correct answer.

Example 31. Find the next number of the series: 7, 23, 47, 119, 167,… **(June 2019)**

(a) 211 (b) 223 (c) 287 (d) 319

Sol. (b) $7 = 3^2 - 2$

$23 = 5^2 - 2$

$47 = 7^2 - 2$

$79 = 9^2 - 2$

$119 = 11^2 - 2$

$167 = 13^2 - 2$

Therefore, the next term is $15^2 - 2 = 223$

Hence, option (b) is the correct answer.

Example 32. Find the odd man out of the following: 13, 14, 18, 27, 32, 43, 68.

(a) 27 (b) 43 (c) 32 (d) 68

Sol. (c) Given: 13, 14, 18, 27, 32, 43, 68

The pattern in the given numbers is:

13 + 1 = 14

14 + 4 = 18

18 + 9 = 27

27 + 16 = 43

43 + 25 = 68

The number 32 does not follow the pattern.

Therefore, the odd one out is 32.

Hence, the correct option is (c).

Example 33. If in a certain language, MADRAS is coded as NBESBT, How is BOMBAY coded in that language? **(Dec, 2019)**

(a) CPNCBX (b) CPNCBZ (c) CPOCBZ (d) CQOCBZ

Sol. (b) M + 1 = N

A + 1 = B

D + 1 = E

R + 1 = S

A + 1 = B

S + 1 = T

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Similarly,

B + 1 = C

O + 1 = P

M + 1 = N

B + 1 = C

A + 1 = B

Y + 1 = Z

i.e. CPNCBZ

Hence, option (b) is the correct answer.

Example 34. Which of the following is an odd one out?

(a) CEHL (b) KMPT (c) OQTX (d) NPSV

Sol. (d) C + 2 = E + 3 = H + 4 = L

K + 2 = M + 3 = P + 4 = T

O + 2 = Q + 3 = T + 4 = X

N + 2 = P + 3 = S + 3 = V

Hence, NPSV is an odd one out.

Therefore, option (d) is the correct answer.

Example 35. Look at this series:2, 1 , $\frac{1}{2}, \frac{1}{4}$,… What number should come next?

(a) $\frac{1}{3}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{1}{16}$

Sol. (b) This is a simple division series: each number is one-half of the previous number. In other terms to say, the number is divided by 2 successively to get the next result:

$2 \div 2 = 1$

$1 \div 2 = \frac{1}{2}$

$\frac{1}{2} \div 2 = \frac{1}{4}$

Next number will be:

$\frac{1}{4} \div 2 = \frac{1}{8}$

Therefore, $\frac{1}{8}$ should come next.

Hence, option (b) is the correct answer.

Example 36. Look at the series: 7, 10, 8, 11, 9, 12, …..What number should come next?

(a) 7 (b) 10 (c) 12 (d) 13

Sol. (b) This is a simple alternating addition and subtraction series. In the first pattern, 3 is added; in the second, 2 is subtracted.

Hence,

$7 + 3 = 10$

$10 - 2 = 8$

$8 + 3 = 11$

$11 - 2 = 9$

$9 + 3 = 12$

$12 - 2 = 10$

10 should come next.

Therefore, option (b) is the correct answer

Example 37. Look at the series: 36, 34, 30, 28, 24,….What number should come next?

(a) 20 (b) 22 (c) 23 (d) 26

Sol. (b) The given series follows the pattern as:

$36 - 2 = 34$

$34 - 4 = 30$

$30 - 2 = 28$

$28 - 4 = 24$

$24 - 2 = 22$

Therefore, 22 should come next.

Hence, option (b) is the correct answer.

Example 38. Look at the series: 22, 21, 23, 22, 24, 23,…What number should come next?

(a) 22 (b) 24 (c) 25 (d) 26

Sol. (c) The given series follows the pattern as:

$22 - 1 = 21$

$21 + 2 = 23$

$23 - 1 = 22$

$22 + 2 = 24$

$24 - 1 = 23$

$23 + 2 = 25$

Thus, 25 should come next.

Therefore, option (c) is the correct answer.

Example 39. Look at the series: 53, 53, 40, 40, 27, 27,… What number should come next?

(a) 12 (b) 14 (c) 27 (d) 53

Sol. (b) The given series follows the pattern as:

$53 - 13 = 40$

$40 - 13 = 27$

$27 - 13 = 14$

Now, 14 will be repeated.

Thus, 14 should come next.

Therefore, option (b) is the correct answer.

Example 40. Look at the series: 21, 9, 21, 11, 21, 13, 21, … What number should come next?

(a) 14 (b) 15 (c) 21 (d) 23

Sol. (b) In this alternating repetition series, the random number 21 is interpolated with every other number into an otherwise simple addition series that increases by 2, beginning with the number 9.

Hence,

9 + 2 = 11

then, 21

11 + 2 = 13

then, 21

13 + 2 = 15

15 should come next.

Therefore, option (b) is the correct answer.

Example 41. Look at the series: 58, 52, 46, 40, 34, … What number should come next?

(a) 26 (b) 28 (c) 30 (d) 32

Sol. (b) The given series follows the pattern as:

58 − 6 = 52

52 − 6 = 46

46 − 6 = 40

40 − 6 = 34

34 − 6 = 28

Thus, 28 should come next.

Therefore, option (b) is the correct answer.

Example 42. Look at the series: 3, 4, 7, 8, 11, 12, … What number should come next?

(a) 7 (b) 10 (c) 14 (d) 15

Sol. (d) The given series follows the pattern as:

3 + 1 = 4

4 + 3 = 7

7 + 1 = 8

8 + 3 = 11

11 + 1 = 12

12 + 3 = 15

Thus, 15 should come next.

Therefore, option (d) is the correct answer.

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Example 43. Look at the series: 8, 22, 8, 28, 8, … What number should come next?

(a) 9 (b) 29 (c) 32 (d) 34

Sol. (d) This is a simple addition series with a random number, 8, interpolated as every other number. In the series, 6 is added to each number except 8, starting from 22 to arrive at the next number.

Hence,

22 + 6 = 28

then 8

28 + 6 = 34

Therefore, 34 should come next.

Hence, option (d) is the correct answer.

Example 44. Look at the series: 31, 29, 24, 22, 17, … What number should come next?

(a) 15 (b) 14 (c) 13 (d) 12

Sol. (a) The given series follows the pattern as:

31 – 2 = 29

29 – 5 = 24

24 – 2 = 22

22 – 5 = 17

17 – 2 = 15

Therefore, 15 should come next.

Hence, option (a) is the correct answer.

Example 45. Look at the series: 2, 4, 6, 8, 10, … What number should come next?

(a) 11 (b) 12 (c) 13 (d) 14

Sol. (b) The given series follows the pattern as:

2 + 2 = 4

4 + 2 = 6

6 + 2 = 8

8 + 2 = 10

10 + 2 = 12

Thus, 12 should come next.

Therefore, option (b) is the correct answer.

Example 46. Look at the series: 201, 202, 204, 207, … What number should come next?

(a) 205 (b) 208 (c) 210 (d) 211

Sol. (d) The given series follows the pattern as:

201 + 1 = 202

202 + 2 = 204

204 + 3 = 207

207 + 4 = 211

Thus, 211 should come next.

Therefore, option (d) is the correct answer.

Example 47. Look at the series: 544, 509, 474, 439, … What number should come next?

(a) 404 (b) 414 (c) 420 (d) 445

Sol. (a) The given series follows the pattern as:

544 – 35 = 509

509 – 35 = 474

474 – 35 = 439

439 – 35 = 404

Thus, 404 should come next.

Therefore, option (a) is the correct answer.

Example 48. Look at the series: 2, 6, 18, 54, … What number should come next?

(a) 108 (b) 148 (c) 162 (d) 216

Sol. (c) The given series follows the pattern as:

2 × 3 = 6

6 × 3 = 18

18 × 3 = 54

54 × 3 = 162

Therefore, 162 should come next.

Hence, option (c) is the correct answer.

Example 49. Find the missing number: 7, 26, 63, 124, 215, ?, 511 **(ICAI)**

(a) 342 (b) 343 (c) 442 (d) 421

Sol. (a) We have, 7, 26, 63, 124, 215, ?, 511

The pattern follows here is:

$2^3 - 1 = 7$

$3^3 - 1 = 26$

$4^3 - 1 = 63$

$5^3 - 1 = 124$

$6^3 - 1 = 215$

$7^3 - 1 = 342$

$8^3 - 1 = 511$

Hence, the correct option is (a).

Example 50. Look at the series: 5.2, 4.8, 4.4, 4, … What number should come next?

(a) 3 (b) 3.3 (c) 3.5 (d) 3.6

Sol. (d) In this simple subtraction series, each number decreases by 0.4 i.e.,

5.2 – 0.4 = 4.8

4.8 – 0.4 = 4.4

4.4 – 0.4 = 4

4 – 0.4 = 3.6

Therefore, 3.6 should come next.

Hence, option (d) is the correct answer.

Example 51. Look at the series: 8, 6, 9, 23, 87, … What number should come next?

(a) 128 (b) 226 (c) 324 (d) 429

Sol. (d) The given series follows the pattern as:

8 × 1 – 2 = 6

6 × 2 – 3 = 9

9 × 3 – 4 = 23

23 × 4 – 5 = 87

87 × 5 – 6 = 429

Therefore, 429 should come next.

Hence, option (d) is the correct answer.

Example 52. The next two terms of the series: 28, 25, 5, 21, 18, 5, 14, ...

(a) 11, 5 (b) 10, 7 (c) 11, 8 (d) 5, 10

Sol. (a) This is an alternating subtraction series with the interpolation of a random number, 5, as every third number. In the subtraction series, 3 is subtracted, then 4, then 3, and so on.

Thus,

28 – 3 = 25

then 5

25 – 4 = 21

21 – 3 = 18

then 5

18 – 4 = 14

14 – 3 = 11

then 5

i.e., 11, 5

Therefore, option (a) is the correct answer.

Example 53. The next two terms of the series: 8, 11, 21, 15, 18, 21, 22,…

(a) 25, 18 (b) 25, 21 (c) 25, 29 (d) 24, 21

Sol. (b) This is an alternating addition series, with a random number, 21, interpolated as every third number. The addition series alternates between adding 3 and adding 4. The number 21 appears after each number arrived at by adding 3.

Hence,

8 + 3 = 11

then 21

11 + 4 = 15

15 + 3 = 18

then 21

18 + 4 = 22

22 + 3 = 25

then 21

i.e. 25, 21

Therefore, option (b) is the correct answer.

Example 54. The next two terms of the series: 9, 16, 23, 30, 37, 44, 51,.....

(a) 59, 66 (b) 56, 62 (c) 58, 66 (d) 58, 65

Sol. (d) Here is a simple addition series, which begins with 9 and adds 7.

Hence,

9 + 7 = 16

16 + 7 = 23

23 + 7 = 30

30 + 7 = 37

37 + 7 = 44

44 + 7 = 51

51 + 7 = 58

58 + 7 = 65

i.e., 58, 65

Therefore, option (d) is the correct answer.

Example 55. The next two terms of the series: 2, 8, 14, 20, 26, 32, 38,...

(a) 2, 46 (b) 44, 50 (c) 42, 48 (d) 40, 42

Sol. (b) This is a simple addition series, which begins with 2 and adds 6.

Hence,

2 + 6 = 8

8 + 6 = 14

14 + 6 = 20

20 + 6 = 26

26 + 6 = 32

32 + 6 = 38

38 + 6 = 44

44 + 6 = 50

i.e., 44,50

Therefore, option (b) is the correct answer.

Example 56. The next two terms of the series: 9, 11, 33, 13, 15, 33, 17, ...

(a) 19, 33 (b) 33, 35 (c) 33, 19 (d) 15, 33

Sol. (a) In this alternating repetition series, a random number, 33, is interpolated every third number into a simple addition series in which each number increases by 2.

Hence,

9 + 2 = 11

then 33

11 + 2 = 13

13 + 2 = 15

then 33

15 + 2 = 17

17 + 2 = 19

then 33

i.e., 19,33

Therefore, option (a) is the correct answer.

Example 57. The next two terms of the series: 2, 3, 4, 5, 6, 4, 8,...

(a) 9, 10 (b) 4, 8 (c) 10, 4 (d) 9, 4

Sol. (d) This is an alternating addition series with a random number, 4, interpolated as every third number. In the main series, 1 is added, then 2 is added, then 1, then 2, and so on.

Hence,

2 + 1 = 3

then 4

3 + 2 = 5

5 + 1 = 6

then 4

6 + 2 = 8

8 + 1 = 9

then 4

i.e.,9,4

Therefore, option (d) is the correct answer.

Example 58. The next two terms of the series: 17, 17, 34, 20, 20, 31, 23, ...

(a) 26, 23 (b) 34, 20 (c) 23, 33 (d) 23, 28

Sol. (d) This is an alternating subtraction series with repetition. There are two different patterns here. In the first, a number repeats itself; then 3 is added to that number to arrive at the next number, which also repeats. This gives the series 17, 17, 20, 20, 23 and so on.

Further, every third number follows a second pattern, in which 3 is subtracted from each number to arrive at the next: 34, 31, 28.

i.e., 23 28

Therefore, option (d) is the correct answer.

Example 59. The next two terms of the series: 6,20,8,14,10,8,12,...

(a) 14, 10 (b) 2, 18 (c) 4, 12 (d) 2, 14

Sol. (d) This is an alternating addition and subtraction series. In the first pattern, 2 is added to each number to arrive at the next; in the alternate pattern, 6 is subtracted from each number to arrive at the next.

Thus,

6 + 2 = 8

8 + 2 = 10

10 + 2 = 12

12 + 2 = 14

Other series:-

20 – 6 = 14

14 – 6 = 8

8 – 6 = 2

i.e., 2 ,14

Therefore, option (d) is the correct answer.

Example 60. The next two terms of the series: 21, 25, 18, 29, 33, 18, ...

(a) 43, 18 (b) 41, 44 (c) 37, 18 (d) 37, 41

Sol. (d) This is a simple addition series with a random number, 18, interpolated as every third number. In the series, 4 is added to each number except 18, to arrive at the next number.

Hence,

21 + 4 = 25

then 18

25 + 4 = 29

29 + 4 = 33

then 18

33 + 4 = 37

37 + 4 = 41

i.e., 37 ,41

Therefore, option (d) is the correct answer.

Example 61. The next two terms of the series: 75, 65, 85, 55, 45, 85, 35, ...

(a) 25, 15 (b) 25, 85 (c) 35, 25 (d) 85, 35

Sol. (b) This is a simple subtraction series in which a random number, 85, is interpolated as every third number. In the subtraction series, 10 is subtracted from each number to arrive at the next.

Hence,

75 – 10 = 65

then 85

65 – 10 = 55

55 – 10 = 45

then 85

45 – 10 = 35

35 – 10 = 25

then 85

i.e., 25 ,85

Therefore, option (b) is the correct answer.

Example 62. The next two terms of the series 11, 16, 21, 26, 31, 36, 41,... are

(a) 47, 52 (b) 46, 52 (c) 45, 49 (d) 46, 51

Sol. (d) In this simple addition series, each number is 5 more than the previous number.

Hence,

11 + 5 = 16

16 + 5 = 21

21 + 5 = 26

26 + 5 = 31

31 + 5 = 36

36 + 5 = 41

41 + 5 = 46

46 + 5 = 51

i.e., 46, 51

Therefore, option (d) is the correct answer.

Example 63. The next two terms of the series 3, 8, 13, 18, 23, 28, 33,... are

(a) 39, 44 (b) 28, 44 (c) 38, 43 (d) 37, 42

Sol. (c) In this simple addition series, each number is 5 greater than the previous number.

Hence,

3 + 5 = 8

8 + 5 = 13

13 + 5 = 18

18 + 5 = 23

23 + 5 = 28

28 + 5 = 33

33 + 5 = 38

38 + 5 = 43

i.e., 38 ,43

Therefore, option (c) is the correct answer.

Example 64. The next two terms of the series: 84,78,72,66,60,54,48 , ...

(a) 44, 34 (b) 42, 36 (c) 42, 32 (d) 40, 34

Sol. (b) In this simple subtraction series, each number is 6 less than the previous number.

Hence,

84 – 6 = 78

78 – 6 = 72

72 – 6 = 66

66 – 6 = 60

60 – 6 = 54

54 – 6 = 48

48 – 6 = 42

42 – 6 = 36

i.e. 42 ,36

Therefore, option (b) is the correct answer.

Example 65. The next two terms of the series: 20, 20, 17, 17, 14, 14, 11,...

(a) 11, 8 (b) 11, 11 (c) 11, 14 (d) 8, 9

Sol. (a) This is a simple subtraction with repetition. It begins with 20, which is repeated, then 3 is subtracted, resulting in 17, which is repeated again then 3 is subtracted to get 11, which is repeated, further 3 is subtracted to get 8.

Therefore, the next two numbers are 11, 8.

Therefore, option (a) is the correct answer.

Example 66. The next two terms of the series: 61, 57, 50, 61, 43, 36, 61,...

(a) 29, 61 (b) 29, 22 (c) 31, 61 (d) 22, 15

Sol. (b) This is an alternating repetition series, in which a random number, 61, is interpolated as every third number into an otherwise simple subtraction series. Starting with the second number, 57, each number (except 61) is 7 less than the previous number.

Hence,

61

57 – 7 = 50

then 61

50 – 7 = 43

43 – 7 = 36

then 61

36 – 7 = 29

29 – 7 = 22

i.e., 29 ,22

Therefore, option (b) is the correct answer.

Example 67. The next two terms of the series: 4, 8, 22, 12, 16, 22, 20, 24 ...

(a) 28, 32 (b) 28, 22 (c) 22, 28 (d) 32, 36

Sol. (c) This is an alternating repetition series, with a random number, 22, interpolated as every third number into an otherwise simple addition series. In the addition series, 4 is added to each number to arrive at the next number.

Hence,

4 + 4 = 8

then 22

8 + 4 = 12

12 + 4 = 16

then 22

16 + 4 = 20

20 + 4 = 24

then 22

24 + 4 = 28

i.e. 22 ,28

Therefore, option (c) is the correct answer.

Example 68. The next two terms of the series: 40, 40, 31, 31, 22, 22, 13 ...

(a) 13, 4 (b) 13, 5 (c) 4, 13 (d) 9, 4

Sol. (a) This is a subtraction series with repetition. Each number repeats itself and then decreases by 9.

Hence,

40

40 – 9 = 31

31

31 – 9 = 22

22

22 – 9 = 13

13

13 – 9 = 4

i.e.13 ,4

Therefore, option (a) is the correct answer.

Example 69. The next two terms of the series: 1, 10, 7, 20, 13, 30, 19, ...

(a) 26, 40 (b) 29, 36 (c) 40, 25 (d) 25, 31

Sol. (c) Here, we have two series patterns. In the first series, 6 is added to each number to arrive at the next. In the second series, 10 is added to each number to arrive at the next.

Hence,

1st series

1 + 6 = 7

7 + 6 = 13

13 + 6 = 19

19 + 6 = 25

2nd series:-

10 + 10 = 20

20 + 10 = 30

30 + 10 = 40

i.e., 40 ,25

Therefore, option (c) is the correct answer.

Example 70. The next two terms of the series: 10, 20, 25, 35, 40, 50, 55, ...

(a) 70, 65 (b) 60, 70 (c) 60, 75 (d) 65, 70

Sol. (d) This is an alternating addition series, in which 10 is added, then 5, then 10, and so on.

Hence,

10 + 10 = 20

20 + 5 = 25

25 + 10 = 35

35 + 5 = 40

40 + 10 = 50

50 + 5 = 55

55 + 10 = 65

65 + 5 = 70

i.e.65 ,70

Therefore, option (d) is the correct answer.

FULL CHAPTER PRACTICE QUESTIONS (PART E)

1. What will be the next term of the series: 7, 23, 47, 119, 167,?

(a) 211 (b) 223 (c) 287 (d) 319

2. Which of the following is the odd one out of the following series?

4, 12, 44, 176, 890

(a) 4 (b) 12 (c) 890 (d) 176

3. Which of the following is the odd one out of the following series?

835, 735, 642, 751, 853, 981

(a) 835 (b) 751 (c) 981 (d) 642

4. Find the odd man out of the series; 22, 33, 66, 77, 121, 279, 594

(a) 33 (b) 77 (c) 279 (d) 594

5. Find the odd man out of the following: 8, 13, 21, 32, 47, 63, 83

(a) 13 (b) 47 (c) 32 (d) 83

6. Find the odd man out of the following: 1, 2, 6, 15, 31, 56, 91

(a) 2 (b) 56 (c) 91 (d) 6

7. Find the odd man out of the following: 2, 3 , $\frac{4}{3}, \frac{1}{2}, \sqrt{3}$

(a) $\sqrt{3}$ (b) 3 (c) $\frac{4}{3}$ (d) $\frac{1}{2}$

8. Find the missing number in the following series: 10, 18, 28, 40, 54, ?, 88

(a) 70 (b) 86 (c) 87 (d) 98

9. If MEKLF is coded as 91782 and LLLJK as 88867, how can IHJED be coded as?

(a) 97854 (b) 64512 (c) 54310 (d) 75632

10. Find the missing number in the following series 7, 26, 63, 214, 215, ?, 511

(a) 342 (b) 343 (c) 441 (d) 421

11. SCD, TEF, UGH _____WKL.

(a) CNM (b) VJI (c) VIJ (d) IJT

12. If GOLD is written as IQNF, how WIND can be written as code?

(a) YKPF (b) VHCM (c) XJOE (d) DNIW

13. If in a certain code language NAME is written as 4258, then what is coded as MEAN?

(a) 2458 (b) 5842 (c) 8524 (d) 5824

14. In certain code, TRIPPLE is written as SQHOOKD, how is DISPOSE written in that code?

(a) CHRONYD (b) CHRONRD (c) CHORNRD (d) DHRONRD

15. In a system 15789 is coded as EGKPT and 2346 is coded as ALUR, how can 23549 be coded?

(a) ALGUT (b) AGLUT (c) LAGUT (d) None

16. If PALAM could be given the code number 43, what code number can be given to SANTACRUZ? **(ICAI)**

(a) 123 (b) 85 (c) 120 (d) 125

17. In a certain code '256' means 'you are good', '637' means 'we are bad' and '358' means 'good and bad'. Which of the following represents 'and' in that code? **(ICAI)**

(a) 5 (b) 5 (c) 8 (d) 3

18. Find the odd one out of the following: 1, 8, 27, 625, 124

(a) 124 (b) 625 (c) 8 (d) 1

19. Find out the next number in the following series: 7, 11, 13, 17, 19, 23, 25, 29?

(a) 30 (b) 31 (c) 32 (d) 33

20. Find odd man out of the following series 15, 21, 63, 81,69

(a) 15 (b) 21 (c) 63 (d) 81

21. March, May, September, December

(a) March (b) September (c) December (d) May

22. Look at the series: 14, 28, 20, 40, 32, 64, ... What number should come next?

(a) 52 (b) 56 (c) 96 (d)128

23. If PALE is coded as 2134 and EARTH is coded as 41590, how is PEARL coded?

(a) 29530 (b) 24153 (c) 25430 (d) 254313

24. Find the missing number in the following series 2, 3, 3, 5, 10, 13, 39, ?, 172, 177

(a) 42 (b) 44 (c) 43 (d) 40

25. If in a certain code language NAME is written as 4258 then what is coded as MEAN?

(a) 2458 (b) 5842 (c) 8524 (d) 5824

26. Find the missing number in the following series 6, 11, 21, 36, 56, ?

(a) 42 (b) 51 (c) 81 (d) 91

27. If A = 1, FAT = 27, FAITH = ?

(a) 44 (b) 45 (c) 46 (d) 36

28. In a certain language, MADRAS is coded NBESBT, how DELHI is coded in that code?

(a) EMMJI (b) EFMIJ (c) EMFIJ (d) JIFEM **(Dec 2019)**

29. Find the missing number in the following series 6, 17, 39, ?, 116

(a) 72 (b) 75 (c) 85 (d) 80

30. Find the odd man out: 835, 734, 642, 751, 853, 981, 532

(a) 751 (b) 853 (c) 981 (d) 532

Answer Key

1. (c)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (a)
11. (b)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (d)
21. (b)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (d)	**26.** (c)	**27.** (a)	**28.** (b)	**29.** (a)	**30.** (a)

PREVIOUS YEAR QUESTION (PYQ'S)

1. 7, 23, 47, 119, 167, ? **(June 2019)**

(a) 211 (b) 223 (c) 287 (d) 319

2. Which of the following is odd one out; 4, 12, 44, 176, 890? **(June 2019)**

(a) 4 (b) 12 (c) 44 (d) 176

3. If in a certain language, MADRAS is coded as NBESBT. How is BOMBAY coded in that language? **(June 2019)**

(a) CPNCBX (b) CPNCBZ (c) CPOCBZ (d) CQOCBZ

4. Which of the following is an odd one? **(June 2019)**

(a) CEHL (b) KMPT (c) OQTX (d) NPSV

5. Find odd one out; 1, 5, 14, 30, 51, 55, 91 **(Dec 2019)**

(a) 14 (b) 55 (c) 51 (d) 91

6. Find odd one out; 5, 10, 17, 27, 37, 50, 65 **(Dec 2019)**

(a) 17 (b) 27 (c) 37 (d) 65

7. Find the missing figures; 4, 16, 36, 64, 100, ? **(Dec 2019)**

(a) 92 (b) 121 (c) 144 (d) 169

8. If 'SYSTEM' be coded as '131625';then 'TERMS' may be coded as **(Dec 2019)**

(a) 62251 (b) 62451 (c) 64951 (d) 62415

9. In a certain language, MADRAS is coded NBESBT, how DELHI is coded in that code? **(Dec 2019)**

(a) EMMJI (b) EFMIJ (c) EMFIJ (d) JIFEM

10. Find the missing value in the series; 0, 2, 3, 6, 10, 17.28, ?, 75. **(Dec 2020)**

(a) 58 (b) 46 (c) 48 (d) 54

11. $\frac{3}{8}, \frac{8}{19}, \frac{18}{41}, ?, \frac{78}{173}$ **(Dec 2020)**

(a) $\frac{38}{85}$ (b) $\frac{83}{38}$ (c) $\frac{31}{38}$ (d) None

12. Find odd man out of the following; 6, 9, 12, 18, 21, 26, 30 **(Dec 2020)**

(a) 21 (b) 30 (c) 26 (d) 9

13. **If in a certain language, HEALTH is coded as IFBMUI, then what is the code for NORTH?** **(Dec 2020)**

(a) OPSUI (b) OPUSI (c) OUSPI (d) OIPSU

14. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, ?$ **(Jan 2021)**

(a) $\frac{9}{32}$ (b) $\frac{10}{17}$ (c) $\frac{11}{34}$ (d) $\frac{12}{35}$

15. Find the missing term;

P 3 C; R 5 F; T 8 I; V 12 L; _____ ? **(Jan 2021)**

(a) Y17O (b) X17M (c) X17O (d) X16O

16. Find out the odd man out in the sequence 8, 27, 64, 125, 216. **(Jan 2021)**

(a) 27 (b) 64 (c) 125 (d) 216

17. In a certain code language BEAT is written as YVZG, then what will be the code for MILD? **(Jan 2021)**

(a) ONRW (b) NOWR (c) ONWR (d) NROW

18. In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How will RIFFLE be written in that code? **(Jan 2021)**

(a) 618892 (b) 689912 (c) 619982 (d) 629981

19. Choose the missing term in the series; 1, 1, 8, 4, 27, ____, 64, 16 **(July 2021)**

(a) 27 (b) 11 (c) 9 (d) 125

20. The wrong term in the series; 225, 196, 169, 144, 121, 100, 77, 64 is **(July 2021)**

(a) 121 (b) 77 (c) 100 (d) 169

21. If DELHI is coded as EFMIJ, then JAIPUR is coded as **(July 2021)**

(a) JQVSBK (b) QVSKBJ (c) BJQVSK (d) KBJQVS

22. If FRAME is coded as 0618011305, then ARISE is coded as **(July 2021)**

(a) 0118091905 (b) 0119091805

(c) 0118190905 (d) 0118091805

23. If CLOCK is coded as 34235, and TIME as 8679, then MOTEL is coded as **(July 2021)**

(a) 27894 (b) 72964 (c) 72894 (d) 77684

24. If MOUSE is coded as 34651 and KEY is coded as 217, then how will YES be coded? **(Dec 2021)**

(a) 715 (b) 517 (c) 175 (d) 571

25. What comes at the last place in R, U, X, A, D, _____ ? **(Dec 2021)**

(a) E (b) F (c) G (d) H

26. The missing term of the series 4, 13, _____, 49, 76 is **(Dec 2021)**

(a) 26 (b) 28 (c) 30 (d) 32

27. Find the odd one from the following: **(Dec 2021)**

(a) Zebra (b) Giraffe (c) Horse (d) Tiger

28. If in a certain code, "THANKS" is written as "SKNTHA", then how is "STUPID" written? **(Dec 2021)**

(a) DIPUTS (b) DISPUT (c) DIPUST (d) DIPSTU

29. 9, 27, 31, 155, 161, 1127, ? **(June 2022)**

(a) 1316 (b) 1135 (c) 1288 (d) 2254

30. What is the missing number in the sequence given below; **(June 2022)**

12, 9, 13.50, 30.375, ?, 341.71875

(a) 91.125 (b) 89.145 (c) 90.475 (d) 92.485

31. Find the missing number; 7, 26, 63, 124, 215, 342, ? **(June 2022)**

(a) 391 (b) 421 (c) 481 (d) 511

32. In a certain code, "TELEPHONE" is written as ENOHPELET, then ALIGATOR is coded as **(June 2022)**

(a) ROTAGILA (b) ROTAGAIL (c) ROTAGILE (d) TOTAGILA

33. In a coded language, if 'EARTH' is written as 34215 and 'VENUS' is written as 73089. What is the code for 'SATURN'? **(June 2022)**

(a) 941012 (b) 941820 (c) 914281 (d) 912418

34. If 'FROZEN' is decoded as 'OFAPSG'. Tick the right option that depicts 'MOLTEN' written in this way? **(Dec 2022)**

(a) OFPOMN (b) OFSMPN (c) OFUMPN (d) OFUNPN

35. Find the odd man out; 34, 105, 424, 2123, 12756 **(Dec 2022)**

(a) 12756 (b) 2123 (c) 424 (d) 34

36. Find the missing in the following series?

3,5,5,19,7,41,9,?,11,109 **(Dec 2022)**

(a) 71 (b) 61 (c) 69 (d) 79

37. In certain code language, if TOUR is written as 1234, CLEAR is written 5678 and SPARE is written as 90847, Find the code for CARE? **(Dec 2022)**

(a) 1247 (b) 4847 (c) 5247 (d) 5847

38. Find the next number in the given sequence? **(Dec 2022)**

11,17,39,85,?,281,447

(a) 133 (b) 143 (c) 153 (d) 163

39. If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, what will be the code for SEARCH? **(Dec 2022)**

(a) 246173 (b) 214673 (c) 216473 (d) 214763

40. The number in place of question mark in; 7, 26, 63, 124, 215, ?, 511 is **(June 2023)**

(a) 342 (b) 343 (c) 441 (d) 421

41. In a certain code, MENTION is written as LNEITNO. How is PRESENT written in that code? **(June 2023)**

(a) QFSFTUM (b) ONESERP (c) QRESTNO (d) OERESTN

42. Out of the following 41,43,47,53,61,71,83,95, the odd man out shall be **(June 2023)**

(a) 95 (b) 83 (c) 71 (d) 53

43. Find the next number in the series: Q1F, S2E, U6D, W21C, ……? **(June 2023)**

(a) Y66B (b) Y44B (c) Y88B (d) Z66B

44. Find the odd man out of the following series; 100, 145, 136, 352, 460, 324, 631, 244 **(June 2023)**

(a) 136 (b) 244 (c) 460 (d) 324

45. 2,3,8, _____, 3968 **(Dec 2023)**

(a) 65 (b) 63 (c) 70 (d) 80

46. Find the odd man out; 16, 25, 36, 72, 144, 196, 225 **(Dec 2023)**

(a) 36 (b) 72 (c) 196 (d) 225

47. In a certain language SIKKIM is written as THLJJL, then TRAINING is coded as **(Dec 2023)**

(a) SQBHOHOH (b) UQBHOHOF (c) UQBJOHOH (d) UQBJOHOM

48. In certain language, PEAR is coded as 7519 and TOIL is coded as 2693, then how DOCTOR be written in that code? **(Dec 2023)**

(a) 463293 (b) 463239 (c) 463269 (d) 463296

49. In certain system of code STATEMENT is coded as TNEMETATS then how the word POLITICAL be coded as? **(Dec 2023)**

(a) LACITILOP (b) LACTILIOP (c) OPILITACL (d) LACATILOP

50. What comes next in the series; FMAMJJASON? **(June 2024)**

(a) DBM (b) DJM (c) DJF (d) DDJ

51. In a certain code INACTIVE is written as VITCANIE. How is COMPUTER written in the same code? **(June 2024)**

(a) PMOCRETU (b) ETUPMOCR (c) UTEPMOCR (d) MOCPETUR

52. In a certain code TEACHER is written as VGCEJGT. How is CHILDREN written in that code? **(June 2024)**

(a) EJKNEGTP (b) EGKNFITP (c) EJKNFGTO (d) EJKNFTGP

53. In a series of letters, which one is the odd one out: BDFH, JLNP, RTVX, ZBDE? **(June 2024)**

(a) BDFH (b) JLNP (c) TVX (d) ZBDE

54. Find the missing value in the series; 51, 52, 60, 87, 151,....., 492. **(June 2024)**

(a) 195 (b) 276 (c) 317 (d) 420

55. Find the next number in the series; 2, 5, 11, 23, 47, _____ **(Sept 2024)**

(a) 84 (b) 95 (c) 98 (d) 105

56. If TAP is coded as SZO, then how is FRIEND coded in same language? **(Sept 2024)**

(a) CMDHQE (b) QEDHCM (c) EQIENE (d) EQHDMC

57. Find the odd man out of the following series; 7, 23, 47, 119, 171, 287 **(Sept 2024)**

(a) 119 (b) 171 (c) 287 (d) 7

58. In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code? **(Sept 2024)**

(a) 318826 (b) 318286 (c) 618826 (d) 338816

59. AZ, GT, MN, ..., YB, EV. The value of blank space (.....) will be; **(Sept 2024)**

(a) JH (b) SH (c) SK (d) TS

60. In a certain code language, 'CLOCK' is coded as '75276' and 'EARTH' is coded as '83491', then 'COAT' is coded as **(Jan 2025)**

(a) 7239 (b) 7329 (c) 7932 (d) 7529

61. Find the missing term of the series 2, 7, 16, 29, ..., 67, 92. **(Jan 2025)**

(a) 46 (b) 39 (c) 43 (d) 62

62. In a certain language, 'MENTION' is written as 'NFOUJPO', the code of 'MYSTIFY' is **(Jan 2025)**

(a) NEOFTJT (b) NZTUJGZ (c) LNEITNO (d) OERESTN

63. Find the missing term CEGI, XVTR, GIKM, _____ **(Jan 2025)**

(a) KMBD (b) TRPN (c) AMNL (d) JLNP

64. Find the odd man out from the following; Marriage, Wedlock, Divorce, Matrimony. **(Jan 2025)**

(a) Wedlock (b) Marriage (c) Divorce (d) Matrimony

65. Identify the next number in the following series; **(May 2025)**

2, 8, 26, 62, 122, 212, ...

(a) 332 (b) 338 (c) 356 (d) 362

66. Find the missing number in the given series; **(May 2025)**

4, 18, ..., 100, 180, 294, 448

(a) 48 (b) 52 (c) 56 (d) 64

67. In a certain code "CH4IR" is written as "GL8MV". How is "1N5T4GR4M" is written in that code? **(May 2025)**

(a) 4HFID8E8N (b) 4P8W7JU8O (c) 5R9X8KV8Q (d) 5KF2EGR4

68. A certain code "564" means "all the best", "736" means "best of luck" and "423" means "all is luck". Which of the following is the code for "luck"? **(May 2025)**

(a) 6 (b) 4 (c) 3 (d) 7

69. Find the odd one out in the series; 2, 5, 10, 17, 26, 37, 50, 64. **(Sept 2025)**

(a) 17 (b) 26 (c) 37 (d) 64

70. Complete the given series; 0, 6, 24, 60, 120, 210, _____. **(Sept 2025)**

(a) 240 (b) 290 (c) 336 (d) 504

71. In the following series, one term is wrong. Find out the wrong term. **(Sept 2025)**

48, 50, 82, 170, 290

(a) 48 (b) 50 (c) 82 (d) 170

72. In a certain code, GLAMOUR is written as IJCNMWP and MISRULE is written as OGUSSNC, then how will MUSICAL be written in that code? **(Sept 2025)**

(a) OSUJECN (b) OSUHACJ (c) OSUJACJ (d) OSUJABJ

73. If in a certain language, MADRAS is coded as NBESBT. How is DELHI coded in that language? **(Sept 2025)**

(a) EMFIJ (b) EFMIJ (c) EFMKJ (d) EFJMI

Answer Key

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(b)	**7.**	(c)	**8.**	(b)	**9.**	(b)	**10.**	(b)
11.	(a)	**12.**	(c)	**13.**	(a)	**14.**	(a)	**15.**	(c)	**16.**	(a)	**17.**	(d)	**18.**	(c)	**19.**	(c)	**20.**	(b)
21.	(d)	**22.**	(a)	**23.**	(c)	**24.**	(a)	**25.**	(c)	**26.**	(b)	**27.**	(d)	**28.**	(d)	**29.**	(b)	**30.**	(a)
31.	(d)	**32.**	(a)	**33.**	(b)	**34.**	(c)	**35.**	(b)	**36.**	(a)	**37.**	(d)	**38.**	(d)	**39.**	(b)	**40.**	(a)
41.	(d)	**42.**	(a)	**43.**	(c)	**44.**	(d)	**45.**	(b)	**46.**	(b)	**47.**	(b)	**48.**	(c)	**49.**	(a)	**50.**	(c)
51.	(b)	**52.**	(d)	**53.**	(d)	**54.**	(b)	**55.**	(b)	**56.**	(d)	**57.**	(b)	**58.**	(a)	**59.**	(b)	**60.**	(a)
61.	(a)	**62.**	(b)	**63.**	(b)	**64.**	(c)	**65.**	(b)	**66.**	(a)	**67.**	(c)	**68.**	(c)	**69.**	(d)	**70.**	(c)
71.	(a)	**72.**	(c)	**73.**	(b)														

SUMMARY

- Number series: Number series is a series of numbers following a particular pattern, and we need to find missing terms while few terms are given.
- Coding and decoding: There are two kinds of coding Letter coding and number coding.
- Letter coding: In this type the real alphabets in a word are replaced by certain other alphabets according to a specific rule to form its code.
- Number coding: Here either letters are given a particular number code, or number is given a particular letter code.
- Odd-man out: When there are few words, numbers or letters are given to you, such that all have a relation except one and you need to find that particular word.

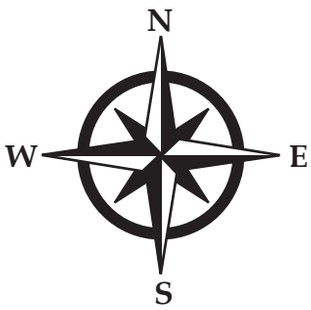
❑❑❑

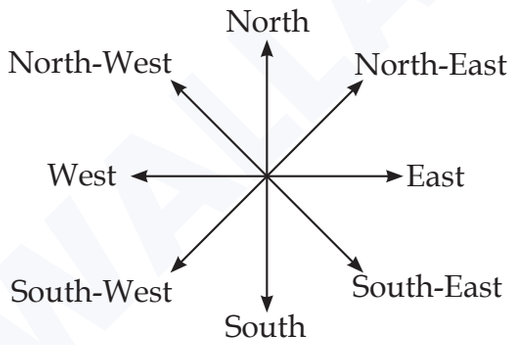
CA WALLAH

CHAPTER

10 Direction Sense Test

- Direction sense is a concept that we have been familiar with since childhood. It involves understanding and navigating the different directions in our surroundings. The four major directions that we commonly know are **North**, **South**, **East**, and **West**.
- In addition to these cardinal directions, we have also established four more directions in between them, known as the intermediate directions. These intermediate directions are **North-East**, **North-West**, **South-East**, and **South-West**.



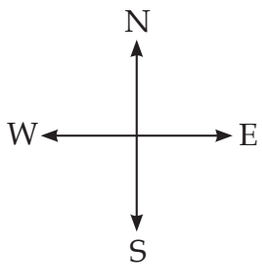


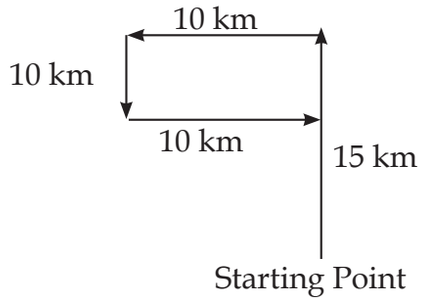
- While questions related to direction sense are generally easy, they can sometimes be confusing if not approached systematically. One effective strategy is to visualize yourself moving in different directions and mentally picture the diagram of the directions mentioned above. By having a clear mental image of the diagram, you can easily determine the relationships and positions of different directions.

Example 1. Laxman went 15 km north then he turned west and covered 10 km. Then he turned south and covered 5 km, finally turning east he covered 10 km. In which direction is he moving now?

(a) East (b) West (c) North (d) South **(ICAI)**

Sol. (a) Following the directions as given in the question, we have





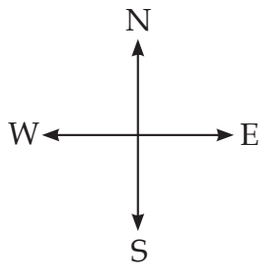
Therefore, he is moving in the East direction.

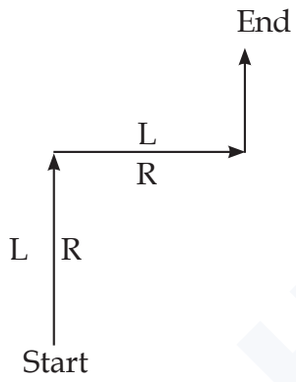
Hence, the correct option is (a).

Example 2. You go north, turn right then go to the left. In which direction are you now?

(a) South (b) West (c) East (d) North

Sol. (d) According to the information given, the directions are:





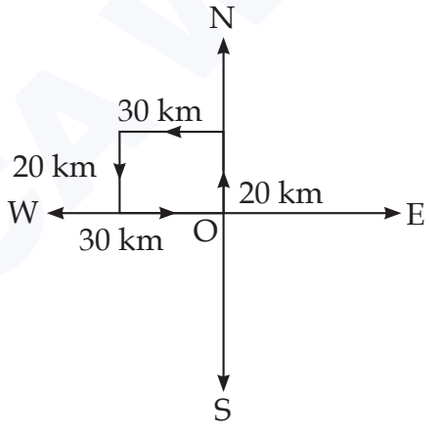
Therefore, he is traveling in the North direction now.

Hence, the correct option is (d).

Example 3. Rohit drove towards the North for 20 km. Then he turned left and drove another 30 km. After a while, he again turned left and drove 20 km and took some rest. Once more he turned left and drove 30 km to reach his town. In which direction is he driving now?

(a) West (b) East (c) North (d) South

Sol. (b) Taking O as the starting point and following the instructions as per the question, we get



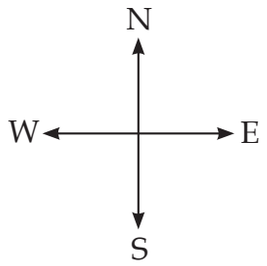
Clearly, he is driving in the East direction.

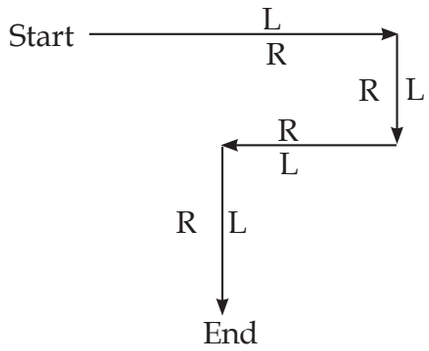
Hence, the correct answer is option (b) i.e. East.

Example 4. A man started to walk east, and after moving a certain distance, he turned to his right. After moving some distance, he turned to his right again. After moving a little, he turned in the end to his left. In which direction was is he going now?

(a) East (b) West (c) North (d) South **(ICAI)**

Sol. (d) According to the given information





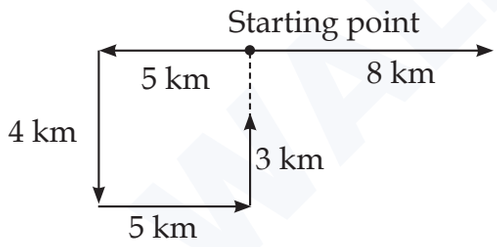
Currently, he is going in the South direction.

Hence, the correct option is (d).

Example 5. Rohan walked 8 km towards the East. Then he turned back and walked 13 km. Again, he turned to his left and walked 4 km. Now, he walked 5 km after turning left. At last, he turned to his left and walked 3 km. How far and in which direction is he from the starting point ?

(a) 3 km West (b) 1 km North (c) 1 km South (d) 4 km South

Sol. (c) Following the direction as per the question, we have



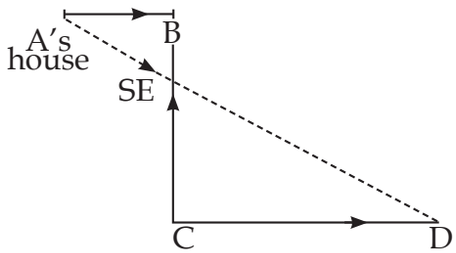
Therefore, he is at distance of (4-3) km =1 km in the South.

Hence, the correct option is (c) i.e., South.

Example 6. Five friends A, B, C, D and E are staying in the same locality. B's house is to the east of A's house and to the north of C's house. C's house is to the west of D's house. D's house is in which direction with respect to A's house? **(July 2021)**

(a) North-East (b) South-East (c) North-West (d) South-West

Sol. (b) According to the given information, the location of D's house with respect to A's house is:



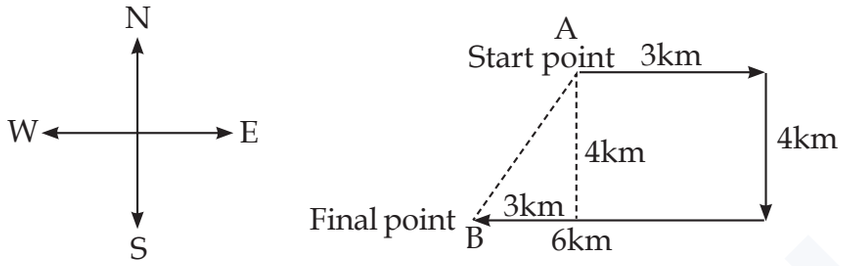
Therefore, D's house is in the South-East direction of A.

Hence, the correct option is (b).

Example 7. Kailash walks 3 km to the East and turns South and walks 4 km. After a while, he turns West and walks 6 km. How far is he from the starting point?

(a) 7 km (b) 5 km (c) 6 km (d) 3 km

Sol. (b) Following the directions as per the question, we get



Here, the initial point is A and the final point is B.

Thus, $AB^2 = 4^2 + 3^2$

$AB^2 = 16 + 9$

$AB^2 = 25$

$AB = 5$

Therefore, Kailash is 5 km away from the starting point.

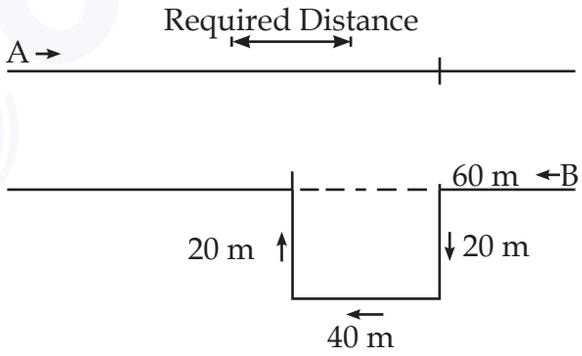
Hence, the correct option is (b).

Example 8. A and B start moving towards each other from two places apart. After walking 60m, B turns left and goes then 20 m then he turns right and goes 40 m. He then turns right again and comes back to the road on which he had started walking. If A and B walk with the same speed, what is the distance between them now? **(July 2021)**

(a) 80 m (b) 70 m (c) 40 m (d) 60 m

Sol. (c) Distance travelled by A on road = 60 + 20 + 40 + 20 = 140 m

Distance traveled by B on road = 60 + 40 = 100 m



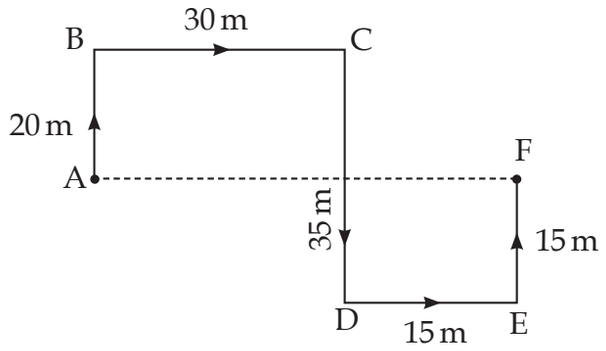
Required difference = 140 – 100 = 40m

Hence, option (c) is correct i.e 40m.

Example 9. Anand walked 20 m towards the north direction. After a while, he turned right and walked 30 m. Again, he turns to his right and walks 35 m. Then he turns left and walks 15 m. At last he turns to his left and walks 15 m. In which direction is Anand from the starting position?

(a) East (b) West (c) North (d) South

Sol. (a) Let the initial position of Anand is at A then following the directions as per the question, we have



Therefore, the final position of Anand is at F which is the East direction from the starting point.

Hence, the correct option is (a) i.e., East.

Example 10. One morning, after sunrise, Vikram and Shailesh were standing in a lawn with their backs towards each other. Vikram's shadow fell exactly towards left hand side. Which direction was Shailesh facing? **(July 2021)**

(a) South-West (b) West (c) South (d) South-East

Sol. (c) We know, Sun rises in the East.

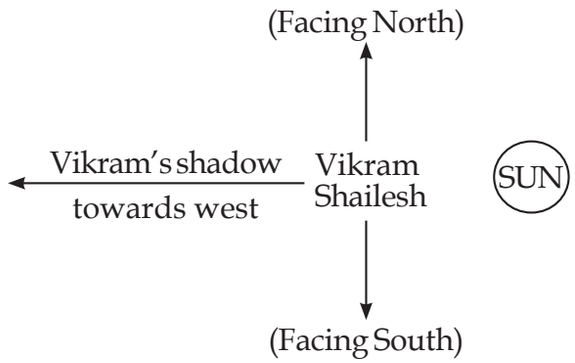
It is given that "one morning after sunrise", therefore, we will mark the Sun in the East direction in the below diagram.

When the Sun is in the East direction, shadow of an object falls in the West.

It is given that Vikram's shadow falls towards left hand side.

This means Vikram's left hand side should be West direction.

Thus, according to the question, the required figure is:



Therefore, Shailesh is facing the South direction.

Hence, the correct option is (c) i.e. South.

PRACTICE QUESTIONS (PART A)

1. From her home Prerna wishes to go to school. From home she goes towards North and then turns left and then turns right, and finally she turns left and reaches school. In which direction her school is situated with respect to her home? **(ICAI)**

(a) North-East (b) North-West (c) South-East (d) South-West

2. Anita travels 15 km to the North. After taking rest for some time, she turned to West and covered 10 km. Then she turned to the South and covered 5 km. At the end, she turned to East and covered 10 km. In which direction is she traveling now?

(a) East (b) West (c) North (d) South

3. Starting from her home, Ankita wishes to go to the library. From her home, she goes towards the South and then turns left, followed by another right turn. Finally, she turns left and reaches the library. In which direction is the library situated with respect to her home?

(a) North (b) North-West (c) South-East (d) South-West

4. Prerna walked 20 km from West towards East. After a while, she once again turned to her right and walked 10 km and again turned right and walked 9 km. Then, she turned to left and walked 5 km and again turned to her left and covered a distance of 12 km. Finally she turned to her left and walked 6 km. In what direction is she moving?

(a) East (b) North (c) West (d) South

5. Karuna walked 10 feet from A to B in the East direction. Then she turned to her right and walked 3 feet. Now again she turned to her right and walked 14 feet. In which direction is she from her starting point A?

(a) North-East (b) South-West (c) South (d) None of these

6. Aditya walks 2 km towards the North. Then he turns to the East and walks 10 km. After this he turns to the North and walks 3 km. Again he turns towards East and walks 2 km. In which direction is he from the starting point?

(a) North-East (b) North-West (c) East (d) North

7. Raman left home one fine morning and cycled 8 km northwards, then he turns right and cycled 5 km straight. Now, he turns right and cycled 12 km and then he turns left and cycled 10 km. Now in which direction Raman is moving?

(a) East (b) West (c) North (d) South

8. Anand ride his bicycle and travels meters towards the East direction. Then he turns to his left and travel meters and again turns to his right and travels some distance. Towards which direction is he now moving?

(a) North (b) East (c) West (d) South

9. Radha moves towards South-East a distance of 7 km, then she moves towards West and travels a distance of 14 km. From here, she moves towards North–West a distance of 7 km and finally she moves a distance of 4 km towards east. How far is she now from the starting point?

(a) 3 km (b) 4 km (c) 10 km (d) 11 km

10. There are four towns P, Q, R and T. Q is to the south-west of P, R is to the east of Q and south east of P, and T is to the north of R in line with QP. In which direction of P is T located?

(a) North (b) North-East (c) East (d) South-East

Answer Key

1. (b) **2.** (a) **3.** (c) **4.** (b) **5.** (b) **6.** (a) **7.** (a) **8.** (b) **9.** (c) **10.** (b)

Let Us Understand the Concept of Rotation and Degrees

When we talk about rotation, it refers to the act of turning or revolving around a central point. In mathematics and geometry, rotations are usually measured in degrees or radians. A complete rotation or a full circle is equal to 360 degrees.

In the context of directions, we often encounter clockwise and anticlockwise rotations. Clockwise rotation refers to the movement or turn in the direction that follows the hands of a clock. It is a rightward or clockwise movement. On the other hand, anticlockwise rotation refers to the movement or turn in the opposite direction, counter clockwise or leftward.

Clockwise
Mean turn right

Anticlockwise
Mean turn left

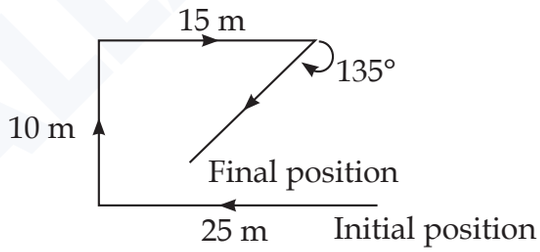
Example 11. Mohit walked 25 m towards the west direction. After a while, he turned to the right and walked 10 m. Again, he then turned to the right and walked 15 m. Now, he turned 135° and walked 30 m. In which direction is he moving now?

(a) South (b) North-East (c) South-West (d) West

Sol. (c) Following the directions as per the information given:

Therefore, he is moving in a South-west direction.

Hence, the correct option is (c) i.e., South-West.



Example 12. Man is facing west. He turns 45° in the clockwise direction and then another 180° in the same direction and then 270° in the anti-clockwise direction. Which direction is he facing now?

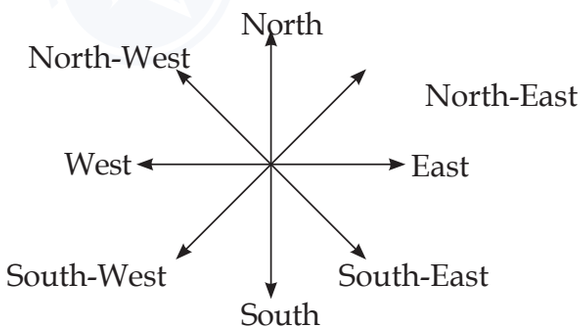
(a) South (b) North–West (c) West (d) South-West

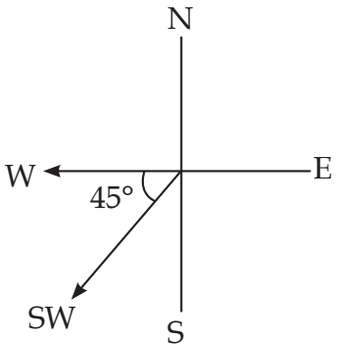
Sol. (d) Clockwise movement =45° + 180° = 225°

Anticlockwise Movement = 270°

Now, 270° – 225° = 45° anticlockwise (∵ 270° is greater than 225°)

Therefore, the final rotation will be 45° anticlockwise.





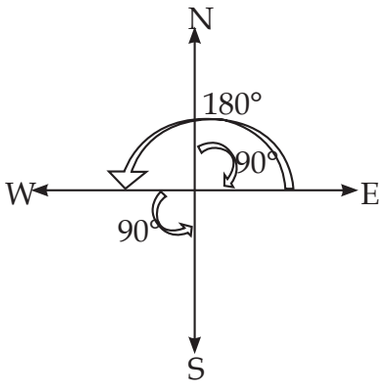
Therefore, he is facing South-West direction.

Hence, the correct option is (d).

Example 13. Ana is facing the north direction. She turns 90° in the clockwise direction, Again after a while she turns 180° in the anticlockwise direction and then another 90° in the same direction. Which direction is she facing now?

(a) East (b) West (c) North (d) South

Sol. (d) The directions of Ana according to the question is as follows:



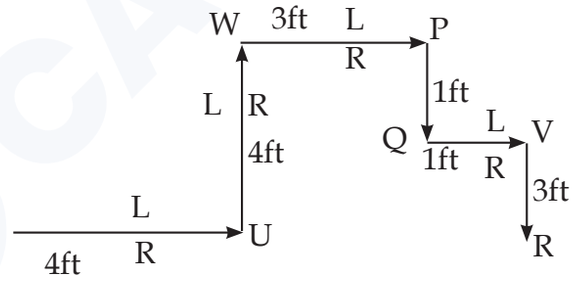
Therefore, Ana is facing the South direction now.

Hence, the correct option is (d) i.e., South.

Example 14. Maya starts at point T, and walks straight to point U which is 4 feet away. She turns left at 90° and walks to W which is 4 feet away, turn 90° right and goes 3 feet to P, turns 90° right and walks 1 feet to Q, turns left at 90° and goes to V, which is 1 feet away and once again turn 90° right and goes to R, 3 feet away. What is the distance between T and R?

(a) 4 feet (b) 5 feet (c) 7 feet (d) 8 feet

Sol. (d) According to the given information:



Distance between T and R = 4 feet + 3 feet + 1 feet = 8 feet

Hence, the correct option is (d).

PRACTICE QUESTIONS (PART B)

1. Sahil started from his school towards the north. After walking a distance of 18 m, he turned to the left and walked 14 m. He then turned to the left again and walked 30 m. After this, he is to turn left at 180 degrees and to cover 12 m. In which direction is he now as per his initial position?

(a) 14 m East (b) 14 m West (c) 14 m South (d) 14 m North

2. Madhuri moved a distance of 75 m towards the north. She then turned to the left and walked for about 25 m, turned left again, and walked 80 m. Finally, she turned to the right at an angle of 45°. In which direction was she moving finally?

(a) South –East (b) South –West (c) North-West (d) North -East

3. A man is facing west. He turns 45 degree in the clockwise direction and then another 180 degrees in the same direction and then 270 degrees in the anticlockwise direction. Find which direction he is facing now?

(a) South-East (b) West (c) South (d) South-West

4. Rohit is facing East direction. He turns 90° in the clockwise direction and then another 180° in the anti-clockwise direction and 270° in clockwise direction. What direction is he facing now?

(a) South-West (b) North-West (c) West (d) South

5. Ankita is facing the East direction. She turns 120° in the clockwise direction and then turns 180° in the anticlockwise direction. Which direction is she facing now?

(a) East (b) West (c) North-East (d) South-East

6. Samar walked 25 m towards the west direction. After a while, he turned to the right and walked 10 m. Again, he then turned to the right and walked 15 m. Now, he turned 135 degrees and walked 30 m. In which direction is he moving now?

(a) South (b) North (c) South-East (d) South-West

Answer Key

1. (b) **2.** (b) **3.** (d) **4.** (c) **5.** (c) **6.** (d)

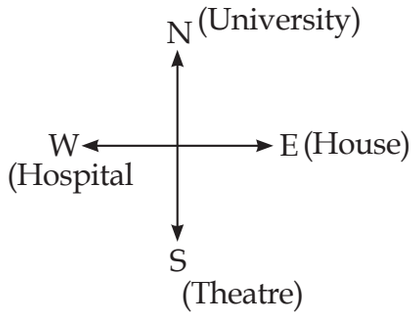
MORE TYPES OF QUESTION

Example 15. Ravi wants to go to the university. He starts from his home which is in the east and comes to a crossing. The road to the left ends in a theatre, and straight ahead is the hospital. In which direction is the university?

(a) North (b) South (c) East (d) West

Sol. (a) Ravi moves westwards, after starting from his house in the East. Then, the theatre, which is to the left, will be in the South.

Then the hospital, which is straight ahead will be towards the West. The only direction left for university is North.



Therefore, the University will be to the North.

Hence, the correct option is (a).

Example 16. A man is performing yoga with his head down and legs up. His face is towards the west. In which direction will his left hand be?

(a) North (b) South (c) East (d) West

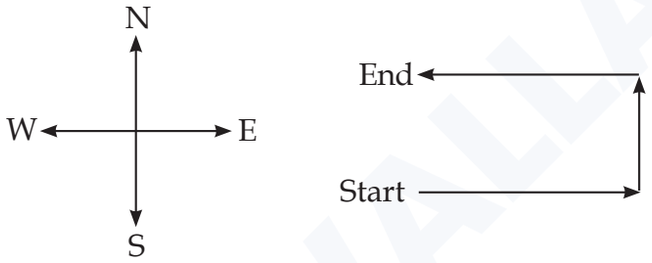
Sol. (a) While doing Yoga in an upside-down position, if a man's face is in the west direction, then his head is in the East direction. The left hand will be in the North direction and the right hand will be in the South direction. Therefore, the man's left hand will be in the North direction.

Hence, the correct option is (a).

Example 17. Raman starts walking in the morning facing the sun. After some time, he turned to the left, and later again he turned to his left. In which direction is Raman moving now?

(a) East (b) West (c) South (d) North

Sol. (b) We know that the sun rises in the East. Thus, following the directions we get



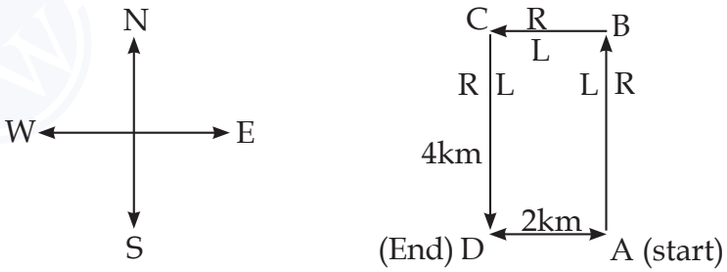
Thus, Raman is moving in the West Direction.

Therefore, the correct option is (b).

Example 18. Sunita rode her Scotty northwards, then turned left and rode few km, and then again rode to her left 4 km. She found herself exactly 2 km west of her starting point. How far did she ride northwards initially?

(a) 2 km (b) 4 km (c) 6 km (d) 5 km

Sol. (b) Following the directions as per the question, we get



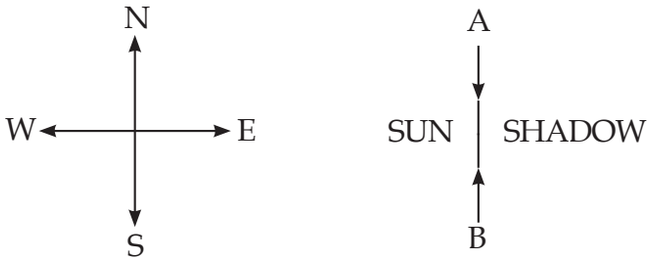
Initial Northwards journey, AB = CD = 4 km.

Hence, the correct option is (b).

Example 19. One evening A & B were standing under the sun talking face to face. The shadow of A fell exactly towards its left hand side. What direction does B face?

(a) North (b) South (c) East (d) West

Sol. (a) In the evening, the sun is in the west direction.



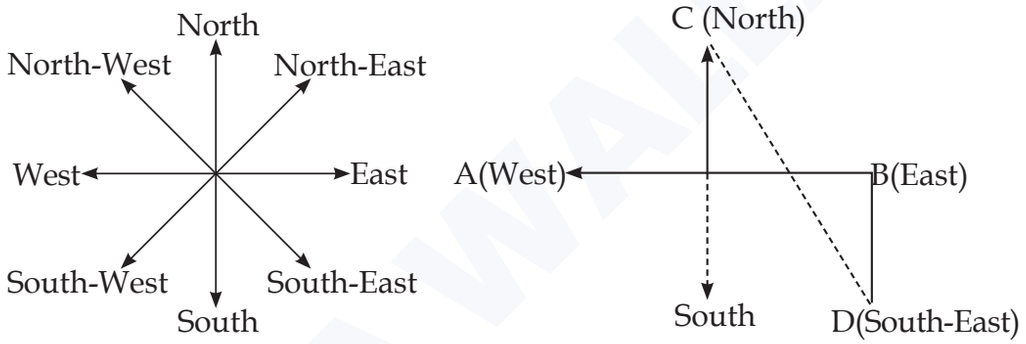
Therefore, B is facing the North direction.

Hence, the correct option is (a).

Example 20. A is located to the West of B. C is located to the North in between A and B. D is exactly to the south of B and also in line with B. In which direction of C is D located?

(a) South (b) South-East (c) West (d) South-West

Sol. (b)



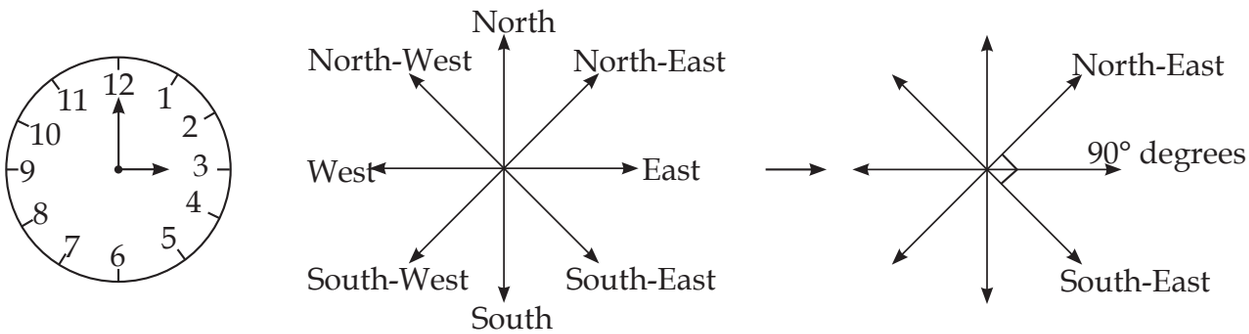
Therefore, D is located in the South-East direction of C.

Hence, the correct option is (b).

Example 21. It is 3 o'clock on a watch. If the minute hand points towards the North-East, then the hour hand will point towards the **(Dec 2022)**

(a) South (b) South-West (c) North-West (d) South-East

Sol. (d) Following the directions as per the question, we get



At 3 o'clock, the angle between the minute and the hour hand is 90 degrees. Now, according to the question, the minute hand points towards the North-East.

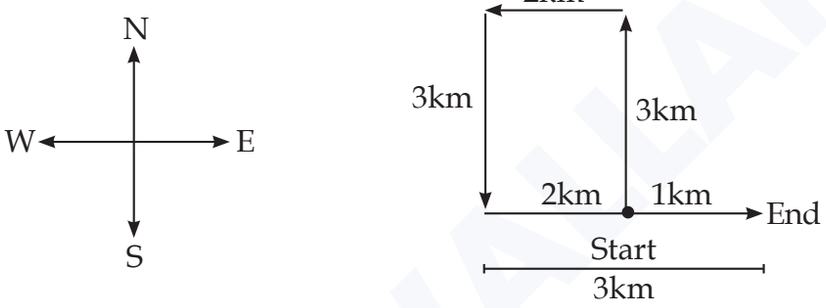
Therefore, when we move clockwise at an angle of 90 degrees, the hour hand will point towards the South-East direction.

Hence, the correct option is (d).

Example 22. Rohan walks a distance of 3 km towards the north then turns to his left and walks for 2 km. He again turns left and walks 3 km. At this point, he turns to his left and walks for 3 km. How many kilometers is he from the starting point?

(a) 1 km (b) 2 km (c) 3 km (d) 5 km

Sol. (a) Following the directions as per the question, we get



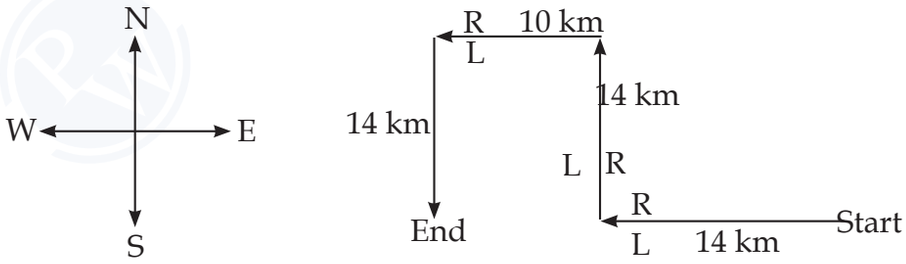
So, it is clear from the above figure that he is 1 km from the starting point.

Hence, the correct option is (a).

Example 23. Namita walks 14 km towards the west then turns to her right and walks 14 km and then turns to her left and walks 10 km. Again turning to her left she walks 14 km. What is the shortest distance (in km) between her starting point and the present position?

(a) 10km (b) 24 km (c) 28 km (d) 38 km

Sol. (b) Following the directions as per the question, we get



Therefore, the shortest distance between her starting point and end point i.e., present position is:

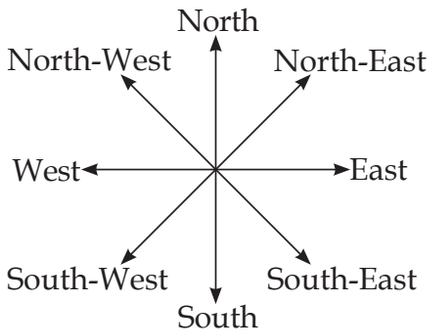
14 km + 10 km = 24 km

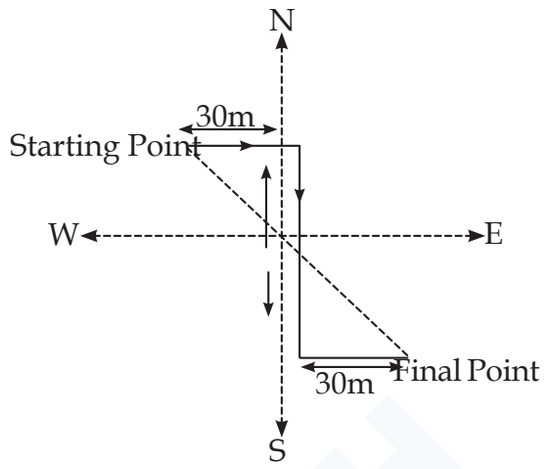
Hence, the correct option is (b).

Example 24. Amit walked 30 meters towards the east, took a right turn and walked 40 meters, then he took a left turn and walked 30 meters. In which direction is he now from the starting point?

(a) North – East (b) East (c) South – East (d) South

Sol. (c)





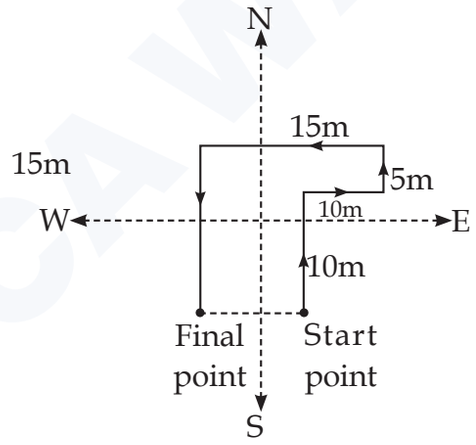
Clearly, the direction of the Amit with respect to the starting point is South East.

Hence, the correct option is (c).

Example 25. A walks 10 meters in front and 10 meters to the right. Then every time turning to his left he walks 5, 15 and 15 meters respectively. How far is he now from his starting point?

(a) 5 meters (b) 10 meters (c) 15 meters (d) 20 meters

Sol. (a) Let A is walking towards North then according to the information given, the directions are:



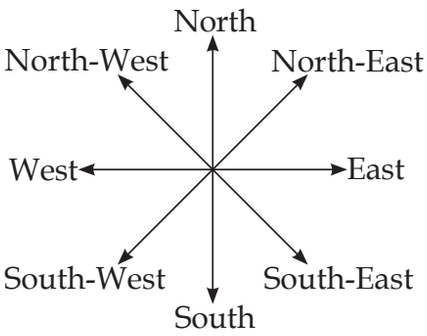
Therefore, he is 5 m from the starting point as shown in the figure. Hence, the correct option is (a)

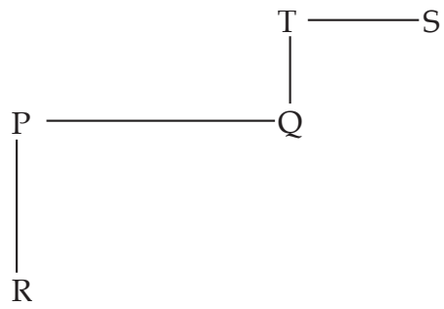
Example 26. Of the five villages P, Q, R, S, and T are situated close to each other, P is to the west of Q, R is to the south of P, T is to the north of Q and S is to the east of T. Then, R is in which direction with respect to S?

(a) North –West (b) South –East

(c) South –West (d) Data inadequate

Sol. (c) Following the information given in the question, we get





Hence, R is in the Southwest direction with respect to S as shown in the figure.

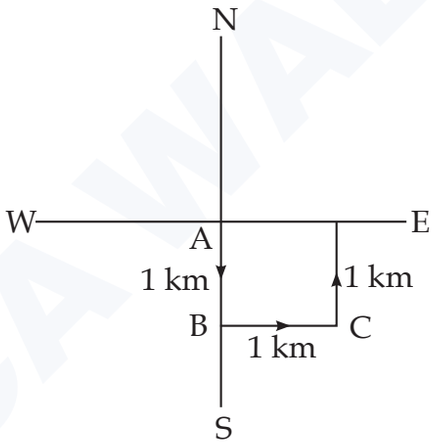
Therefore, the correct option is (c).

Exdample 27. Mohan starts from point A and walks 1 km towards south, turns left and walks 1 km. Then he turns left again and walks 1 km. Now he is facing _____ direction. **(ICAI)**

(a) East (b) West (c) North (d) South-West

Sol. (c) Given, A be the initial position of Mohan, then

According to the question,



Mohan walks 1 km towards the south and reaches to point B. He turns left, which means he moves towards the east side and reaches to point C. He again turns left and walks 1 km towards the North direction.

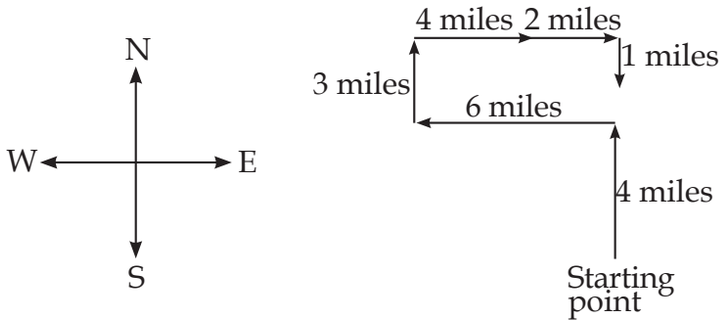
Thus, Mohan is facing the north side.

Hence, the correct answer is option (c).

Example 28. A man starts from a point, walks 4 miles towards north and turns left and walks 6 miles, turns right and walks for 3 miles and again turns right and walks 4 miles and takes rest for 30 minutes. He gets up and walks straight 2 miles in the same direction and turns right and walks one mile. What is the direction he is facing?

(a) North (b) South (c) South-East (d) West **(ICAI)**

Sol. (b) Following the directions as per the question, we get



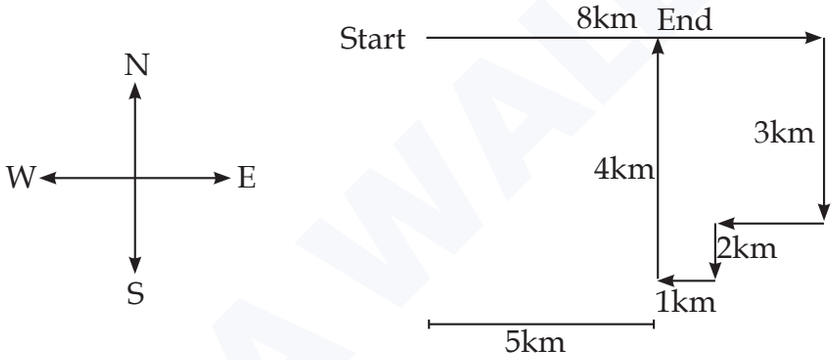
Therefore, he is facing in the South direction.

Hence, the correct option is (b)

Example 29. A boy starts from home in the early morning and walks straight for 8km facing the sun. Then, he takes a right turn and walks straight for 3km. Then, he turns right again and walks for 2km, and then turns left and walks for 1km. Then, he turns right, travels 1 km, and then turns right and travels for 4km straight. How far is he from the starting point?

(a) 5 km (b) 6 km (c) 2 km (d) 4 km

Sol. (a)



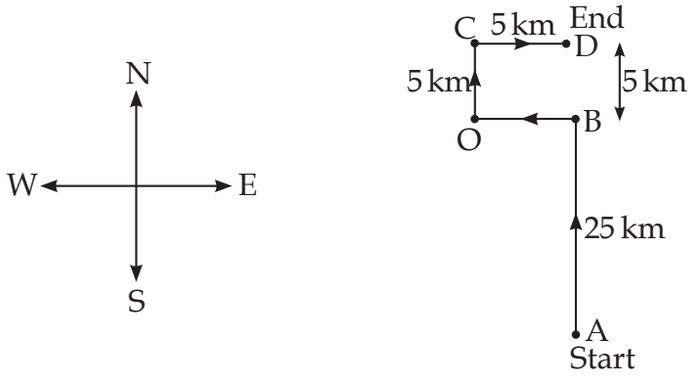
Therefore, he is 8 km-2 km-1 km = 5 km from the starting point.

Hence, the correct option is (a).

Example 30. Raju drives 25 km north and turns left and travels 5 km and reaches point 'O'. He then turns right and covers another 5km. Afterward, he turns to the east and drives 5 km. How much distance does he has to travel to go back to the starting point?

(a) 30 km (b) 20 km (c) 25 km (d) 35 km

Sol. (a)



The distance Raju has to travel = DB + BA = 5km + 25 km = 30 km

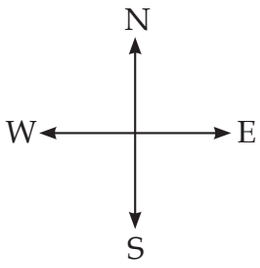
Therefore, he has to travel 30 km to go back to the starting point.

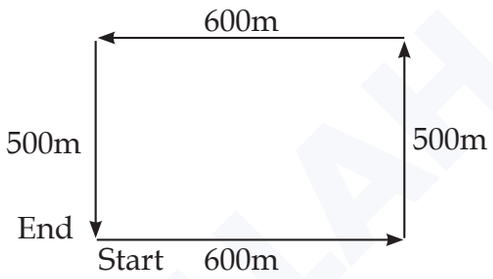
Hence, the correct option is (a).

Example 31. Pinky walks a distance of 600 m towards the east, turns left and moves 500m, then turns left and walks 600m, and then turns left again and moves 500m and halts. At what distance (in m) is she from the starting point?

(a) 0 (b) 600 (c) 500 (d) 2200

Sol. (a) According to the information given,





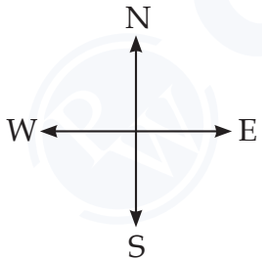
As we can see that the starting point coincides with the endpoint. Hence, Pinky is at zero distance from the starting point.

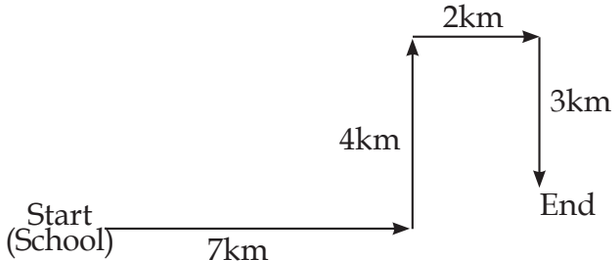
Hence, the correct option is (a).

Example 32. Pratap starts from school and walks 7km towards the east. He takes a left and walks 4km, then takes a right and walks 2km, and again takes a right and walks 3km. Which direction is he facing now?

(a) South (b) North (c) East (d) West

Sol. (a) According to the information given,





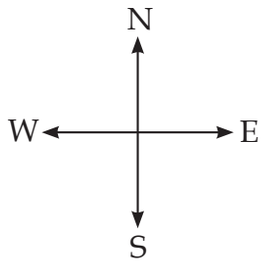
Therefore, Pratap is facing the South direction.

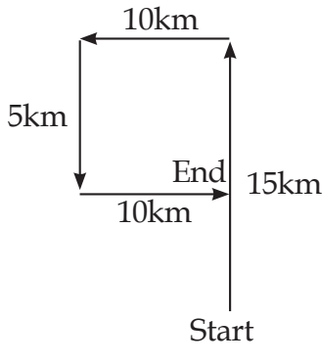
Hence, the correct option is (a).

Example 33. Laxman went 15 km to the north, then he turned west and covered 10 km. Then he turned south and covered 5 km, finally turning to east he covered 10 km. In which direction he is moving now?

(a) East (b) West (c) North (d) South **(May, 2018)**

Sol. (a) According to the information given, the directions are:



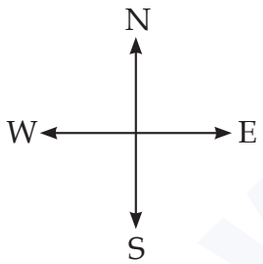


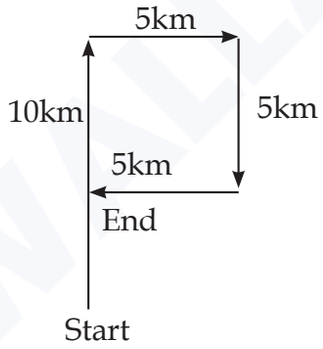
Therefore, Laxman is moving in the East direction now.
Hence, the correct option is (a).

Example 34. A man is facing east, then he turns left and goes 10 meters then turns right and goes 5 meters then goes 5 meters to the south and from there 5 meters to west. In which direction is he from his original place?

(a) East (b) West (c) North (d) South **(May, 2018)**

Sol. (c) According to the information given, the directions are:



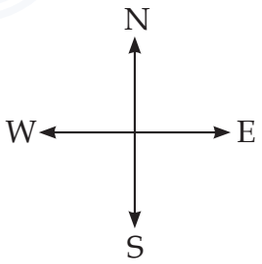


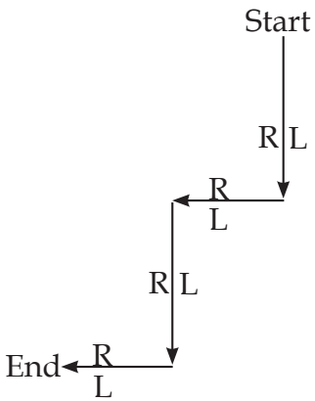
Therefore, the man is in the North direction from his original place.
Hence, the correct option is (c).

Example 35. X walks southwards and then turns right then left and then right. In which direction is he moving now? **(May, 2018)**

(a) South (b) North (c) West (d) South West

Sol. (c) According to the information given, the directions are:





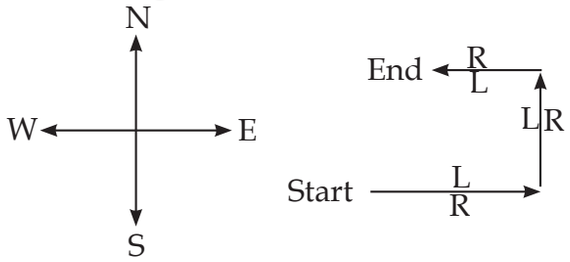
Therefore, X is moving in the West direction now.
Hence, the correct option is (c).

Example 36. Mr. Raman starts walking in the morning facing the sun. After some time, he turned to the left, later again he turned to his left. In which direction is Raman moving now?

(May, 2018)

(a) East (b) West (c) South (d) North

Sol.(b) According to the information given, the directions are:



Therefore, Raman is moving in the West direction.

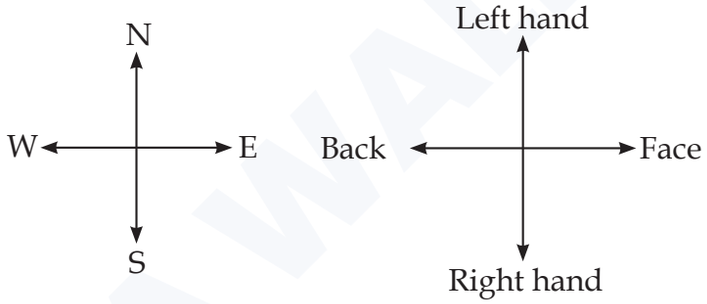
Hence, the correct option is (b).

Example 37. I stand with my right hand extended side-ways towards the south. Towards which direction will my back be?

(May, 2018)

(a) North (b) West (c) East (d) South

Sol.(b) According to the information given, the directions are:



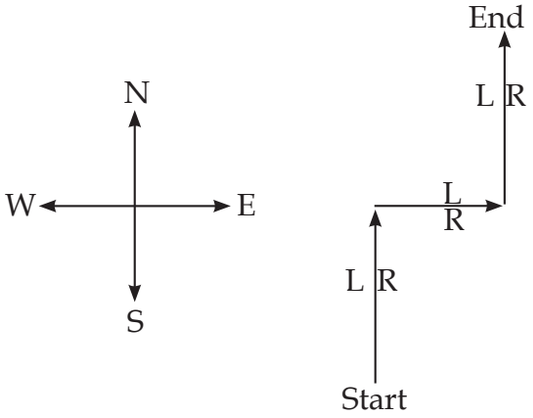
Therefore, my back will be in the West direction.

Hence, the correct option is (b).

Example 38. You go north, turn then right then go to the left. In which direction are you now?

(a) South (b) West (c) East (d) North

Sol.(d) According to the information given, the directions are:



Therefore, I am now in the North direction.

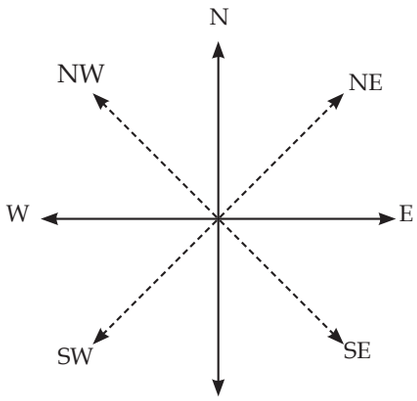
Hence, the correct option is (d).

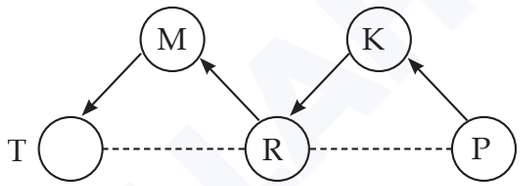
Example 39. K is a place which is located 2 km away in the north-west direction from the capital P. R is another place that is located 2 km away in the south-west direction from K. M is another place and that is located 2 km away in the north-west direction from R. T is yet another place that is located 2 km away in the south-west direction from M. In which direction is T located in relation to P? **(ICAI)**

(a) South-West (b) North-West

(c) West (d) North

Sol. (c) Following the directions as per the question, we get





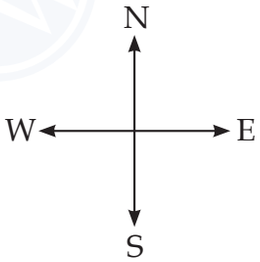
Therefore, T is located in the west direction in relation to P.

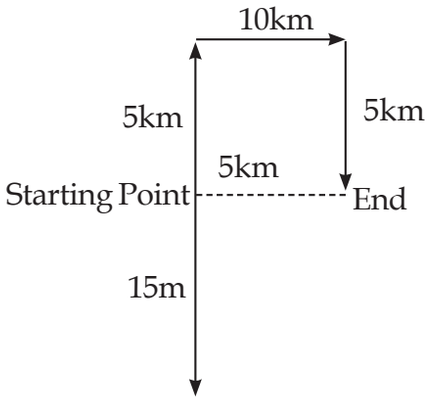
Hence, the correct answer is option (c).

Example 40. Anoop starts walking towards south after walking 15 meters he turns towards north. After walking 20 meters he turns towards east and walks 10 meters. He then turns towards south and walks 5 meters. In which direction is he from the original position?

(a) North (b) South (c) East (d) West **(Nov, 2018)**

Sol. (c) Following the directions as per the question, we get





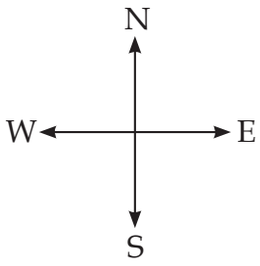
Therefore, Anoop is in the East direction from the original position.

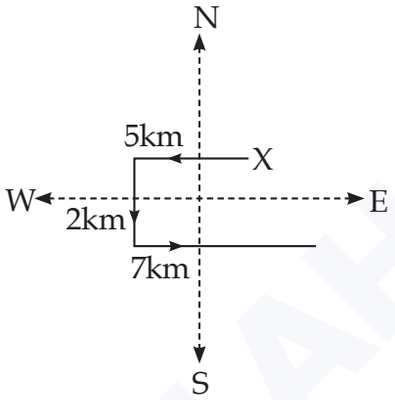
Hence, the correct option is (c).

Example 41. Rahim started from point X and walked straight 5 km. West, then turned left and walked straight 2 km. and again turned left and walked straight 7 km. In which direction is he from point X?

(a) North – East (b) South- West (c) South-East (d) North-West **(Nov, 2018)**

Sol. (c) Following the directions as per the question, we get





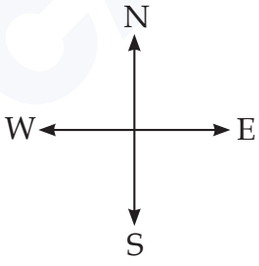
Therefore, he is in the South East direction from X.

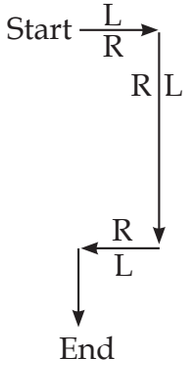
Hence, the correct option is (c).

Example 42. A man started to walk east, after moving a certain distance, he turned to his right. After moving some distance, he turned to his right again. After moving a little he turns now to his left, currently he is going in _____ direction.

(a) East (b) West (c) North (d) South **(Nov, 2018)**

Sol. (d) Following the directions as per the question, we get





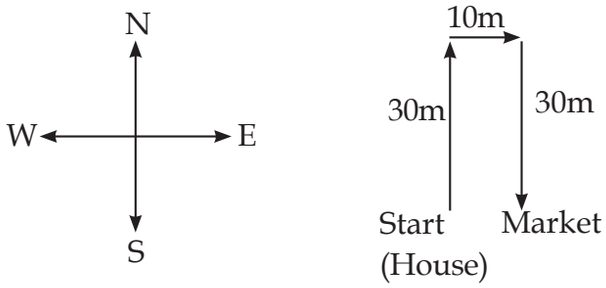
Currently, he is in the South direction.

Hence, the correct option is (d).

Example 43. Manu wants to go to the market. He starts from his house towards the north and reaches a crossing after 30 m. He turns towards east, goes 10 m till the second crossing and turns again, moves towards south straight for 30 m where the marketing complex exits. In which direction is the market from his house? **(Nov 2018)**

(a) North (b) South (c) East (d) West

Sol. (c) Following the directions as per the question, we get



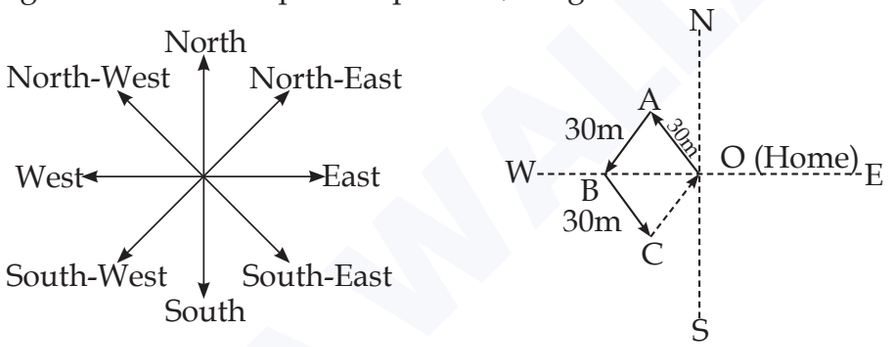
Therefore, the market is in the East direction of Manu's house

Hence, the correct option is (c).

Example 44. Sangeeta leaves from her home, she first walks 30 meters in North–West direction and then 30 meters in south west direction, next she walks 30 meters in south east direction. Finally, she turns towards her house. In which direction is she moving? **(June, 2019)**

(a) North – West (b) North – East (c) South – East (d) South – West

Sol. (b) Following the directions as per the question, we get



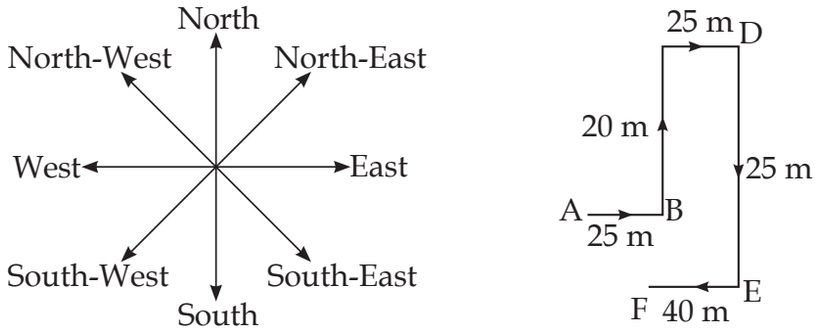
So, as Sangeeta is moving towards her house, she is moving towards point 'O' from point 'C' which is in the 'north-east' direction.

Hence, the correct option is (b).

Example 45. When a person faces north and walks 25m right and she turns left and walks 20m and she again turns right and walks 25m, turns right 25m and turns right and walks 40m , in which direction is he now from his starting point? **(June, 2019)**

(a) North –west (b) North-east (c) South –east (d) South –west

Sol. (c) Following the directions as per the question, we get



Therefore, she is in the South East direction from her starting point.

Hence, the correct option is (c).

Example 46. Y is in the East of X, which is in the North of Z. If P is in the South of Z, then in which direction of Y, is P?

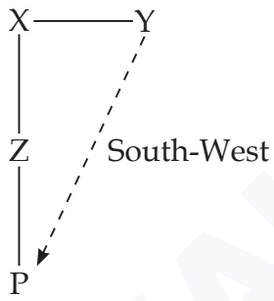
(a) North (b) South

(c) South-East (d) South-West

Sol.(d) Given; Y is in the East of X, which is in the North of Z

Also, P is in the South of Z

Thus, it can be represented as;

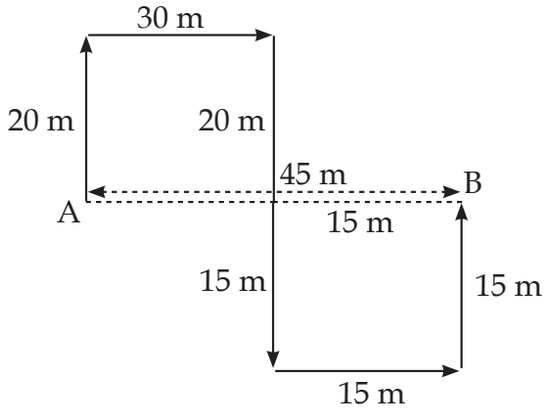


Therefore, P is in the South-West direction of Y.

Example 47. Rajni walked 20 m towards the North. Then she turned right and walks 30 m. Then she turns right and walks 35 m. Then she turns left and walks 15 m. Finally she turns left and walks 15 m. In which direction and how many metres is she from the starting position?

(a) 15 m West (b) 30 m East (c) 30 m West (d) 45 m East

Sol. (d) According to question, we have



where, A is the initial point and B is the final point. Therefore, Rajni is 45 metres East from the initial point.

FULL CHAPTER PRACTICE QUESTIONS

1. Laxman went 15 km to the North, then he turned West and covered 10 km. Then he turned south and covered 5 km, finally turning to East he covered 10 km. In which direction is he now moving?

(a) East (b) West (c) North (d) South

2. Sandesh starts driving from point A and drives 12 km toward North. He takes a right turn and drives 20 km. Now drive 8 km after taking a left turn. Finally, he takes a left turn and drives 20 km and stops at point B. How far is point A with respect to point B?

(a) 20 km (b) 10 km (c) 5 km (d) 9 km

3. Sohan drove 15 km to the west from his house, then turned left and drove 20km. Again, turned east and drove 25 km and finally turning left covered 20 km. How far is he from his house?

(a) 40 km (b) 80 km (c) 5 km (d) 10 km

4. Roza walked 25 m towards the south, took a right turn and walked 15m. She then took a left turn and walked 25 m. She is in which direction from starting point:

(a) S-E (b) South (c) S-W (d) N-W

5. Swati starts from a point, walks 2 km towards North, turns toward her right and walks 2 km, turns right again and walks. In which direction is she going now?

(a) North (b) S-E (c) South (d) West

6. After walking 6 km, I turned right and covered a distance of 2 km, then turned left and covered a distance of 10 km. In the end, I was moving towards the north. From which direction did I start my journey?

(a) North (b) South (c) East (d) West

7. Rashmi leaves from her home. She first walks 30 m in N-W direction and then 30 m in S-W direction. Next she walks 30 m in S-E direction. Finally, she turns towards her house. In which direction is she moving?

(a) N-E (b) N-W (c) S-E (d) S-W

8. I am facing south. I turn right and walk 20 m. Then I turn right again and walk 10 m. Then I turn left and walk 10 m and then turn right and walk 20 m. Then I turn right again and walk 60 m. In which direction I am from the starting point?

(a) North (b) E-N (c) East (d) N-E

9. A walks 3 km north and then he turns left and goes 2 km. He again turns left and goes 3km. He turns right and walks straight. In which direction is he walking now?

(a) East (b) West (c) North (d) South

10. A walks Southwards, then turns right, then left and then right. In which direction is he from the straight point?

(a) South (b) East (c) South-West (d) North

11. A boy started walking towards the West. He turns right and again he turns right and then turns left at last. Towards which direction is he walking now?

(a) West (b) North (c) South (d) East

12. A person Moves 4 km towards west, then turns right and moves 3 km and turns right and moves 6 km , which direction he is moving?

(a) East (b) West (c) North (d) South

13. I started walking down a road in the morning facing the Sun. After walking for some time I turned to my left. Then I turned to my right. In which direction was I going then?

(a) East (b) West (c) North (d) south

14. You go North, turn right, then right again and then go left. In which direction are you now?

(a) South (b) East (c) West (d) North

15. Suman starts from a point, walks 2 km towards North, turns towards her right and walks 2 km. turns right again and walks. What is the direction she is facing now?

(a) East (b) West (c) South (d) North

16. Ramu starts walking in the morning facing the sun. After some time, he turned to the left. Later, he turned to his left. In which direction Ramu is moving now?

(a) East (b) West (c) South (d) North

17. Mr. A walks Southwards and then turn right, then left and then right. In which direction is he from his starting point ?

(a) South-West (b) North (c) South (d) South-East

18. One morning, Rita started to walk toward the sun. After walking a while, she turns to her left and again to her left. She again turned right. Which direction is she facing?

(a) East (b) West (c) North (d) South

19. One day Nita left home and cycled 10 km Southwards, turned right and cycled 5 km and turned right and cycled 10 km and turned left 10 km. How many kilometers will she have to cycle to reach her home straight?

(a) 10 km (b) 15 km (c) 20 km (d) 25 km

20. A located to the West of B. C is located to the north in between A and B. D is exactly to the south of B and also in line with B. In which direction of C is D located?

(a) South (b) S-E (c) West (d) S-W

21. Gopal started walking 2 km straight from his school. Then he turned right and walked 1 km. Again, he turned right and walked 1 km to reach his house. If his house is southeast from his school, then in which direction did Gopal start walking from the school?

(a) East (b) West (c) South (d) North

22. A taxi driver commenced his journey from a point and drove 10 km toward north and turned to his left and drove another 5 km. After waiting to meet a friend here, he turned to his right and continued to drive another 10 km. He has covered a distance of 25 km so far, but in which direction would he be now?

(a) South (b) North (c) East (d) South-east

23. Laxman went 15 km to the North, then he turned West and covered 10 km. Then he turned south and covered 5 km. Finally turning to the East he covered 10 km. In which direction is he from his house?

(a) East (b) West (c) North (d) South

24. A car traveling from the south covers a distance of 8 km, then turns right and runs another 9 km and again turns to the right and stops. Which direction does it face now?

(a) South (b) North (c) West (d) East

25. Five boys A, B, C, D, and E are sitting in a park in a circle. A is facing South-West, D is facing South-East, B, and E are right opposite A and D respectively and C is equidistant between D and B. Which direction is C facing?

(a) West (b) South (c) North (d) East

26. Hema starts from his house and walks 3 km towards the north. Then he turns right and walks 2 km and then turns right and walks 5 km, then turns right and walks 2 km, and then again turns right and walks 2 km. Which direction is he facing now?

(a) North (b) South (c) West (d) East

27. Sikha starts walking north and after a while, she turns to her right. After walking some distance, she turns to his left and walks a distance of 1 km. She then turns to her left again. In which direction is she moving now?

(a) North (b) West (c) East (d) South

28. Roy walks 2 km to the East, then turns North-West and walks 3 km. Then he turns South and walks 5 km. Then again, he turns West and walks 2 km. Finally, he turns North and walks 6 km. In which direction, is he from the starting point?

(a) South-West (b) South-East (c) North-West (d) North-East

29. Kunal starts walking North, then turns left and covers some distance, then he turns right and walks. After some time, he turns to his right and then turns left. In which direction he is walking now?

(a) East (b) South (c) North (d) South-East

30. X walks southwards and then turns right, then left, and then right. In which direction is he moving now?

(a) South (b) North (c) West (d) South-West

Answer Key

1. (a)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (d)	**9.** (b)	**10.** (c)
11. (b)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (b)
21. (a)	**22.** (b)	**23.** (c)	**24.** (a)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (c)	**30.** (c)

PREVIOUS YEAR QUESTION (PYQ'S)

1. Sangeeta leaves from her home, she first walks 30 meters in North–West direction and then 30 meters in south west direction, next she walks 30 meters in south east directions. Finally, she turns towards her house. In which direction is she moving? **(June 2019)**

(a) North –West (b) North –East (c) South –East (d) South-West

2. When a person faces north and walk 25 m and he turn right and walk 20 m and again turns left and walk 25 m and turns right 25 m and turns right and walks 40 m. In which direction is he now from his starting point? **(June 2019)**

(a) North-West (b) North-East (c) South-East (d) South-West

3. Madhuri moves a distance of 75 metres towards North. She then turned to her left and walked about 25 m, turned left again and walked 80 m. Finally she turned to the right at an angle of 45°. In which direction was she moving finally? **(June 2019)**

(a) South-East (b) South-West (c) North-West (d) North-East

4. A person is facing towards the North. He moves 70° clockwise direction. Again he is moving 300° clockwise direction. Now, in which direction is he presently facing? **(June 2019)**

(a) North-West (b) South-East (c) North-East (d) South-West

5. A man started walking West. He turned right. Then again right and finally turned left. Towards which direction was he walking now? **(Dec 2019)**

(a) North (b) South (c) West (d) East

6. A starts from a point and walks 2 km North, then turns left and walks 1 km, then again turns left and walks 2 km. Point out the direction in which he is going now? **(Dec 2019)**

(a) East (b) West (c) North (d) South

7. A man is moving on cycle and move 4 km South then turns left and move 2 km and turns again to the right to move to go more. In which direction is he moving? **(Dec 2019)**

(a) North (b) West (c) East (d) South

8. If Mohan travels towards North from his house then turn to left, then to South covering equal distance in each direction to reach Sohan's house. In which direction Mohan's house is from Sohan's house now? **(Dec 2019)**

(a) East (b) South (c) North (d) West

9. If Shyam sees the rising sun behind the tower and the setting sun behind the Railway station from his house. What is the direction of the tower from the Railway station? **(Dec 2019)**

(a) South (b) North (c) West (d) East

10. A man takes his dog for a walk whose house is facing East. He walks first towards west and then walks towards South. In which direction he has to walk now to reach home? **(Dec 2019)**

(a) North-East (b) West (c) South (d) North-West

11. Rahim faces towards the North. Turning to his right, he walks 25 m. He then turns to his left and walks 30 m. Next he moves 25 m to his right, he then turns to his right again and walks 55 m. Finally he turns to the right and moves 40 m. In which direction is he now from the starting point? **(Dec 2020)**

(a) South-West (b) South (c) North-West (d) South-East

12. A man can walk by having long, medium and short steps. He can cover 60 meters by 100 long steps, 100 meters by 200 medium steps and 80 meters by 200 short steps. He starts walking by 5000 long steps, then he turns left and walks by taking 6000 medium steps. He then turns right and walks by taking 2500 short steps. How far (in meters) is he away from his starting point **(Dec 2020)**

(a) 5000 m (b) 4000 m (c) 6000 m (d) 7000 m

13. One day, Ram left home and cycled 10 km Southwards, then he turns right and cycled 5 km, then he turns right and cycled 10 km and then he turns left and cycled 10 km. How many kilometers will he have to cycle to reach his home straight? **(Dec 2020)**

(a) 15 km (b) 10 km (c) 20 km (d) 25 km

14. You are facing North-East and moved forward 10 cm and turned left for 7.5 cm, what is your position? **(Dec 2020)**

(a) North from initial (b) South from initial

(c) East from initial (d) None

15. A man is facing West. He turns 45° in the clockwise direction and then another in the same direction and then 270° in the anti-clockwise direction. In which direction is he facing now? **(Jan 2021)**

(a) South-West (b) North-West (c) West (d) South

16. One day Ram left home and bi-cycled 10 km southwards, turned right and travelled 5 km and turned right and went 10 km, he turned left and went 10 km, how many kilometers he has to cycle to reach his home straight? **(Jan 2021)**

(a) 10 (b) 15 (c) 20 (d) 25

17. Ms. N walks 10 km towards the North. From there, she walks 6 km towards the South. Then she walks 3 km towards the East. How far and in which direction is she with reference to her starting point? **(Jan 2021)**

(a) 4 km West (b) 6 km West (c) 3 km East (d) 5 km North-East

18. A and B start moving towards each other from two places 200 m apart. After walking 60 m, B turns left and goes 20 m, then he turns right and goes 40 m. He then turns right again and comes back to the road on which he had started walking. If A and B walk with the same speed, what is the distance between them now? **(July 2021)**

(a) 80 m (b) 70 m (c) 40 m (d) 60 m

19. There are four towns P, Q, R and T. Q is to the South-West of P, R is to the East of Q and South-East of P, and T is to the North of R in line with QP. In which direction of P is T located? **(July 2021)**

(a) North (b) North-East (c) East (d) South-East

20. Five friends A, B, C, D and E are staying in the same locality. B's house is to the East of A's house and to the North of C's house. C's house is to the West of D's house. D's house is in which direction with respect to A's house? **(July 2021)**

(a) North-East (b) South-East (c) North-West (d) South-West

21. One morning, after sunrise Vikram and Shailesh were standing in a lawn with their backs towards each other. Vikram's shadow fell exactly towards the left hand side. Which direction was Shailesh facing? **(July 2021)**

(a) South-West (b) West (c) South (d) South-East

22. A person walks 1 km towards West and then he turns to South and walks 5 km. Again, he turns to the West and walks 2 km. After this, he turns to the North and walks 9 km. How far is he from the starting point? **(Dec 2021)**

(a) 3 km (b) 4 km (c) 5 km (d) 7 km

23. Daily in the morning the shadow of a clock tower installed on Railway station falls on the high rise Mall and in the evening the shadow of the same mall falls on the Clock Tower installed on Railway station exactly. So, in which direction is the Clock Tower to the Mall? **(Dec 2021)**

(a) Eastern side (b) Western side (c) Northern side (d) Southern side

24. R's office is 4 km in East direction from his home and club is 4 km in North Direction from his home. On midway from office to club, R starts moving towards his home. In which direction is he facing his back? **(Dec 2021)**

(a) South-East (b) North-West (c) North-East (d) South-West

25. A man starts from a point and walks 4 miles towards North and turns left and walks 6 miles, turns right and walks for 3 miles and again turns right and walks 4 miles and takes rest for 30 minutes. He gets up and walks straight 2 miles in the same direction and turns right and walks one mile. What is the direction he is facing? **(Dec 2021)**

(a) North (b) South (c) South-East (d) West

26. The hour hand of a clock is in the West direction when time is 3'o clock. What is the direction of minute hand when time is 6 : 45 pm? **(Dec 2021)**

(a) East (b) West (c) North (d) South

27. If P × Q means P is to the south of Q; P + Q means P is to the north of Q; P % Q means P is to the East of Q; P – Q means P is to the West of Q; then in case of A % B + C – D, D is in which direction with respect to B? **(June 2022)**

(a) North-West (b) South-East (c) North-East (d) South-West

28. I am facing west, turning to the left I go 20 m, then turning to the left I go for 20 m and turning to the right I go 20 m, then again turning to the right I go for 40 m and then again I go 40 m to the right. In which direction am I from my original position? **(June 2022)**

(a) North (b) West (c) South (d) East

29. A, B, C, D, E, F, G, H and I are nine poles. C is 2 km East of B. A is 1 km north of B and H is 2 km south of A. G is 1 km West of H while D is 3 km East of G and F is 2 km North of G. I is situated right in the middle of B and C while E is just in the middle of H and I. The distance between B and I is **(June 2022)**

(a) 1 km (b) 1.41 km (c) 2 km (d) 3 km

30. Starting from the point, Rani walked 12 m South, she turned left and walked 10 m, she again turned left and walked 12 m, then she turned right and walked 5 m, how far is she now and in which direction from the starting point? **(June 2022)**

(a) 27 m towards East (b) 5 m towards East

(c) 10 m towards West (d) 15 m towards East

31. Puru was driving his car and at a circle there was a direction pole, which was showing all the four correct directions. But due to the wind, it turns in such a manner that now North pointer is showing West. Puru went in the wrong direction thinking that he was travelling East. In what direction he was actually travelling? **(June 2022)**

(a) West (b) East (c) North (d) South

32. Radha moves towards the South-East at a distance of 7 km, then she moves towards West and travels a distance of 14 km. From here she moves towards the North-West and travels a distance of 7 km and finally she moves a distance of 4 km towards East. How far is she now from the starting point? **(Dec 2022)**

(a) 3 km (b) 4 km (c) 10 km (d) 11 km

33. P, Q, R and S are playing a game of carrom. P, R and S, Q are partners. 'S' is to the right of 'R'. If 'R' is facing West, then 'Q' is facing which direction? **(Dec 2022)**

(a) South (b) North (c) East (d) West

34. One morning a boy starts walking in a particular direction for 5 km and then takes a left turn and walks another 5 km thereafter he again takes left turn and walks another 5 km and at last he takes right turn walks 5 km. Now he sees his shadow in front of him. What direction he did start initially? **(Dec 2022)**

(a) South (b) North (c) West (d) East

35. It is 3'o clock in a watch. If the minute hand points towards the North-East then the hour hand will point towards the **(Dec 2022)**

(a) South (b) South-West (c) North-West (d) South-East

36. A man is facing west. He turns 45 degree in the clock-wise direction and then another 180 degrees in the same direction and then 270 degrees in the anti-clockwise direction. Find which direction he is facing now? **(Dec 2022)**

(a) South-East (b) West (c) South (d) South-West

37. Deepika starts walking straight towards the East. After walking 65 m, she turns to the left and walks 25 m straight. Again she turns to be left and walks a distance of 40 m. At what distance and in which direction currently is she from the initial point? **(June 2023)**

(a) 33.35 m in North-East (b) 35.35 m in South-West

(c) 25 m in North (d) 25 m in West

38. Mr. Kartik puts his time piece on the table in such a way that at 6 : 00 pm, hours hand points to North. In which direction the minute hand will point at 9 : 15 pm? **(June 2023)**

(a) South-East (b) East (c) West (d) South-West

39. Shrikant is facing East and turns 120° in the clockwise direction and then turns 180° in the anti-clockwise direction. Which direction is Shrikant facing now? **(June 2023)**

(a) East (b) North-East (c) North (d) South-West

40. Kamal walks 10 km north from there he walks 6 km towards south then he walks 3 km towards East. How far and in which direction is he with reference to the starting point? **(Dec 2023)**

(a) 5 km South-East (b) 7 km North-East

(c) 5 km North-East (d) 7 km South-West

41. Mr X walks northwards. After a while, he turns to his right and a little further to his left. Finally, after walking 500 meters, he turns to his left again. In which direction is he moving now? **(Dec 2023)**

(a) North (b) South (c) East (d) West

42. Sunitha walked 30 m towards the East, took a right turn and walked 40 m, then she took a left turn and walked 30 m. In which direction is she now from the starting point? **(Dec 2023)**

(a) North-East (b) East (c) South-East (d) South

43. Mr X walks 14 km towards the North. From there he walks 8 km South. Then he walks 8 km towards the West. How far and in which direction is he with reference to his starting point? **(Dec 2023)**

(a) 10 km North-West (b) 10 km West

(c) 7 km East (d) 7 km West

44. A car starts from a point, runs 20 kms towards North, turns right and runs 35 kms, turns right again and runs. Which is the direction now it is facing? **(June 2024)**

(a) East (b) South (c) North (d) West

45. If Shyam sees the rising sun behind the tower and setting sun behind the Railway station from his house, then what is the direction of the tower from the Railway station?

(June 2024)

(a) West (b) South (c) North (d) East

46. Shyam walks 12 m South from his house, turns left and walks 20 m, again turns left and walks 45 m, then turns right and walks 10 m to reach coffee shop. In which direction is coffee shop from his house? **(June 2024)**

(a) South-East (b) East (c) North-East (d) South

47. Roy walks 2 kms to West, then turns and starts walking South-East. He walks 3 kms. Then he turns North and walks 5 kms. Then again, he turns East and walks 2 kms. Finally, he turns South and walks 6 kms. In which direction is he from the starting point? **(June 2024)**

(a) South-West (b) North-East (c) South-East (d) North-West

48. Ram started walking from A, 200 m towards North to reach B. Then he turned right and walks 300 m to reach C. Then he turns right and walks 350 m to reach D. Then he turns left and walks 150 m to reach E. Finally, he turns left and walks 150 m. He arrived at the point F. What is the distance between point A and F? **(June 2024)**

(a) 450 m (b) 250 m (c) 300 m (d) 400 m

49. In a multi-storey building on one floor there are six flats in two rows facing East and West and they are allotted to A, B, C, D, E, and F. B gets an East facing flat, which is not next to D. F and D gets diagonally opposite flat. A gets a West facing flat and E gets an East facing flat. Who's flat is between A and F? **(Sept 2024)**

(a) B (b) C (c) D (d)F

50. Balkrishna is Ritik's neighbour and his house is 200 meters away in the north-west direction from Ritik's house. Jayendra is Ritik's neighbour and his house is located 200 meters away in the south-west direction from Ritik's house. Girdhari is Jayendra's neighbour and he stays 200 meters away in the south east direction from Jayendra's house. Ritik is Girdhari's neighbour and his house is located 200 meters away in north-east direction from Girdhari's house. Then where is the position of Ritik's house in relation to Balkrishna's? **(Sept 2024)**

(a) South-East (b) South-West (c) North (d) North-East

51. If Ajay stands on his head with his face towards North, in which direction will his left hand point? **(Sept 2024)**

(a) North-East (b) North (c) East (d) North-West

52. One morning after sunrise, A and B were talking to each other face to face very closely at a crossing point. If B's shadow was exactly to the right of A, in which direction was B facing?

(Sept 2024)

(a) East (b) West (c) North (d) South

53. If Kiran put her time-piece on the table in such a way that at 6:00 PM, hour hand points to East. In which direction the minute hand will be at 9:30 PM? **(Sept 2024)**

(a) South-East (b) North-West (c) East (d) West

54. Anil started walking 5 km towards North, then he turned left and walked 3 km. Again he turned left and walked 5 km. Then the total number of km he walked is **(Jan 2025)**

(a) 8 km (b) 13 km (c) 3 km (d) 5 km

55. Raju started walking 10 km towards East from his home. He turned right and walked 5 km to the South to reach his school. In which direction is his school from his home? **(Jan 2025)**

(a) North-East (b) North-West (c) South-East (d) South-West

56. A man starts walking 10 km to the North. He turns right and walks 5 km, then turns right again and walks 10 km. In which direction is man now from the starting point? **(Jan 2025)**

(a) West (b) East (c) North (d) South

57. A started walking from his house & walk 4 km North side then turns right and walk 3 km. If he turns right again, what is the direction now? **(Jan 2025)**

(a) West (b) North (c) East (d) South

58. In the morning Anika started walking from a point where her shadow falls in front of her. She walked 2 km and then turned left and walked 2 km. Again she turned left and walked 2 km. In which direction is she now facing? **(Jan 2025)**

(a) West (b) East (c) South (d) North

59. Rajni walked 20 m towards the North. Then she turned right and walks 30 m. Then she turns right and walks 35 m. Then she turns left and walks 15 m. Finally she turns left and walks 15 m. In which direction and how many meters is she from the starting position? **(May 2025)**

(a) 15 m West (b) 30 m East (c) 30 m West (d) 45 m East

60. A man is facing North-West. He turns 90° in the clockwise direction, then 180° in the anticlockwise direction and then another 90° in the same direction. Which direction is he facing now? **(May 2025)**

(a) South (b) South-West (c) South-East (d) East

61. Two cars start from the opposite places on a highway, 150 km apart. First car runs for 25 km and takes a right turn and then runs 15 km. It then turns left and then runs for another 25 km and then takes the direction back to reach the main road. In the mean time, due to minor break down the other car has run only 35 km along the main road. What would be the distance between two cars at this point? **(May 2025)**

(a) 65 km (b) 75 km (c) 80 km (d) 85 km

62. Some boys are sitting in three rows all facing North such that A is in the middle row. P is just to the right of A but in the same row. Q is just behind of P, while R is in the North of A. In which direction of R is Q? **(May 2025)**

(a) South (b) South-West (c) North-East (d) South-East

63. At a crossing, there was a direction pole, which was showing all the four correct directions. But due to the wind, it turns in such a manner that now West pointer is showing South. Harish went in the wrong direction thinking that he was travelling East. In which direction was he actually travelling? **(May 2025)**

(a) South (b) North (c) West (d) East

64. Starting from point A, PQ walked 40 metres South. She turned left and walked 40 metres. She then turned left and walked 40 metres. She again turned left and walked 60 metres and reached point B. How far and in which direction is the point B from the point A? **(Sept 2025)**

(a) 20 metres West (b) 20 metres East

(c) 30 metres West (d) 10 metres West

65. A hunter is chasing a deer, by running 200 metres in east direction, turns to his right, runs 100 metres and turns to his right, runs 90 metres. Turning to his left, he runs 50 metres and then turns to left, runs 120 metres. Finally, he turns to left and runs 60 metres. He finds the deer in front of him at 100 metres. In which direction is the hunter standing with respect to the deer now? **(Sept 2025)**

(a) South (b) North (c) East (d) West

66. Two persons, P and Q, start walking from a meeting point towards North. After walking 100 metres, P turns left and Q turns right. P, after walking 50 metres, takes a left turn and walks 150 metres. But Q walks 30 metres, turns to his right and walks 90 metres. What is the shortest distance between P and Q now in metres? **(Sept 2025)**

(a) 80 (b) 90 (c) 100 (d) 110

67. Suresh started walking 4 km west from his office. Then he turned right and walked 2 km. Again he turned right and walked 2 km to reach his house. In which direction is Suresh's house from his office? **(Sept 2025)**

(a) South (b) South-East (c) North-West (d) East

68. Mr. PQR walked 50 metres towards east, took a right turn and walked 40 metres. Then he took a left turn and walked 30 metres. In which direction is he now from the starting point? **(Sept 2025)**

(a) North-East (b) East (c) South-East (d) South

Answer Key

1.	(b)	**2.**	(b)	**3.**	**(c)**	**4.**	(c)	**5.**	(a)	**6.**	(d)	**7.**	(d)	**8**	(a)	**9.**	(d)	**10.**	(a)
11.	(d)	**12.**	(a)	**13.**	**(a)**	**14.**	(d)	**15.**	(a)	**16.**	(b)	**17.**	(d)	**18.**	(c)	**19.**	(b)	**20.**	(b)
21.	(c)	**22.**	(c)	**23.**	**(a)**	**24.**	(c)	**25.**	(b)	**26.**	(a)	**27.**	(b)	**28.**	(b)	**29.**	(a)	**30.**	(d)
31.	(c)	**32.**	(c)	**33.**	**(b)**	**34.**	(a)	**35.**	(d)	**36.**	(d)	**37.**	(a)	**38.**	(c)	**39.**	(b)	**40.**	(c)
41.	(d)	**42.**	(c)	**43.**	**(a)**	**44.**	(b)	**45.**	(d)	**46.**	(c)	**47.**	(c)	**48.**	(a)	**49.**	(b)	**50.**	(a)
51.	(c)	**52.**	(c)	**53.**	**(c)**	**54.**	(b)	**55.**	(c)	**56.**	(d)	**57.**	(d)	**58.**	(b)	**59.**	(d)	**60.**	(c)
61.	(a)	**62.**	(d)	**63.**	**(b)**	**64.**	(a)	**65.**	(a)	**66.**	(c)	**67.**	(c)	**68.**	(c)				

SUMMARY

- Picturising the directions through a diagram is the best way to solve the directions.
- Pythagoras theorem : In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides i.e. $h^2 = p^2 + b^2$
- When we are in the North, turning left is West and turning right is East.
- When we are in the West, turning left will be South, and turning right will be North.
- When we are in the East, turning to left will be North, and turning to right will be South.
- When we are in the South, turning to the left is East and turning to the right is West
- If we move to the right, the entire direction will be clockwise. So, every right movement will lead to the East, South, and West directions, which are clockwise when starting from the right point, at North.
- Similarly, if the movement begins from the left, the entire directions that follow would be anti-clockwise. So, if you turn to the left from North, the directions would be West, South, and East, which are in a reverse pattern.

CA WALLAH

CHAPTER

11 Seating Arrangements

INTRODUCTION

In these questions, you have to arrange a group of persons fulfilling certain conditions. Here we can classify these problems into 4 types:

1. Linear Arrangement
2. Two row sequence Arrangement
3. Circular Arrangement
4. Polygon Arrangement

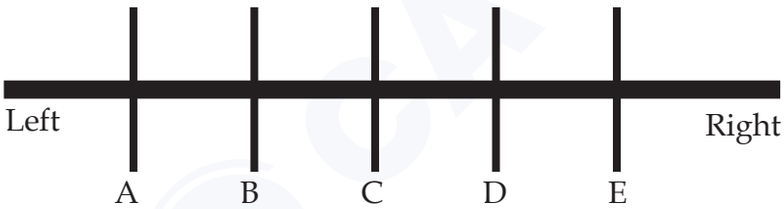
LINEAR ARRANGEMENT

Here the arrangement of the persons is linear i.e., you have to arrange them in a line. Here generally a single row of arrangement is formed.

There will be two types of questions for linear arrangement.

1. When you don't know the direction of facing
2. When you know the direction of facing, (for Example: Upward, Downward, East, North, South, West etc.)

(a) When direction of facing is not clear, then we take our self as below:



Now, there are a few conclusions which we have to make from above. But first let us know some definition:

1. **Immediate left**: It refers to the position or object located just to the left side of the reference point, without any other object in between.

   E.g.: A is immediate left to B.

2. **Immediate right**: It refers to the position or object located just to the right side of the reference point, without any other object in between.

   E.g.: D is immediate right to C.

3. **To the left**: It means moving towards the left side of the reference point, which can include objects located at different distances or positions.

   E.g.: A, B, C are to the left of D and E.

4. **To the right:** It means moving towards the right side of the reference point, which can include objects located at different distances or positions.

   E.g.: B, C, D and E are to the right of A.

5. **In between**: It refers to objects or positions that are located between two specified points or objects.

   E.g.: B is between A and C.

   Now, the conclusion from the above diagram are:

   - B, C, D, E are right of A but only B is to the immediate right of A.
   - D, C, B, A are right of E but only D is to the immediate left of E.
   - C, D, E are right of B but Only C is immediate right of B
   - C is in the middle of line.
   - A and E are sitting at the extreme end of line.

Example 1. In a park, there are five trees labeled A, B, C, D, and E. Tree A is to the right of Tree B. Tree C is to the left of Tree D. Also, Tree D is to the right of Tree E and Tree B is to the left of Tree C. Which tree is in the middle?

(a) Tree B (b) Tree A (c) Tree D (d) Tree C

Sol. (d) Let's analyze the given information step by step to determine which tree is in the middle:

Tree A is to the right of Tree B i.e., B A

Tree C is to the left of Tree D i.e., C D

Tree D is to the right of Tree E i.e., E D

Tree B is to the left of Tree C i.e., B C

Thus, the final arrangement:

E B C D A

From the arrangement, we can see that Tree C is in the middle position.

Therefore, the tree in the middle is C.

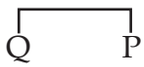
Hence, the correct option is (d) i.e.,Tree C.

Example 2. There are five houses P, Q , R, S, T. P is the immediate right of Q and T is the immediate left of R and immediate right of P. Q is on the right of S. Which house is located at the extreme left end?

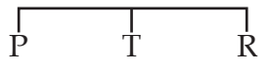
(a) P (b) R (c) S (d) T

Sol. (c) According to the question,

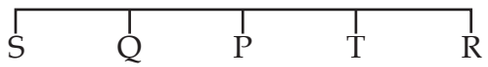
P is immediate right of Q



T is immediate left of R and immediate right of P i.e.,



Now, Q is in the right of S, thus the final arrangement will be:



Therefore, house S is located at the extreme left end.

Hence, the correct option is (c) i.e. S.

Example 3. Five students P, Q, R, S, and T, are positioned in a row facing North. Among them, S is seated between T and Q, and Q is immediately to the left of R. P is immediately to the left of T. Who is positioned at the second right?

(a) S (b) T (c) R (d) Q

Sol. (d) According to the question,

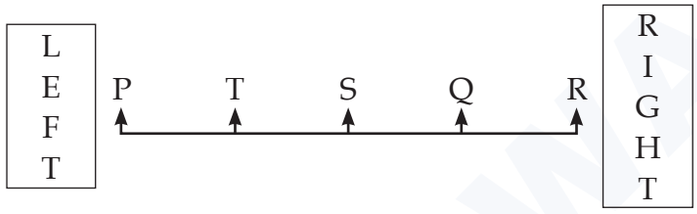
Students are sitting in a row facing North.

S is seated between T and Q i.e. T S Q

Q is seated immediately to the left of R i.e. Q R

P is seated immediately to the left of T i.e. P T

Thus, the final sitting arrangement:



As shown in figure, Q is seated at second right.

Hence, the correct option is (d) i.e., Q.

Example 4. Five students A, B, C, D and E are standing in a row. D is on the right of E, B is on the left of E but on the right of A. D is next to C on his left. The student in the middle is

(a) B (b) E (c) C (d) A **(Nov 2018)**

Sol. (b) According to the question,

D is on the right of E → E, D

B is on the left of E but on the right of A → A, B, E, D

D is next to C on his left → A, B, E, C, D

The arrangement which we get from the above data is; → A, B, E, D, C

Clearly, student E is in the middle of this arrangement.

Hence, the correct option is (b) i.e, E.

Example 5. In a conference, seven participants labeled A, B, C, D, E, F, and G are seated in a row. The following statements are given:

(I) C is seated to the left of B, but on the right of D.

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(II) A is seated to the right of B.

(III) F is seated to the right of E, but to the left of D.

(IV) H is seated to the left of E.

Which person is seated at the extreme right end?

(a) H (b) D (c) A (d) None of these

Sol. (c) Considering the seven participants facing in North direction, then

According to the question,

'C' is seated left to 'B' but on the right to 'D' i.e. D C B

'A' is seated right to 'B' i.e. D C B A

'F; is seated right to 'E' but left to 'D' i.e.

E F D C B A

'H' is seated to the left to 'E', then the final arrangement of the seven members in row will be:

H E F D C B A

Therefore, A is seated at the extreme right end.

Hence, the correct option is (c) i.e., A.

Example 6. P, Q, R, S and T are sitting in a line facing West. P and Q are sitting together. R is sitting at the south end and S is sitting at the North end. T is neighbor of Q and R. Who is sitting in the middle? **(Dec 2022)**

(a) P (b) Q (c) R (d) S

Sol. (b) Given, P, Q, R, S and T are seated in a line facing West.

As, R is sitting at the north end and S is sitting at the south end

Thus, the arrangement is:

R

—

—

—

S

Also, T is neighbour of R and Q i.e.,

R

T

Q

—

S

Since, P and Q are seated together thus the final arrangement is:

S

P

Q

T

R

Therefore, Q is sitting in the middle.

Hence, the correct option is (b).

Example 7. Five friends A, B, C, D, and E are sitting on a bench.

I. A is sitting next to B.

II. C is sitting next to D.

III. D is not sitting with E.

IV. E is the last end of the bench.

V. C is on the 2nd position from the right.

VI. A is on the right of B and E.

VII. A and C are sitting together.

In which position A is sitting?

(a) Between B and D

(b) Between B and C

(c) Between E and D

(d) Between C and E

Sol. (b) Since, A is sitting next to B and A is on the right of B i.e., B – A

Also, C is sitting next to D i.e., C - D or D - C

Since A is on right of B and E and also E is the last end of the bench, we get

E – B – A

Also, A and C are sitting together and D is not sitting with E.

Thus, E – B – A – C – D, which is the required arrangement

Clearly, A is sitting between B and C.

Hence, option (b) is correct.

Example 8. In a college party, 5 girls are sitting in a row. F is to the left of M and to the right of O. R is sitting to the right of N but to the left of O. Who is sitting in the middle?

(a) O (b) R (c) P (d) M

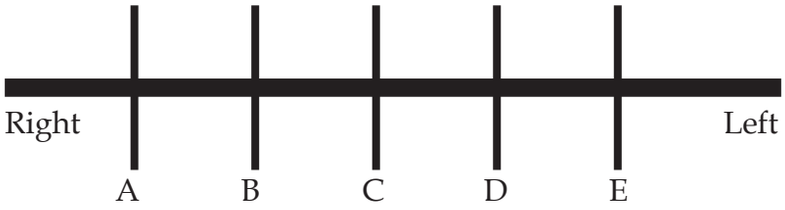
Sol. (a) Arranging the positions of girls according to given arrangement, we observe that F is to the left of M and to the right of O i.e., O – F – M will sit together, Also, R is sitting to the right of N but to the left of O i.e., N – R – O – F – M will be the final sitting arrangement.

Therefore, O is sitting in the middle,

Hence, the correct option is (a).

When you know the direction of facing. (For example – upward, Downward, East, north, south, west etc.)

If it faced South, then



Example 9. P, Q, R, S, T, U , V and W are sitting in a row facing south.

a. P is fourth to the right of T

b. W is fourth to the left of S

c. R and U, which are not at the ends, are neighbours of Q and T respectively.

d. W is next to the left of P and P is the neighbour of Q.

Who are sitting at the extreme ends?

(a) T and S (b) T and P (c) P and S (d) Q and T

Sol. (a) Given: P, Q, R, S, T, U, V and W are sitting in a row facing south

Now, P is fourth to the right of T i.e, P __ __ __ T

W is next to the left of P and P is the neighbour of Q i.e.,

Q P W __ __ T

W is fourth to the left of S i.e., S __ Q P W __ __ T

R and U are neighbours of Q and T respectively i.e., S R Q P W __ U T

Thus, the position of V and the final arrangement is

S R Q P W V U T

Therefore, T and S are sitting at extreme end.

Hence, option (a) is correct.

Example 10. Five boys are standing in a row facing East. Pavan is left of Tavan. Vipin and Chavan to the left of Nakul. Chavan is between Tavan and Vipin. Vipin is fourth from the left, then how far is Tavan to the right? **(ICAI)**

(a) First (b) Second (c) Third (d) Fourth

Sol. (d) From the given information, we get

Pavan is left of Tavan, i.e., Pavan - Tavan,

Chavan is between Tavan and Vipin and Vipin is fourth from the left i.e., Pavan - Tavan – Chavan - Vipin

Vipin and Chavan to the left of Nakul i.e., Pavan – Tavan - Chavan - Vipin - Nakul

Therefore, Tavan is fourth from the right.

Hence, the correct option is (d).

Example 11. Five boys A_1, A_2, A_3, A_4, A_5 are sitting in a stair in the following way:

(i) A_5 is above A_1

(ii) A_4 is under A_2

(iii) A_2 is under A_1

(iv) A_4 is between A_2 and A_3

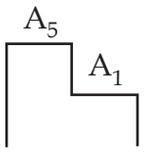
Who is in the lowest position of the stair? **(ICAI)**

(a) A_1 (b) A_3 (c) A_5 (d) A_2

Sol. (b) Arrangement according to question is:

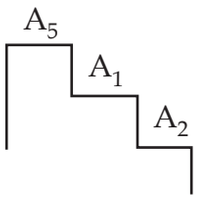
Since, A_5 is above A_1 (from i)

i.e.

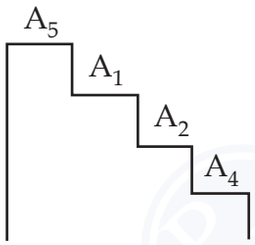


Now, A_2 is under A_1 (from iii)

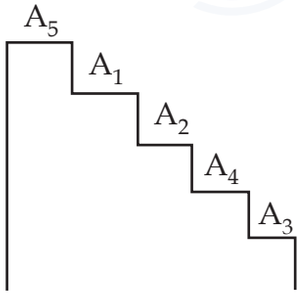
i.e.



Also, A_4 is under A_2



Now, A_4 is between A_2 and A_3, thus



Therefore, A_3 is at the lowest position of the stair.

Hence, the correct option is (b).

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Example 12. In a residential complex, five families named A, B, C, D, and E are living in a multi-storeyed building. The following statements are given:

Family A lives in a flat above Family B.

Family C lives in a flat below Family D.

Family B lives in a flat above Family D.

Family E lives in a flat below Family C.

Which family lives in the middle?

(a) Family A (b) Family B (c) Family C (d) Family D

Sol. (d) According to the question,

Family A lives in a flat above Family B i.e.,

Family A
Family B

Also, Family C lives in a flat below Family D and Family B lives in a flat above Family D i.e.,

Family B
Family D
Family C

Now, Family E lives in a flat below Family C thus the final arrangement is:

Family A
Family B
Family D
Family C
Family E

Therefore, family D lives in the middle.

Hence, the correct option is (d) i.e., Family D.

- **Double Row Arrangement:** In these questions, there will be two groups of persons. You have to arrange one group in one row and the other group in the other row. The persons in these rows normally face each other.

Right A B C Left

Left P Q R Right

Example 13. Six persons A, B, C, D, E and F are sitting in two rows. Three in each row.

1. E is not at the end of any row
2. D is second to the left of F

3. C, the neighbour of E, is sitting diagonally opposite to D
4. B is the neighbour of F.

Which of the following are in one of the two rows? **(ICAI)**

(a) D, B, and F (b) C, E and B
(c) A, E and F (d) F, B and C

Sol. (a) As per the given information,

Six persons A, B, C, D, E and F are sitting in two rows, three in each row.

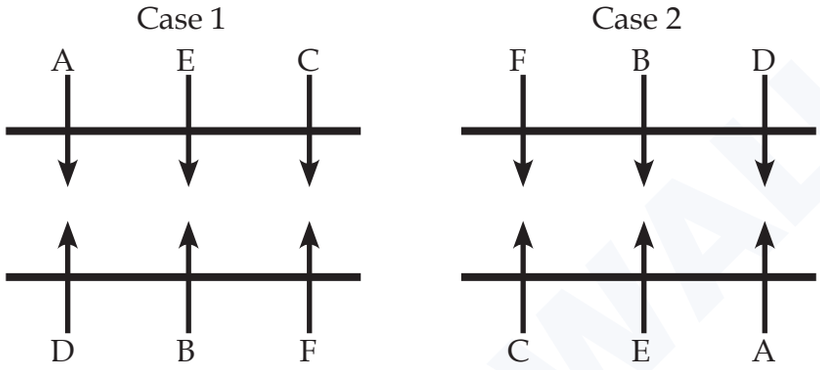
E is not at the end of any row

D is second to the left of F

C, the neighbour of E, is sitting diagonally opposite to D

B is the neighbour of F.

Thus, two cases will be possible that is shown below:



Therefore, D, B and F are in are in one of the two rows

Hence, the correct option is (a) i.e., D, B and F.

Example 14. Eight persons P to W are sitting in front of one another in two rows. Each row has four persons. P is between U and V and facing North. Q, who is to the immediate left of M is facing W. R is between T and M and W is to the immediate right of V.

Who is to the immediate right of R? **(ICAI)**

(a) M (b) U (c) M or T (d) None of these

Sol. (d) According to the question,

P is between U and V and facing North.

So, the arrangement can be: U P V or V P U

It is given that W is to the immediate right of V.

So, north facing persons sit in the following manner: **U P V W**

Now, the other row will be south facing.

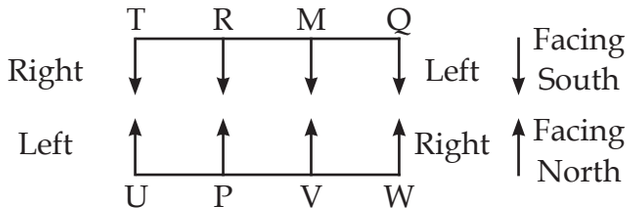
As, W is opposite to Q who is to the immediate left of M

i.e., **_ _ M Q**

Next condition is, R is between T and M.

Thus, South facing persons sit in the following order: **T R M Q**

Arrangement according to the question is as follows:



Clearly, T is to the immediate right of R.

Hence, the correct option is (d) i.e., None of these.

PRACTICE QUESTIONS (PART A)

1. There are five different houses, A to E, in a row. A is to the right of B and E is to the left of C and right of A, B is to the right of D. Which of the houses is in the middle?

(a) A (b) B (c) C (d) D

2. In a classroom, five students named Alice, Bob, Charlie, Dave, and Emma are seated in a row facing the front. Here, Charlie is sitting between Dave and Emma, and Dave is to the immediate left of Bob. Alice is to the immediate left of Emma. Who is in the middle?

(a) Charlie (b) Bob (c) Emma (d) Dave

3. In a queue, five individuals are standing in a line. One of the two individuals at the extreme ends is a doctor, and the other is an engineer. An artist is standing to the right of the student. An architect is positioned to the left of the engineer. The student is standing between the doctor and the artist. Counting from the left, at which place is the architect?

(a) 2nd (b) 3rd (c) 4th (d) None of these

4. Six individuals named A, B, C, D, E, and F are seated in front of one another facing each other in two rows. Each row has three people. The following statements provide their seating arrangement:

(I) A is seated between D and F and is facing South.

(II) B, who is to the immediate left of E, is facing F.

(III) C is seated to the immediate right of E.

Who is sitting in front of C?

(a) E (b) B (c) D (d) C

5. Six friends A, B, C, D, E and F are sitting in a row facing towards North. C is sitting between A and E. D is not at the end. B is sitting at immediate right of E. F is not at the right end but D is sitting at 3rd left of E. How many persons are there to the right of D?

(a) One (b) Two (c) Three (d) Four

6. A, P, R, X, S and Z are sitting in a row. S and Z are in the centre. A and P are at the ends. R is sitting to the left of A. Who is to the right of P?

(a) A (b) X (c) S (d) Z

Answer Key

1. (a) **2.** (a) **3.** (d) **4.** (c) **5.** (d) **6.** (b)

❑ Circular Arrangement:

In the circular arrangement, we need to arrange around a circular table and check if the arrangement is to the left or right or facing outwards or inwards.

Example 15. Four girls A, B, C, D are sitting around a circle facing the centre. B and C infront of each other, which of the following is definitely true?

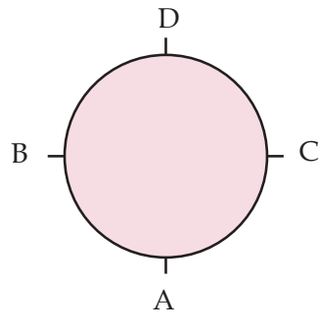
(a) A and D infront of each other

(b) A is not between B and C

(c) D is left of C

(d) A is left of C

Sol. (a) According to the given information,



Hence, the correct option is (a) i.e., A and D infront of each other.

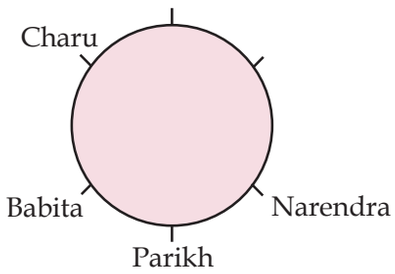
Example 16. Parikh is sitting between Narendra and Babita, Charu is to the left of Babita, Pankaj is sitting between Charu and Ashma, they all sitting around a circle facing the center then who is sitting to the right of Babita?

(a) Parikh (b) Ashma (c) Charu (d) Narendra

Sol. (a) Consider the arrangement of six people: Parikh, Narendra, Babita, Charu, Pankaj, Ashma sitting around a circle facing the center.

According to the question,

Parikh is sitting between Narendra and Babita and Charu is to the left of Babita;



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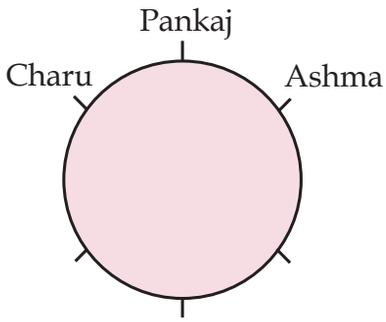
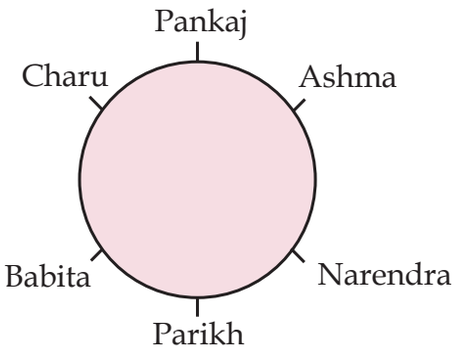
Pankaj is sitting between Charu and Ashma;



Therefore, the final arrangement will be:



Clearly, Parikh is sitting to the right of Babita.

Hence, the correct option is (a) i.e., Parikh.

Example 17. Bunty, Dev, Manav, Kavya, Payal, Qasturba, Wasir and Himmat are sitting around a circle facing at the center. Manav is to the immediate right of Bunty who is 4th to the right of kavya. Payal is 2nd to the left of Bunty and is 4th to the right of Wasir. Qasturba is 2nd to the right of dev who is 2nd to the right of Himmat.

Who is 3rd to the right of Bunty?

(a) Wasir (b) Manav

(c) Himmat (d) None of these

Sol. (c) According to the information given we get that,

Starting from Manav his right is Bunty, so arrangement will be Manav, Bunty

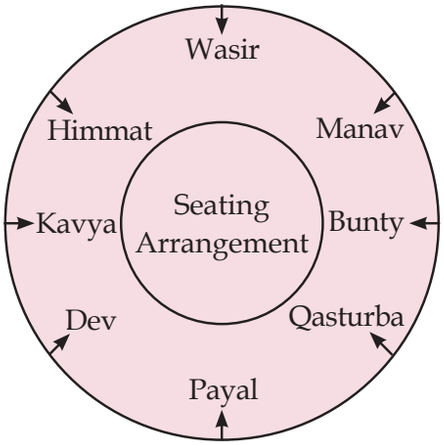
Also bunty is 4th to the right of kavya so arrangement becomes, Kavya, __ , __ , Manav, Bunty

Payal is 2nd to the left of Bunty and is 4th to the right of Wasir, So Wasir must be 2 right of Kavya

Therefore, the arrangement becomes Kavya, __ , Wasir , Manav, Bunty, __ , Payal , __

Also, Qasturba is 2nd to the right of dev who is 2nd to the right of Himmat.

Thus, the final arrangement is:



Therefore, the third right of Bunty is Himmat.

Hence, the correct option is (c).

Example 18. Eight executives H, I, J, K, L, M, N and P are sitting around a circular table for a meeting. M is third to the right of J and second to the left of K. H is to the immediate left of I. P is to the immediate right of K, L is third to the left of N. Find who will be a neighbour to I?

(a) H and M (b) L and K (c) H and J (d) M and J

Sol. (a) According to given conditions,

M is third to the right of J & second to the left of K so J, __ , __ , M , __ , K,

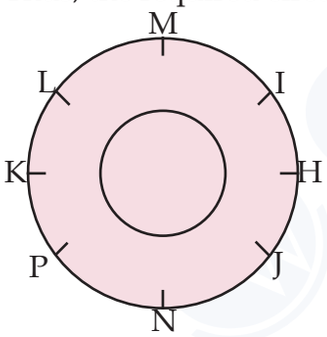
P is to immediate right of K so, J, __ , __ , M , __ , K, P

H and I are together so, it will be

J, H , I , M , __ , K, P

Now, L is third to the left of N

Thus, the required circular arrangement is given as:



Therefore, H and M are neighbours of I.

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART B)

1. Six friends are seated in a circular arrangement, all facing the center. Deepa is positioned between Prakash and Pankaj. Priti is located between Mukesh and Lalit and Mukesh is sitting right to Priti. Prakash and Mukesh are seated opposite each other. Who is sitting immediate left of Prakash ?

(a) Mukesh (b) Deepa (c) Pankaj (d) Lalit

2. Six persons A, B, C, D, E and F are standing in a circle. B is between C and D and A is between E and C. F is to the right of D. Who is between A and F?

(a) B (b) C (c) D (d) E

3. Six girls, named P, Q, R, S, T, and V, are seated in a circle, all facing the center.

The following statements are given:

A. T is not seated between Q and S but instead, between two other individuals.

B. P is positioned immediately to the left of V.

C. R is located four seats to the right of P.

Which of the following statements is not true?

(a) V is seated just to the right of P.

(b) T is seated just to the right of V.

(c) R is positioned second to the left of T.

(d) P is seated second to the right of R.

4. Eight girls are sitting around a square table, they are facing centre,

- A sits 6th to the left of B, who sits third to the right of F.
- E sits 2nd to the left of F.
- G sits in between A and B.
- C sits immediate left of F, who is immediate left of D.

(I) Who sits second to the left of D?

(a) E (b) A (c) C (d) F

(II) What is the position of E with the Respect to H?

(a) 2nd to the left (b) 2nd to the right

(c) Fourth to the right (d) Third to the left

5. Shyam, Sathish, Aman and Pavan are playing cards. Amar is to the right of Sathish, who is to the right of Shyam. Who is to the right of Amar?

(a) Satish (b) Amar (c) Pavan (d) Shyam

Answer Key

1. (b) **2.** (d) **3.** (c) **4.** (I) - (c) (II) - (c) **5.** (c)

DIRECTION (Q19 – Q23):

I. A, B, C, D, E, F, G, and H are sitting in a row facing North.

II. A is fourth to the right of E.

III. H is fourth to the left of D.

IV. C and F, who are not at the ends are neighbours of B and E, respectively

V. H is next to the left of A and A is the neighbour of B.

Example 19. Who are sitting at the ends?

(a) E and C (b) E and D (c) G and D (d) None of these

Sol. (b) According to given conditions we get

A is fourth to the right of E, order is: E ___ ___ ___ A

Now, H is next to the left of A and A is the neighbours of B, order is E ___ ___ H A B

H is fourth to the left of D: E ___ ___ H A B ___ D

Also, C and F, who are not at the ends are neighbours of B and E

Arrangement is given as: EFGHABCD

Therefore, E and D are sitting at the ends.

Hence, the correct option is (b).

Example 20. Which of the following statements is not true?

(a) H is second to the right of F.

(b) E is fourth to the left of A.

(c) D is fourth to the right of H.

(d) None of these

Sol. (d) From the above arrangement given: EFGHABCD

Options (a), (b) and (c) are true.

Therefore, option (d) is correct.

Example 21. Who is/are the neighbour(s) of D?

(a) F alone (b) C alone (c) B and C (d) Cannot be determined

Sol. (b) From the above arrangement given: EFGHABCD

Clearly, C alone is a neighbour of D.

Hence, option (b) is correct.

Example 22. Which of the following statements is not true?

(a) G is the neighbours of H and F (b) B is the next to the right of A

(c) E is at left end (d) D is next to the right of B

Sol. (d) From the above arrangement given: EFGHABCD

Out of all the options, option (d) i.e., D is next to the right of B is not true.

Hence, option (d) is correct.

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Example 23. What is the position of F?

(a) Next to the right of E. (b) Next to the right of G.

(c) Sixth to the right of D. (d) Between G and H.

Sol. (a) From the above arrangement given: EFGHABCD

Clearly, we can see that F is next to the right of E.

Hence, option (a) is correct.

DIRECTION (Q24 – Q27): Seven friends P, Q, R, S, T, U and V are standing in a straight line facing north but not necessarily in the same order.

- U stands second from the left end of the line.
- Only two people stand between U and T.
- Only one person stands between P and R.
- V stands third to the left of R.
- Neither R nor U is an immediate neighbors of S.

Example 24. Which of the following pairs stand at the extreme ends of the line?

(a) Q, S (b) Q, P (c) Q, T (d) V, S

Sol. (d) According to the given conditions:

U stands second from the left end of the line and only two people stand between U and T i.e.,

__ U __ __ T __ __

V stands third to the left of R, so possibilities are:

V U __ R T __ __ 'OR' __ U __ V T __ R 'OR' __ U V __ T R __

Only one person stands between P and R so the above 2nd arrangement cannot be possible, thus the possibilities are:

V U __ R T P __ 'OR' __ U V P T R __

Neither R nor U is an immediate neighbours of S.

So, the possible arrangement is:

V U Q R T P S

Therefore, V and S are at the extreme ends of the line.

Hence, option (d) is correct.

Example 25. What is the position of V with respect to U?

(a) Immediate left (b) Second to the right

(c) Third to the right (d) Immediate right

Sol. (a) According to the above arrangement i.e., V U Q R T P S

V is immediate left of U.

Hence, option (a) is correct.

Example 26. Four of the following five form a group as per the given arrangement. Which of the following does not belong to the group?

(a) R Q (b) T P (c) V T (d) U V

Sol. (c) According to the above arrangement i.e., V U Q R T P S Clearly, V and T do not form a group. Hence, option (c) is correct.

Example 27. Which of the following is TRUE regarding Q?

(a) P stands second to the left of Q.

(b) S is one of the immediate neighbour of Q.

(c) Only one person stands between Q and T.

(d) Q stands at one of the extreme ends of the line.

Sol. (c) According to the above arrangement i.e., V U Q R T P S

Now, clearly only one person stands between Q and T i.e., R.

Hence, option (c) is correct.

Example 28. In a gathering seven members are sitting in a row. 'C' is sitting left to 'B' but on the right to 'D'. 'A' is sitting right to 'B', 'F' is sitting right to 'E' but left to 'D'. 'H' is sitting left to 'E'. Find the person sitting in the middle. **(ICAI)**

(a) C (b) D (c) E (d) F

Sol. (b) Considering the seven members facing in North direction, then

According to the question,

'C' is sitting left to 'B' but on the right to 'D' i.e. D C B

'A' is sitting right to 'B' i.e. D C B A

'F; is sitting right to 'E' but left to 'D' i.e.

E F D C B A

'H' is sitting to the left to 'E', then the final arrangement of the seven members in row will be:

H E F D C B A

Therefore, D is sitting in the middle.

Hence, the correct option is (b).

Example 29. Six children A, B, C, D, E and F are standing in a row. B is between F and D. E is between A and C. A does not stand next to either F or D. C does not stand next to D. F is between which of the following pairs of children? **(ICAI)**

(a) B and E (b) B and C (c) B and D (d) B and A

Sol. (b) According to question,

B is between F and D i.e., F B D

E is between A and C i.e., A E C

A does not stand next to either F or D and C does not stand next to D.

$\Rightarrow$ C stands next to F.

$\therefore$ The arrangement of six children:

A E C F B D

Clearly, F is standing between B and C.

Hence, the correct option is (b)

Example 30. Five senior citizens are living in a multi-storeyed building. Mr. Manu lives in a flat above Mr. Ashokan, Mr. Lokesh in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat above Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives in the topmost flat?

(May 2018)

(a) Mr. Lokesh (b) Mr. Gaurav (c) Mr. Manu (d) Mr. Rakesh

Sol. (c) Mr. Manu lives in a flat above Mr. Ashokan

Mr. Manu
Mr. Ashokan

Mr. Lokesh in a flat below Mr. Gaurav

Mr. Gaurav
Mr. Lokesh

Mr. Ashokan lives in a flat above Mr. Gaurav

Mr. Manu
Mr. Ashokan
Mr. Gaurav
Mr. Lokesh

Mr. Rakesh lives in a flat below Mr. Lokesh, thus the order of flats would be:

Mr. Manu
Mr. Ashokan
Mr. Gaurav
Mr. Lokesh
Mr. Rakesh

Therefore, Mr. Manu lives in the topmost flat.

Hence, the correct option is (c).

Example 31. There are eight books kept one over the other. Two books are on Organisation Behaviour, two books on TQM, three books on Industrial Relations and one book is on Economics. Counting from the top, the second, fifth and sixth books are on Industrial Relations. Two books on Industrial Relations are between two books on TQM. One book of Industrial Relations is between two books on Organizational Behaviour while the book above the book of Economics is a book of TQM. Which book is the last book from the top? **(ICAI)**

(a) Economics (b) TQM

(c) Industrial Relations (d) Organizational Behaviour

Sol. (a) According to the question,

The second, fifth and sixth books are on Industrial Relations, thus

2. Industrial Relations
5. Industrial Relations
6. Industrial Relations

Since, two books on Industrial Relations are between two books on TQM, thus

2. Industrial Relations
4. TQM
5. Industrial Relations
6. Industrial Relations
7. TQM

Now, one book of Industrial Relations is between two books on Organizational Behavior, thus

1. Organizational Behavior
2. Industrial Relations
3. Organizational Behavior
4. TQM
5. Industrial Relations
6. Industrial Relations
7. TQM

Also, the book above the book of Economics is a book of TQM

1. Organizational Behavior
2. Industrial Relations
3. Organizational Behavior
4. TQM
5. Industrial Relations
6. Industrial Relations
7. TQM
8. Economics

Therefore, Economics is the last book from the top.

Hence, the correct option is (a)

FULL CHAPTER PRACTICE QUESTIONS

Directions (Q1 – Q4): A, B, C, D, E, F and G are sitting in a row facing North:

I. F is to the immediate right of E

II. E is 4th to the right of G

III. C is the neighbour of B and D

IV. Person who is third to the left of D is at one of ends

1. Who is at immediate right of D? **(July 2021)**

(a) E and F only (b) G, B, and C

(c) E, F and A (d) G and B only

2. Who are neighbours of F?

(a) A and E (b) E and A (c) D and E (d) B and D

3. The third person from the right extreme end is

(a) E (b) D (c) C (d) F

4. Person immediately to the left of B is

(a) C (b) G (c) F (d) D

5. Four children are sitting in a row. A is occupying a seat next to B but not next to C. If C is not sitting next to D? Who is occupying the seat adjacent to D?

(a) B (b) B and A (c) Not enough information (d) A

6. Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary. Who is sitting immediate right to Reeta? **(Jan 2021)**

(a) Seema (b) Rani (c) Bindu (d) Mary

7. Five persons are sitting on a bench to be photographed, S is to the left of N and to the right of B. M is to the right of N. R is between N and M. Who is sitting immediate right to R?

(a) B (b) N (c) M (d) S

8. Six friends P, Q. R, S, T and U are sitting around the hexagonal table each at one corner and are facing the centre of the hexagonal. P is second to the left of U. Q is neighbour of R and S. T is second to the left of S. Which one is sitting opposite to P?

(a) R (b) Q (c) T (d) S

9. 5 children are sitting in a row. S is sitting next to P but not T. K is sitting next to R, who is sitting on the extreme left. T is not sitting next to K. Who are sitting adjacent to S? **(Dec 2020)**

(a) K & P (b) R & P (c) only P (d) P & T

10. In a college party, 5 girls are sitting in a row. F is to the left of M and to the right of O. R is sitting to the right of N but to the left of O, who is sitting in the middle?

(a) O (b) R (c) P (d) M

11. A is seated between D and F at a round table. C is seated opposite to D. E is round adjust to D. Who sit opposite to B?

(a) A (b) D (c) C (d) F

Directions (Q12 – Q15): Four Indian, A, B, C and D and four Chinese E, F, G and H are sitting in a circle around a table facing each other in a conference. No two Indians or Chinese are sitting side by side, C who is sitting between G and E is facing D, F is between D and A and facing G, H is to the right of B.

12. Who is sitting left of A? **(Jan 2021)**

(a) E (b) F (c) G (d) H

13. Who is sitting third right of D?

(a) A (b) C (c) G (d) E

14. Person sitting between B & C is

(a) G (b) H (c) A (d) E

15. Looking clockwise, Person sitting second next to H is

(a) A (b) G (c) E (d) B

16. Five boys A, B, C, D and E are sitting in a row. A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end?

(a) D (b) A (c) E (d) B **(June 2019)**

17. Five senior citizens are living in a Multi-Storeyed building. Mr. Muan lives in a flat above Mr. Ashokan, Mr. Lokesh in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat below Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives in the top most flat? **(ICAI)**

(a) Mr. Lokesh (b) Mr. Gaurav

(c) Mr. Muan (d) Mr. Rakesh

Directions (Q18 – Q21): Statement: In a straight line there are six person sitting in a row. B is between F and D. E is between A and C. A does not stand next to F or D, C does not stand next to D.

18. F is between which of the following persons?

(a) B and E (b) B and C (c) B and D (d) B and A

19. Who are sitting at extreme ends?

(a) B and A (b) D and E (c) D and A (d) B and E

20. Who is fourth from the left extreme end?

(a) B (b) C (c) B (d) F

21. Which group is followed from given information?

(a) F C E (b) F E A (c) D F E (d) F A E

Directions (Q22 – Q25): Six persons A, B, C, D, E & F are sitting in two rows. Three in each row such that

I. E is not at the end of any row

II. D is second to the left of F

III. C, the neighbor of E, is sitting diagonally opposite to D

IV. B is neighbor of F

22. What will be the arrangement?

(a) AEC & DBF (b) FBE & CDA (c) CBE & ABF (d) DBC & AEF

23. Who is sitting diagonal to A?

(a) B (b) F (c) D (d) C

24. Who is sitting between A & C?

(a) B (b) E (c) D (d) F

25. Who is sitting between D and F?

(a) B (b) E (c) D (d) F

Answer Key

1. (c)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (a)
11. (a)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (c)	
20. Both (b) and (d)			**21.** (a)	**22.** (a)	**23.** (b)	**24.** (b)	**25.** (a)		

PREVIOUS YEAR QUESTION (PYQ'S)

1. Four girls are seated for a photograph. Shikha is left of Reena. Manju is to the right of Reena. Reeta is between Reena and Manju. Who is second from left in photograph? **(June 2019)**

(a) Reena (b) Manju (c) Reeta (d) Shikha

2. In a straight line there are six persons sitting in a row. B is between F and D. E is between A and C. A does not stand next to F or D, C does not stand next to D. F is between which of the following persons? **(June 2019)**

(a) B and E (b) B and C (c) B and D (d) B and A

3. 5 children are sitting in a row. S is sitting next to P but not T. K is sitting next to R. K is not sitting on extreme end. T is not sitting next to K.Who are adjacent to S? **(June 2019)**

(a) K and P (b) R and P (c) Only P (d) P and T

4. Five boys A, B, C, D and E are sitting in a row. A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end? **(June 2019)**

(a) D (b) A (c) E (d) B

5. 5 persons are standing in a line. One of the 2 persons at the extreme ends is a professor and the other a businessman. An advocate is standing to the right of student. An author is to the left of the businessman. The student is standing between the professor and advocate. Counting from the left, the author is at which place? **(Dec 2019)**

(a) 2nd (b) 3rd (c) 4th (d) None of these

6. Parikh is sitting between Narendra and Babita, Charu is to the left of Babita, Pankaj is sitting between Charu and Ashma, they all sitting around a circle facing the center then who is sitting to the right of Babita? **(Dec 2019)**

(a) Parikh (b) Ashma (c) Charu (d) Narendra

7. A, B, C, D, E and F are sitting around a round table. A is between E and F, E is opposite to D, and C is not in either of the neighboring seats of E. Who is opposite to B? **(Dec 2020)**

(a) C (b) D (c) F (d) None of these

8. Six persons A, B, C, D, E and F are sitting in a circle. B is between C and D and A is between E and C. F is to the right of D. Who is between A and F? **(Dec 2020)**

(a) B (b) C (c) D (d) E

9. Five boys A, B, C, D and E are sitting in a row. A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end? **(Dec 2020)**

(a) D (b) A (c) E (d) B

10. 5 children are sitting in a row. S is sitting next to P but not T. K is sitting next to R. K is not sitting on the extreme end. T is not sitting next to K. Who is sitting adjacent to S? **(Dec 2020)**

(A) K and P (B) R and P (C) Only P (D) P and T

11. Six children A, B, C, D, E and F are sitting in a row. B is between F and D. E is between A and C. However, A does not sit next to F or D. C does not sit next to D. Then, F is sitting between ____ & _____? **(Dec 2020)**

(a) B and C (b) E and C (c) B and D (d) None

12. A is seated between D and F at a round table. C is seated opposite to D. E is round adjust to D. Who sits opposite to B? **(Jan 2021)**

(a) A (b) D (c) C (d) F

13. Four Indian, A, B, C and D and four Chinese E, F, G and H are sitting in a circle around a table facing each other in a conference. No two Indians or Chinese are sitting side by side, C who is sitting between G and E is facing D, F is between D and A and facing G, H is to the left of B. Who is sitting to the left of A? **(Jan 2021)**

(a) E (b) F (c) G (d) H

14. Five friends A, B. C, D and E are sitting on a beach. A is sitting next to B; C is sitting next to D; D is not sitting with E; E is at the left end of the bench. C is on second position from the right; A is on the right side of B who is to the right side of E. A and C are sitting together. What is the position of B? **(Jan 2021)**

(a) Second from right (b) Centre

(c) Extreme left (d) Second from left

15. A, B, C, D and E are sitting on a bench. A is sitting next to B, C is sitting next to D, D is not sitting with E; E is at the left end of bench. C is on second position from the right; A is to the right of B who is to the right side of E. A and C are sitting together. What is the position of B? **(Jan 2021)**

(a) Between C and D (b) Between D and E

(c) Between E and A (d) Between B and D

16. A, B, C and D are playing cards, A and B are partners. D faces towards the North. If A faces west, then who faces south? **(Jan 2021)**

(a) C (b) B (c) D (d) Data is inadequate

17. Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary. Who is sitting immediate right to Reeta ? **(Jan 2021)**

(a) Seema (b) Rani (c) Bindu (d) Mary

18. Six friends P, Q. R, S, T and U are sitting around the hexagonal table each at one corner and are facing the centre of the hexagonal. P is second to the left of U. Q is neighbour of R and S. T is second to the left of S. Which one is sitting opposite to S? **(July 2021)**

(a) R (b) P (c) Q (d) T

19. A, B, C, D, E, F and G are sitting in a row facing North.

I. F is to the immediate right of E

II. E is 4th to the right of G

III. C is the neighbour of B and D

IV. Person who is third to the left of D is at one of ends

Who are to the right of D? **(July 2021)**

(a) E and F only (b) G, B and C

(c) E, F and A (d) G and B only

20. Six children named P, Q, R, S, T and U are sitting in a row; Q is between U and S; T is between P and R; P does not sit next to either U or S; B does not sit next to S. So, U is sitting between the pairs of _____ of children. **(Dec 2021)**

(a) Q and T (b) Q and R (c) Q and S (d) Q and P

21. Five persons A, B, C, D and E are sitting in a row. A sit left to C and c sit left to B. E sits right to B, D sits in between E and B. Who is sitting in the middle? **(Dec 2021)**

(a) B (b) C (c) E (d) D

22. Four ladies A, B, C and D and four gentlemen E, F, G and H are sitting in a circle around a table facing each other.

I. No two Ladies or Gentlemen are sitting side by side.

II. C, who is sitting between G and E, facing D.

III. F is between D and A and facing G.

IV. H is to the right of B.

Who is immediate neighbour of B? **(Dec 2021)**

(a) G and H (b) E and F (c) E and G (d) A and B

23. Persons M, N, O, P, Q, R, S and T are sitting on a compound wall facing North, O sits fourth left of S; P sits second to the right of S; only two people sit between P and M; N and R are immediate neighbours of each other. N is not an immediate neighbour of M; T is not neighbour of P. How many persons are seated between M and Q? **(Dec 2021)**

(a) One (b) Two (c) Three (d) Four

24. In a line , P is sitting 13th from left . Q is sitting 24th from right and 3rd left from P .How many people are sitting in a line? **(Dec 2021)**

(a) 34. (b) 31 (c) 32 (d) 33

25. Six friends Surya, Bhanu, Dinkar, Ravi, Suraj and Dinesh Are sitting in a circle and are facing the centre of the circle, Dinesh is between Dinkar and Suraj. Bhanu is between Ravi and Surya. Dinkar and Ravi are opposite to each other.

Who are the immediate neighbours of Ravi? **(June 2022)**

(a) Suraj and Dinesh (b) Dinkar and Bhanu

(c) Surya and Dinesh (d) Bhanu and Suraj

26. Eight persons E, F, G, H, l, J, K and L are seated around a square table, facing table - two on each side. J is between L and F; G is between I and F; H a lady member is second to the left of J; F a male member is seated opposite to E, a lady member There is a lady member between F and I. who among the following is to the immediate left of F? **(June 2022)**

(a) G (b) I (c) J (d) H

27. Five persons are sitting on a bench to be photographed, S is to the left of N and to the right of B. M is to the right of N. R is between N and M. Who is sitting immediate right to R? **(June 2022)**

(a) B (b) N (c) M (d) S

28. Six friends P, Q, R, S, T and U are sitting around the hexagonal table each at one corner and are facing the center of the hexagonal. P is second to the left of U. Q is neighbour of R and S. T is second to the left of S. Which one is sitting opposite to P? **(June 2022)**

(a) R (b) Q (c) T (d) S

29. Six persons A, B, C, D, E and F are sitting in two rows with three persons in each row. Both rows are in front of each other. E is not at the end of the any row and D is second left to the F, C is neighbor of E and diagonally opposite to D if B is neighbour F who is in front of C then who is sitting diagonally to F? **(Dec 2022)**

(a) C (b) E (c) A (d) D

30. P, Q, R, S and T are sitting in a line facing West. P and Q are sitting together. R is sitting at south end and S is sitting at North end. T is neighbor of Q and R. Who is sitting in the middle? **(Dec 2022)**

(a) P (b) Q (c) R (d) S

31. Seven friends A, B, C, D, E, F & G are watching movie sitting in a row. E is sitting at one extreme end. C is sitting second to E. B is sitting between A & C. G is not sitting at any extreme end. A is not at any extreme end. D is sitting immediate to F.
Who is sitting in the middle? **(June 2023)**

(a) G (b) D (c) C (d) B

32. Five boys Ajay, Brijmohan, Chandru, Dheeraj and Ehsan are sitting in a park in a circle facing the centre. Ajay is facing south-West, Dheeraj is facing South-East, Brijmohan and Ehsan are right opposite Ajay and Dheeraj, respectively and Chandru is equidistant between Dheeraj and Brijmohan. Which direction is Chandru facing? **(June 2023)**

(a) West (b) South (c) North (d) East

33. Pran, Qomal, Ravi, Shalu, Trilok, Urvi, Vasu and Walter are sitting in a row facing North.

(I) Pran is fourth to the right of Trilok

(II) Walter is fourth to the left of Shalu

(III) Ravi and Urvi, which are not at the ends, are neighbours of Qomal and Trilok, respectively.

(IV) Walter is immediate left of Pran and Pran is the neighbours of Qomal.

Identify who are sitting at the extreme ends? **(June 2023)**

(a) Pran and Walter (b) Trilok and Urvi

(c) Trilok and Shalu (d) Shalu and Pran

34. Six friends – A, B, C, D, E, and F are sitting around a circular table facing towards the entre of the circle. E is not sitting between B and D. A sits to the left of F and C is fourth to the right of A. D is immediate right of E.

Who sits second to right to F? **(June 2023)**

(a) C (b) A (c) D (d) B

35. A, B, C, D, E, F, G and H are sitting in a circle facing centre. D is 3rd to left of A. E sits to immediate right of A. B is 3rd to left of D. G is second to right of B. C is neighbour of B. C is 3rd to left of H. Who sits exactly between F and E? **(Dec 2023)**

(a) C (b) E (c) H (d) A

Answer the following Question :(Q 36 & 37)

Nine friends, J, K, L, M, N, O, P, Q and R, are sitting in a row. L is to the right of M and is at the third place to the right of N, K is at one end of the row. Q is seated adjacent to both O and P. O is at the third place to the left of K. J is right next to L and the left of O. **(Dec 2023)**

36. Who is immediate left to M?

(a) N (b) R (c) L (d) J

37. Who is sitting at the center of the row?

(a) I (b) J (c) O (d) Q

38. Five people are living in a five story building. Mr. Mahesh lives in a flat above Mr. Ashok. Mr. Lokesh lives in a flat below Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who possibly lives on the ground floor? **(Dec 2023)**

(a) Mr. Rakesh (b) Mr. Lokesh (c) Mr. Mahesh (d) Mr. Gaurav

39. Seven friends O, P, Q, R, S, T And U are watching movies sitting in a row , S is sitting at one extreme end. Q is sitting second to the right of S. P is sitting between O and Q. Both U and O are not sitting at the extreme end. R is sitting in the immediate left of T. Who is sitting in the middle? **(Dec 2023)**

(a) O (b) P (c) Q (d) U

40. A, B, C, D, E, and F are standing in a circle talking, facing inward. E is right to C who is standing 3 places away from D. A is standing 2 seats away from F. Which of the following has to be true? **(June 2024)**

(a) A is standing opposite to C (b) A is standing next to E

(c) A is standing next to D (d) A is standing next to B

41. Eight friends A, B, C, D, E, F, G and H are sitting around a circular table facing the center. A is sitting two places to the right of B, who is sitting directly opposite to D. C is sitting to the immediate left of B and to the immediate right of E. F, who is opposite to G, is not sitting next to A. Who is sitting to the immediate left of H? **(June 2024)**

(a) A (b) B (c) C (d) D

42. The persons named U, V, W, X, Y and Z are sitting along the circumference of a round table. They are facing the center of the round table. Given: X is the third left of Z and U is the second right of X; W is third to Y's right. Then V is sitting immediate left to **(June 2024)**

(a) U (b) W (c) X (d) Y

43. 5 persons are standing in a line. The 2 persons at the extreme ends are a professor and a business man. An advocate is standing to the right of student. An author is to the left of the business man. Counting from the Professor's end, the author is at which place? **(June 2024)**

(a) 2nd (b) 3rd (c) 4th (d) 5th

44. Five players named as A, B, C, D and E are sitting on a bench, facing south, and are waiting to be interviewed by a selector. The person C is an immediate neighbor of both A and B. The person A is the fourth person from right end. If E is to the right of B, then where is E sitting? **(June 2024)**

(a) Fifth from right end (b) Fourth from right end

(c) Fifth from left end (d) Forth from left end

45. Six persons A, B, C, D, E and F are sitting in a row in a straight line. B is between F & D. E is between A & C. A does not sit next to F or D; C does not sit next to D. F is between which of the following persons? **(Sept 2024)**

(a) B & E (b) B & C (c) B & D (d) B & A

46. In a school cultural committee meeting, four girls Dipti, Aruna, Chandra, Bindu and four boys Gautam, Faneesh, Harendra, Eshaan are sitting in a circle around a table, facing each other as under: **(Sept 2024)**

(i) No two girls or boys are sitting side by side.

(ii) Chandra, who is sitting between Gautam and Eshaan, is facing Dipti.

(iii) Faneesh is between Dipti and Aruna and facing Gautam,

(iv) Harendra is to the right of Bindu.

Identify the person whom Eshaan is facing.

(a) Faneesh (b) Bindu (c) Gautam (d) Harendra

Note: Read the following information carefully and answer the questions (47 to 48) given below:

Six persons B, D, C, M, J and K are split into groups of three each and are made to stand in two rows in such a way that a person in one row is exactly facing a person in the other row. M is not at the ends of any row and is to the right of J, who is facing C. K is to the left of D, who is facing M.

47. Who is to the immediate left of B? **(Sept 2024)**

(a) M (b) J (c) D (d) K

48. Which of the following persons are in the same row? **(Sept 2024)**

(a) BDC (b) BMJ (c) MJK (d) MJD

49. During an interview, seven applicants sitting in a row are awaiting their turn. Chandresh is sitting left to Kuldeep but on the right to Diksha, Reshma is sitting right to Kuldeep, Priyanka is sitting right to Gayatri but left to Diksha. Himani is sitting left to Gayatri. The person sitting in the middle must be: **(Sept 2024)**

(a) Chandresh (b) Diksha (c) Gayatri (d) Priyanka

50. Five persons A, B, C, D and E are sitting in a circle facing centre. C is sitting immediate left of E. A is sitting in between E and D. Who is sitting between B and A? **(Jan 2025)**

(a) D (b) C (c) E (d) B

51. Eight persons A, B, C, D, E, F, G and H are sitting in two rows opposite to each other. Each row has 4 persons. B and C are sitting opposite side. C is sitting in between E and D. H is sitting immediate left of E. F and H are sitting at diagonally opposite position. G is sitting extreme end. Who is sitting in front of E? **(Jan 2025)**

(a) G (b) F (c) B (d) A

52. Five friends A, B, C, D and E are sitting in a row facing east. A is sitting between C & D. B is second to the left of C. Who is sitting at the south end? **(Jan 2025)**

(a) B (b) E (c) C (d) D

53. Five persons A, B, C, D & E sitting on a bench. A is immediate right of B. E is immediate left of C and immediate right of A. B is to the right of D. Which person is sitting in the middle of bench? **(Jan 2025)**

(a) E (b) B (c) A (d) D

54. In a class, there are seven students (including boys and girls) A, B, C, D, E, F and G. They sit on three benches I, II and III, such that atleast two students on each bench and atleast one girl on each bench. C who is a girl student, does not sit with A, E and D. F, the boy student sits with only B. A sits on the bench I with his best friends. G sits on the bench III. E is the brother of C. Which of the following is the group of girls? **(May 2025)**

(a) BCG (b) BFC (c) BCD (d) CDF

55. Five boys A, B, C, D and E are sitting in a row. A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end? **(May 2025)**

(a) D (b) A (c) B (d) E

56. Six friends P, Q, R, S, T and U are sitting around the hexagonal table each at one corner and are facing the centre of the hexagonal. P is second to the left of U. Q is neighbour of R and S. T is second to the left of S. Which one is sitting opposite to P? **(May 2025)**

(a) R (b) Q (c) T (d) S

57. A, B, C, D and E are sitting on a bench. A is sitting next to B, C is sitting next to D, D is not sitting with E who is on the left end of the bench. C is on the second position from the right. A is to the right of B and E. A and C are sitting together. In what position A is sitting? **(May 2025)**

(a) Between B and D (b) Between B and C

(c) Between E and D (d) Between C and E

58. Six persons A, B, C, D, E and F are sitting in two rows with 3 persons in each row facing same side. C is sitting in the middle of first row. B is sitting in the left of C. E is also sitting in the middle. A is sitting in the right of E. F is sitting in the right of C. Who is not sitting at one of the ends of any row? **(Sept 2025)**

(a) A (b) E (c) D (d) F

59. Six persons A, B, C, D, E and F are sitting in a circle facing the centre. A is facing D. C is left of A and right of E. D is between B and E. F is right of A. Which one of the following statements is incorrect? **(Sept 2025)**

(a) F and C are neighbours. (b) F is between A and B.

(c) F and A are neighbours. (d) B is left of D.

60. Eight friends with names P, Q, R, S, T, U, V and W are sitting on a bench and facing North. U is sitting between S and V; Q is sitting between W and P. T is third to the left of V; W is third left of R; V is sitting at one of the corners. Who is sitting immediate right of W? **(Sept 2025)**

(a) P (b) Q (c) R (d) T

61. Six people, A, B, C, D, E and F are sitting in a hexagonal shape. All the sides of the hexagon so formed are of same length. A is not adjacent to B or C; D is not adjacent to C or E; B and Care adjacent; F is in the middle of D and C. Which of the following is not a correct neighbour pair? **(Sept 2025)**

(a) A and F (b) D and F (c) B and E (d) C and F

62. Seven persons with names T, B, L, A, P, Z and R are standing in a row, facing East. It is given that T is to the right of R; A is between B and Z, L, who is third to the left of B, is at the end; R is fourth to the right of L. Who is at the immediate right of T? **(Sept 2025)**

(a) Z (b) B (c) P (d) R

Answer Key

1.	(c)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(a)	**7.**	(c)	**8.**	(d)	**9.**	(c)	**10.**	(a)
11.	(a)	**12.**	(a)	**13.**	(b)	**14.**	(d)	**15.**	(c)	**16.**	(a)	**17.**	(d)	**18.**	(b)	**19.**	(c)	**20.**	(b)
21.	(a)	**22.**	(a)	**23.**	(a)	**24.**	(d)	**25.**	(d)	**26.**	(c)	**27.**	(c)	**28.**	(d)	**29.**	(c)	**30.**	(b)
31.	(d)	**32.**	(d)	**33.**	(c)	**34.**	(c)	**35.**	(d)	**36.**	(b)	**37.**	(b)	**38.**	(a)	**39.**	(b)	**40.**	(c)
41.	(a)	**42.**	(c)	**43.**	(c)	**44.**	(c)	**45.**	(b)	**46.**	(d)	**47.**	(a)	**48.**	(b)	**49.**	(b)	**50.**	(a)
51.	(d)	**52.**	(d)	**53.**	(c)	**54.**	(c)	**55.**	(d)	**56.**	(d)	**57.**	(b)	**58.**	(b)	**59.**	(a)	**60.**	(b)
61.	(a)	**62.**	(c)																

SUMMARY

- We have to arrange a group of person according to given conditions, and we classify the arrangement into 4 types:
  - Linear arrangement
  - Two row arrangement
  - Circular arrangement
  - Polygon arrangement
- **Linear arrangement:** Here we have to arrange given people in a line.
  - Here, either we know the direction of facing; in this case we consider all possibilities.
  - Or we know the direction of facing such as (facing upward, downward, east, west, …)
- **Two row arrangement:** There will be two groups of persons. Here, we arrange one group in one row and another group in another row. and persons in these rows face each other.
- **Circular arrangement:** In the circular arrangement, we need to arrange around a circular table and check if the arrangement is to the left or right or facing outwards or inwards.

❑❑❑

CHAPTER

12

Blood Relations

- Blood relation refers to the connection between people based on their shared family ancestry, not through marriage or other relationships.
- In blood relation questions, certain information about the members of a family is given, and based on that information, you need to determine the relationship between specific family members.

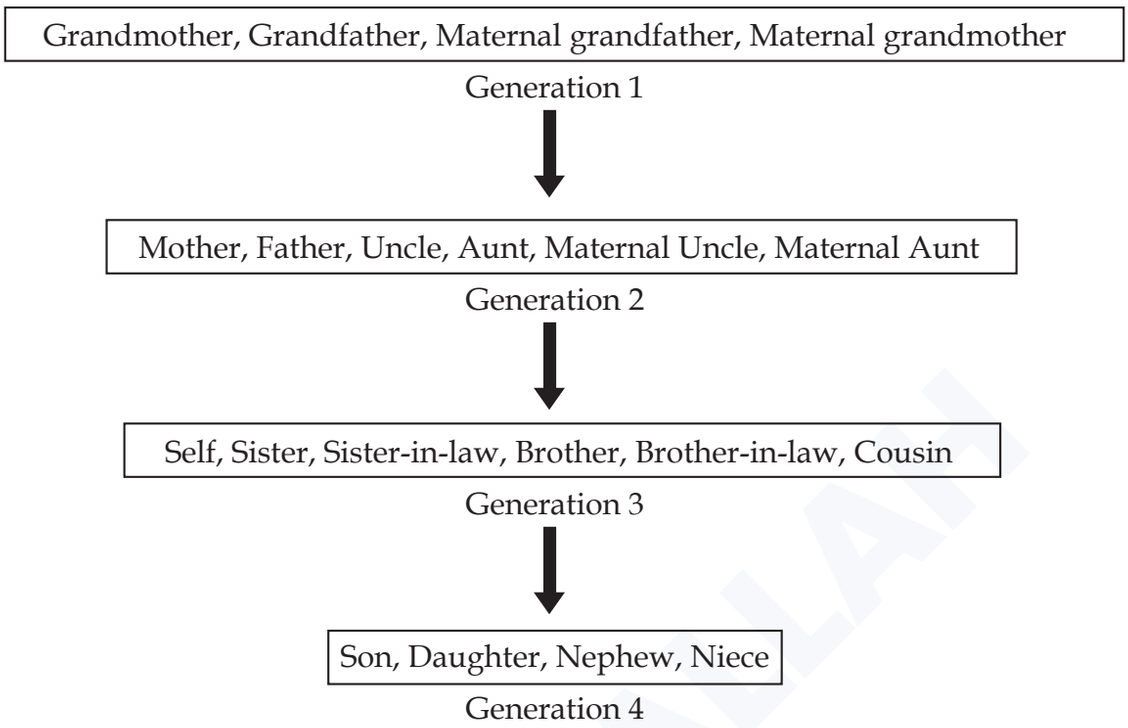
S.No	Let's see some Relations:
1.	Mother's or Father's Father: Grandfather (Maternal Grandfather/Paternal Grandfather)
2.	Mother's or Father's Mother: Grandmother (Maternal Grandmother/Paternal Grandmother)
3.	Mother's or Father's Brother: Uncle
4.	Mother's or Father's Sister: Aunt
5.	Wife's Father or Husband's Father: Father-in-law
6.	Wife's Mother or Husband's Mother: Mother-in-law
7.	Son's Wife: Daughter-in-law
8.	Daughter's Husband: Son-in-law
9.	Husband's or Wife's Brother: Brother-in-law
10.	Husband's or Wife's Sister: Sister-in-law
11.	Brother's or Sister's Daughter: Niece
12.	Brother's or Sister's Son: Nephew
13.	Uncle's or Aunt's Son or Daughter: Cousin

Note: Husband and Wife can be referred to as "spouse" only.

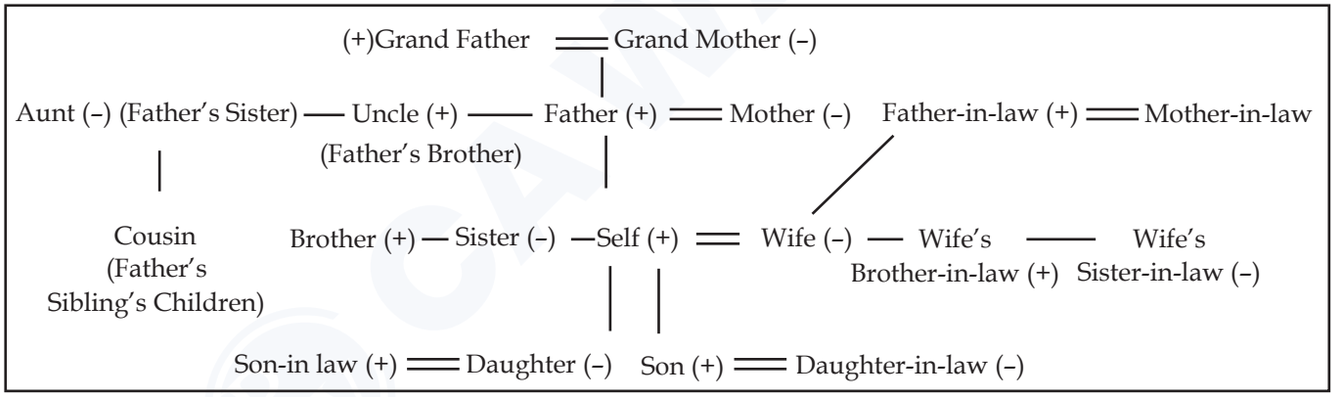
Now, if we directly attempt to solve the questions, it can become complicated and confusing. Therefore, it is essential to follow a systematic approach.

Symbols	Representation
+	Male
−	Female
→ or ←	Same generation
↑ or ↓	One generation gap
=	Husband/Wife

Also keep in mind about different generation



Now if all above is clear we can draw the main family diagram called as Family Tree.



—	Sibling's Relation
=	Husband – Wife Relation
|	Next generation
(+) Male	(–) Female

By using all of the above, we can solve questions of blood relatives.

SOME TIPS BEFORE WE START

- **Break the question into small parts:** Long statements can be confusing. To avoid confusion, break down each statement into smaller sub-statements and analyse them individually before proceeding to the next part of the question.

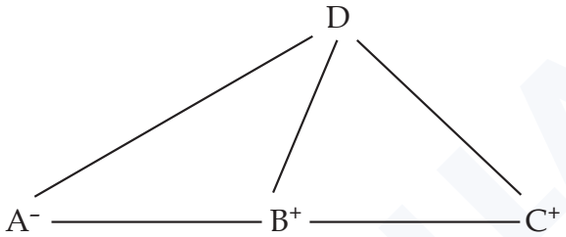
- **Avoid assuming gender based on names:** It's important not to make assumptions about a person's gender solely based on their name. Read the question carefully and gather information about relationships before drawing any conclusions about gender.
- **Draw clear diagrams:** Visual representation can greatly aid in solving blood relation questions. Draw a clear and organized family tree diagram as you solve the question. Place individuals of the same generation at the same level, creating a hierarchical structure. This will help you visualize and understand the relationships more easily.

BASED ON PUZZLES

Example 1. A is the sister of B. B is the brother of C, C is the son of D. How is D related to A?

(a) Son (b) Mother (c) Daughter (d) Uncle

Sol. (b) Presenting the given relations in diagram, we get



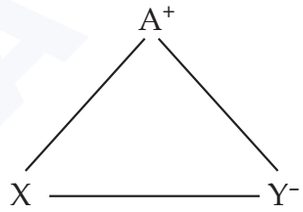
Therefore, D can be mother or father of A.

Hence, according to options, D is mother of A.

Example 2. X and Y are the children of A. A is the father of X but Y is not his son. How is Y related to A?

(a) Son (b) Daughter (c) Sister (d) Brother

Sol. (b) Presenting the given relations in diagram, we get



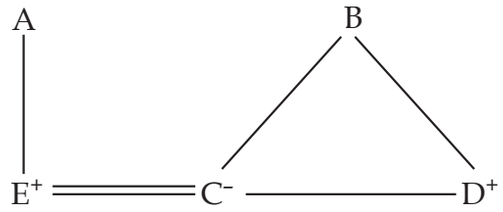
Since, Y is not A's son.

Therefore, Y is daughter of A.

Example 3. E is the son of A. D is the son of B. E is married to C. C is B's daughter. How is D related to E?

(a) Brother (b) Uncle (c) Father-in-law (d) Brother-in-law

Sol. (d) Presenting the given relations in diagram, we get



Therefore, D is brother-in-law of E.

Example 4. Given that

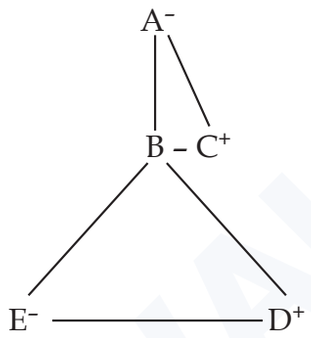
1. A is mother of B.
2. C is son of A.
3. D is brother of E.
4. E is daughter of B.

The grandmother of D is

(a) A (b) B (c) C (d) E

Sol. (a) According to given relation, we have

1. A is mother of B.
2. C is son of A.
3. D is brother of E.
4. E is daughter of B.

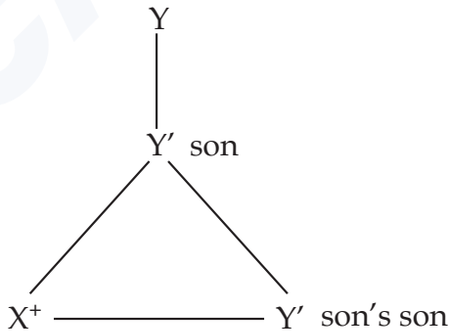


Therefore, A is grandmother of D.

Example 5. If X is brother of son of Y's son, then how is X related to Y?

(a) Brother (b) Cousin (c) Grandson (d) Son

Sol. (c) Presenting the given relations in diagram, we get



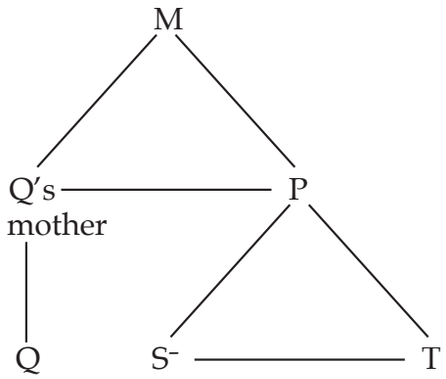
Therefore, X is grandson of Y.

Example 6. Q's mother is sister of P and daughter of M. S is daughter of P and sister of T. How is M related to T?

(a) Father (b) Grandmother

(c) Grandfather or Grandmother (d) Grandfather

Sol. (c) Presenting the given relations in diagram, we get



where, – → female

Therefore, M can be Grandfather or Grandmother of T.

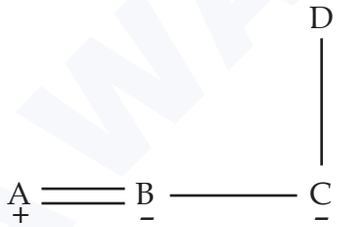
Example 7. If A is the husband of B, B is the sister of C, and C is the daughter of D, then what is the relationship of D to A?

(a) Father-in-law (b) Father (c) Uncle (d) Brother

Sol. (a) According to the given information, we have

Since B is the sister of C and C is the daughter of D, it means that B and C are siblings (sisters) and both are the children of D.

Also, A is husband of B making him son-in-law of D.



Therefore, D is the father-in-law of A.

Hence, the correct option is (a).

Example 8. A, B and C are sisters. D is the brother of E and E is the daughter of B. How is A related to D?

(a) Sister (b) Cousin (c) Niece (d) Aunt

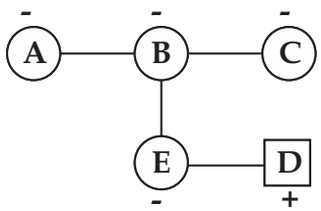
Sol. (d) According to the given information,

E is the daughter of B and D is the brother of E.

Thus, D is the son of B.

Also, A is the sister of B.

Then the relation figure is:



where, the positive sign represents a male and negative sign represents a female.

Therefore, A is the maternal aunt of D.

Hence, the correct option is (d).

Example 9. Given that:

I. A is the mother of B.

II. C is the son of A.

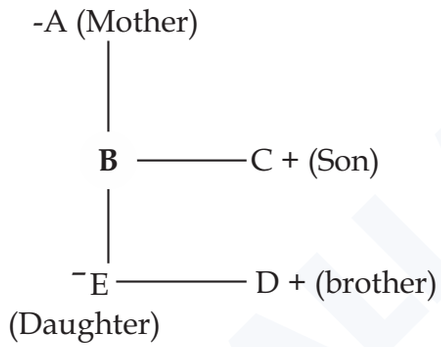
III. D is the brother of E.

IV. E is the daughter of B.

Who is grandmother of D? **(ICAI)**

(a) A (b) B (c) C (d) D

Sol. (a) According to the given relations, we have



Therefore, A is grandmother of D.

Hence, the correct option is (a).

Example 10. If P is the husband of Q and R is the mother of S and Q. What is R to P?

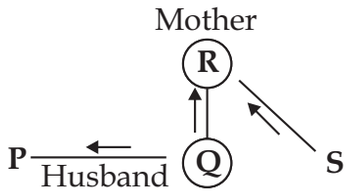
(a) Mother (b) Sister (c) Aunt (d) Mother-in-law **(ICAI)**

Sol. (d) Given,

P is the husband of Q

R is the mother of S and Q

The above situation can be represented as:



where the circle around the alphabet indicates that person is a lady.

Therefore, P is the son-in-law of R or R is the mother-in-law of P.

Hence, the correct answer is option (d) i.e., Mother-in-law.

Example 11. A and B are married couples. X and Y are brothers. X is the brother of A. How is Y related to B?

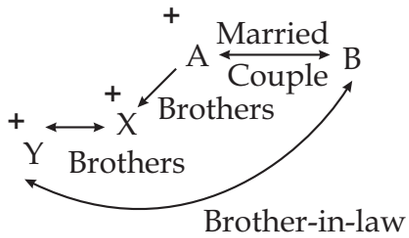
(a) Brother-in-law (b) Brother (c) Cousin (d) None of these

Sol. (a) Given,

A and B are husband and wife, X and Y are the brothers, and X is the brother of A.

So, Y is also the brother of A.

Then the relation figure is:



where, the positive sign represents a male.

Therefore, Y is the brother-in-law of B.

Hence, the correct option is (a).

Example 12. I. F is the brother of A.

II. C is the daughter of A.

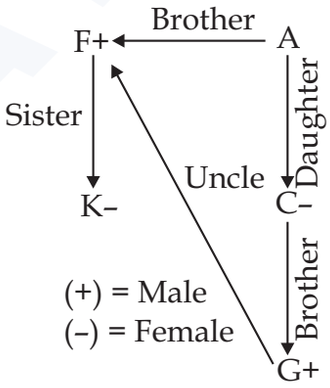
III. K is the sister of F.

IV. G is the brother of C.

Who is the uncle of G? **(ICAI)**

(a) A (b) C (c) F (d) K

Sol. (c) According to the question,



From the diagram, it is clear that F is the uncle of G.

Hence, the correct option is (c).

Example 13. E is the son of A, D is the son of B, E is married to C. C is B's daughter. How is D related to E?

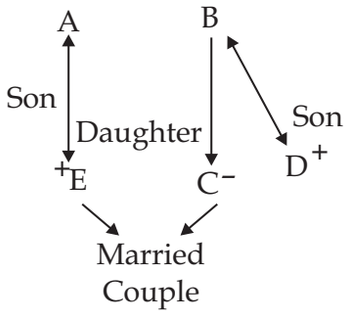
(a) Brother (b) Uncle (c) Father-in-law (d) Brother-in-law

Sol. (d) Given: C is D's daughter and D is B's son.

This means, D is the brother of C.

Since, E is married to C.

Then the relation figure is:



where, positive sign represents the person is a male and negative sign represents person is a female.

Therefore, D is the brother-in-law of E.

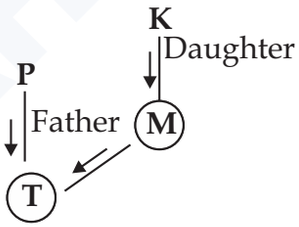
Hence, the correct option is (d).

Example 14. P is the father of T. T is the daughter of M. M is the daughter of K. What is P to K ? **(ICAI)**

(a) Father (b) Father-in-law

(c) Brother (d) Son-in-law

Sol. (d) The given information can be represented as:



where the circle around the alphabet indicates that person is a lady.

Thus, M is the mother of T and wife of P.

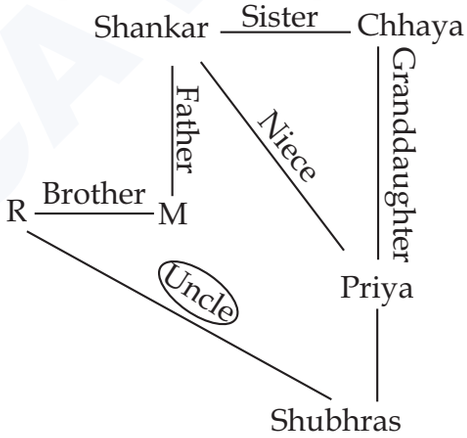
Therefore, P is the son-in-law of K.

Hence, the correct option is (d) i.e., Son-in-law.

Example 15. Ram and Mohan are brothers; Shankar is Mohan's father. Chhaya is Shankar's sister. Priya is Shankar's niece. Shubhra is Chhaya's granddaughter. Then Ram is Shubhra's **(Nov, 2018)**

(a) Brother (b) Uncle (c) Cousin (d) Nephew

Sol. (b) Make flow diagram according to the situations given,



Thus, from the diagram Ram is Shubhra's Uncle.

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART A)

1. A is the father of D. D is the son of E. E is the daughter of F. What is A to F?

(a) Grandson (b) Great Grandson

(c) Son (d) None of these

2. Ravi is the brother of Anu. Sita is the sister of Mohan. Anu is the daughter of Sita. How is Ravi related to Sita?

(a) Nephew (b) Father (c) Son (d) Brother

3. P is Q's brother. Q is R's father. R and S are sisters. How is P related to S?

(a) Son (b) Uncle (c) Father (d) Grandson

4. Aryan is the brother of Bhavana. Deepak is the brother of Chetna. Bhavana is the daughter of Deepak. How is Aryan related to Chetna?

(a) Nephew (b) Father (c) Son (d) Brother

5. C is wife of B. E is the son of C, A is the brother of B and father of D. What is the relationship of E to D?

(a) Mother (b) Sister (c) Brother (d) Cousin

6. If X is the wife of Y and Z is the father of W and Y. What is Z to X?

(a) Father (b) Brother (c) Uncle (d) Father-in-law

7. A boy goes to see a movie and sees a man sitting to his left and finds out that the man is his relative. The man is the husband of the sister of his mother. How is the man related to the boy?

(a) Nephew (b) Brother (c) Uncle (d) Father

Answer Key

1. (d) **2.** (c) **3.** (b) **4.** (a) **5.** (d) **6.** (d) **7.** (c)

BASED ON DIALOGUE OR CONVERSATION

Similarly, few dialogues and in parallel relations to it:

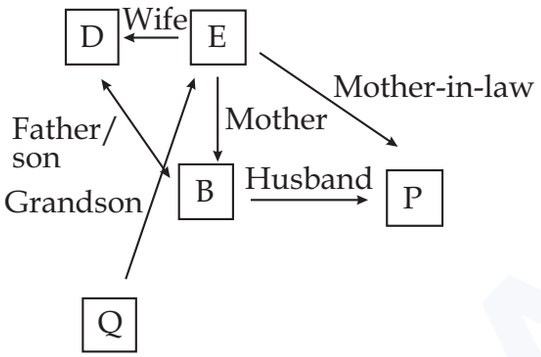
- Only son of my Grandfather: My Father
- Only son of my Grandmother: My Father
- Only daughter of my Grandmother: My Mother
- Only daughter of my Grandfather: My Mother
- Sister of my Mother: My Aunt
- Son /daughter of my Husband: My Son /Daughter
- Son /Daughter of my Wife: My Son /Daughter
- Only Daughter of my Grandfather's only Son: My Sister/Self
- Grandmother of my Father's only Son: My Grandmother
- Father of my Daughter's Father: My Father/Father-in-law
- Father of my Son's Father: My Father/Father-in-law
- My Son's Sister: My Daughter
- Daughter-in-law of Grandmother of my Father's only Son: My Mother
- A is the Father of B but B is not the Son of A: B is Daughter of A

Example 16. B is the husband of P. Q is the only grandson of E, who is the wife of D and mother–in-law of P. How is B related to D?

(a) Nephew (b) Cousin (c) Son –in-law (d) Son

Sol. (d) Given: B is the husband of P and E is the mother-in-law of P.

According to question the relation figure is:



From the diagram, it is clear that B is the son of D.

Hence, the correct option is (d).

Example 17. Maya introduces Rajesh as the son of the only daughter of her mother's brother. How is Maya related to Rajesh?

(a) Cousin (b) Sister (c) Aunt (d) Niece

Sol. (c) In the given scenario, Maya introduces Rajesh as the son of the only daughter of her mother's brother.

i.e., Maya's mother's brother has a daughter, and that daughter is Rajesh's mother. As Maya is the daughter of Rajesh's grandmother, thus Maya is Rajesh's Aunt.

Therefore, Maya is Rajesh's aunt.

Hence, the correct option is (c) i.e., Aunt.

Example 18. Pointing to a photograph, a man said "His mother's husband's sister is my aunt," Then what is the relation between a man and him? **(June 2019)**

(a) Son (b) Uncle (c) Nephew (d) Brother

Sol. (d) According to the question,

His mother's husband's sister implies his paternal aunt.

Now, man (Photograph) and he are brothers.

Hence, the correct answer is option (d).

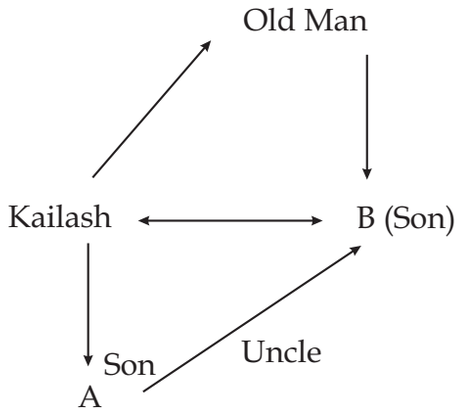
Example 19. Pointing to the old man Kailash said "his son is my son's uncle". How is Kailash related to an old man? **(June 2019)**

(a) Brother (b) Either son or son in law

(c) Father (d) Grand father

Sol. (b) According to the question,

Make a flow diagram along to the scenarios,



We can clearly see; Kailash is either the son or son-in-law.

Hence, the correct answer is option (b).

Example 20. Sanjay's mother said to him, "My brother has a son whose daughter is Tina." How is Tina related to Sanjay?

(a) Cousin (b) Sister (c) Aunt (d) Niece

Sol. (d) In the given scenario, Sanjay's mother states that her brother has a son whose daughter is Tina.

This means that Tina is the daughter of Sanjay's cousin.

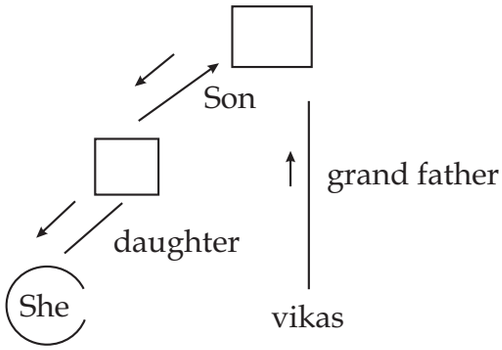
Therefore, Tina is Sanjay's niece.

Hence, the correct option is (d) i.e., Niece.

Example 21. Pointing to a photograph Vikas said "She is the daughter of my grandfather's only son". How is the lady in the photograph related to Vikas in the photograph? **(ICAI)**

(a) Father (b) Brother (c) Sister (d) Mother

Sol. (c) Depicting the given information, we get



Vikas's grandfather's only son is Vikas's father.

So, the daughter of Vikas's father is Vikas's sister.

Thus, the lady is the sister of Vikas.

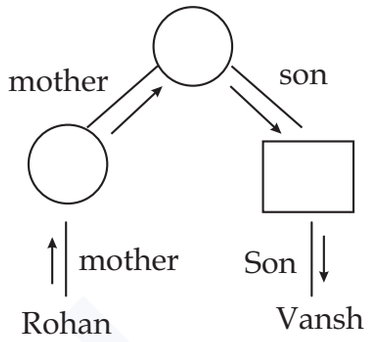
Hence, the correct option is (c) i.e., Sister.

Example 22. Rohan's mother said to Rohan "My mother has a son whose son is Vansh". Rohan is related to Vansh as

(a) Uncle (b) Cousin (c) Nephew (d) Grandfather

Sol. (b) According to the question,

Make a flow diagram according to the conditions given,



Therefore, Rohan is the cousin of Vansh.

Hence, the correct answer is option (b) i.e., Cousin.

Example 23. Kalyani is the mother-in-law of Veena, who is the sister-in-law of Ashok. Dheeraj is the father of Sandeep, the only brother of Ashok.

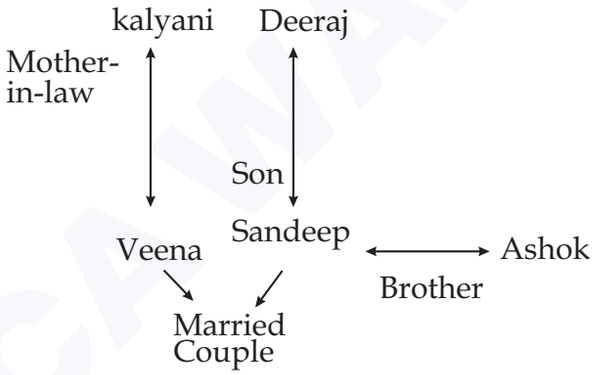
How is Kalyani related to Ashok?

(a) Mother (b) Wife (c) Mother-in-law (d) Aunt

Sol. (a) According to question,

Kalyani is Veena's mother-in-law and Ashok is Veena's brother-in-law.

Thus, Ashok is Kalyani's son.



Therefore, Kalyani is the mother of Ashok.

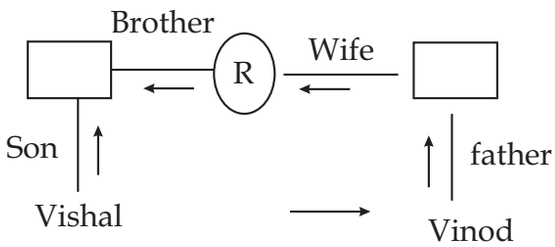
Hence, the correct option is (a).

Example 24. Vinod introduces Vishal as the son of the only brother of his father's wife. How is Vinod related to Vishal? **(May 2018)**

(a) Cousin (b) Brother (c) Son (d) Uncle

Sol. (a) As per the given information,

Wife of Vinod's father is mother of Vinod Only brother of Vinod's mother is maternal uncle of Vinod.



Therefore, Vinod is the cousin of Vishal.

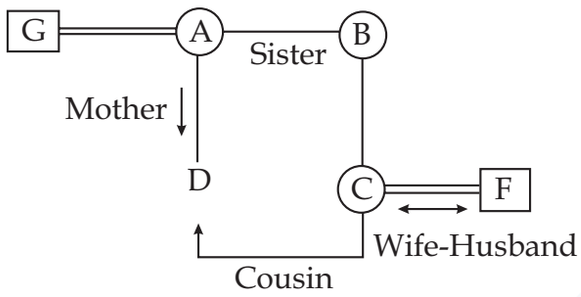
Hence, the correct answer is option (a).

CAWALAH

Example 25. A and B are sisters. A is the mother of D. B has a daughter C who is married to F. G is the husband of A. How is C related to D? **(ICAI)**

(a) Cousin (b) Niece (c) Aunt (d) Sister-in-law

Sol. (a) The given information can be depicted as:



So, B is the aunt of D and the mother of C.
Therefore, C is the cousin of D.
Hence, the correct option is (a) i.e., Cousin.

Example 26. Rahul and Robin are brothers. Promod is Robin's father. Sheela is Pramod's sister. Prema is Promod's niece. Shubha is Sheela's grand-daughter. How is Rahul related to Shubha?

(a) Brother (b) Cousin (c) Uncle (d) Nephew **(ICAI)**

Sol. (c) Depicting the given information, we get

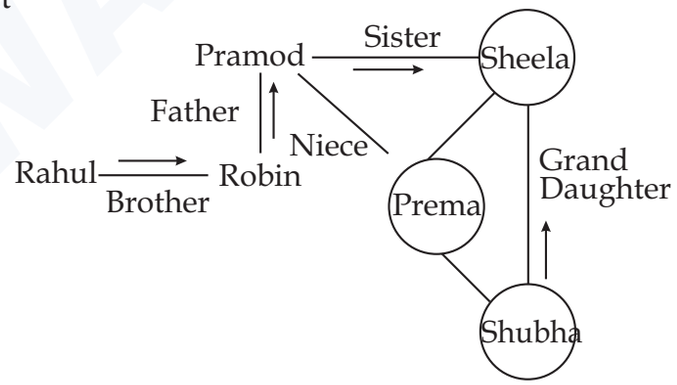
Rahul and Robin are brothers and Pramod is Robin's father which implies that Rahul is also the son of Pramod.

Also, Sheela is Pramod's sister and Prema is Pramod's niece which implies that Prema is the daughter of Sheela.

Since, Shubha is Sheela's grand-daughter.

Therefore, Rahul is the uncle of Shubha.

Hence, the correct option is (c).



Example 27. Suresh introduces a man as "He is the son of the woman who is the mother of the husband of my mother". How is Suresh related to the man? **(ICAI, May 2018)**

(a) Uncle (b) Son (c) Cousin (d) Grandson

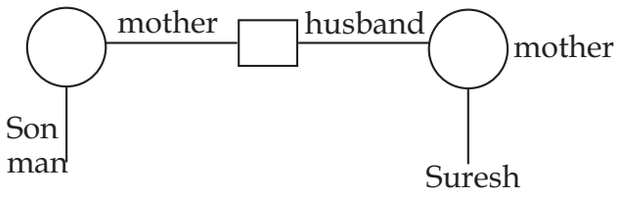
Sol. (b) According to the question,

The husband of my mother is my father (Suresh) and the mother of my father is the mother of the man, so the man is the brother of the father of Suresh.

Thus, the brother of my father (Suresh) is my uncle.

Therefore, Suresh is the son of the man.

Hence, the correct option is (b) i.e., Son



Example 28. Pointing to a lady, Sahil said, "She is the daughter of the women who is the mother of the husband of my mother". Who is the lady to Sahil?

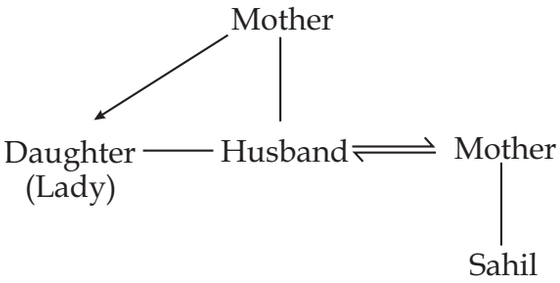
(a) Aunt (b) Sister (c) Daughter (d) Sister-in-Law

Sol. (a) According to the question,

Mother's husband i.e., Father

Mother of the husband of my mother i.e., Grandmother.

Grandmother's daughter i.e., Father's sister or paternal aunt.



Thus, the lady is Sahil's Aunt.

Hence, the correct answer is option (a) i.e., Aunt.

PRACTICE QUESTIONS (PART B)

1. Pointing to a picture, Abhishek said, she is the mother of my son's wife's daughter. How is the lady related to Abhishek?

(a) Uncle (b) Cousin (c) Daughter (d) None

2. Ramu's mother said to Ramu,"My mother has a son whose son is Achyut". How is Achyut relation to Ramu? **(ICAI)**

(a) Uncle (b) Cousin (c) Brother (d) Nephew

3. Sanjay's mother said to him, "My brother has a son whose daughter is Tina." How is Tina related to Sanjay?

(a) Cousin (b) Sister (c) Aunt (d) Niece

4. Pointing to a man, a lady said, ''His mother is the only daughter of my mother''. How is the lady related to the man?

(a) Mother (b) Daughter (c) Sister (d) Aunt

5. A woman said, "The man in the picture is my nephew's maternal grandfather." How is the man in the picture related to the woman?

(a) Brother (b) Son (c) Grandfather (d) Father

6. Looking at the portrait of a man, Ashok said, 'his mother is the wife of my father's son. Brother and sisters I have none'. At whose portrait was Ashok looking?

(a) His cousin (b) His nephew (c) His uncle (d) His son

7. Lalita said to Tina, "You are the daughter-in-law of the grandmother of my father's only son". How is Lalita related to Tina?

(a) Aunt (b) Sister (c) Mother (d) Indeterminable

8. Pointing to a photograph, Sonia said, "His mother's only daughter is my mother". How does Sonia relate to that man? **(June 2019)**

(a) Nephew (b) Sister (c) Wife (d) Niece

9. Pointing to a photograph, Amar said, "I have no brother or sister but that man's father is my father's son." Whose photograph was it?

(a) His son's (b) His father's (c) His nephew's (d) None of these

10. Pointing to a man in a photograph, a woman said, "the father of his brother is the only son of my grandfather", how is the woman related to the man in the photograph?

(a) Mother (b) Aunty (c) Daughter (d) Sister **(Nov, 2018)**

Answer Key

1. (d) **2.** (b) **3.** (d) **4.** (a) **5.** (d) **6.** (d) **7.** (d) **8.** (d) **9.** (a) **10.** (d)

SYMBOLICALLY CODED RELATION

- In symbolically coded relations questions, the relationships between certain members of the family are indicated by symbols like @, £, $, %, &, +, –, etc. The candidate is required to understand the codes/symbols assigned to the relationships and decipher the other relationship expressed to such codes or express another given relationship in the symbolic code form.

Example 29. Read the following information carefully and answer the questions given below:

1. A + B means A is the son of B;

2. A – B means A is the wife of B;

3. A × B means A is the brother of B;

What does P + R - Q mean?

(a) Q is the uncle of P (b) Q is the son of P

(c) Q is the father of P (d) Q is the brother of P

Sol. (c) Here, P + R - Q means,

P is the son of R, who is the wife of Q.

Therefore, Q is the father of P.

Hence, the correct option is (c).

Example 30. Read the following carefully and answer the questions given below:

1. A $ B means A is the father of B;

2. A # B means A is the sister of B;

3. A * B means A is the daughter of B and

4. A @ B means A is the brother of B.

Which of the following indicates that M is the wife of Q ?

(a) Q $ R # T @ M (b) Q $ R @ T # M

(c) Q $ R * T # M (d) Q $ R @ T * M

Sol. (d) Here, Q $ R → Q is the father of R

Also, R @ T→ R is the brother of T

This means, Q is the father of T.

Now, T * M → T is the daughter of M.

Then, M is the mother of T.

Therefore, M is the wife of Q.

Hence, the correct option is (d).

Example 31. Read the following information and answer the following question:

1. A $ B means A is mother of B
2. A # B means A is father of B
3. A @ B means A is husband of B
4. A % B means A is daughter of

Which of the following expressions indicate 'R is the sister of H'?

(a) H $ D @ F # R (b) H % D @ F $ R

(c) R $ D @ F # H (d) R % D @ F $ H

Sol. (d) Here, R % D means R is the daughter of D.

So, D @ F means D is the husband of F.

Then, F $ H means F is the mother of H.

Therefore, D and F have children R and H.

R is the sister of H.

Hence, the correct option is (d).

Answer the below question based on the following information:

1. A + B means A is the mother of B.
2. A - B means A is the sister of B.
3. A * B means A is the father of B.
4. A ? B means A is the brother of B.

Example 32. Which of the following means Q is the grandfather of P?

(a) P + N * M * Q (b) Q * N * M + P

(c) Q ? M ? N * P (d) None of these

Sol. (d) Option (a): P + N * M * Q

Here, P is the mother of N who is father of M and M is father of Q.

Therefore, P is great grandmother of Q.

Option (b): Q * N * M + P

Q is the father of N who is the father of M and M is mother of P.

Therefore, Q is great grandfather of P .

Option (c): Q ? M ? N * P

Here, Q is the brother of M who is the brother of N and N is the father of P.

Therefore, Q is the uncle of P.

Hence, the correct option is (d).

Example 33. If P + Q means P is the mother of Q.

P ÷ Q means P is the father of Q.

P – Q means P is the sister of Q.

Then which of the following relationship shows that M is the daughter of R? **(Nov 2018)**

(a) R ÷ M + N (b) R + N + M (c) R + M ÷ N (d) None of these

Sol. (a) According to the question,

P + Q means P is the mother of Q

P ÷ Q means P is the father of Q

P – Q means P is the sister of Q

According to the options given,

Option (a): R ÷ M + N:

R ÷ M indicates R is the father of M.

M + N indicate M is the mother of N

Thus M is a female making M daughter of R.

From the above scenario, R ÷ M + N shows that M is the daughter of R.

Hence, the correct answer is option (a).

Example 34. Which of the following means that N is the maternal uncle of M?

(a) N ? P – L + E – M (b) N – Y + A ? M

(c) M – Y * P – N (d) N ? C + F * M

Sol. (a) N ? P → N is the brother of P

P - L → P is the sister of L

L + E → L is the mother of E

E - M → E is the sister of M.

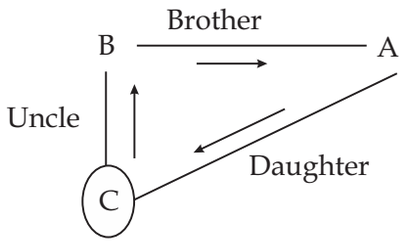
Therefore, L is the mother of M, P is the maternal aunt of M and N is the maternal uncle of M

Hence, the correct option is (a).

Example 35. A reads a book and finds the name of the author familiar. The author 'B' is the paternal uncle of C. C is the daughter of A. How is B related to A? **(May 2018)**

(a) Brother (b) Sister (c) Father (d) Uncle

Sol. (a) According to the question,



B is paternal uncle of C and C is daughter of A

Thus, there are two possibilities:

(I.) If A is father of C then B is brother of A.

(II.) If A is mother of C then B is brother-in-law of A.

Out of the given options, we are given with 'brother' as a option.

The diagram according to the given scenarios,

Thus, B is the brother of A.

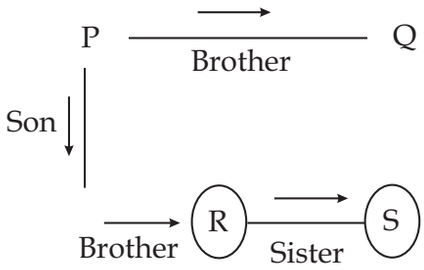
Hence, the correct answer is option (a).

Example 36. P and Q are brother's, R and S are sisters. P's son is R's brother. How is Q related to R? **(May, 2018)**

(a) Uncle (b) Brother (c) Father (d) Grandfather

Sol. (a) According to the question,

Make the flow diagram establishing the relationship,



Thus, Q is the uncle of R.

Hence, the correct answer is option (a).

Example 37. Six persons are seen together in a group. They are A, B, C, D, E and F, B is the brother of D, but D is not brother of B. F is brother of B, C and A are married together, F is son of C, but C is not mother of F, E is brother of A, then the number of female members in the group is **(Nov 2018)**

(a) 1 (b) 2 (c) 3 (d) 4

Sol. (b) According to the question,

B is the brother of D thus B is a male.

D is not a brother of B thus D is a female.

F is the brother of B thus F is a male.

C and A are married together, i.e., one is male and other is female.

F is the son of C, but C is not the mother of F. C is the father of F i.e., C is a male.

E is the brother of A thus E is a male.

i.e., B, F, C, E are males.

Therefore, there are a total of 6 members, 4 are males, 2 are females.

Hence, the correct answer is option (b).

PRACTICE QUESTIONS (PART C)

1. Pointing to a photograph, Sheela said, he is the only son of my grandfather's only son. How is the boy in the photograph related to Sheela?

(a) Brother (b) Cousin (c) Son (d) Cannot be determined

2. Pointing to a girl, Prasan said, she is the only granddaughter of my wife's grandfather's only child. How is the girl related to Prasan?

(a) Sister (b) Niece (c) Wife (d) Cannot be determined

3. P is the sister of Q. R is the father of S, who is the brother of Q. R married to T. How is Q related to T?

(a) Son (b) Daughter

(c) Either son or daughter (d) Data inadequate

4. G is the father of K, who is the brother of B. K is married to U. B is the daughter of C. D is the father of U and M is the only son of D. How is U related to C?

(a) Daughter (b) Daughter-in-law

(c) Grand Daughter (d) Son–in–law

5. K is married to S. P is a brother of B. S is a mother of N, who is a brother of B. How is B related to K?

(a) Son (b) Daughter

(c) Either son or daughter (d) None of these

6. Pointing to a girl, Mala said, 'she is the granddaughter of my grandfather's only son'. How is the girl related to Mala?

(a) Niece (b) Sister (c) Daughter (d) Cannot be determined

7. Pointing to a boy, Ram said, 'He is the only son of my grandfather's son'. How is the boy related to Ram?

(a) Brother (b) Son (c) Cousin (d) Data inadequate

Directions (Q8 – Q9): These are seven members A, C, D, E, F, G and H in a family. There are two fathers, one mother, two sisters and four brothers. E is the sister–in–law of D. G is a daughter of C. F is the brother of E. A is a grandfather of G. E is a mother of H?

8. How many are related to A?

(a) Grandson (b) Granddaughter

(c) Son (d) Cannot be determined

9. How many male members in the family?

(a) 4 (b) 5 (c) 3 (d) Data inadequate

Directions (Q10 – Q12): M is the grandson of G. D is the husband of G. K is married to the son of S. T have two children of different gender. E is the daughter of K's brother N is brother–in–law of the son of S. G has one only one child. D is the father of N.

10. How is N related to G?

(a) Son (b) Son–in–law (c) Grandson (d) Brother

11. If Q is married to N, then how is Q related to M?

(a) Father (b) Mother (c) Sister (d) Grandmother

12. If U is the son of T, then how is U related to N?

(a) Brother (b) Uncle (c) Cousin (d) Brother–in–law

Directions (Q13 – Q15): A family consists of six members P, Q, R, X, Y and Z. Q is the son of R but R is not the mother of Q. P and R are married couple. Y is the brother of R. X is the daughter of P and Z is the brother of P.

13. Who is the brother–in–law of R?

(a) P (b) Z (c) Y (d) X

14. How many female members are there in the family?

(a) One (b) Two (c) Three (d) Four

15. Which of these is a pair of brothers?

(a) P and X (b) P and Z (c) Q and X (d) R and Y

Directions (Q16 – Q17): There are six children taking part in an essay competition, namely A, B, C, D, E, and F. A and E are brothers. F and D are the sisters of E. C is the only son of A's uncle. B is brother of C's father.

16. How is D related to A?

(a) Uncle (b) Sister (c) Niece (d) Cousin

17. How many male competitors are there?

(a) 6 (b) 5 (c) 4 (d) 3

Directions (Q18 – Q20):

1. P × Q means P is the brother of Q.
2. P ÷ Q means Q is the mother of P.
3. P – Q means P is father of Q.
4. P + Q means Q is the sister of P.

18. Which of the following means M is the daughter of T?

(a) M + N ÷ J – T (b) T – J × R + M

(c) M – J × T ÷ K (d) M + W × R ÷ T

19. How is K related to R in the expression R ÷ T + K?

(a) Daughter (b) Sister (c) Niece (d) None of these

20. Which of the following means D is grandfather of W?

(a) D – K × T – W (b) D ÷ K × T ÷ W

(c) D – K × T ÷ W (d) D – K × T ÷ W

21. "Rahul said, 'She is the daughter-in-law of my father's only child.' How is Rahul related to the girl?"

(a) Uncle (b) Father-in-law (c) Brother (d) None of these

22. A woman said, "The man in the picture is my nephew's maternal grandfather." How is the man in the picture related to the woman?

(a) Brother (b) Son (c) Grandfather (d) Father

23. When Rani saw Vinit, she recollected that "He is the brother of my grandfather's son". How is Rani related to Vinit?

(a) Aunt (b) Daughter (c) Sister (d) Niece

24. A prisoner introduced a boy who came to visit him to the jailor as "Brothers and sisters, I have none, he is my father's son's son". Who is the boy ?

(a) Nephew (b) Son (c) Cousin (d) Uncle

25. Suresh introduce a man as "he is the son of the woman who is the mother of the husband of my mother". How is Suresh is related to the man?

(a) Brother-in-law (b) Son (c) Brother (d) Nephew

26. P and Q are brothers. R and S are sisters. P's son is R's brother. How is Q related to R ?

(a) Uncle (b) Brother (c) Father (d) Grandfather

Answer Key

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (a)
11. (b)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (a)
21. (b)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (b)	**26.** (a)				

PREVIOUS YEAR QUESTION (PYQ'S)

1. Pointing to a picture, Sumit said, she is mother of my son's wife's daughter. How is lady related to Sumit? **(June 2019)**

(a) Uncle (b) Cousin (c) Daughter (d) None

2. Pointing to a photograph, a man said "His mother's husband's sister is my aunt". Then what is relation between a man and he? **(June 2019)**

(a) Son (b) Uncle (c) Nephew (d) Brother

3. Pointing to the old man, Kailash said, "his son is my son's uncle." How is Kailesh related to old man? **(June 2019)**

(a) Brother (b) Either son or son-in-law

(c) Father (d) Grand father

4. Pointing in a photograph, Sonia said, "His mother's only daughter is my mother". How is Sonia related to that man? **(June 2019)**

(a) Nephew (b) Sister (c) Wife (d) Niece

5. Read the following information carefully to answer the given questions.

Six members of a family namely A, B, C, D, E and F are travelling together. B is the son of C but C is not the mother of B. A and C are married couples. E is the brother of C. D is the daughter of A. F is the brother of B. **(Dec 2019)**

I. How many male members are there in the family?

(a) 3 (b) 2 (c) 4 (d) 1

II. How many children does A have?

(a) 1 (b) 2 (c) 3 (d) 4

III. What is the relation of E to D?

(a) Uncle (b) Brother (c) Father (d) None of these

IV. Who is the mother of B?

(a) C (b) D (c) F (d) A

6. Vicky introduces John as the son of the only brother of his father's wife. How is Vicky related to John? **(Dec 2020)**

(a) Cousin (b) Son (c) Brother (d) Uncle

7. Point out to a lady, Sohil said she is the daughter of woman. Who is the mother of the husband of my mother. Who is the lady to Sohil? **(Dec 2020)**

(a) Sister (b) Aunt (c) Daughter (d) Sister-in-law

8. Pointing towards a person a man said to a woman, "His mother is the only daughter of your father". How is the woman related to that person? **(Dec 2020)**

(a) Daughter (b) Mother (c) Sister (d) Wife

9. Pointing to a lady in a photograph, Ram said "Her son's father is the son-in-law of my mother". How is Ram related to the lady? **(Jan 2021)**

(a) Aunt (b) Cousin (c) Sister (d) Mother

10. P is the brother of Q and R, S is the mother of R. T is the father of P. Which of the following statements cannot be definitely true? **(Jan 2021)**

(a) S is the mother of P (b) P is son of S

(c) T is husband of S (d) Q is son of T

11. A girl introduced a boy as the son of the daughter of the father of her uncle. The boy is girl's **(Jan 2021)**

(a) Son (b) Brother (c) Son-in-law (d) Uncle

12. Pointing to a lady, Sahil said, "She is the daughter of the women who is the mother of the husband of my mother". Who is the lady to Sahil? **(Jan 2021)**

(a) Aunt (b) Sister (c) Daughter (d) Sister-in-law

13. Pointing towards "A", "B" said; "Your mother is the younger sister of my mother". "A" is related to "B" as **(July 2021)**

(a) Uncle (b) Cousin (c) Nephew (d) Father

14. Shyam's mother said to Shyam, "my mother has a son whose son is Ram". Shayam is related to Ram as **(July 2021)**

(a) Uncle (b) Cousin (c) Nephew (d) Grandfather

15. Amit said, "This girl is the wife of the grandson of my mother". How does Amit relate to the girl? **(July 2021)**

(a) Father-in-law (b) Grandson (c) Father (d) Son

16. A is the son of C; C and Q are sisters; Z is the mother of Q and P is the son of Z. Which of the following statements is true? **(July 2021)**

(a) A and P are cousins (b) C and P are sisters

(c) P is the maternal uncle of A (d) A is the maternal uncle of P

17. D is the daughter of E. A is the son of D. C is brother of A and B is sister of A. F is brother of D. How is F related to B? **(Dec 2021)**

(a) Father-in-law (b) Uncle (c) Brother (d) Mother-in-law

18. Introducing a boy, a girl said, "He is the son of the daughter of the father of my uncle." Who is the boy to the girl? **(Dec 2021)**

(a) Brother (b) Nephew (c) Uncle (d) Son-in-law

19. It is given that, "A is the mother of B; B is the sister of C; C is the father of D". How is A related to D? **(Dec 2021)**

(a) Mother (b) Grandmother (c) Aunt (d) Sister

20. R told to M as, "the girl I met at the beach, was the youngest daughter of the brother-in-law of my friend's mother". How is the girl related to R's friend? **(Dec 2021)**

(a) Cousin (b) Daughter (c) Niece (d) Aunt

21. P, Q, R, S, T, U are 6 members of a family in which there are two married couples. T, a teacher is married to a doctor who is mother of R and U, Q the lawyer is married to P. P has one son and one grandson. Of the two married ladies, one is a housewife. There is also one student and one male engineer in the family. Which of the following is true about the granddaughter of the family? **(Dec 2021)**

(a) She is a lawyer (b) She is an engineer

(c) She is a student (d) She is a doctor

22. A woman going with a boy is asked by another woman about the relationship between them. The woman replied, "My maternal uncle and the uncle of his maternal uncle is the same". How is the lady with that boy?

(a) Grandmother and grandson (b) Mother and son

(c) Brother and sister (d) Aunt and nephew

23. If X+ Y means X is the mother of Y; X- Y means X is the brother y; X % Y means X is the father of Y and X x Y means X is the sister of Y, Which of the following shows that A is the maternal uncle of B? **(June 2022)**

(a) B + D x C -A (b) B - D % A (c) A - C + D x B (d) A + C x D - B

24. B and C are siblings. M has two children, and he is son of E, who is father-in-law of H. H has only one son. C is not granddaughter of E. How's B related to E? **(June 2022)**

(a) Daughter (b) Son (c) Granddaughter (d) Grandson

25. Ravi is a son of Aman's father's sister. Sahil is the son of Divya who is the mother of Gaurav and grandmother of Aman. Ashok is the father of Tanya and grandfather of Ravi Divya is the wife of Ashok.

How is Ravi related to Divya? **(June 2022)**

(a) Nephew (b) Son (c) Grandson (d) Father

26. Rani told Jaya, "The girl I met yesterday at the beach was the youngest daughter of the brother-in-law of my friend's mother." How is the girl related to Rani's friend?

(June 2022)

(a) Cousin (b) Daughter (c) Mother (d) Aunt

27. Suresh's sister is the wife of Ram, Ram is Rani's brother. Ram's father is Madhur, Sheetal is Ram's grandmother, Rema is sheetal's daughter –in-law. Rohit is Rani's brother's son. Who is Rohit to Suresh? **(Dec 2022)**

(a) Brother-in-law (b) Son

(c) Brother (d) Nephew

28. There are six children playing football namely A,B,C,D,E and F, A & E are brothers, F is sister of E, C is the only son of A's uncle, B & D are daughter of the brother of C's father. How D is related to A? **(Dec 2022)**

(a) Uncle (b) Cousin (c) Niece (d) Sister

29. In a joint family, there are father, mother, 3 married sons and one unmarried daughter. Out of the sons, two have 2 daughters each and one has a son only. How many female members are there in the family? **(Dec 2022)**

(a) 3 (b) 6 (c) 9 (d) 8

30. When Rani saw Vinit, she recollected that "He is the brother of my grandfather's son". How is Rani related to Vinit? **(Dec 2022)**

(a) Aunt (b) Daughter (c) Sister (d) Niece

31. Annanya is mother of Satya and Shyam is the son of Bhima, Shiva is brother of Annanya. If Satya is sister of Shyam, How Bhima is related to Shiva? **(Dec 2022)**

(a) Son (b) Cousin

(c) Brother-in-law (d) Son-in-law

32. Suman is daughter-in-law of Rakesh and sister-in-law of Rajesh, Ramesh is the son of Rakesh and only brother of Rajesh. Find the relation of Suman with Ramesh.

(Dec 2022)

(a) Sister-in-law (b) Cousin (c) Aunt (d) Wife

33. Pointing to a man in the photograph, Khushi says, "This man's son's sister is my mother –in-law," How is the Khushi's husband related to the man in the photograph?

(Dec 2022)

(a) Grandson (b) Son (c) Son in law (d) Cousin

34. P, Q, R, S, T and U are 6 members of a family in which there are two married couples. T, a teacher is married to a doctor, who is mother of R and U. Q, the lawyer is married to P. P has one son and one grandson. Of the two married ladies one is a housewife. There is also one student and one male engineer in the family. Which of the following is true about the grandson of the family? **(June 2023)**

(a) He is a lawyer (b) He is an engineer

(c) He is a student (d) He is a doctor

35. Neelam, who is Deepak's dauhter, says to Deepika "your mother-in-law Rekha is younger of Ramlal, who is my grandfather." How Neelam is related to Deepika? **(June 2023)**

(a) Sister (b) Niece (c) Sister-in-law (d) Aunt

36. Pointing to a photograph, a woman says "This man's son's sister is my mother-in-law". How the woman's husband related to the man in the photograph? **(June 2023)**

(a) Grandson (b) Nephew (c) Son (d) Son-in-law

37. If A is the brother of B, B is the daughter of C and D is the father of A, then how is C related to D? **(June 2023)**

(a) Husband (b) Wife (c) Granddaughter (d) Grandfather

38. Based on the statements given below; find out who is the uncle of P?

(a) K and J are brothers (b) K's sister is M

(c) P and N are siblings (d)N is the daughter of J **(June 2023)**

(a) K (b) J (c) N (d) M

39. There are six persons A, B, C, D, E and F in a family. A and B are a married couple and A is a male member. D is the only son of C, who is the brother of A, E is the sister of D. B is the daughter-in-law of F whose husband has died. Who is the mother of C? **(June 2023)**

(a) A (b) E (c) D (d) F

40. Pointing to a lady, Suresh said"She is the mother of my son's wife's daughter". How is Suresh related to lady? **(Dec 2023)**

(a) Uncle (b) Cousin

(c) Daughter-in-law (d) Father-in-law

41. When Mr. P saw Mr. Q he recalled, " He is the son of the father of my daughter's mother." Who is Mr. Q to Mr. P? **(Dec 2023)**

(a) Brother (b) Cousin (c) Nephew (d) Brother-in-law

42. Showing the man playing the cricket, Ms. P said, " He is the brother of my uncle's daughter." Who is the man to Ms. P? **(Dec 2023)**

(a) Son (b) Cousin (c) Uncle (d) Brother-in-law

43. In a family, there are six members A,B,C,D,E and F. A & B are married couples. A being the male member. D is only son of C and is brother of A. E is sister of D. B is daughter-in-law of F, whose husband has died. How is C related to B? **(Dec 2023)**

(a) Brother (b) Nephew (c) Brother-in-law (d) Sister-in-law

44. Introducing a boy , Akshat said, "His mother is the only daughter of my mother-in-law." How is Akshat related to the boy? **(Dec 2023)**

(a) Uncle (b) Father (c) Brother (d) Husband

45. Pointing to a lady, a man said, "The son of her only brother is the brother of my wife ". How is lady is related to man ?

(a) Mother-in-law (b) Sister of father-in-law

(c) Mother of Father-in-low (d) Cousin

46. There are six persons A, B, C, D, E and F in a family. B is the son of C but C is not the mother of B, A and C are a married couple. E is the brother of C. D is the daughter of A. F is the brother of B. Who is the mother of B? **(Dec 2023)**

(a) A (b) E (c) D (d) F

47. A is B's sister, C is B's mother, D is C's father, E is D's mother. How is A related to D? **(June 2024)**

(a) Grandmother (b) Grandfather (c) Daughter (d) Grand Daughter

48. Pointing to a photograph, Ms. X says, "This man's son's sister is my sister-in-law." How is Ms. X's husband related to the man in the photograph? **(June 2024)**

(a) Son (b) Grandson (c) Brother (d) Nephew

49. X, a male, introduces Y saying, "He is the husband of the granddaughter of the father of my father". How is Y related to X? **(June 2024)**

(a) Brother (b) Father (c) Brother-in-law (d) Father-in-law

50. Consider the relations: Raja is husband of Devi; Karan is the father of Gopal; Arjun is the son of Shoba; Ashok is brother of Shoba; Arjun's father Karan is the son-in-law of Devi. How is Raja related to Ashok? **(June 2024)**

(a) Father (b) Uncle (c) Son-in-law (d) Grandfather

51. A is B's Father. C is B's mother. D is C's Son. E is A's mother. Then how is A related to D? **(Sept 2024)**

(a) Uncle (b) Grandson (c) Grand Daughter (d) Father

52. Pointing to a man in photograph, a woman said "The father of his brother is only son of my grandfather", then how is that woman related to the man in photograph? **(Sept 2024)**

(a) Daughter (b) Sister (c) Mother (d) Aunty

53. A family consists of six members P, Q, R, S, T & U. There are two married couples. Q is a doctor and father of T; U is grandfather of R and is a contractor; S is grandmother of T and is a house-wife. There is one doctor, one contractor, one Professor, one house-wife and two students in the family. Find who is the husband of P. **(Sept 2024)**

(a) T (b) S (c) R (d) Q

54. P is the son of Q while Q & R are sisters to one another. T is the mother of R. If S is son of T, how S is related to P? **(Sept 2024)**

(a) Brother (b) Cousin (c) Maternal uncle (d) Nephew

55. Sandhya is the daughter-in-law of Shailesh and sister-in-law of Rajan. Manak is son of Shailesh and only brother of Rajan. Then, how Sandhya is related to Manak? **(Sept 2024)**

(a) Sister-in-law (b) Aunty (c) Cousin (d) Wife

56. L is wife of N, P is son of N, K is brother of N and father of O. What is the relationship of P and O? **(Jan 2025)**

(a) Brother (b) Uncle (c) Cousin (d) Nephew

57. C is sister of B, D is father of A. A is brother of B. D and E are married couple. How is C related to E? **(Jan 2025)**

(a) Son (b) Daughter (c) Mother (d) Father

58. B is daughter of A. C is brother of B. C is the only son of D. C and E are married couple. F is the only son of E. Then how is F related to A? **(Jan 2025)**

(a) Father (b) Grandson (c) Brother (d) Uncle

59. Given that X is mother of Y. Z is son of X. A is brother of B. B is daughter of Y. Who is grandmother of A? **(Jan 2025)**

(a) Y (b) X (c) A (d) B

60. Pointing to a photohraph, Rajesh said, "He is Aarav and he is the son of the only daughter of the father of my brother", how is Rajesh related to the Aarav referred in the photograph? **(May 2025)**

(a) Nephew (b) Brother (c) Father (d) Maternal uncle

61. Study the following information carefully: **(May 2025)**

- A3P means A is the mother of P.
- A4P means A is the brother of P.
- A9P means A is the husband of P.
- A5P means A is the daughter of P.

Which of the following means that K is the mother-in-law of M?

(a) M9N3K4J (b) M9N5K3J (c) K5J9M3N (d) K3J9N4M

62. Q, W, E, R, T and Y are members of a family consisting of two children, one of whom is T is a boy. Q and R are brothers and Q is an engineer. E is a doctor married to one of the brothers. W is married to R, and Y is their only child. How T is related to Q? **(May 2025)**

(a) Father (b) Brother (c) Nephew (d) Son

63. K is the son of A's mother's sister. Q is daughter of D, who is the father of G and grandfather of A. P is the daughter of H, who is grandmother of K. D is husband of H and G is husband of L. How is P related to Q? **(May 2025)**

(a) Mother (b) Sister (c) Daughter (d) Cousin

64. If "A # B" means A is father of B, "A * B" means A is brother of B, "A @ B" means A is mother of B, then which of the following is correct about G @ T # P? **(May 2025)**

(a) G is mother of P (b) P is father of T

(c) T is son of G (d) P is brother of T

65. Ms. X told Ms. Y, "The girl I met yesterday at the market was the youngest daughter of the brother-in-law of my friend's mother." How is the girl related to Ms. X's friend?

(Sept 2025)

(a) Cousin (b) Daughter (c) Niece (d) Aunt

66. There are six members, named as, A, B, C, D, E and F. It is given that E is the brother of A's husband; F is the mother of E; B is the daughter of D and A and is the granddaughter of C. How is C related to E? **(Sept 2025)**

(a) Son (b) Father (c) Brother (d) Grandfather

67. A girl introduced a boy as the son of daughter of the father of her uncle. The boy is girl's

(Sept 2025)

(a) Father (b) Son (c) Uncle (d) Cousin/Brother

68. A and B are Brothers. C and D are Sisters. A's son is D's Brother. How is B related to C?

(Sept 2025)

(a) Father (b) Brother (c) Grandfather (d) Uncle

69. It is given that a person named as A is married to B; the person E is a brother of D and the person B is the mother of C whose siter is D. How is D related to A? **(Sept 2025)**

(a) Sister (b) Aunt (c) Daughter (d) Granddaughter

Answer Key

1. (d)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (i) (c), (ii) (c), (ii) (a), (iv) (d)					**6.** (a)	**7.** (b)	
8. (b)	**9.** (c)	**10.** (d)	**11.** (b)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (b)		
18. (a)	**19.** (b)	**20.** (a)	**21.** (c)	**22.** (b)	**23.** (c)	**24.** (c)	**25.** (c)	**26.** (a)	**27.** (d)		
28. (d)	**29.** (c)	**30.** (d)	**31.** (c)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (c)	**36.** (a)	**37.** (b)		
38. (a)	**39.** (d)	**40.** (d)	**41.** (d)	**42.** (b)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (d)		
48. (a)	**49.** (c)	**50.** (a)	**51.** (d)	**52.** (b)	**53.** (d)	**54.** (c)	**55.** (d)	**56.** (c)	**57.** (b)		
58. (b)	**59.** (b)	**60.** (d)	**61.** (b)	**62.** (d)	**63.** (b)	**64.** (c)	**65.** (a)	**66.** (b)	**67.** (d)		
68. (d)	**69.** (c)										

SUMMARY

- Any relation in the world which either by birth or by marriage is called a Blood Relation.
- Different ways in which the blood relation is described :
  - Dialogue/ Conversation Based - One person describes his/her relation with another person (this may or may not be related to the person with whom the conversation is being made).

- Based on Puzzles - To make the questions complex, blood relation questions are also being asked in the form of a puzzle. A piece of brief information about multiple people being interrelated is given and sub-questions based on the same may be asked.
- Symbolically coded relations: the relationships between certain members of the family are indicated by symbols like £, $. %, &. +. - etc.

- Family tree is the best way to represent blood relations and related answers to them. The following points help in drawing a family tree.
- All the family members of the upper generation are represented above in the family tree. **E.g.:** father, mother, uncle, aunt etc. The logic can be extended by representing the grandparents above the parents in the family tree. **E.g.:** grandfather and grandmother.
- Similarly the relation between two members of the family is shown by connecting a double-headed arrow.
- All the family members of the same generation are represented in the middle of the family tree. **E.g.:** brothers, Sisters, cousins, wife, husband, etc.
- All the family members of the next generation are represented below in the family tree. **E.g.:** Daughter, son, niece, nephew.

❑❑❑

CA WALLAH

PART

C

Statistics

CHAPTER

13

Statistical Description of Data and Sampling

Unit-1: Statistical Description of Data

WHAT ARE STATISTICS AND WHY WE NEED TO STUDY IT?

Statistics refers to the collection, analysis, interpretation, presentation, and organization of data. It involves using mathematical techniques to draw meaningful conclusions and make informed decisions based on data. The study of statistics is important because it provides us with the tools and methods to analyze and understand data, which is crucial in various fields.

Where have we heard this word Statistics?

We often encounter the word "statistics" in different contexts. **E.g.:**, we may hear it when referring to the weather report, which is based on statistical analysis of historical weather data to predict future conditions. Similarly, when someone mentions that the Indian population will be the highest by 2025, this statement is likely based on statistical projections and demographic data analysis. Furthermore, when discussing business goals, such as the next year's target for a company, statistics can be used to analyze past performance and set realistic objectives.

HISTORY OF STATISTICS

Book	Author
Artha shastra	Kautilya
Ain I Akbari	Abu Fazl

"Written in 300 BC, Kautily's Arthashastra is the oldest and one of the most exhaustive treatises on governance, administration and economics."

Ain-i-Akbari is a 16th century document.

- **Written by:** Akbar's court historian Abu'l Fazl in Persian language.
- **Deals with:** the administration of Mughal Emperor Akbar.
- **Background:** Beginning in 1589, Abu'l Fazl

Language	Word
Latin	Status
Italian	Statista
French	Statistique
German	Statistik

There are two main types of statistics:

a) **Descriptive Statistics:** This involves summarizing and describing the features of a dataset. [Ex; Mean, Median, Mode, SD]

b) **Inferential Statistics:** This involves using a sample of data to make predictions or inferences about a larger population.

MEANING OF STATISTICS

- When defined in Plural sense: Statistics refer to the collection of qualitative and quantitative data in a way that allows us to draw meaningful conclusions from it.
- When defined in Singular sense: Statistics represent a scientific method used to collect, organize, and analyze data, ultimately leading to drawing statistical inferences about important characteristics. It can be considered the "science of counting" or the "science of averages."

APPLICATION OF STATISTICS

Statistics is applicable in every field where understanding and analysis of data are crucial. In the context of commerce, economics, business management, and industry, statistics provides the means to analyze economic data, make informed decisions, develop theories, conduct surveys, project future outcomes, and maximize profit. It offers various tools and techniques for data analysis, allowing businesses to gain a competitive advantage and make effective strategies based on statistical insights.

ECONOMICS

1. **Many branches of economics,** such as time series analysis, index number analysis, and demand analysis, are essentially branches of statistics. These branches use statistical methods to analyze economic data and draw meaningful conclusions.
2. **Econometrics** is another important application of statistics in economics. It involves the quantitative application of statistical and mathematical models using data to develop theories or test existing hypotheses in economics.
3. **Socio-economic surveys** play a crucial role in understanding various socio-economic factors. Statistical methods are used to conduct these surveys and analyze the derived data.
4. Regression analysis is used in economics to project future outcomes, such as sales, production, and prices, based on statistical analysis of past data.

BUSINESS MANAGEMENT

1. **How do managers work today?** In modern business management, decisions are no longer based solely on instincts. Managers now rely on quantitative data analysis, combined with statistical methods and operational research techniques, to make calculative and informed decisions.
2. **Sampling helps us to create criteria for making strategy**. In complex business environments like e-commerce, statistics plays a vital role in formulating strategies at every point by analyzing relevant data.
3. **Statistical decision theory** is employed to analyze complex strategies and assist businesses in making fruitful decisions by considering the merits and demerits of different options.

STATISTICS IN COMMERCE AND INDUSTRY:

1. **Getting an Edge in a competitive business environment.** In the competitive business environment, statistics can provide a significant edge. Businesses collect, compare, and analyze data on sales, production, profit, and other relevant factors to gain insights and formulate effective strategies.
2. **To Maximize Profit:** Maximizing profit is a common goal in commerce and industry. Statistical analysis is used to examine data on previous sales, wages, raw materials, and competitor products. By comparing and analyzing this data, businesses can identify opportunities to maximize profit.
3. **Various tools used in Commerce and Industry of Statistics,** including measures of central tendency (such as mean, median, and mode), measures of dispersion (such as range and standard deviation), sampling techniques, correlation and regression analysis, and index number and time series analysis. These tools help in analyzing data, identifying patterns, and making informed decisions.

LIMITATIONS OF STATISTICS

1. **Study of Quantitative data only:** Statistics studies only such facts as can be expressed in numerical terms. It does not study qualitative phenomena like honesty, friendship, wisdom, health, patriotism, justice, etc.
2. **Study of Aggregates only:** Statistics studies only the aggregates of quantitative facts. It does not study statistical facts relating to any particular. **E.g.:** It may be a statistical fact that your class teacher earns 50,000 per month. But if you want to find statistics of average salary of different schools, you will study aggregates of salary of each school
3. **Homogeneity of Data, an essential Requirement:** To compare data, it is essential that statistics are uniform in quality. Data of diverse qualities and kinds cannot be compared. **E.g.:** we cannot compare the statistics of banana and air pressure in tyre
4. **Results are true only on an Average:** Most statistical findings are true only as averages. They are not always valid under all conditions. For instance, if it is said that per capita income in India is 50,000 per annum, it does not mean that the income of each and every Indian is 50,000 per annum. Some may have more and some may have less.
5. **Results may prove to be wrong:** In order to understand the conclusions precisely, it is necessary that the circumstances and conditions under which these conclusions have been drawn are also studied. Otherwise, they may prove to be wrong.
6. **Can be used only by the Experts:** Statistics can be used only by those persons who have special knowledge of statistical methods. Those who are ignorant about these methods cannot make sensible use of statistics. In the words of Yule and Kendall, "Statistical methods are the most dangerous tools in the hands of an inexpert.

COLLECTION OF DATA

On the nature of data, we can define data as follows:

- **Quantitative Data:** This refers to data that represents numeric values of quantitative data include measurements, counts, and numerical ratings.

- **Qualitative Data:** This refers to data that represents characteristics or qualities of qualitative data include descriptive information, opinions, and categorical variables.

  If we want to analyze qualitative information using statistics, it needs to be converted into quantitative information by assigning a numeric description to the given characteristic. Quantitative data is collected in the form of variables.

WHAT IS VARIABLE?

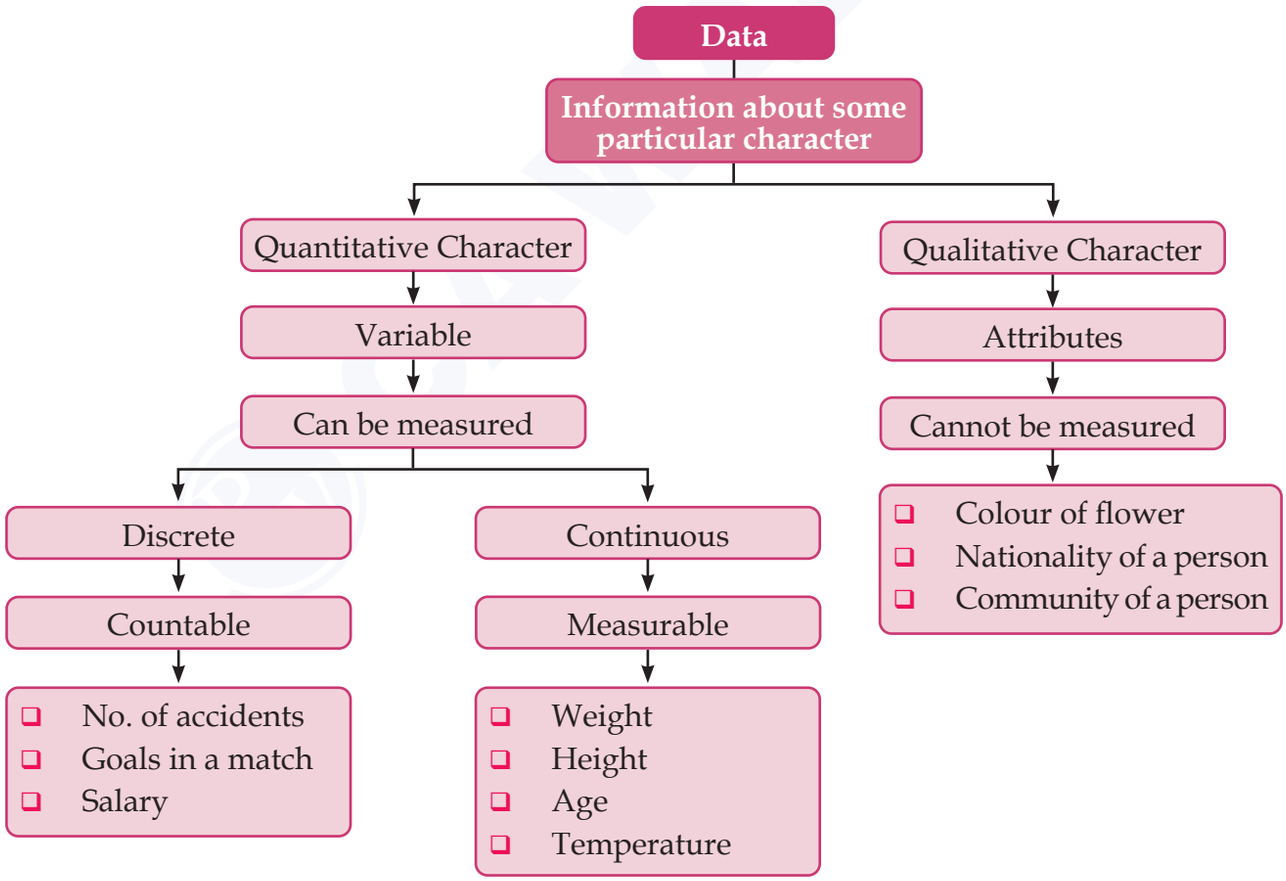
A variable is a characteristic or attribute that can take on different values. It is the unit of measurement for collecting data.

TYPES OF VARIABLES

1. Discrete Variable
2. Continuous Variable

- **Discrete Variable:** A discrete variable is one that can only take on specific, countable values. **E.g.:** the number of children in a family, the number of students in a class, or the number of cars in a parking lot.
- **Continuous Variable:** A continuous variable is one that can take on any value within a given interval or range. It can have decimal values and is not limited to specific points.

  E.g.: height, weight, temperature, or time.



Example 1. What is the title of the book written by Kautilya where statistical records are mentioned?

(a) Ain-i-Akbari (b) Artha shastra (c) Mahabharata (d) Rigveda

Sol. (b) Artha Shastra is the title of the book written by Kautilya where statistical records are mentioned.

Example 2. Which of the following statements is false?

(a) Statistics is derived from the Latin word 'Status'

(b) Statistics is derived from the Italian word 'Statista'

(c) Statistics is derived from the French word 'Statistik'

(d) None of these

Sol. (c) We know,

Status	Latin
Statista	Italian
Statistik	German
Statstique	French

Therefore, Statistics is derived from the Italian word 'Statista' is a true statement.

Example 3. How can "statistics" be defined in the plural sense?

(a) As data that is qualitative or quantitative, collected for statistical analysis.

(b) As the science of averages and counting.

(c) As a scientific method for presenting data.

(d) As a theoretical approach to statistical inference.

Sol. (a) We know, In the Plural sense

Statistics refer to the collection of qualitative and quantitative data in a way that allows us to draw meaningful conclusions from it.

Hence, option (a) is correct.

Example 4. Gender of a person is:

(a) Attribute (b) Discrete variable

(c) Continuous variable (d) Variable

Sol. (a) We know that,

A qualitative characteristic is known as an attribute.

Therefore, the gender of a person is an example of an attribute.

Example 5. Number of books on a shelf is:

(a) Attribute (b) Discrete variable

(c) Continuous variable (d) Variable

Sol. (b) We know that,

When a variable assumes a finite or a countably infinite number of isolated values, it is known as a discrete variable.

Thus, Number of books on a shelf is a discrete variable.

Example 6. The data collected on the height of a group of students after recording their heights with a measuring tape are

(a) Primary date (b) Secondary data

(c) Discrete data (d) Continuous data

Sol. (a) The data collected on the height of a group of students after recording their heights with a measuring tape are Primary data.

Hence, option (a) is the correct answer.

Example 7. Profits made by XYZ Bank which is a blue chip company in different years refer to:

(a) An attribute (b) A discrete variable

(c) A continuous variable (d) None of these

Sol. (c) We know, when a variable can assume any value from a given interval, it is known as a continuous variable.

Example 8. The marital status of employees in a company (Single, married, Divorced) is:

(a) An attribute (b) A discrete variable

(c) A continuous variable (d) None of these

Sol. (a) We know that,

A qualitative characteristic is known as an attribute.

Therefore, the marital status of employees in a company is an example of an attribute.

Example 9. Statistics is defined in terms of numerical data in the **(ICAI)**

(a) Singular sense (b) Plural sense

(c) Either (a) or (b) (d) Both (a) and (b)

Sol. (c) The definition of statistics can be expressed in both the singular and plural sense. In the singular sense, statistics refers to the discipline or field of study that deals with the collection, analysis, interpretation, presentation, and organization of data. In the plural sense, statistics refers to the numerical data or facts that are collected and analyzed for various purposes.

Therefore, the definition of statistics can be understood in either the singular sense or the plural sense.

Hence, the correct option is (c).

Example 10. Statistics is concerned with **(ICAI)**

(a) Qualitative information (b) Quantitative information

(c) (a) or (b) (d) Both (a) and (b)

Sol. (d) Statistics is a field that deals with the collection, analysis, interpretation, presentation, and organization of data. It is concerned with both qualitative and quantitative information.

Qualitative information refers to non–numerical data that describes qualities or attributes, such as gender, occupation, or type of vehicle.

Quantitative information, on the other hand, involves numerical data that can be measured or counted, such as age, height, income, or the number of items sold.

Statistics provides methods and techniques to analyze and summarize both qualitative and quantitative data, allowing for meaningful interpretations and informed decision–making.

Hence, the correct option is (d).

Example 11. Annual income of a person is **(ICAI)**

(a) An attribute (b) A discrete variable

(c) A continuous variable (d) (b) or (c)

Sol. (b) We know that,

When a variable assumes a finite or a countably infinite number of isolated values, it is known as a discrete variable.

Therefore, Annual income of a person is a discrete variable.

Hence, the correct option is (b).

Example 12. Marks of a student is an example of **(ICAI)**

(a) An attribute (b) A discrete variable

(c) A continuous variable (d) None of these

Sol. (b) A discrete variable is one that can take on specific, separate values with no intermediate values between them. In the case of marks, they are typically represented as whole numbers or specific grades, such as 85, 90, or A, B, C, etc. Each mark represents a distinct category or value, and there are no fractional or continuous values between them.

Therefore, marks of a student can be considered as a discrete variable.

Hence, the correct option is (b).

Example 13. Drinking habit of a person is **(ICAI)**

(a) An attribute (b) A variable

(c) A discrete variable (d) A continuous variable

Sol. (a) Attributes refer to qualities or characteristics that describe individuals or objects. In this case, the drinking habit is a qualitative characteristic that describes a person's behavior or preference. It does not have numerical value or magnitude associated with it, making it an attribute rather than a variable.

Therefore, the drinking habit of a person can be categorized as an attribute.

Hence, the correct option is (a).

Example 14. Statistics is defined in terms of numerical data in the **(ICAI)**

(a) Singular sense (b) Plural sense

(c) Either (a) or (b) (d) Both (a) and (b)

Sol. (c) The definition of statistics can be expressed in both the singular and plural sense.. In the singular sense, statistics refers to the discipline or field of study that deals with the collection, analysis, interpretation, presentation, and organization of data. In the plural sense, statistics refers to the numerical data or facts that are collected and analyzed for various purposes.

Therefore, the definition of statistics can be understood in either the singular sense or the plural sense.

Hence, the correct option is (c).

Example 15. Statistics is concerned with **(ICAI)**

(a) Qualitative information (b)Quantitative information

(c) (a) or (b) (d) Both (a) and (b)

Sol. (d) Statistics is a field that deals with the collection, analysis, interpretation, presentation, and organization of data. It is concerned with both qualitative and quantitative information.

Qualitative information refers to non-numerical data that describes qualities or attributes, such as gender, occupation, or type of vehicle.

Quantitative information, on the other hand, involves numerical data that can be measured or counted, such as age, height, income, or the number of items sold.

Statistics provides methods and techniques to analyze and summarize both qualitative and quantitative data, allowing for meaningful interpretations and informed decision-making. Hence, the correct option is (d).

Example 16. Annual income of a person is **(ICA)**

(a) An attribute
(b) A discrete variable
(c) A continuous variable
(d) (b) or (c)

Sol. (b) We know that,

When a variable assumes a finite or a countably infinite number of isolated values, it is known as a discrete variable.

Therefore, Annual income of a person is a discrete variable.

Hence, the correct option is (b).

Example 17. Marks of a student is an example of **(ICAI)**

(a) An attribute
(b) A discrete variable
(c) A continuous variable
(d) None of these

Sol. (b) A discrete variable is one that can take on specific, separate values with no intermediate values between them. In the case of marks, they are typically represented as whole numbers or specific grades, such as 85, 90, or A, B, C, etc. Each mark represents a distinct category or value, and there are no fractional or continuous values between them.

Therefore, marks of a student can be considered as a discrete variable. Hence, the correct option is (b).

On the basis of source of data, we can define data as follows:

- **Primary Data:** Primary data refers to data collected by the investigator for their own purpose, from the beginning to the end of the research.
  - These are collected from the source of origin.
  - Primary data is considered original and specific to the research being conducted of primary data collection methods include surveys, interviews, experiments, and observations.
- **Secondary Data:** Secondary data refers to data that already exists and has been collected for some other purpose.
  - These data are considered second–hand as they have been collected by someone else.
  - Secondary data can be obtained from published or unpublished reports, articles, books, databases, or other sources of secondary data include government reports, academic studies, industry statistics, and historical records.

COLLECTION OF PRIMARY DATA CAN BE DONE BY 4 WAYS

- Interview method
- Mailed questionnaire method
- Observation method
- Questionnaires filled and sent by enumerators.

1) INTERVIEW METHOD

- Direct Interview Method or Personal Interview Method
- Indirect Interview Method
- Telephonic Interview Method

This method involves direct interaction between the investigator and the respondent. There are different types of interview methods, such as direct interview, indirect interview, and telephonic interview. Direct interview provides more accurate data, while telephonic interview allows for faster data collection over a wider area. However, the telephonic interview method may have a higher number of non–responses.

E.g.: Let's say a school wants to gather feedback from its students regarding their satisfaction with the school facilities and extracurricular activities. They decide to use the interview method to collect data directly from the students.

Note:

- In the first two methods data collected will be more accurate but if you have to collect data faster and at a wider area then you need to collect data by Telephonic Interview method.
- The number of non–responses is maximum for this third method of data collection

2) MAILED QUESTIONNAIRE METHOD

In this method, questionnaires are mailed to the informants. The questionnaire is accompanied by a letter explaining the purpose of the enquiry and ensuring the confidentiality of the information. The informants fill out the questionnaires and return them to the investigator.

This method allows for covering a wide area, but it may also result in a higher number of non–responses.

E.g.: Suppose a local government wants to gather feedback from residents about their satisfaction with public transportation services in the city. They decide to use the mailed questionnaire method to reach a wide range of residents.

3) OBSERVATION METHOD

This method involves direct observation by the investigator to collect information. It is considered one of the best methods for data collection, but it can be time–consuming and covers only a small area.

E.g.: A researcher is interested in studying the playground behavior of children in a local park. They choose to use the observation method to gather data directly by observing the children during their playtime.

4) QUESTIONNAIRES FILLED AND SENT BY ENUMERATORS

Under this method, enumerators are appointed to approach the informants and fill out the questionnaires. This method is suitable for covering a wider range of respondents but can be costly.

E.g.: A market research company wants to collect data on consumer preferences for a new product. They decide to use the questionnaires filled and sent by enumerators method to gather information from a diverse group of respondents.

SOURCES OF SECONDARY DATA

Secondary data refers to data that has been previously collected by someone else for a different purpose but can be utilized for the current research or analysis. Here are some important sources of secondary data.

There are many sources of getting secondary data. Some important sources are listed below:

- **International sources:** These include organizations like the World Health Organization (WHO), International Labour Organization (ILO), International Monetary Fund (IMF), World Bank, and others. These global institutions collect and provide data on various topics such as health, labor, economy, and development.
- **Government sources:** Governments often collect extensive data on various aspects of society and the economy. Statistical agencies, such as the Central Statistical Office (CSO) in different countries, publish reports and statistical abstracts that provide valuable data. Ministries and departments related to agriculture, education, labor, and more also release official statistics.
- **Private and quasi-government sources:** Private organizations and quasi-governmental institutions also generate and maintain datasets relevant to specific sectors. These sources can include research institutes, industry associations, academic institutions, and specialized agencies.
- **Unpublished sources:** Researchers and experts often conduct studies or collect data that may not be widely available. These unpublished sources can include research papers, reports, surveys, and studies conducted by various institutions, researchers, or organizations.

SCRUTINY OF DATA

Scrutiny of data refers to the process of carefully examining the collected data to ensure its accuracy, reliability, and consistency. It involves reviewing the data for any errors, inconsistencies, or outliers that may affect the validity of the analysis.

Researchers scrutinize the data by performing data cleaning and validation procedures. This includes checking for missing values, outliers, data entry errors, and logical inconsistencies. By carefully reviewing the data, researchers can ensure the quality and integrity of the dataset before proceeding with analysis.

Example 18. Which of the following is an example of qualitative data?

(a) Temperature in degrees Celsius.

(b) Number of cars in a parking lot.

(c) Rating of customer satisfaction (Excellent, Good, Average, Poor).

(d) Height in centimeters.

Sol. (c) Qualitative data refers to non–numerical information that describes qualities or characteristics. It is subjective and categorical in nature.

Therefore, Rating of customer satisfaction is an example: of qualitative data.

Hence, the correct option is (c).

Example 19. Data collected on religion from the census reports are **(ICAI)**

(a) Primary data (b) Secondary data

(c) Sample data (d) (a) or (b)

Sol. (b) Data collected on religion from census reports are considered secondary data. Secondary data refers to data that has been collected by someone else or from existing sources for a purpose other than the current research or study. In this case, the census reports have already been conducted by a designated authority, and the data collected from those reports are used for various analyses and studies.

Therefore, Data collected on religion from the census reports are Secondary data.

Hence, the correct option is (b).

Example 20. The weights of a group of individuals were recorded using a weighing scale. Based on this information, identify the nature of the data collected.

(a) Primary data (b) Secondary data

(c) Discrete data (d) Continuous data

Sol. (a) The weights recorded using a weighing scale represent measurements that are directly collected from the individuals themselves. Therefore, the data is considered "Primary data".

Hence, the correct option is (a).

Example 21. Which of the following methods is considered the most efficient for collecting primary data quickly? **(ICAI)**

(a) Personal interview (b) Indirect interview

(c) Telephone interview (d) Direct observation

Sol. (c) The quickest method to collect primary data is through a telephone interview. This method involves conducting interviews with respondents over the phone, where the interviewer asks questions and records the responses.

Hence, the correct option is (c).

Example 22. The most effective approach for data collection during a natural disaster is

(a) Personal interview (b) Indirect interview

(c) Questionnaire method (d) Direct observation method.

Sol. (a) During a natural disaster, conducting personal interviews is often the most effective method for data collection. This involves directly interacting with individuals who have experienced the disaster and gathering information through face–to–face conversations. Personal interviews allow for detailed and nuanced responses, providing valuable insights into the impact of the disaster, the needs of the affected individuals, and their perspectives on the situation. It enables researchers or relief workers to ask specific questions, probe further for details, and gain a deeper understanding of the situation on the ground.

Hence, the correct option is (a).

Example 23. When studying historical events and analyzing past records, which method of data collection is commonly used to extract information?

(a) Personal interview
(b) Secondary data analysis
(c) Questionnaire method
(d) None of these

Sol. (b) By conducting secondary data analysis, researchers can extract valuable information from these existing sources and analyze it to gain insights into the historical events being studied. This method saves time and resources compared to conducting personal interviews, administering questionnaires, or conducting case study analyses, especially when studying events that have already occurred.

Hence, the correct option is (b).

Example 24. Which data collection method involves gathering information from a large sample of individuals by mailing them questionnaires to be completed and returned?

(a) Telephone interview method
(b) Mailed questionnaire method
(c) Direct interview method
(d) All of these

Sol. (b) The mailed questionnaire method is specifically designed to reach a large number of individuals dispersed across different locations. Questionnaires are sent via mail, allowing respondents to complete them at their convenience and return them by mail. This method is often used when a wide geographic area needs to be covered, making it an effective approach for reaching a diverse population.

Hence, the correct option is (b).

Example 25. Which method of data collection involves recording information from documents, reports, or other existing sources?

(a) Personal interview
(b) Secondary data analysis
(c) Questionnaire method
(d) Direct observation method

Sol. (b) The method of data collection that involves recording information from documents, reports, or other existing sources is called secondary data analysis.

However, secondary data analysis is specifically focused on using existing data sources such as published reports, official records, surveys, or databases to analyze and draw conclusions.

Hence, the correct option is (b).

Example 26. Some important sources of secondary data are **(ICAI)**

(a) International and Government sources
(b) International and primary sources
(c) Private and primary sources
(d) Government sources.

Sol. (a) We know that,

International sources refer to data collected and maintained by international organizations such as the United Nations, World Bank, or World Health Organization. These organizations collect data from various countries and provide global datasets on topics such as economy, development, health, education, and more.

Government sources, as mentioned earlier, are also crucial sources of secondary data. Government agencies at national, regional, or local levels collect and publish data on a wide range of subjects, including demographics, employment, trade, crime, and public services. These datasets are often reliable and comprehensive, making them valuable for research and analysis purposes.

Therefore, International and government sources are important sources of secondary data.

Hence, the correct option is (a).

PRESENTATION OF DATA

- After collecting and verifying the quality of data, it should be presented in a clear and concise manner.
- Effective data presentation is important for effectively communicating findings and facilitating understanding.
- Common methods of data presentation include tables, charts, graphs, and visualizations.
- Researchers select the appropriate presentation format based on the nature of the data and the research objectives.
- The presentation should emphasize the essential features of the data to make it easier for the audience to interpret and derive meaningful insights.

CLASSIFICATION OF DATA OR ORGANIZING OF DATA

Classification of data involves organizing raw data into meaningful groups or classes based on their characteristics. This process enables researchers to draw conclusions and identify patterns or relationships within the data.

Data can be classified based on various attributes such as age groups, income brackets, geographical regions, or product categories. By categorizing data into relevant classes, researchers can analyze and interpret data more effectively.

E.g.: A market researcher conducting a survey on customer preferences for a product might classify the responses into different age groups (e.g., 18–25, 26–35, 36–45) to understand how preferences vary across different demographic segments.

Classification of data helps in summarizing and simplifying complex data sets, making it easier to identify trends, make comparisons, and draw meaningful conclusions.

OBJECTIVES OF CLASSIFICATION

1. **Simplification and Briefness:** Classification aims to simplify complex data by grouping them into categories or classes. It provides a condensed and organized representation of data, making it easier to analyze and interpret.
2. **Comparability:** Classification enables the comparison of data within and across different categories or classes. It allows for identifying patterns, trends, and relationships between different groups of data.
3. **Statistical Analysis:** Classification facilitates statistical analysis by organizing data into meaningful groups. It enables researchers to apply various statistical techniques and methods to explore the characteristics and relationships within each category or class.

4. **Makes data more understandable:** Classification makes data more understandable by presenting them in a structured manner. It provides a clear framework for data interpretation and helps in deriving insights and making informed decisions.

DATA MAY BE CLASSIFIED AS

1. **Data Chronological or Temporal or Time Series:** This classification arranges data based on their time intervals or chronological order. It helps in studying patterns and trends over time.

   E.g.: Classifying monthly sales data of a product over the past year into different time intervals such as quarters or seasons.

2. **Geographical or Spatial Series Data:** This classification groups data based on their geographical or locational differences. It helps in analyzing variations across different regions or areas.

   E.g.: Classifying population data of different cities or states into regional groups such as North, South, East, and West.

3. **Qualitative or Ordinal Data:** This classification categorizes data based on their qualities or attributes. It involves assigning data to specific categories or classes based on subjective characteristics.

   E.g.: Classifying survey responses into categories such as "Satisfied," "Neutral," and "Dissatisfied" based on customer satisfaction levels.

4. **Quantitative or Cardinal Data:** This classification organizes data into classes or groups based on their numerical values. It involves creating intervals or ranges to represent different levels or quantities.

   E.g.: Classifying test scores of students into different grade ranges such as A, B, C, etc., based on their numerical value ranges.

FREQUENCY DATA

Frequency of a particular data value is the number of times the data value occurs and data in which we can count frequency is called as frequency data

E.g.: Recording the number of times a specific word appears in a document or the number of customers who visited a store on different days of the week.

NON-FREQUENCY DATA

Non-frequency data refers to data where the individual values and their specific identities are important and need to be preserved.

E.g.: Recording the names and addresses of customers in a database for a marketing campaign, where each customer's information is unique and distinct.

THERE ARE GENERALLY THREE FORMS OF PRESENTATION OF DATA

Textual or Descriptive Presentation

In the textual presentation, data is described using paragraphs of text. This method is commonly used in official reports, where the activities, plans, or programs of a project are explained in detail, with relevant facts and figures inserted within the text. This form of presentation is suitable when the amount of data is relatively small.

However, statisticians generally prefer other methods as it can be dull, monotonous, and difficult to compare different fields of data.

E.g.: A project report describing the progress, achievements, and future plans of a company, including relevant statistical data within the narrative.

Tabular Presentation

Data is organized in rows and columns in a tabular presentation. A statistical table provides a systematic and structured representation of data in a concise format.

Considerations when creating a table:

- Designing of Table: Ensuring a clear and organized layout with appropriate column headings and row labels.
- Comparable Data: Presenting data in a format that allows for easy comparison and analysis.
- Beautiful Presentation: Enhancing the visual appeal of the table through proper formatting, font usage, and gridlines.
- Descriptive Notes: Including explanatory notes or footnotes to provide additional context or clarify any assumptions made.

1) Table Number
2) Title (which explains content of tables)
3) Headnote (Given in brackets information about the units)
4) Caption – Describes column & Subcolumns
5) Stubs – Row head
6) Body of Table – Numeric information
7) Source of Data
8) Footnote – Specific feature which is not explained

Box Head ⇓ Upper part of table includes column & Sub-coloumn number

Here's an example of a tabular presentation depicting the sales performance of a company across different product categories:

Product Category	Q1 Sales (in INR)	Q2 Sales (in INR)	Q3 Sales (in INR)
Electronics	5,00,000	4,50,000	5,50,000
Apparel	3,50,000	4,00,000	3,80,000
Home Goods	2,80,000	3,20,000	3,00,000
Beauty	2,00,000	1,80,000	2,20,000

DESCRIPTION OF EACH CONSIDERATION

1. **Designing of Table:** The table is designed with clear column headings ("Product Category," "Q1 Sales," "Q2 Sales," "Q3 Sales") and appropriate row labels for each product category.
2. **Comparable Data:** The data is presented in a format that allows easy comparison of sales across different quarters for each product category. This enables analysis and identification of trends or variations.

3. **Beautiful Presentation:** The table is properly formatted, with consistent alignment, appropriate font usage, and visible gridlines, making it visually appealing and easy to read.
4. **Descriptive Notes:** No specific descriptive notes are provided in this example, but they could be included below the table to explain any abbreviations, define terms, or provide additional context if needed.

The tabulation method is generally preferred over textual presentation because

1. It allows for easy comparison of data across different categories or variables.
2. It provides a structured and organized format that facilitates data analysis.
3. It presents data in a concise and condensed manner, making it easier to grasp key information.
4. It allows for the inclusion of mathematical calculations and statistical measures within the table.

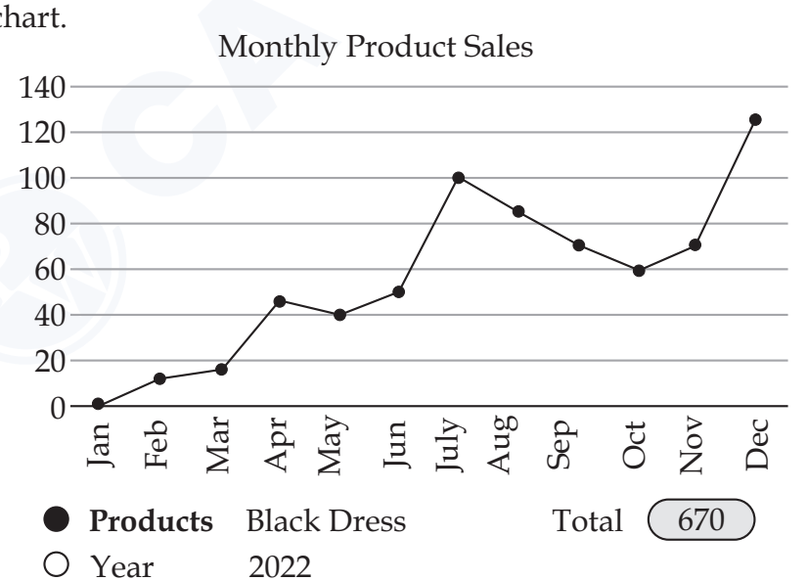
DIAGRAMMATIC REPRESENTATION OF DATA

Diagrammatic presentation of data is a highly effective method of visually representing information in a concise manner. It involves the use of various types of diagrams to convey data effectively. In this context, we will focus on three commonly used diagrams.

LINE DIAGRAM OR HISTORIAGRAM

A line diagram, also known as a historiagram, is used to represent data that changes over time. It involves plotting pairs of values on a graph. By connecting these points with lines, we can visualize the trend or pattern of the data over time.

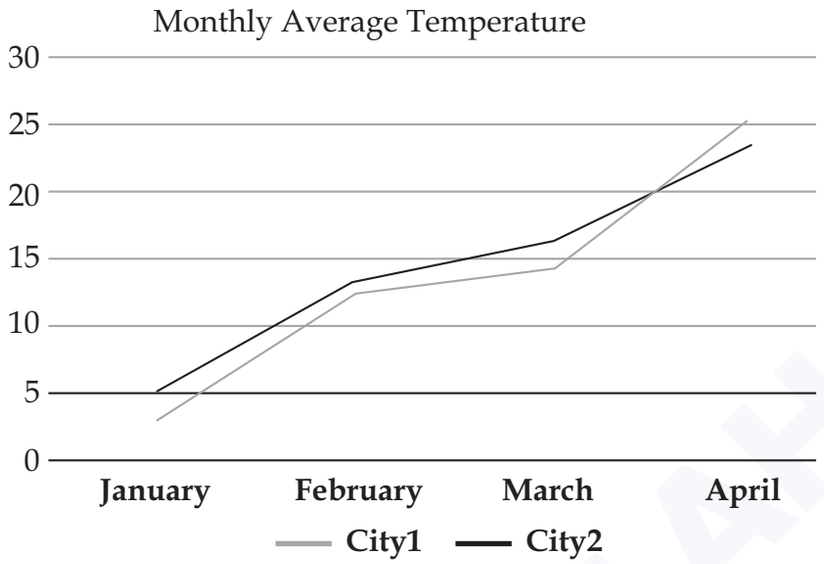
- If we have to represent multiple data (similar data in same unit) varying with time, we can use Multiple line chart.
- If we have to represent multiple data (with different unit) varying with time, we can use Multiple axis chart.



E.g.:

- **Line Chart:** Suppose we have data on the monthly product (Black dress) sale in the year 2022. We can use a line diagram to plot the sale (y-axis) against the corresponding months (x-axis), allowing us to observe the sale trend over time.

MULTIPLE LINE CHART:

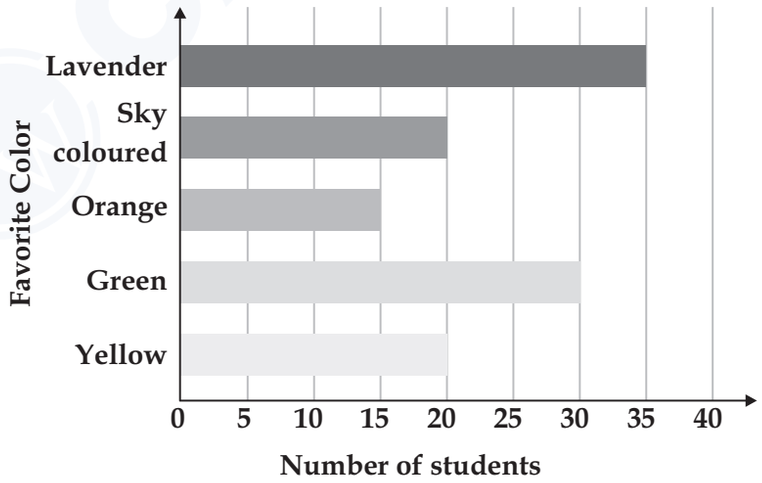


BAR DIAGRAM

A bar diagram is a popular choice for comparing different categories or variables. It involves the use of bars to represent the magnitude of each category or variable. There are different types of bar diagrams depending on their orientation and purpose.

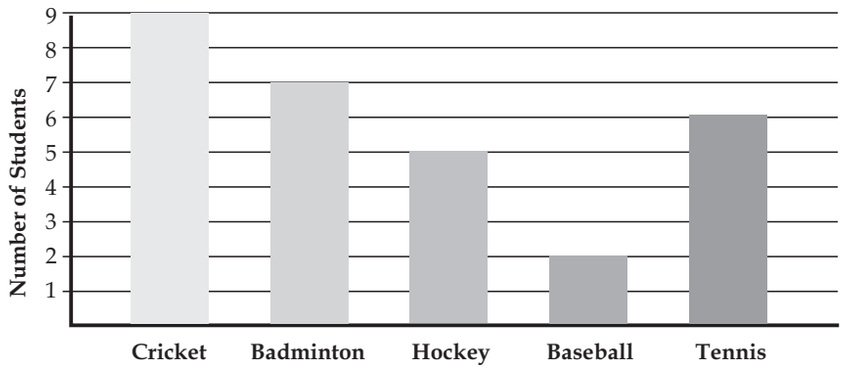
- **Horizontal Bar Diagram:** In a horizontal bar diagram, the categories or variables are represented on the y-axis, while the corresponding values or frequencies are shown on the x-axis. The lengths of the horizontal bars represent the magnitude of the variables.

E.g.:



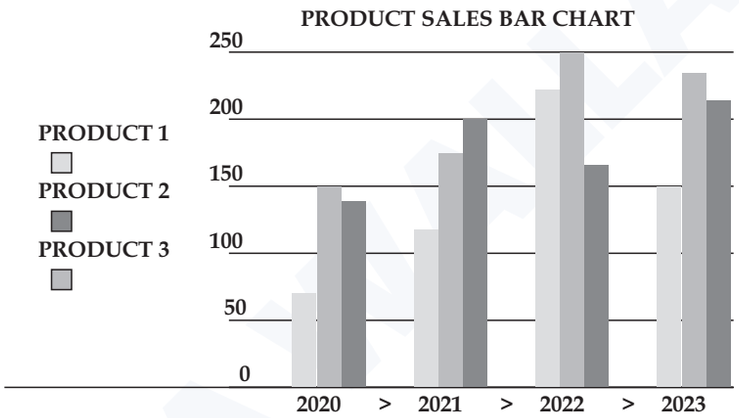
Vertical Bar Diagram: In a vertical bar diagram, the categories or variables are represented on the x-axis, while the corresponding values or frequencies are shown on the y-axis. The lengths of the vertical bars represent the magnitude of the variables.

E.g.:



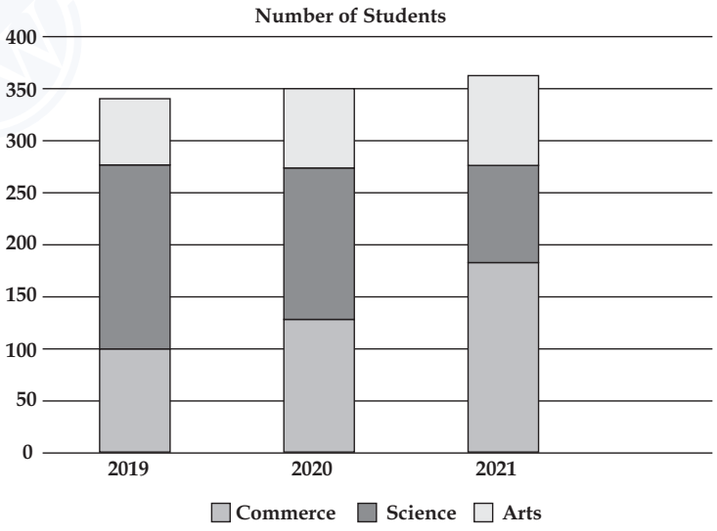
- **Multiple or Grouped Bar Diagrams:** Multiple or grouped bar diagrams are used when we need to represent multiple datasets that have the same unit and vary with time. It allows for easy comparison between the different datasets.

E.g.:



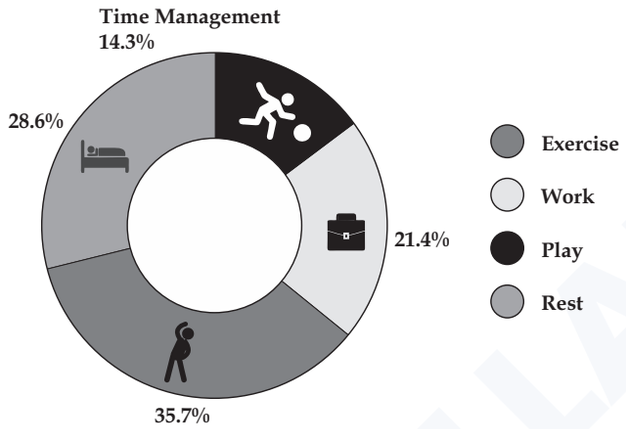
- **Component or Sub-divided Bar Diagrams:** Component or sub-divided bar diagrams are used when we need to represent multiple datasets that have different units but still vary with time. This type of diagram uses multiple axes to accommodate the different units.

E.g.:



- **Pie Chart:** A Pie chart is used to represent data as a circular chart divided into sectors. Each sector represents a specific category or variable, and its size or angle represents the proportion or percentage of that category. Pie charts are especially useful for illustrating proportions or composition.

  E.g.: The below Pie chart represents the time management. The size of each sector in the pie chart corresponds to the proportion of time dedicated to that specific category, providing a clear visual overview of how time is distributed across these four essential aspects of life.



In summary, diagrammatic representation of data provides an effective way to present information visually. Line diagrams show trends over time, bar diagrams compare variables, and pie charts depict proportions or compositions.

Example 27. Two companies A and B got monthly profit in lakhs for year 2015, 2016, 2017, 2018, 2019 are 5, 6, 8, 10, 20 and 3, 2, 7, 18, 12 respectively. Represent the above information in Tabular form.

Sol.

Monthly Profit (in Lakhs)			
Year	Companies		Total
	A	B	
2015	5	3	8
2016	6	2	8
2017	8	7	15
2018	10	18	28
2019	20	12	32
Total	49	42	

Footnote: A and B represents companies

FREQUENCY DISTRIBUTION

- Frequency distribution is a method of organizing data to provide insights into how often certain values occur.
- It can be classified in two ways: as frequency data classified by categories or as frequency data classified by intervals.

Frequency represents the number of times an observation occurs in the data.

E.g.: in the list of numbers 1, 2, 3, 4, 6, 9, 9, 8, 5, 1, 1, 9, 9, 0, 6, 9, the frequency of the number 9 is 5. Frequency distribution involves tabulating data in a table, where the total frequency is distributed among different classes or intervals. The classes or intervals should be mutually exclusive and cover the entire range of data.

UNGROUPED FREQUENCY DISTRIBUTION

When the data consists of discrete variables, we can create an ungrouped frequency distribution table. This table presents the frequency counts for each unique value in the dataset.

E.g.: Suppose we have a dataset representing the number of pets owned by individuals: 2, 4, 1, 3, 2, 4, 2, 1, 3, 4. The ungrouped frequency distribution table would show the frequency count for each unique value:

Number of Pets	Frequency
1	2
2	3
3	2
4	3

GROUPED FREQUENCY DISTRIBUTION

When the data consists of continuous variables, we can create a grouped frequency distribution table. This table categorizes the data into intervals or classes and displays the frequency count for each interval.

E.g.: Suppose we have a dataset representing the heights (in centimeters) of a group of individuals: 156, 168, 174, 160, 162, 170, 168, 172, 158, 164. To create a grouped frequency distribution, we can group the data into intervals (e.g., 150–160, 161–170, 171–180) and count the frequency in each interval:

Heights (in cm)	Frequency
150 – 160	2
160 – 170	5
170 – 180	3

HOW TO MAKE A FREQUENCY DISTRIBUTION TABLE FOR DISCRETE VARIABLE:

I. Find the largest and smallest observations and obtain the difference between them.

II. Form a number of classes depending on the number of isolated values assumed by a discrete variable.

III. Present the class in a table known as frequency distribution table

IV. Apply 'tally mark' i.e., a stroke against the occurrence of a particulars value in a class.

V. Count the tally marks and present these numbers in the next column, known as frequency column, and finally check whether the total of all these class frequencies tally with the total number of observations.

E.g.: Let's say you survey a number of households and find out how many pets they own. The results are 3, 0, 1, 4, 4, 1, 2, 0, 2, 2, 0, 2, 0, 1, 3, 1, 2, 1, 1, 3. Let's distribute data in frequency table.

How to make a frequency distribution table for continuous variable:

Creating a Frequency Distribution Table for Discrete Variables

To create a frequency distribution table for a discrete variable, follow these steps:

1. Identify the largest and smallest observations in the dataset and calculate the range by finding the difference between them.
2. Determine the number of classes or categories to be used in the frequency distribution table. This depends on the number of distinct values assumed by the discrete variable.
3. Present the classes in a table, known as the frequency distribution table. The classes should cover the entire range of values and be mutually exclusive.
4. Use tally marks (vertical strokes) to represent the occurrence of each value within a class. Each value gets a tally mark against the corresponding class.
5. Count the tally marks and record the frequencies in the next column of the frequency distribution table. This column represents the frequency of each class.
6. Finally, verify whether the total of all class frequencies matches the total number of observations in the dataset.

Example 28. Following are the heights (in cm) of B. Com students of St. Xavier's College.

161, 150, 154, 165, 168, 161, 154, 162, 150, 151, 162, 164, 171, 165, 158, 154, 156, 172, 160, 170, 153, 159, 161, 170, 162, 165, 166, 168, 165, 164, 154, 152, 153, 156, 158, 162, 160, 161, 173, 166, 161, 159, 162, 167, 168, 159, 158, 153, 154, 159

Construct a frequency distribution of heights, taking class length as 5.

Sol. Here,

Smallest weight: 150 cm

Largest weight: 173 cm

Range: 173–150=23

$$\text{Number of classes} = \frac{\text{Range}}{\text{Class length}} = \frac{23}{5} = 4.6 \text{ (round up to 5 classes)}$$

Therefore, the frequency distribution table is as follow:

Heights (in cm)	Frequency
150 – 154	12
155 – 159	9
160 – 164	14
165 – 169	10
170 – 174	5
	50

SOME IMPORTANT TERMINOLOGIES

- **Class Limit:** The class limit refers to the minimum and maximum values that define a class interval. The lower–class limit (LCL) is the minimum value, and the upper–class limit (UCL) is the maximum value within a class interval.

  E.g.: in the class interval 10–20, 10 is the lower–class limit, and 20 is the upper–class limit.

- **Class boundary:** Class boundaries are used to establish a common boundary between adjacent class intervals when there is a difference between the upper class limit of one interval and the lower class limit of the next interval. By using class boundaries, we ensure a smooth transition and maintain consistency in data representation.

  Please note that for overlapping class interval– Class boundaries and Class limits are same.

- **Lower class boundary (LCB)** $= \text{LCL} - \frac{D}{2}$

- **Upper class boundary (UCB)** $= \text{UCL} - \frac{D}{2}$ where, D is difference between the LCL of the next class interval and UCL of the given class interval.

  E.g.: Let's consider the following class intervals for a dataset: 10–15,15–20,20–25,25–30.

  Here, the class intervals are overlapping thus Class boundaries and Class limits are same.

  E.g.: Let's consider the following class intervals for a dataset: 10–14, 15–19, 20–24, 25– 29.

  Here, the class intervals are non–overlapping.

  Thus, first class limits: 10–15

  And class boundaries of first Class

  $$\text{LCB} = 10 - \frac{1}{2} = 9.5, \quad \text{HCB} = 14 + \frac{1}{2} = 14.5$$

  Thus, the first class boundary is 9.5–14.5.

- **Class Midpoint:** The class midpoint (or class mark) is a specific point in the center of the class interval in a frequency distribution table.

  Mid–Point = (LCL + UCL) / 2 = (LCB + UCB) / 2

  E.g.: For class interval 10–14

  $$\text{Mid–point} = \frac{10+14}{2} = \frac{24}{2} = 12$$

- **Width or Size of Class Interval:** The class width is the difference between the upper–class boundary to the lower–class boundary of consecutive classes

  E.g.: Width of the interval 10–14 is 14 – 10 = 4

Cumulative Frequency Cumulative frequency is defined as a running total of frequencies.

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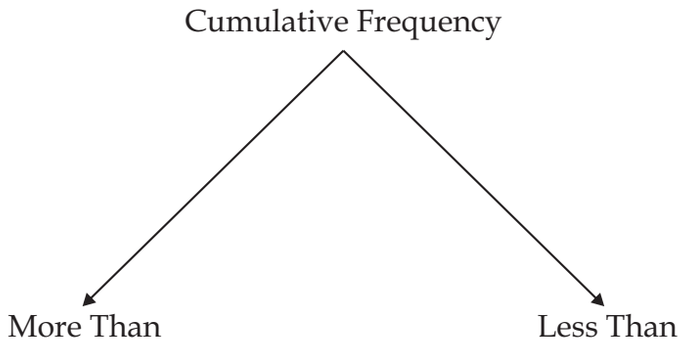


LESS THAN CUMULATIVE FREQUENCY

Heights (in cm)	Frequency	Less than Cumulative frequency
150 – 154	12	12
155 – 159	9	12 + 9 = 21
160 – 164	14	21 + 14 = 35
165 – 169	10	35 + 10 = 45
170 – 174	5	45 + 5 = 50
	50	

MORE THAN CUMULATIVE FREQUENCY

Heights (in cm)	Frequency	More than Cumulative frequency
150 – 154	12	50
155 – 159	9	50 – 12 = 38
160 – 164	14	38 – 9 = 29
165 – 169	10	29 – 14 = 15
170 – 174	5	15 – 10 = 5
	50	

- **Frequency Density of Class Interval:** Frequency density is a measure that helps to standardize the frequencies of different class intervals. It is calculated by dividing the frequency of a class interval by its width or size. The frequency density gives us an idea of the concentration of data within each class interval, taking into account the varying widths of the intervals.

  Frequency Density = Frequency / Width

- **Relative Frequency or Percentage Frequency of Class Interval:** Relative frequency, also known as percentage frequency, is the proportion of the total frequencies that each class interval represents. It is calculated by dividing the frequency of a class interval by the total number of observations and multiplying by 100 to express it as a percentage.

  Formula: Relative Frequency = (Frequency / Total number of observations) × 100

GRAPHICAL REPRESENTATION OF FREQUENCY DISTRIBUTION

HISTOGRAM

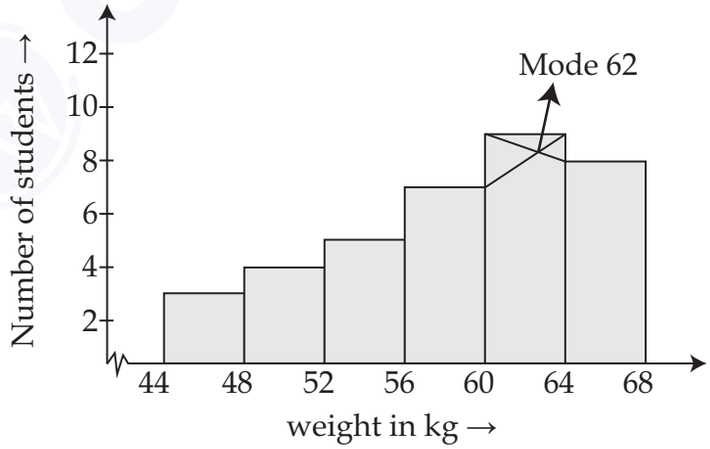
- This is very convenient way to represent a frequency distribution. Comparison among class interval is possible in this mode of diagrammatic representation
- A two–dimensional graphical representation of a continuous frequency distribution is called a histogram.
- In histogram, the bars are placed continuously side by side with no gap between adjacent bars.
- That is, in histogram rectangles are erected on the class intervals of the distribution. The areas of rectangle are proportional to the frequencies.
- We can also find the mode from Histogram

Let's draw histogram for the following data:

FREQUENCY DISTRIBUTION OF WEIGHT OF 36 BBA STUDENT

Weight in kg (class interval)	Tally marks	No. of students (frequency)
44-48	𝍸	3
49-52	𝍸 I	4
52-56	𝍸 𝍸	5
56-60	𝍸 𝍸 II	7
60-64	𝍸 𝍸 IIII	9
64-68	𝍸 𝍸 III	8
Total		36

HISTOGRAM



FREQUENCY POLYGON

As name says, we have to plot frequencies on the graph. For single frequency distribution we can do it easily. By plotting points and joining the points with the line.

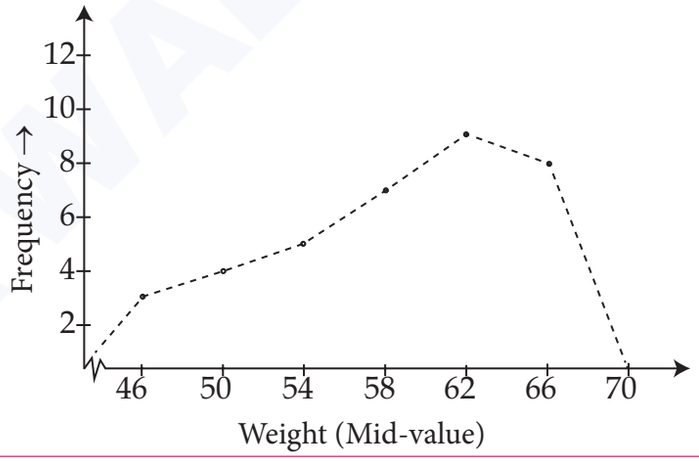
- But for grouped frequency distribution, frequency polygon can be drawn only if the class interval is uniform. Then we take the value of x_i as the midpoint of the class intervals and f_i being the frequencies of the classes respectively we can draw frequency polygon by plotting (x_i, f_i) and then joining it with the line.

  E.g.: Let's draw for the below table

FREQUENCY DISTRIBUTION OF WEIGHTS OF 36 BBA STUDENTS

Weight in kg (class interval)	Tally marks	No. of students (frequency)
44-48	III	3
48-52	IIII	4
52-56	~~IIII~~	5
56-60	~~IIII~~ II	7
60-64	~~IIII~~ IIII	9
64-68	~~IIII~~ III	8
Total	-	36

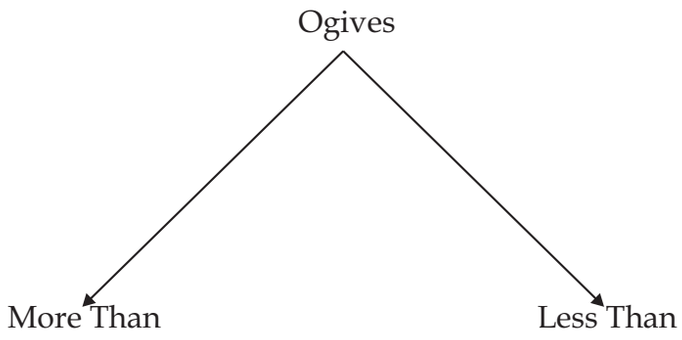
Mid-point	Frequency	Frequency Polygon
46	3	
50	4	
54	5	
58	7	
62	9	
66	8	



OGIVES OR CUMULATIVE FREQUENCY GRAPH

By plotting cumulative frequency against the respective class boundary, we get ogives.

As Cumulative Frequency Graph can be drawn as less than type and more than type. Therefore



E.g.: Let's draw ogive for the below table

Frequency distribution of weights of 36 BBA Students

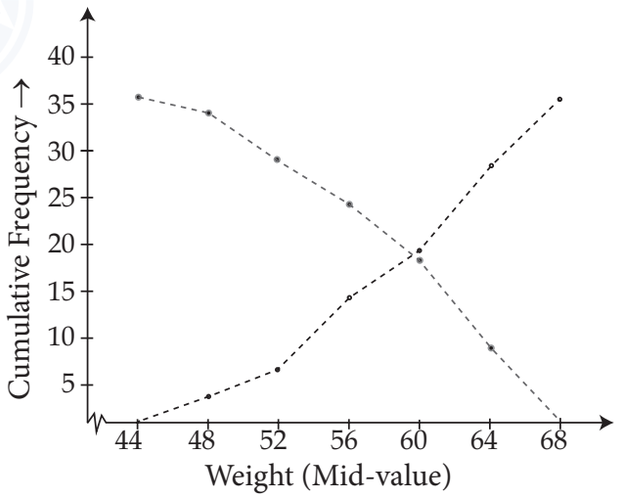
Weight in kg (class interval)	Tally marks	No. of students
44-48	III	3
49-53	IIII	4
54-58	𝍸	5
59-63	𝍸 II	7
64-68	𝍸 IIII	9
69-73	𝍸 III	8
Total	-	36

Less than Ogive

Weight	No. of students	Cumulative Frequency
Less than 48	3	3
Less than 52	4	7
Less than 56	5	12
Less than 60	7	19
Less than 64	9	28
Less than 68	8	36
Total	36	

More than Ogive

Weight	No. of students	Cumulative Frequency
More than 44	3	36
More than 48	4	36 – 3 = 33
More than 52	5	36 – 3 – 4 = 29
More than 56	7	36 – 3 – 4 – 5 = 24
More than 60	9	36 – 3 – 4 – 5 – 7 = 17
More than 64	8	36 – 3 – 4 – 5 – 7 – 9 = 8
Total	36	



Note: Ogives help us to find out Median. If a perpendicular is drawn from the point of intersection of the two ogives on the horizontal axis, then the x–value of this point gives us the value of median

FREQUENCY CURVE

A frequency curve is a graphical representation of the frequency distribution of a dataset. It is a smooth curve obtained by joining the midpoints of the upper side of each vertical bar in a histogram. The frequency curve depicts the relationship between the frequency density on the vertical axis and the class boundary on the horizontal axis. The total area under the frequency curve is taken to be unity, meaning the graph represents the relative frequencies of the data.

There are four main types of frequency curves:

1. **BELL SHAPED CURVE:** The bell–shaped curve, also known as the normal distribution, exhibits a symmetrical shape. It starts with a low frequency value, increases to a maximum at the center, and then decreases to a low frequency value at the other extreme. The bell–shaped curve is commonly observed in natural phenomena and human attributes, such as the distribution of heights, weights, exam scores, or profits.

   E.g.: The distribution of human heights in a population often follows a bell–shaped curve. In this distribution, the majority of individuals tend to cluster around the average height, with fewer individuals at the extreme ends (very short or very tall).

2. **U SHAPED CURVE:** The U–shaped curve represents a situation where the frequency is minimum near the central part and gradually increases to the maximum at the two extremities. This type of curve indicates a pattern where there is a higher concentration of observations at the extremes and fewer observations in the middle.

   E.g.: The traffic flow in a metropolitan city during office timings often exhibits a U–shaped curve. The traffic volume is minimal during non–peak hours, increases significantly during rush hours, and then decreases again during the late evening.

3. **J–SHAPED CURVE:** The J–shaped curve starts with a minimum frequency and gradually increases to reach its maximum frequency at the other extremity. This type of curve suggests a pattern where there are initially fewer observations, but as we move towards the extreme, the frequency increases rapidly.

   E.g.: The traffic on roads between 5 am to 10 am in a busy city like Delhi often follows a J–shaped curve. Initially, there is minimal traffic in the early morning, but as people start commuting to work, the traffic volume gradually increases, peaking during the morning rush hour.

4. **MIXED CURVE:** In a mixed curve, different portions of the distribution may resemble a bell–shaped curve, a U–shaped curve, a J–shaped curve, or other shapes. This indicates that the dataset is a combination of different subgroups or populations, each with its own distribution pattern.

   E.g.: Let's consider a dataset representing the heights of individuals from two different populations, such as male and female. If we combine the heights of both populations in a single frequency distribution, the resulting frequency curve may exhibit characteristics of both a bell–shaped curve and a U–shaped curve. The female heights may follow a bell–shaped curve, while the male heights may show a U–shaped curve. The overall distribution would then be a mixed curve, reflecting the characteristics of both populations.

Example 29. Sweetness of sweet dish is **(Jan 2021)**

(a) An attribute (b) A discrete variable

(c) A continuous variable (d) A variable

Sol. (a) As we know, Attribute is a quality or trait that is assessed for every observation (record) and is variable among observations.

A variable's trait or property is described by an attribute. A sweet dish's characteristic is sweetness.

Therefore, Sweetness of a sweet dish is an attribute.

Hence, the correct option is (a).

Example 30. The accuracy and consistency of data can be verified by **(ICAI)**

(a) Internal checking (b) External checking

(c) Scrutiny (d) Both (a) and (b).

Sol. (c) Scrutiny refers to the careful examination and analysis of data to ensure its accuracy and consistency. It involves a thorough review of the data, checking for any errors, inconsistencies, or discrepancies. This process may involve comparing the data against predetermined criteria, conducting data validations, and performing data quality checks.

Therefore, the accuracy and consistency of data can be verified by Scrutiny.

Hence, the correct option is (c).

Example 31. There were 200 employees in an office in which 150 were married. Total male employees were 160 out of which 120 were married. What was the number of female unmarried employees? **(July 2021)**

(a) 30 (b) 40 (c) 50 (d) 10

Sol. (d) Make a data table according to the conditions given in the question,

	Male	Female	Total
Married	120	30	150
Unmarried	40	10	50
Total	160	40	200

From the data table, we see unmarried female employees are 10.

Hence, the correct option is (d).

Example 32. The distribution of scores in a test is an example of the frequency distribution of

(a) A discrete variable (b) A continuous variable

(c) An attribute (d) (a) or (c)

Sol. (a) The distribution of scores in a test is an example: of a frequency distribution, which shows the frequency or count of each score. In this case, the scores are discrete because they represent distinct categories or values.

For example, if the test scores range from 0 to 100, each score represents a separate category or value.

Therefore, the distribution of scores in a test is an example of the frequency distribution of a discrete variable.

Hence, the correct option is (a).

Example 33. Which of the following is suitable for the graphical representation of a Cumulative frequency distribution? **(Jan 2021)**

(a) Frequency polygon (b) Histogram

(c) Ogive (d) Pie chart

Sol. (c) As we know, Ogive is defined as the cumulative frequency distribution graph of a series.

The Ogive is a cumulative distribution graph that displays data values on the horizontal axis and either cumulative relative frequencies, cumulative frequencies, or cumulative percent frequencies on the vertical axis.

Thus, Ogive is suitable for the graphical representation of a Cumulative frequency distribution.

Hence, the correct option is (c).

Example 34. Which method is commonly used to present detailed information?

(a) Charts (b) Graphs (c) Tables (d) Visualizations

Sol. (c) Tables are commonly used to present detailed information in a structured and organized format. They consist of rows and columns, allowing for clear representation of data. Tables are particularly useful when presenting numerical data or when comparing multiple variables or categories. They provide a concise and systematic way to present information, making it easier for readers to interpret and analyze the data.

Hence, the correct option is (c).

Example 35. The column headings of a table are known as:

(a) Body (b) Stub (c) Box–head (d) Caption

Sol. (d) The table under consideration is divided into caption, Box–head, Stub and Body where,

Caption is the upper part of the table, describing the columns and sub–columns, if any.

Therefore, the column headings of a table are known as Caption.

Hence, the correct option is (d).

Example 36. The number of "Frequency distribution" is

(a) Two (b) one (c) five (d) four

Sol. (b) "Frequency distribution" refers to a tabular or graphical representation of data that shows the frequency of each value or class interval in a dataset. It presents the data in a systematic and organized manner, allowing for easy analysis and interpretation. There is typically only one frequency distribution for a given dataset, which summarizes the distribution of values or intervals and their corresponding frequencies.

Therefore, the number of "Frequency distribution" is one.

Hence, the correct option is (b).

Example 37. (class frequency)/(width of the class) is defined as

(a) Frequency density
(b) Frequency Distribution
(c) both
(d) none

Sol. (a) We know that,

(Class frequency)/(width of the class) represents the frequency density of a class in a frequency distribution.

Frequency density is a measure that accounts for the width of each class interval. It is calculated by dividing the frequency of the class by the width of the class interval. Frequency density is used to compare the densities of different classes and provides a more meaningful representation of the distribution of data.

Hence, the correct option is (a).

Example 38. Tally Marks determines **(ICAI)**

(a) class width
(b) class boundary
(c) class limits
(d) class frequency

Sol. (d) Tally marks are used to determine the frequency or count of observations within each class of a frequency distribution. Each tally mark represents one observation, and by counting the tally marks, we can determine the frequency or number of observations in a particular class. Tally marks are a visual representation of the frequency and help in organizing and summarizing the data in a frequency distribution.

Therefore, Tally Marks determines class frequency.

Hence, the correct option is (d).

Example 39. Most of the Commonly used distributions provide **(Jan 2021)**

(a) Bell – Shaped
(b) U Shaped
(c) J – Shaped Curve
(d) Mixed Curve

Sol. (a) As we know, A bell curve, commonly known as the normal distribution, is a typical type of distribution for a variable.

The graph that is used to represent a normal distribution has a symmetrical bell–shaped curve, hence the name "bell curve."

All other potential occurrences are symmetrically distributed around the mean, resulting in a downward–sloping curve on either side of the peak, while the highest point on the curve, or the top of the bell, represents the most probable event in a series of data (its mean, mode, and median in this case).

The standard deviation of the bell curve serves as a measure of its width.

Hence, the correct option is (a).

Example 40. Unequal widths of classes in the frequency distribution can create difficulties in the construction of:

(a) Ogive
(b) Frequency polygon
(c) Histogram
(d) None of the above

Sol. (c) Unequal widths of classes in the frequency distribution can make it challenging to accurately represent the data in a histogram. Histograms require equal–width intervals to ensure the bars accurately depict the distribution of data. Unequal class widths may result in distorted representations or inaccurate interpretations of the data when constructing a histogram.

Hence, the correct option is (c).

Example 41. The number of types of cumulative Frequency is **(ICAI)**

(a) One (b) Two (c) Three (d) Four

Sol. (b) There are two types of cumulative Frequency

Less than cumulative Frequency It represents the cumulative frequency of values that are less than or equal to a given class or value.

Greater than cumulative Frequency It represents the cumulative frequency of values that are greater than or equal to a given class or value.

Hence, the correct option is (b).

Example 42. Classes with zero Frequency are called **(ICAI)**

(a) nil class (b) Empty class (c) class (d) none

Sol. (b) Classes with zero frequency, meaning there are no data points falling within that class interval, are referred to as empty classes. These classes are still included in the overall class structure but do not contain any data.

Hence, the correct option is (b).

Example 43. For calculating class frequencies, it is important that these classes are

(a) Mutually exclusive (b) Non–overlapping

(c) Independent (d) none

Sol. (b) Class intervals in a frequency distribution should be non–overlapping to ensure that each data point falls into only one class. This ensures accuracy and prevents ambiguity in determining the class frequencies.

Hence, the correct option is (b).

Example 44. The value exactly at the middle would ever be included in a class interval are called

(a) class mark (b) mid value (c) both (d) none

Sol. (c) The class mark or midpoint of a class interval is calculated as the average of the lower and upper limits of the interval. It represents the central value within the interval and is often used as a representative value for that interval.

E.g.: If a class interval is defined as 20–30, the class mark would be $\frac{20+30}{2} = 25$

Hence, the correct option is (c).

Example 45. For the non–overlapping classes 1–20,21–40,41–60,61–80,81–100, the class mark of the class 61–80 is

(a) 10.5 (b) 70.5 (c) 90.5 (d) None

Sol. (b) Class mark: Corresponding to a class interval, this may be defined as the sum of the lower class limit and upper class limit divided by 2.

Thus, the class mark of the class 61–80 is:

$$\text{Class mark} = \frac{\text{LCL} + \text{UCL}}{2} = \frac{61+80}{2} = 70.5$$

Hence, the correct option is (b).

Example 46.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	10	12	8	15	6

For the class 20-30, cumulative frequency is

(a) 22 (b) 8 (c) 51 (d) 30

Sol. (d) According to given data,

Class	0-10	10-20	20-30	30-40	40-50
Frequency	10	12	8	15	6
Cumulative Frequency	10	22	30	45	51

Therefore, for the class 20-30, cumulative frequency is 30.

Hence, the correct option is (d).

Example 47. The curve obtained by joining the points, whose x – coordinates are the upper limits of the class – intervals and y coordinates are corresponding cumulative frequencies is called

(a) Ogive (b) Histogram

(c) Frequency Polygon (d) Frequency curve

Sol. (a) An ogive is a graphical representation of the cumulative frequency distribution. It is obtained by plotting points using the upper limits of the class intervals on the x-axis and the corresponding cumulative frequencies on the y-axis. The curve formed by joining these points is called an ogive. It is used to observe the cumulative frequency distribution and analyze the pattern of data.

Hence, the correct option is (a).

Example 48. In Histogram, the classes are taken

(a) Overlapping (b) Non – overlapping

(c) Both (d) None

Sol. (a) In a histogram, the classes are typically represented by adjacent bars, where the intervals or ranges of values overlap with each other. Each data point can fall within multiple overlapping intervals, allowing for a more detailed representation of the distribution. Overlapping classes in a histogram provide a smoother and continuous representation of the data, allowing for better visualization of the frequency distribution.

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART A):

1. Histogram is used for finding

(a) Mode (b) Mean (c) First quartile (d) None

2. Ogive graph is used for finding

(a) Mean (b) Mode (c) Median (d) None

3. _____ series is continuous.

(a) Open ended (b) Exclusive (c) Close ended (d) Unequal class intervals

4. Histogram can be shown as

(a) Ellipse (b) Rectangle (c) Hyperbola (d) Circle

5. Which of the following graphs is suitable for cumulative frequency distribution?

(a) Ogive (b) Histogram (c) G.M (d) A.M

6. For the class 20-30, cumulative frequency is:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	20	8	3

(a) 10 (b) 26 (c) 30 (d) 41

7. An ogive is a graphical representation of

(a) Cumulative frequency distribution

(b) A frequency distribution

(c) Ungrouped data

(d) None

8. The number of times a particular item occurs in a class interval is called its:

(a) Mean (b) Frequency

(c) Cumulative Frequency (d) None of the above

9. A suitable graph for representing the portioning of total into sub parts in statistics is:

(a) A Pie chart (b) A pictograph

(c) An ogive (d) Histogram

10. Data are said to be _____ if the investigator himself is responsible for the collection of the data.

(a) Primary data

(b) Secondary data

(c) Mixed of primary and secondary data

(d) None of these

11. Histogram is useful to determine graphically the value of _____ for the collection of the data.

(a) Arithmetic mean (b) Median

(c) Mode (d) None of these

12. The following Frequency distribution is classified as

X	12	17	24	36	45
Y	2	5	3	8	9

(a) Continuous distribution (b) Discrete distribution
(c) Cumulative frequency distribution (d) None of these

13. Frequency density is used in the construction of
(a) Histogram (b) Ogive
(c) Frequency polygon (d) None when the classes are of unequal width

14. Divided bar charts is considered for
(a) Comparing different components of a variables
(b) The relation of different components to the table
(c) (a) or (b)
(d) (a) and (b)

15. Frequency density corresponding to a class interval is the ratio of
(a) Class frequency to total frequency
(b) Class frequency to the class length
(c) Class length to class frequency
(d) Class frequency to the cumulative

16. 'Stub' of a table is the
(a) Left part of the table describing the columns
(b) Right part of the table describing the columns
(c) Right part of the table describing the rows
(d) Left part of the table describing the rows.

17. The point of intersection of less than ogive and greater than ogive curve gives us:
(a) Mean (b) Mode (c) Median (d) None of the above

18. Find the number of observations between 250 – 300 from the following data:

Value more than:	200	250	300	500
No. of observation:	56	38	15	0

(a) 38 (b) 23 (c) 15 (d) None of the above

19. The chart that uses logarithm of variables is known as:
(a) Ratio chart (b) Line chart
(c) Multiple line chart (d) Component line chart

20. Data Collection on religion from census reports are:
(a) Primary data (b) Secondary data
(c) Sample data (d) (a) or (b)

21. Difference between the maximum & minimum value of a given data is called
(a) width (b) size (c) range (d) None

22. The most appropriate diagram to represent the data relating to the monthly expenditure on different items by a family is

(a) Histogram (b) Pie-diagram

(c) Frequency polygon (d) Line graph

23. What is a exclusive series ?

(a) In which both upper and lower limit are not included in class frequency.

(b) In which lower limit is not included in class frequency.

(c) In which upper limit is not included in class frequency

(d) None of the above

24. In indirect oral investigation:

(a) Data is not capable of numerical expression

(b) Not possible or desirable to approach informant directly

(c) Data is collected from the books

(d) None of these

25. If the class intervals are 10 – 14, 15 – 19, 20 – 24, …, then the first class boundary is:

(a) 9.5 – 14.5 (b) 10 – 15 (c) 9 – 15 (d) 10.5 – 15.5

26. The following data related to the marks of group of students

Marks No of Students	
More than 70%	7
More than 60%	18
More than 50%	40
More than 40%	60
More than 30%	75
More than 20%	100

How many students have got marks less than 50%?

(a) 60 (b) 82 (c) 40 (d) 53

27. Out of 1000 persons, 25 percent were industrial workers and the rest were agricultural workers. 300 persons enjoyed world cup matches on T.V. 30 percent of the people who had not watched world cup matches were industrial workers. What is the number of agricultural workers who had enjoyed world cup matches on TV?

(a) 230 (b) 250 (c) 240 (d) 260

28. A pie diagram is used to represent the following data.

Source	Customers	Excise	Income Tax	Wealth Tax
Revenue in Millions:	120	180	240	180

The central angles corresponding to Income Tax and Wealth Tax are

(a) 90°, 120° (b) 120°, 90° (c) 60°, 120° (d) 90°, 60°

29. A sample study of the people of an area revealed that total number of women were 40% and the percentage of coffee drinkers were 45 as a whole and the percentage of male coffee drinkers was 20. What was the percentage of female non-coffee drinkers?

(a) 10% (b) 15% (c) 18% (d) 20%

30. In 2000, out of total of 1,750 workers of a factory 1,200 were members of a trade union. The number of women employed was 200 of which 175 did not belong to a trade union. In 2004, there were 1,800 employees who belong to a trade union 50 who did not belong to trade union. Of all the employees in 2004, 300 were women of whom only 8 were non-trade members. On the basis of this information, the ratio of female member of the trade union in 2000 and 2004 is :

(a) 292 : 25 (b) 8 : 175 (c) 175 : 8 (d) 25 : 292

31. The population of roses in Salt Lake City is an example of **(ICAI)**

(a) A finite population. (b) An infinite population.

(c) A hypothetical population. (d) An imaginary population.

32. As the sample size increases, standard error **(ICAI)**

(a) Increases (b) Decreases

(c) Remains constant (d) Decreases proportionately.

33. Which sampling method involves dividing the population into clusters and then randomly selecting entire clusters for the sample?

(a) Simple random sampling (b) Systematic sampling

(c) Cluster sampling (d) Stratified sampling

34. A parameter is a characteristic of **(ICAI)**

(a) Population (b) Sample

(c) Both (a) and (b) (d) (a) or (b)

35. A statistic is **(ICAI)**

(a) A function of sample observations (b)A function of population units

(c) A characteristic of a population (d)A part of a population

36. From a group of 30 individuals, a committee of 4 is to be formed without replacement. How many different committees can be formed?

(a) 435 (b) 27,405 (c) 27,300 (d) 29,160

37. A box contains 6 distinct items. If 3 items are randomly selected with replacement, how many different sets of items can be chosen?

(a) 216 (b) 120 (c) 729 (d) 15

38. In a survey of 50 households, a sample of 10 households is randomly chosen without replacement. How many different samples are possible?

(a) 1,102,100 (b) 102,722,500 (c) 102,722 (d) 10,000

39. A population consists of 8 distinct elements. If a sample of 3 elements is drawn with replacement, how many different samples can be formed?

(a) 512 (b) 64 (c) 1,024 (d) 125

40. If a random sample of size two is drawn without replacement from a population containing the units a, b, c, and d, what are the possible samples? **(ICAI)**

(a) (a, b), (a, c), (a, d)

(b) (a, b), (b, c), (c, d)

(c) (a, b), (b, a), (a, c), (c, a), (a, d), (d, a)

(d) (a, b), (a, c), (a, d), (b, c), (b, d), (c, d)

Answer Key

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(a)	**6.**	(c)	**7.**	(a)	**8.**	(b)	**9.**	(a)	**10.**	(a)
11.	(c)	**12.**	(b)	**13.**	(a)	**14.**	(d)	**15.**	(a)	**16.**	(d)	**17.**	(c)	**18.**	(b)	**19.**	(a)	**20.**	(b)
21.	(c)	**22.**	(b)	**23.**	(c)	**24.**	(b)	**25.**	(a)	**26.**	(a)	**27.**	(d)	**28.**	(b)	**29.**	(b)	**30.**	(d)
31.	(b)	**32.**	(b)	**33.**	(c)	**34.**	(a)	**35.**	(a)	**36.**	(b)	**37.**	(a)	**38.**	(b)	**39.**	(a)	**40.**	(d)

PREVIOUS YEAR QUESTION (PYQ'S)

1. Which of the following graph is suitable for cumulative frequency distribution? **(June 2019)**

(a) Ogive (b) Histogram (c) Hyperbola (d) A.M

2. Histogram can be shown as **(June 2019)**

(a) Ellipse (b) Rectangle (c) Hyperbola (d) Circle

3. _____ series is continuous. **(June 2019)**

(a) Open ended (b) Exclusive (c) Close ended (d) Unequal class intervals

4. Ogive graph is used for finding **(June 2019)**

(a) Mean (b) Mode (c) Median (d) None

5. Histogram is used for finding **(June 2019)**

(a) Mode (b) Mean (c) First quartile (d) None

6. Histogram is used for presentation of the following type of series. **(Dec 2019)**

(a) Time series (b) Continuous frequency series

(c) Discrete series (d) Individual series

7. The graphical representation of cumulative frequency distribution is called **(Dec 2019)**

(a) Histogram (b) Pie Chart

(c) Frequency polygon (d) Ogive

8. In how many cases 4 or more accidents occur? **(Dec 2019)**

No. of accidents	0	1	2	3	4	5	6	7
Frequency	36	27	33	29	24	27	18	9

(a) 96 (b) 133 (c) 78 (d) 54

9. The difference between upper limits and lower limits of a class called **(Dec 2019)**

(a) Class interval (b) Class boundaries

(c) Mid-value (d) Frequency

10. The average of salaries in a factory is ₹47,000. The statement that the average salary is ₹47,000 is **(Dec 2020)**

(a) Descriptive statics (b) Inferential

(c) Detailed (d) Undetailed

11. Statistics cannot deal with _____ data. **(Dec 2020)**

(a) Quantitative (b) Qualitative (c) Textual (d) Attribute

12. Sweetness of a sweet dish is _____. **(Dec 2020, June 2022)**

(a) Attribute (b) Discrete variable

(c) Continuous variable (d) variable

13. Census reports are used as a source of _____ data. **(Dec 2020)**

(a) Secondary (b) Primary (c) Organize (d) Confidential

14. Type of cumulative frequencies are _____ . **(Dec 2020)**

(a) 1 (b) 2 (c) 3 (d) 4

15. You are an Auditor of a firm and the firm earns a profit ₹67,000. You are stated to them the annual profit is ₹67,000. This is _____ type of statistics. **(Dec 2020)**

(a) Descriptive (b) Detailed (c) Non-detailed (d) Inferential

16. The _____ are used usually when we wants to examine the relationship between two variables. **(Dec 2020)**

(a) Bar graph (b) Pie chart (c) Line chart (d) Scatter plot

17. A bar chart is drawn for **(Dec 2020)**

(a) Continuous data (b) Nominal data

(c) Time series data (d) Comparing different components

18. A tabular presentation can be used for **(Jan 2021)**

(a) Continuous series data (b) Nominal data

(c) Time series data for longer period (d) Primary data

19. A variable with qualitative characteristic is known as **(Jan 2021)**

(a) Quality variable (b) An attribute

(c) A discrete variable (d) A continuous variable

20. The accuracy and consistency of data can be verified by **(Jan 2021)**

(a) Scrutiny (b) Internal checking

(c) External checking (d) Double checking

21. From a histogram one cannot compute the approximate value of **(Jan 2021)**

(a) Mode (b) Standard deviation

(c) Median (d) Mean

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22. The left part of a table providing the description of rows is called **(Jan 2021)**

(a) Caption (b) Box-head (c) Stub (d) Body

23. Mode can be obtained from _____. **(Jan 2021)**

(a) Frequency polygon (b) Histogram

(c) Ogive (d) All of the above

24. Most of the commonly used distributions provide a **(Jan 2021)**

(a) Bell-shaped (b) U-shaped (c) J-shaped curve (d) Mixed curve

25. Which of the following is suitable for the graphical representation of a cumulative frequency distribution? **(Jan 2021)**

(a) Frequency polygon (b) Histogram

(c) Ogive (d) Pie chart

26. Sweetness of sweet dish is **(Jan 2021)**

(a) An Attribute (b) A discrete variable

(c) A continuous variable (d) A variable

27. There were 200 employees in an office in which 150 were married. Total male employees were 160 out of which 120 were married. What was the number of female unmarried employees? **(July 2021)**

(a) 30 (b) 40 (c) 50 (d) 10

28. Data collected on religion from the census reports are **(July 2021)**

(a) Primary data (b) Unclassified data

(c) Sample data (d) Secondary data

29. Which of the following diagrams is the most appropriate to represent various heads in total cost? **(July 2021)**

(a) Pie chart (b) Bar graph

(c) Multiple line chart (d) Scatter plot

30. In a graphical representation of data, the largest numerical value is 4 and the smallest numerical value is 25. If classes desired are 4 then the width of class interval is **(July 2021)**

(a) 45 (b) 5 (c) 20 (d) 7.5

31. In graphical representation of data, ideographs are also called as **(July 2021)**

(a) Picto-graphs (b) Asymmetry graphs

(c) Symmetry graphs (d) Tabulation

32. _____ means separating items according to similar characteristics grouping them into various classes. **(July 2021)**

(a) Classification (b) Editing (c) Separation (d) Tabulation

33. Frequency density of a class interval is the ratio of **(July 2021)**

(a) Class frequency to the total frequency

(b) Class length to class frequency

(c) Class frequency to the cumulative frequency

(d) Frequency of that class interval to the corresponding class length

34. A graph that uses vertical bars to represent data is called a **(July 2021)**

(a) Line graph (b) Scatter plot (c) Vertical graphs (d) Bar

35. There were 200 employees in an office in which 150 were married. Total male employees were 160 out of which 120 were married. What was the number of female unmarried employees? **(July 2021)**

(a) 30 (b) 40 (c) 50 (d) 10

36. Data collected on religion from the census reports are **(July 2021)**

(a) Primary data (b) Unclassified data

(c) Sample data (d) Secondary data

37. In a study about the male and female students of Commerce and Science departments of a college in 5 years, the following data were obtained; **(Dec 2021)**

1995	2000
70% female students	75% female students
65% read Commerce	40% read science
20% of male students read Science	50% of female students read commerce
3000 total No. of students	3600 total No. of students

After combining 1995 and 2000 if x denotes the ratio of female commerce student to female Science student and y denotes the ratio of male commerce student to male Science student, then

(a) $x = y$ (b) $x > y$ (c) $x < y$ (d) $x \leq y$

38. A national Institute arrange its students data in accordance with different States. This arrangement of data is known as **(Dec 2021)**

(a) Temporal data (b) Geographical data

(c) Ordinal data (d) Cardinal data

39. A student marks in five subject S1, S2, S3, S4 and S5 are 86, 79, 90, 88 and 89. If we need to draw a Pie chart to represent these marks, then what will be the Central angle for S3?

(a) 103.2° (b) 75°

(c) 105.6° (d) 94.8°

40. The following data relate to the marks of a group of students:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
No. of students	15	38	65	84	100

How many students got marks more than 30?

(a) 65 (b) 50 (c) 35 (d) 43

41. The following data relate to the marks of 48 students in Statistics:

56	10	54	38	21	43	12	22
48	51	39	26	12	17	36	19
48	36	15	33	30	62	57	17
5	17	45	46	43	55	57	38
43	28	32	35	54	27	17	16
11	43	45	2	16	46	28	45

What are the frequency densities for the class intervals 30-39, 40-49, 50-59?

(a) 0.20, 0.50, 0.90 (b) 0.70, 0.90, 1.10

(c) 0.1875, 0.1667, 0.2083 (d) 0.90, 1.00, 0,80

42. Multiple axis line chart is considered when **(Dec 2021)**

(a) There is more than one time series

(b) The units of the variables are different.

(c) In any case.

(d) If there are more than one time series and unit of variables are different.

43. Median of a distribution can be obtained from **(June 2022)**

(a) Frequency polygon (b) Histogram

(c) Less than type ogives (d) Pie chart

44. By plotting cumulative frequency against the respective class boundary, we get **(June 2022)**

(a) Histogram (b) Polygon

(c) Pie chart (d) Ogives

45. Types of research data are **(June 2022)**

(a) Organized data and Unorganized data

(b) Qualitative data and Quantitative data

(c) Processed data and Unprocessed data

(d) Discrete data and Continuous data

46. The collected information on which of the following characteristics do not form data? **(June 2022)**

(a) The number of files audited are 'less than 6', 'between 5 and 10' and 'more than 9'

(b) The number of files audited are 'very less', 'moderate' and 'very large'

(c) The number of files audits in a file

(d) The number of auditors who audited a file

47. Histograms are drawn only when **(June 2022)**

(a) Frequencies in various class intervals are equal

(b) Class intervals are equal

(c) Class intervals are unequal

(d) For less than type cumulative frequencies

48. Which one of the following is not a mode of presentation of data? **(June 2022)**

(a) Textual presentation (b) Tabular presentation

(c) External presentation (d) Diagrammatic presentation

49. Which one of the following is a continuous variable? **(June 2022)**

(a) The quantum of days to get a cure from illness

(b) The quantum of oxygen cylinders used to treat a patient

(c) The quantum of drug injected into a patient

(d) The quantum of tablets prescribe to a patient

50. Which one of the following is source of primary data?

(a) Government records

(b) Research Articles

(c) Journals

(d) Questionnaire filled by Enumerators

51. Which is the left part of the table providing the description of the rows?

(a) Captain (b) Box head (c) Stub (d) Body

52. The suitable formula for computing the number of class intervals is:

(a) 3.322 logN (b) 0.322 logN

(c) 1 +3.322 logN (d) 1-3.322 logN

53. Ogive for more than type and less than type distributions intersect at

(a) Mean (b) Median

(c) Mode (d) Origin

54. In a frequency distribution, the relative frequency of the class is **(Dec 2023)**

(a) The ratio of class frequency to the total number of classes

(b) The ratio of class frequency to the total frequency

(c) The ratio of class frequency to the total number of data points

(d) The ratio of class mid-point to the class frequency

55. The frequency of visitors in an office is given below:

Time	9 am - 11 am	11 am - 1 pm	1 pm - 3 pm	3 pm - 5 pm
Frequency	5	18	7	12

Find the cumulative frequency of visitors for the time 11 am - 1 pm. **(Dec 2023)**

(a) 5 (b) 23 (c) 18 (d) 30

56. Consider the following data where class length is given as 5. Calculate the number of class intervals. **(Dec 2023)**

59, 68, 78, 57, 44, 73, 40, 60, 70, 47

(a) 5 (b) 6 (c) 7 (d) 8

57. Frequency density corresponding to a class interval is the ratio of **(Dec 2023)**

(a) Class frequency to class length

(b) Class frequency to total frequency

(c) Class frequency to Cumulative frequency

(d) Class length to class frequency

58. If two variables x and y are related as $2x - y = 3$, if the median of x is 10, then what is the median of y. **(Dec 2023)**

(a) 4 (b) 7 (c) 5 (d) 6

59. The mean of set of 20 observations is 18.3. The mean is reduced by 0.6 when a new observation is added to the set. The new observation is **(Dec 2023)**

(a) 17.6 (b) 18.9 (c) 5.7 (d) 24.6

60. If the mean of 5 observations $x + 1$, $x + 3$, $x + 5$, $x + 7$ and $x + 9$ is 15, then $x =$ **(Dec 2023)**

(a) 10 (b) 12 (c) 8 (d) 11

61. The mean of the first three terms is 17 and the mean of next four terms is 21. Calculate the mean of seven terms. **(Dec 2023)**

(a) 18.28 (b) 19.78 (c) 19.58 (d) 19.28

62. A less than ogive curve is drawn by plotting **(June 2024)**

(a) Less than Cumulative frequencies on the vertical axis

(b) More than Cumulative frequencies on the vertical axis

(c) Highest frequencies on the vertical axis

(d) Lowest frequencies on the vertical axis

63. An ogive is used to represent **(June 2024)**

(a) The frequency of each data point

(b) The number of data points falling below a specific value

(c) The proportion of data points falling below a specific value

(d) The relationship between two variables

64. Two frequency distributions are given to you. To compare them visually, the best diagram to be drawn on the same sheet is **(June 2024)**

(a) Pie Chart (b) Histogram

(c) Frequency polygon (d) Bar chart

65. A histogram and a pie chart represent the same data on monthly expenses of a household. Which statement is most likely true? **(June 2024)**

(a) The histogram only shows the frequency of each expense category, while the pie chart shows the proportion of each category

(b) Both histogram and pie chart show the frequency of each expense category

(c) Both histogram and pie chart show the proportion of each expense category

(d) None of these

66. According to the empirical rule, if the data form a "bell-shaped" distribution, then the maximum and minimum frequencies occur at _____ and _____ respectively. **(June 2024)**

(a) Middle, left end (b) Middle, right end

(c) End, Middle (d) Middle, ends

67. The following set of data cannot be represented in a table **(June 2024)**

(a) The heights of students described in centimeters

(b) The weights of students described in kilograms

(c) The amount of rainfall opined a "medium", "average", "heavy", etc

(d) The number of bills per day cleared by an auditor in a month

68. The Secondary data is collected by: **(Sept 2024)**

(a) International source like World Bank.

(b) Observation method.

(c) Interview method.

(d) Mailed questionnaire method.

69. For the non-overlapping classes 25-34, 35-44, 45-54, 55-64 the class mark of the class 35-44 is **(Sept 2024)**

(a) 40.5 (b) 39.5 (c) 35.0 (d) 44.0

70. Out of 1000 persons, 40% are female, others are male. In a marriage function, 300 persons enjoyed the song. 30% of the people who had not enjoyed the song were female. What is the number of male, who did not enjoy the song in the function? **(Sept 2024, Jan 2025)**

(a) 180 (b) 120 (c) 360 (d) 490

71. The distribution of commuters coming to a Metro station from early morning hours to peak morning hours follows which type of frequency curve? **(Sept 2024)**

(a) Bell shaped curve (b) J-shaped curve

(c) U-shaped curve (d) Mixed curve

72. Series in which frequencies are continuously added corresponding to each class interval in the series: **(Sept 2024)**

(a) Frequency (b) Cumulative frequency series

(c) Deviation (d) Mid value

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73. The Ogive can be used for making **(Sept 2024)**

(a) short term projection (b) medium term projection

(c) long term projection (d) group frequency distribution

74. Numerical data presented in descriptive form are called: **(Sept 2024)**

(a) Classified presentation (b) Tabular presentation

(c) Textual presentation (d) Graphical presentation

75. What type of data is most appropriate for representing using a Pie chart? **(Sept 2024)**

(a) Continuous data (b) Categorical data

(c) Ordinal data (d) Interval data

76. What is the range of a data set? **(Sept 2024)**

(a) The difference between the highest and lowest values in the data set

(b) The difference between the mean and median of the data set

(c) The number of data points in the data set

(d) The standard deviation of the data set

77. If the class intervals of certain data are 10-14, 15-19, 20-24, then the first class boundaries is **(Sept 2024)**

(a) 9.5-14.5 (b) 10-14 (c) 10-15 (d) 10.5-15.5

78. For the non-overlapping classes 25-34, 35-44, 45-54, 55-64 the class mark of the class 35-44 is **(Jan 2025)**

(a) 40.5 (b) 39.5 (c) 35.0 (d) 44.0

79. In tabular presentation of data, stub is **(Jan 2025)**

(a) Right part of the table providing the description of the row

(b) Left part of table, which provide the description of rows

(c) Left part of the table providing the description of columns

(d) Right part of the table providing the description of columns

80. Ogive is used to find **(Jan 2025)**

(a) Median (b) Mean (c) Mode (d) Range

81. In pie chart, if a category represents 25% of the total data, what will be the angle of corresponding sector? **(Jan 2025)**

(a) 45° (b) 90° (c) 60° (d) 75°

82. Frequency density corresponding to a class interval for the continuous frequency distribution, is the ratio of **(May 2025)**

(a) class frequency to the total frequency

(b) class frequency to the class length

(c) class length to the class frequency

(d) class frequency to the cumulative frequency

83. The curve obtained by joining the points, whose X co-ordinates are the upper limits of the class intervals and Y co-ordinates are corresponding cumulative frequencies, is called **(May 2025)**

(a) Ogive (b) Histogram
(c) Frequency polygon (d) Frequency curve

84. The following data relate to the wages of a group of workers: **(May 2025)**

Wages (in ₹)	Below 100	Below 200	Below 300	Below 400
No. of workers	15	38	65	90

How many workers got wages more than ₹300?

(a) 25 (b) 65 (c) 90 (d) 27

85. The mode of a continuous frequency distribution can be determined graphically from **(May 2025)**

(a) By using Histogram (b) By using frequency polygon
(c) By using ogive (d) By using frequency curve

86. A frequency curve which starts with a minimum frequency and then gradually reaches its maximum frequency at the other extremity is known as **(May 2025)**

(a) Bell shaped curve (b) Mixed curve
(c) U-shaped curve (d) J-shaped curve

87. In tabulation, source of data, if any, is shown in the **(May 2025)**

(a) Footnote (b) Body (c) Stub (d) Caption

88. Number of students in a college is an example of **(Sept 2025)**

(a) An attribute (b) A discrete variable
(c) A continuous variable (d) A constant

89. For manifold classification, this method of presentation of data cannot be recommended: **(Sept 2025)**

(a) Textual presentation (b) Tabular presentation
(c) Bar Diagram (d) Pie Chart

90. The most commonly used distribution is _____ in which the maximum frequency is at the central part and the frequency decreases when one moves away from the central part on either the left side or the right side. **(Sept 2025)**

(a) Bell-shaped curve (b) U-shaped curve
(c) J-shaped curve (d) Mixed curve

91. An investigator collects information on salaries received by 1000 persons. From this collection, the data on women are extracted. Now the data is called _____ data. **(Sept 2025)**

(a) Primary (b) Secondary (c) Census (d) Ordinal

92. Two sales-persons present their numbers of sales per week for a month. An appropriate diagram that can be drawn for this data is _____. **(Sept 2025)**

(a) Histogram (b) Pie chart (c) Ogive (d) Adjacent bar chart

Answer Key

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(a)	**6.**	(b)	**7.**	(d)	**8.**	(c)	**9.**	(a)	**10.**	(a)
11.	(c)	**12.**	(a)	**13.**	(a)	**14.**	(b)	**15.**	(a)	**16.**	(d)	**17.**	(d)	**18.**	(b)	**19.**	(b)	**20.**	(a)
21.	(b)	**22.**	(c)	**23.**	(b)	**24.**	(a)	**25.**	(c)	**26.**	(a)	**27.**	(d)	**28.**	(d)	**29.**	(a)	**30.**	(b)
31.	(d)	**32.**	(a)	**33.**	(d)	**34.**	(d)	**35.**	(d)	**36.**	(d)	**37.**	(c)	**38.**	(b)	**39.**	(b)	**40.**	(c)
41.	(d)	**42.**	(d)	**43.**	(c)	**44.**	(d)	**45.**	(b)	**46.**	(b)	**47.**	(b)	**48.**	(c)	**49.**	(c)	**50.**	(d)
51.	(c)	**52.**	(c)	**53.**	(b)	**54.**	(b)	**55.**	(b)	**56.**	(d)	**57.**	(a)	**58.**	(b)	**59.**	(c)	**60.**	(a)
61.	(d)	**62.**	(a)	**63.**	(b)	**64.**	(b)	**65.**	(a)	**66.**	(d)	**67.**	(c)	**68.**	(a)	**69.**	(b)	**70.**	(d)
71.	(b)	**72.**	(b)	**73.**	(d)	**74.**	(c)	**75.**	(b)	**76.**	(a)	**77.**	(a)	**78.**	(b)	**79.**	(b)	**80.**	(a)
81.	(b)	**82.**	(b)	**83.**	(a)	**84.**	(a)	**85.**	(a)	**86.**	(d)	**87.**	(a)	**88.**	(b)	**89.**	(a)	**90.**	(a)
91.	(b)	**92.**	(d)																

SUMMARY

- Statistics deals with the aggregates. An individual, to a statistician has no significance except the fact that it is a part of the aggregate.
- Statistics is concerned with quantitative data. However, qualitative data also can be converted to quantitative data by providing a numerical description to the corresponding qualitative data.
- We can broadly classify data as
  - Primary
  - Secondary
- Scrutiny of Data : Data scrutiny focuses on inference, the process of deriving a conclusion based solely on what is already known by the researcher. Data needs to be curated, cleansed, enriched and translated into actionable insights before it can be put to work doing something meaningful.
- Mode of Presentation of Data
  (a) Textual presentation;
  (b) Tabular presentation or Tabulation;
  (c) Diagrammatic representation.
- The types of diagrams:
  (a) Line diagram or Histogram
  (b) Bar diagram
  (c) Pie chart
- Frequency Distribution of a Variable
  (a) Find the largest and smallest observations and obtain the difference between them, known as Range,
  (b) In the case of a discrete variable, Form a number of classes depending on the number of isolated values assumed by a discrete variable.

(c) In case of a continuous variable, find the number of class intervals using the relation, No. of class Interval × class length = Range.

(d) Cumulative frequency – less than and more than types

- Graphical Representation of Data
  - (i) Histogram or Area diagram;
  - (ii) Frequency Polygon;
  - (iii) Ogives or cumulative Frequency graphs.
- Frequency Curve
  - (a) Bell-shaped curve;
  - (b) U-shaped curve;
  - (c) J-shaped curve;
  - (d) Mixed curve.
- Lower Class Boundary (LCB) = Lower Class Length – $\frac{D}{2}$,

  Upper Class Boundary = Upper Class Length + $\frac{D}{2}$
- Mid–point $= \frac{LCB + UCB}{2}$ or $\frac{LCL + UCL}{2}$
- Frequency density of a class interval = frequency of that class interval/corresponding class length
- Relative frequency and percentage frequency of a class interval = Class frequency/total frequency

❑❑❑

CA WALLAH

Unit-2: Sampling

INTRODUCTION TO SAMPLING

Sampling is a fundamental technique in statistics that enables researchers to draw meaningful conclusions about a large population by studying a smaller, manageable subset. This approach acknowledges that it is often impractical, if not impossible, to collect data from an entire population. Instead, it provides a systematic way to select a representative group, known as the sample, which is carefully chosen to reflect the characteristics of the larger population.

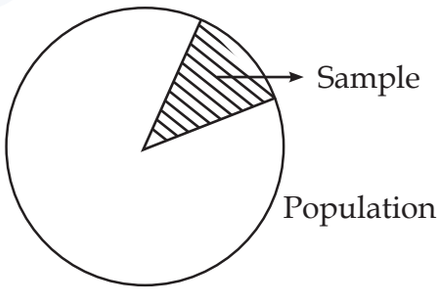
Sampling is essential for various fields, including market research, scientific experiments, public opinion polling, and quality control, as it offers an efficient means of making inferences about a broad range of phenomena.

For example, when a person goes to market to buy rice, firstly, he takes some rice in his hand from a bag containing rice and then examining it, he takes the decision whether to buy it or not.

POPULATION AND SAMPLE:

1. **Population:** The population is the entire group or collection of individuals, objects, or data that the researcher wants to draw conclusions about. It represents the larger entity under study and can be finite or infinite, depending on the research context.
2. **Sample:** A sample is a subset of the population selected for a research study. It is chosen to represent the population because studying the entire population may be impractical or too costly. The characteristics and behaviors of the sample are analyzed to make inferences about the entire population. The process of selecting a sample should be done systematically to ensure it is a representative and unbiased reflection of the population.

S.No.	Population	Sample
1.	Students of the school	Commerce students of grade 12
2.	All books in a library	Book



Also, the number of elements in a sample is called **Sample Size.**

3. **Parameter:** A parameter is a characteristic of a population, determined by considering all units within that population. Statistical inferences about population parameters are made by analyzing sample observations drawn from the same population.

Example: Population mean = μ

Population variance = σ^2

Here,

- When we are interested in **'Population Mean'** then it is represented as:

$$\mu = \frac{\sum_{a=1}^{n} x_a}{N}$$

Where N denotes the population size and x_a represents a^{th} member of the population.

- When we are interested in **'Population Proportion'** then it is represented as: $P = \frac{x}{N}$

Where, X represents the count possessing the required attribute.

- Another important parameter namely the poluation variance, to be denoted by σ^2 is given by:

$$\sigma^2 = \frac{\Sigma (X_a - \mu)^2}{N}$$

Also, $SD = \sigma = \sqrt{\frac{\sum (X_a - \mu)^2}{N}}$

PRINCIPLES OF SAMPLE SURVEY:

A sample survey is a research method where data is collected from a subset (sample) of individuals or items from a larger group (population). It's used to make inferences about the entire population by studying the characteristics and behaviors of the selected sample.

Basic principles of Sample survey comparison the components such as:

- **Law of Statistical Regularity:** This principle is based on the notion that in large and random samples, patterns and regularities emerge. It means that in sufficiently large samples, the characteristics of the sample start to mirror the characteristics of the population it's drawn from.
- **Principle of Inertia:** This principle suggests that, unless there's a good reason to the contrary, researchers often prefer not to change their methods or sample size. It helps in maintaining consistency and comparability in survey results over time.
- **Principle of Optimization:** This principle focuses on maximizing the information gained from a sample while minimizing the cost and effort of collecting data. It's about finding the right balance between sample size and data quality.
- **Principle of Validity:** Validity is all about measuring what you intend to measure. In sampling surveys, this principle emphasizes the importance of using methods and questions that accurately capture the information you're interested in. It's essential to ensure that your survey results are meaningful and truly represent the characteristics of the population.

SAMPLING V/S CENSUS:

1. Speed: Sampling takes less time
2. Cost: Sampling is least expensive
3. Accuracy & Reliability : Census gives more Accurate & Reliable results unless there is no bias or error in collecting information

STATISTICS

It is defined as a statistical measure of sample observation.

Example; sample mean = $\bar{x}$

sample variance = S^2

It is a function of of sample observations. If the sample observations are denoted by $x_1, x_2, x_3, \ldots x_n$, then a statistics T may be expressed as $T = f(x_1, x_2, x_3, \ldots x_n)$.

A statistic is used to estimate a particular population parameter. The estimates of population mean, variance and population proportion are given by

$$\bar{x} = \hat{\mu} = \frac{\sum x_i}{n}$$

$$S_2 = \widehat{\sigma^2} = \frac{\sum (x_i - \bar{x})^2}{n}$$

and $p = \hat{p} = \frac{x}{n}$

Here x (in the last formula) denotes the number of units in the sample in possession of the attribute under discussion.

SAMPLING

The process of selecting a sample from the given population is called Sampling.

Broadly it is of three types:

(a) Probability sampling

(b) Non-probability sampling

(c) Mixed sampling

(A) PROBABILITY SAMPLING:

1. Simple Random Sampling (SRS)

- Probability-based; each member has a fixed chance to be part of the sample.
- Units selected independently; can be with or without replacement.
- Effective for small, homogenous populations.

Example; There are 1000 students in a college. You assign a number to every student & then randomly select a sample of 100 students

2. Stratified Sampling:

- Suitable for large, heterogeneous populations.
- Divides population into strata; minimal variation within strata.
- Enhances precision, provides representation for all sub-populations.

Example: He say there are 1000 employees out of which 800 are males & 200 females. A sample of 100 employees reflecting gender balance of the company is made by Dividing the employees in males & females then selecting 80 male & 20 female employer.

3. Multi-Stage Sampling: (Cluster Sampling)

- Complex design with multiple stages of sampling units.
- **E.g.:** State, district, police station, household.
- Cost-effective, flexible, and covers a large population, but may be less accurate.

Example: You want to study the behavior of Govt. employees of Nagar Nigam. Nagar Nigam Dept. of each state is a cluster (Sample). Then you select any 3 Department of Nagar Nigam & then select 20 members from each Department. Thus sample of 60 item is formed.

4. Systematic Sampling: (Quasi)

- Units selected at regular intervals after a random start.
- Convenient and less time-consuming; suitable with an updated sampling frame.
- Prone to bias if there is an undetected periodicity in the sampling frame.

Example: There are 1000 employees in a company, they are assigned numbers from 1 to 1000 we randomly select number '6' from first 10 numbers & after that every 10th person is selected that is 6, 16, 26, 36,...

(B) NON-PROBABILITY SAMPLING:

1. Purposive or Judgment Sampling

- Non-probabilistic; dependent on the discretion of the sampler.
- Subjective, varies among individuals; unsuitable for statistical hypothesis testing.

Example: Indian Idol

2. Convenience Sampling:

- Those elements are selected which are easily accessible to researcher.

Example: Asking your students to complete survey regarding services provided by universities.

3. Volunteer response sampling:

- People who are themselves ready to conduct the survey collect the sample Data

4. Snow ball sampling:

- First select some members, then with the help of them select some more & process continues.

(C) MIXED SAMPLING:

1. Combines probabilistic and pre-decided rules.
2. Systematic sampling is an example.

Simple Random Sampling (SRS) Considerations:

1. Effective if the population is not very large, sample size is reasonable, and population is homogenous.
2. Free from sampler's biases; foundational for tests of significance.

Stratified Sampling Considerations:

1. Advisable for large, heterogeneous populations with prior information available.
2. Involves proportional allocation or Bowley's allocation for sample sizes.

Multi-Stage Sampling Coverage:

1. Extensive coverage, computational labor savings, and cost-effectiveness.
2. Adds flexibility but may be less accurate compared to stratified sampling.

Systematic Sampling Drawback:

1. Prone to bias if periodicity in the sampling frame exists.
2. No statistical inference about population parameters due to non-probabilistic nature.

TYPES OF ERRORS IN SAMPLE SURVEY:

There are two types of errors in sample survey:

(a) Sampling errors (b) Non-sampling errors

- Sampling errors are discrepancies between the characteristics of a sample and the larger population, usually due to random chance.
  - (a) **Errors arising out of defective sampling** occur when the sampling method used is flawed, leading to an unrepresentative sample.
  - (b) **Errors arising out due to substitution** involve issues when intended survey participants are replaced with others, impacting data accuracy.
  - (c) **Errors owing to faulty demarcation of units** result from misidentifying or misclassifying survey units, affecting the overall sample.
  - (d) **Errors owing to the wrong choice of statistic** occur when an inappropriate statistical method is used, leading to misleading or inaccurate results.
  - (e) **Variability in the population** is an error related to the natural differences that exist within a population. It arises when the sampled units exhibit greater diversity than expected, leading to less precise estimates.
- Non-sampling errors are inaccuracies in survey data that aren't a result of the sampling process. They encompass a wide range of errors, including coverage errors, nonresponse errors, measurement errors, and processing errors. These errors can be caused by factors such as undercoverage, nonresponse from participants, errors in data collection, and mistakes during data processing. Non-sampling errors can significantly affect the quality and reliability of survey results.

Understanding Sampling Distribution and Standard Error:

Imagine starting with a group of N items. By taking several samples of a fixed size (let's call it n), we can explore the variability in our results.

- Total number of samples with replacement = $(N)^n$
- Total number of Sample without Replacement = ${}^{N}C_{n}$

Sampling Fluctuation:

- Value of Sample Statistics may be different in different samples, this variation is called sampling fluctuation

Sampling Distribution

- We can make many samples of same size with a given population.
- Sample statistic will be different in each sample.
- If this sample statistic is considered as a Random variable we can make probability distribution..
- This probability Distribution is known as sampling Distribution

If we manage to gather all possible values of a statistic (let's call it T) from samples of a fixed size, along with their probabilities, we can create a probability distribution. This distribution, treating the statistic as a random variable, is the sampling distribution of the statistic. Similar to theoretical probability distributions, this sampling distribution has key characteristics. The mean of the statistic in its sampling distribution is known as the "Expectation," and the standard deviation of the statistic (T) is called the "Standard Error (SE)" of T. The SE acts as a measure of precision achieved through sampling and is inversely proportional to the square root of the sample size. This implies that as the sample size increases, the SE decreases, indicating increased precision.

SE is inversely proportional to the square root of sample size.

$$SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}$$ for simple random sampling with replacement

$$= \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$$ for simple random sampling without replacement

STANDARD ERROR FOR PROPORTION

$$SE(p) = \sqrt{\frac{Pq}{n}}$$ for simple random sampling with replacement

$$= \sqrt{\frac{Pq}{n}}\sqrt{\frac{N-n}{N-1}}$$ for simple random sampling without replacement

The factor $\sqrt{\frac{N-n}{N-1}}$ is known as finite population correction (fpc) or finite population multiplier and may be ignored as it tends to 1. If the sample size (n) is very large or the population under consideration is infinite when the parameters are unknown, they may be replaced by the corresponding statistic.

Example 1. Sampling can be described as a statistical procedure **(ICAI)**

(a) To infer about the unknown universe from a knowledge of any sample

(b) To infer about the known universe from a knowledge of a sample drawn from it

(c) To infer about the unknown universe from a knowledge of a random sample drawn from it

(d) Both (a) and (b).

Sol. (c) Sampling involves drawing conclusions about an entire population based on information obtained from a randomly selected sample. This option accurately describes the purpose of statistical sampling.

Hence, the correct option is (c).

Example 2. The basis for making a statistical decision about an unknown universe is:

(a) Sample observations (b) A sampling frame

(c) Sample survey (d) Complete enumeration.

Sol. (a) Sample observations. Statistical decisions about an unknown universe are typically made based on observations from a representative sample rather than examining the entire population.

Example 3. Which sampling method provides flexibility in the sampling process?

(a) Simple random sampling (b) Multistage sampling

(c) Stratified sampling (d) Systematic sampling

Sol. (c) Stratified sampling offers flexibility by dividing the population into subgroups (strata) and then sampling from each stratum separately.

Example 4. Which sampling method is most impacted if the sampling frame reveals an undetected periodicity?

(a) Simple random sampling (b) Stratified sampling

(c) Multistage sampling (d) Systematic sampling

Sol. (d) Systematic sampling is sensitive to periodic patterns in the sampling frame, as it involves selecting every kth element from a list.

Example 5. If a random sample of size 2 with replacement is taken from the population containing the units 3, 6 and 1, then the samples would be **(ICAI)**

(a) (3,6),(3,1),(6,1)

(b) (3,3),(6,6),(1,1)

(c) (3, 3),(3, 6),(3, 1),(6,6),(6,3),(6,1),(1,1),(1,3),(1,6)

(d) (1,1),(1,3),(1,6),(6,1),(6,2),(6,3),(6,6),(1,6),(1,1)

Sol. (c) Given units: 3, 6 and 1

Since, a random sample of size 2 with replacement is taken from the population thus the required samples would be (3, 3),(3, 6),(3, 1),(6, 6),(6, 3),(6, 1),(1, 1),(1, 3),(1, 6)

Example 6. Which sampling method is at the discretion of the sampler?

(a) Systematic sampling (b) Simple random sampling

(c) Purposive sampling (d) Quota sampling

Sol. (c) In purposive sampling, the sampler exercises judgment or purpose in selecting specific elements from the population.

Example 7. If from a population with 25 members, a random sample without replacement of 2 members is taken, the number of all such samples is **(ICAI)**

(a) 300 (b) 625 (c) 50 (d) 600

Sol. (a) Given,

Total members = 25

No. of sample taken = 2

Since, the number of ways to choose 2 members from a population of 25 members without replacement is $^{25}C_2 = \frac{25!}{(25-2)!\times 2!} = \frac{25!}{23!\times 2!} = 300$

Hence, the correct option is (a).

Example 8. A population comprises 5 members. The number of all possible samples of size 2 that can be drawn from it with replacement is **(ICAI)**

(a) 100 (b) 15 (c) 125 (d) 25

Sol. (d) Since, the sampling is done with replacement, each member can be chosen again so the number of possible samples is given by n^K where n is population size and k is the sample size.

According to the given problem,

The required ways = $5^2 = 25$

Hence, the correct option is (d).

Example 9. In the following sampling distribution, the mean and variance is given by

x_i	1	2	3
P_i	0.2	0.5	0.3

(a) 2.1, 4.9 (b) 2.1, 0.49 (c) 4.41, 0.49 (d) None

Sol. (b) According to given data, we have

x_i	p_i	p_ix_i	$p_ix_i^2$
1.	0.2	0.2	0.2
2.	0.5	1.0	2.0
3.	0.3	0.9	2.7
	1	2.1	4.9

Mean of expectation = $\Sigma(x) = \Sigma p_ix_i = 2.1$

Variance = $\Sigma p_ix_i^2 - (\Sigma p_ix_i)^2$

$= 4.9 - (2.1)^2$

$= 4.9 - 4.4$

$= 0.49$

Example 10. In a population of 3 items SD is $\sqrt{\frac{8}{3}}$. If sample of 2 items are made

(i) with Replacement, find standard error of mean

(ii) without replacement, find statement error of mean

Sol.

i) with replacement

$N = 3, n = 2, \sigma = \sqrt{\frac{8}{3}} = \frac{2\sqrt{2}}{\sqrt{3}}$

$S.E(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{2\sqrt{2}}{3} \times \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{3}}$

ii) without replacement

$N = 3, n = 2, \sigma = \sqrt{\frac{8}{3}} = \frac{2\sqrt{2}}{\sqrt{3}}$

$S.E(\bar{x}) \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$

$= \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{1}{\sqrt{2}} \times \sqrt{\frac{3-2}{3-1}}$

$= 2\sqrt{\frac{2}{3}} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \sqrt{\frac{2}{3}}$

PRACTICE QUESTIONS (PART A):

1. The population of roses in Salt Lake City is an example of **(ICAI)**

(a) A finite population (b) An infinite population

(c) A hypothetical population (d) An imaginary population.

2. As the sample size increases, standard error **(ICAI)**

(a) Increases (b) Decreases

(c) Remains constant (d) Decreases proportionately.

3. Which sampling method involves dividing the population into clusters and then randomly selecting entire clusters for the sample?

(a) Simple random sampling (b) Systematic sampling

(c) Cluster sampling (d) Stratified sampling

4. A parameter is a characteristic of **(ICAI)**

(a) Population (b) Sample

(c) Both (a) and (b) (d) (a) or (b)

5. A statistic is **(ICAI)**

(a) A function of sample observations

(b) A function of population units

(c) A characteristic of a population

(d) A part of a population

6. From a group of 30 individuals, a committee of 4 is to be formed without replacement. How many different committees can be formed?

(a) 435 (b) 27,405 (c) 27,300 (d) 29,160

7. A box contains 6 distinct items. If 3 items are randomly selected with replacement, how many different sets of items can be chosen?

(a) 216 (b) 120 (c) 729 (d) 15

8. In a survey of 50 households, a sample of 10 households is randomly chosen without replacement. How many different samples are possible?

(a) 1,102,100 (b) 102,722,500

(c) 102,722 (d) 10,000

9. A population consists of 8 distinct elements. If a sample of 3 elements is drawn with replacement, how many different samples can be formed?

(a) 512 (b) 64 (c) 1,024 (d) 125

10. If a random sample of size two is drawn without replacement from a population containing the units a, b, c, and d, what are the possible samples? **(ICAI)**

(a) (a, b), (a, c), (a, d)

(b) (a, b), (b, c), (c, d)

(c) (a, b), (b, a), (a, c), (c, a), (a, d), (d, a)

(d) (a, b), (a, c), (a, d), (b, c), (b, d), (c, d)

11. Which sampling is based on the discretion of the sampler? **(June 2024)**

(a) Systematic (b) Multi-stage (c) Stratified (d) Purposive

12. Which of the following is not a type of sampling? **(June 2024)**

(a) Probability (b) Non-Probability

(c) Stand-alone (d) Mixed

13. What is the purpose of stratified random sampling? **(Sept 2024)**

(a) To ensure that every individual in the population has an equal chance of being selected.

(b) To divide the population into subgroups and then randomly sample from each subgroup.

(c) To select individuals based on the availability and convenience.

(d) To select a fixed percentage of the population without any specific criteria.

14. Exit polls are an example of which method of collecting data? **(Sept 2024)**

(a) Random sampling (b) Investigation

(c) Census (d) Quota Sampling

15. A population comprises 7 members. The number of all possible samples of size 3 that can be drawn from it with replacement is **(Jan 2025)**

(a) 343 (b) 216 (c) 21 (d) 125

16. Standard Error (SE) and square root of sample size are **(Jan 2025)**

(a) Equal (b) Directly proportional

(c) Inversely proportional (d) Not equal

17. Non-probability Sampling is known as **(Jan 2025)**

(a) Simple Random Sampling (b) Stratified Sampling

(c) Purposive or Judgement Sampling (d) Cluster Sampling

18. A sample of 100 people is taken from a population of 1000. The sample mean height is 170 cm with a standard deviation of 10 cm. What is the standard error of mean? **(Jan 2025)**

(a) 1.0 cm (b) 0.5 cm (c) 1.58 cm (d) 10 cm

19. Which sampling technique is most appropriate when a person wants to ensure that subgroups are proportionally represented? **(Jan 2025)**

(a) Simple Random Sampling (b) Stratified Sampling

(c) Multipstage Sampling (d) Systematic Sampling

20. A population comprises 5 members. The number of possible samples of size 2, that can be drawn from it with replacement is **(May 2025)**

(a) 100 (b) 15 (c) 125 (d) 25

21. Which of the following statements about simple random sampling is not true? **(May 2025)**

(a) Simple random sampling ensures that each unit in the population has an equal chance of being selected.

(b) In simple random sampling with replacement, each selected unit is replaced to the population before the next unit is drawn.

(c) Simple random sampling is highly effective when the population is very large and heterogeneous.

(d) In a simple random sampling without replacement, a unit is selected, it will never be selected again.

22. The law of statistical regularity says that **(May 2025)**

(a) Sample drawn from the population under discussion possesses the characteristics of population.

(b) A large sample drawn at random from the population would possess the characteristics of the population.

(c) A large sample drawn at random from the population would possess the characteristics of then population on an average.

(d) An optimum level of efficiency can be attained at a minimum cost.

23. Sampling method which belongs to the category of Mixed sampling is **(Sept 2025)**

(a) Systematic Sampling (b) Simple Random Sampling

(c) Stratified Sampling (d) Multi-stage Sampling

24. After a singing competition on a live TV show, the winner is selected according to the number of likes each candidate has received on a messaging app. This method of data collection is known as _____ sampling. **(Sept 2025)**

(a) Random (b) Probabilistic (c) Purposive (d) Multi-Stage

25. Which of the followings is not a basic principle of sample survey? **(Sept 2025)**

(a) Principle of Inertia (b) Principle of Optimization

(c) Principle of Large Numbers (d) Law of Statistical Regularity

Answer Key

1. (b)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (d)
11. (d)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (d)
21. (c)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (c)					

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CA WALLAH

CHAPTER

14

Measures of Central Tendency and Dispersion

Unit-1: Measures of Central Tendency

WHAT IS CENTRAL TENDENCY?

Let's take an example, if you talk about the weight or height of students in your class, what would you observe?

As we can see the majority of data tends to cluster towards the center which we call the central tendency and the value which represents the center is called as measure of central tendency.

In the simplest cases, the measure of central tendency is an average of a set of measurements.

That's why, when our marks in exams come – we see average marks of students or when we go to work – we see the average salary of employees.

Central Tendency refers to the tendency of data to cluster around a central or typical value. It represents the value that best represents the center of a distribution. In other words, it gives us an idea of where most of the data points are concentrated.

LET'S SEE SOME DIFFERENT MEASURES OF CENTRAL TENDENCY

- **Arithmetic Mean (AM):** It is also known as the average, which is the sum of all the values divided by the total number of values.
- **Median (Me):** It is the middle value in a sorted dataset. It divides the dataset into two equal halves, with half of the values below and half above the median.
- **Mode (Mo):** It is the value that occurs most frequently in a dataset. It represents the value that appears with the highest frequency.
- **Geometric Mean (GM):** It is the nth root of the product of n values. It is often used for calculating growth rates or when dealing with exponential data.
- **Harmonic Mean (HM):** It is the reciprocal of the arithmetic mean of the reciprocals of a set of values. It is useful when dealing with rates, ratios, or averages of rates.

  Now, let's understand all the above measures of central tendency in detail.

ARITHMETIC MEAN

$$\text{Arithmetic Mean (Mean)} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

Let's variable x has n values: $x_1, x_2, x_3, x_4, x_5, \ldots\ldots x_n$, then Arithmetic Mean is denoted by $\bar{x}$, which is equal to $\frac{x_1 + x_2 + \ldots.. + x_n}{n}$

Example 1. The following figures give the marks of 10 students in a class test:
Marks obtained: 12, 8, 17, 13, 15, 9, 18, 11, 6, 1. Find the arithmetic mean.

(a) 8 (b) 9 (c) 10 (d) 11

Sol. (d) To find Arithmetic Mean,

As we know, Arithmetic Mean is given by the formula,

$$\text{A.M} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Given Observations: 12 8 17 13 15 9 18 11 6 1

Here, number of observations = 10

$$\text{Arithmetic Mean (A.M)} = \frac{12+8+17+13+15+9+18+11+6+1}{10} = \frac{110}{10} = 11$$

Hence, the correct option is (d).

Example 2. Following are the Monthly salary in multiple of 1000 INR of a sample of 10 employees: 41, 32, 45, 65, 67, 78, 90, 75, 33, 44. Compute the mean wage (in 1000 INR).

(a) 40 (b) 45 (c) 57 (d) 59

Sol. (c) As per the definition,

$$\text{A.M} = \frac{x_1 + x_2 + ... + x_n}{n} \text{ or A.M} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Given Observations: 41, 32, 45, 65, 67, 78, 90, 75, 33, 44

Here, number of observations = 10

According to the formula, Arithmetic mean (A.M). is given by

$$\text{A.M} = \frac{x_1 + x_2 + ... + x_n}{n}$$

$$\bar{x} = \frac{41+32+45+65+67+78+90+75+33+44}{10}$$

$$\bar{x} = \frac{570}{10}$$

$$\bar{x} = 57$$

Hence, the correct option is (c).

ANOTHER METHODS TO FIND MEAN

1. **Direct method:** If $x_1, x_2, x_3, ... x_n$ be n observations with respective frequencies $f_1, f_2, f_3, ... f_n$, then

$$\text{Mean} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + ... + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

2. **Step-deviation method:** When the data values are quite large, the step-deviation method is used to find the mean.

$$\text{Mean} = a + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h$$

where, a = assumed mean

x_i = class-marks = $\frac{\text{upper limit class} + \text{Lower limit class}}{2}$

$d_i = x_i - a$ = deviation from i^{th} class

$u_i = \frac{x_i - a}{h}$ and h = length of class

Example 3. In a test of 10 marks, 100 students get marks as below:

Marks Obtained	Number of Students
1	2
2	3
3	5
4	10
5	30
6	10
7	18
8	12
9	6
10	4

What is the average marks obtained?

(a) 4.89 (b) 5.89 (c) 5.79 (d) 6.99

Sol. (b) Let Marks obtained by students be x_i and frequency be represented as f_i

Let the assumed mean, a = 5

Using Shortcut method,

Marks Obtained (x_i)	Number of Students (f_i)	$x_i \times f_i$
1	2	2
2	3	6
3	5	15
4	10	40
5	30	150
6	10	60
7	18	126
8	12	96
9	6	54
10	4	40
	$\sum f_i = 100$	$\sum = 589$

Now, Arithmetic mean will be given as,

$A.M = \frac{589}{100} = 5.89$

Hence, the correct option is (b).

Example 4. The following data give the daily earnings (in ₹) of 20 workers in a factory:

Daily Earnings (in ₹)	100	140	170	200	250
No. of Workers	5	2	6	4	3

Calculate the average daily earnings using:

(I) Direct Method

(II) Step-deviation Method.

(a) 167.5 (b) 165.85 (c) 510.70 (d) 166.5

Sol. (I) As per the definition direct method is given as,

$A.M = \frac{x_1 + x_2 + \ldots + x_n}{n}$ or $A.M = \frac{\text{Sum of all observations}}{\text{Number of observations}}$

Number of observations = 20

Observations = 100, 100, 100, 100, 100, 140, 140, 170, 170, 170, 170, 170, 170, 200, 200, 200, 250, 250, 250

Now, Arithmetic Mean (A.M) will be,

$A.M = \frac{5\times100+2\times140+6\times170+4\times200+3\times250}{20} = \frac{3350}{20} = 167.5$

(II) As per the definition, step-deviation method is given as,

$\text{Mean} = a + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h$

Daily Earnings (x_i)	Number of Workers (f_i)	$x_i - a$	$u_i = \frac{xi - a}{h}$ where, h = 10	$f_i \times u_i$
100	5	100 – 170 = -70	–7	–35
140	2	140 – 170 = -30	–3	–6
170	6	170 – 170 = 0	0	0
200	4	200 – 170 = 30	3	12
250	3	250 – 170 = 80	8	24
	$\Sigma f_i = 20$			$\Sigma f_i \times u_i = -5$

$\text{Mean} = a + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h = 170 + \frac{(-5)}{20} \times 10 = 170 - 2.5 = 167.5$

Hence, the correct option is (a).

Example 5. Compute the mean weight of your friends' group:

Weight in kg	41 – 45	46 – 50	51 – 55	56 – 60	61 – 65	66 – 70
No. of Friends:	4	3	9	8	7	2

(a) 55 (b) 54.2727 (c) 56.7272 (d) 59.2882

Sol. (b) Represent the given data in tabular form,

Weight in kg	Number of Friends (f_i)	Mid Points (x_i)	$f_i \times x_i$
41 – 45	4	43	$4 \times 43 = 129$
46 – 50	3	48	$3 \times 48 = 144$
51 – 55	9	53	$9 \times 53 = 477$
56 – 60	8	58	$8 \times 58 = 464$
61 – 65	7	63	$7 \times 63 = 441$
66 – 70	2	68	$2 \times 68 = 136$
	$\Sigma f_i = 33$		$\Sigma f_i x_i = 1791$

Now, Mean will be given as, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\bar{x} = \frac{1791}{33} = 54.2727$

Hence, the correct option is (b).

Example 6. Given below is the distribution of marks obtained by 140 students in an examination:

Marks	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of Students	7	15	18	25	30	20	16	7	2

(a) 67.5 (b) 56.85 (c) 51.070 (d) 50.714

Sol. (d) Represent the given data in tabular form,

Marks	Number of Students (f_i)	Mid Points (x_i)	$f_i \times x_i$
10 - 19	7	14.5	$7 \times 14.5 = 101.5$
20 - 29	15	24.5	$15 \times 24.5 = 367.5$
30 - 39	18	34.5	$18 \times 34.5 = 621$
40 - 49	25	44.5	$25 \times 44.5 = 1112.5$
50 - 59	30	54.5	$30 \times 54.5 = 1635$
60 - 69	20	64.5	$20 \times 64.5 = 1290$
70 - 79	16	74.5	$16 \times 74.5 = 1192$
80 - 89	7	84.5	$7 \times 84.5 = 591.5$
90 - 99	2	94.5	$2 \times 94.5 = 189$
	$\Sigma f_i = 140$		$\Sigma f_i x_i = 7100$

Now, Mean will be given as, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\bar{x} = \frac{7100}{140}$

$\bar{x} = 50.714$

Hence, the correct option is (d).

Example 7. Calculate mean from the following data:

Marks	No. of Students
0 - 10	4
10 - 20	16
20 - 30	40
30 - 40	76
40 - 50	96
50 - 60	112
60 - 70	120
70 - 80	125

(a) 50.20 (b) 53.60 (c) 51.70 (d) 66.5

Sol. (b) According to the given data, we get the following:

Marks	Number of Students (f_i)	Mid Points (x_i)	$f_i \times x_i$
0-10	4	5	$4 \times 5 = 20$
10 - 20	16	15	$16 \times 15 = 240$
20 - 30	40	25	$40 \times 25 = 1000$
30 - 40	76	35	$76 \times 35 = 2660$
40 - 50	96	45	$96 \times 45 = 4320$
50 - 60	112	55	$112 \times 55 = 6160$
60 - 70	120	65	$120 \times 65 = 7800$
70 - 80	125	75	$125 \times 75 = 9375$
	$\Sigma f_i = 589$		$\Sigma f_i x_i = 31575$

Now, Mean will be given as,

$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\bar{x} = \frac{31575}{589}$

$\bar{x} = 53.607 \approx 53.60$

Hence, the correct option is (b).

Example 8. The mean of the following frequency distribution is 50 but the frequencies f_1 and f_2 in classes 20 - 40 and 60 - 80 are missing. Find the missing frequencies.

Class	0 - 20	20 - 40	40 – 60	60 – 80	80 – 100	Total
Frequency	17	f_1	32	f_2	19	120

(a) 28, 24 (b) 26, 26 (c) 22, 30 (d) 32, 20

Sol. (a) Given: Mean $(\bar{x})$ = 50

$\Sigma f_i = 120$

Represent the data in tabular form,

Classes	Frequency (f_i)	Mid Points (x_i)	$f_i \times x_i$
0 - 20	17	10	$17 \times 10 = 170$
20 - 40	f_1	30	$f_1 \times 30 = 30\, f_1$
40 – 60	32	50	$32 \times 50 = 1600$
60 – 80	f_2	70	$f_2 \times 70 = 70\, f_2$
80 – 100	19	90	$19 \times 90 = 1710$
	$\Sigma f_i = 68 + f_1 + f_2$		$\Sigma f_i d_i = 3480 + 30f_1 + 70f_2$

According to the question, $\Sigma f_i = 120$

$\Rightarrow 120 = 68 + f_1 + f_2$

$\Rightarrow f_1 + f_2 = 52$ ...(i)

Now, Mean will be given as, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$$50 = \frac{3480 + 30f_1 + 70f_2}{120}$$

$6000 = 3480 + 30f_1 + 70f_2$

On simplifying it, we get

$3f_1 + 7f_2 = 252$ ...(ii)

On solving, multiply equation (i) by 3 and subtract (i) and (ii), we get

$3f_1 + 3f_2 = 156$

$3f_1 + 7f_2 = 252$

$- \quad - \quad -$

$-4f_2 = -96$

$$f_2 = \frac{96}{4}$$

$f_2 = 24$

Put $f_2 = 24$ in (i), we get

$f_1 + 24 = 52$

$f_1 = 52 - 24$

$f_1 = 28$

So, $f_1 = 28$, $f_2 = 24$

Hence, the correct option is (a).

PROPERTIES OF ARITHMETIC MEAN

1. If all the observation of the dataset are constants, let 'a', then the arithmetic mean will also be equal to 'a'.

   E.g.: If the weight of each student of group of 6 students of grade 10 is 60 kg, then the mean weight will also be 60 kg.

2. The algebraic sum of deviations of the given set of observations from their arithmetic mean is zero.

   That is, $(x - \bar{x}) = 0$ or for a frequency distribution $\Sigma f(x - \bar{x}) = 0$

   E.g.: For the given set of observations 2, 3, 4, 5, 6 then, $\bar{x} = \frac{2+3+4+5+6}{5} = 4$

   Thus, $\Sigma (x - \bar{x}) = (2 - 4) + (3 - 4) + (4 - 4) + (5 - 4) + (6 - 4)$

   $= -2 - 1 + 0 + 1 + 2 = 0$

   i.e., $\Sigma (x - \bar{x}) = 0$

3. If each observation of a dataset is increased or decreased by a constant 'k', then the arithmetic mean of new data will also gets increased or decreased by the same constant 'k'.

   E.g.: The arithmetic mean of 5 observations: 5, 10, 15, 20 and 25 is 15. If each observations is increased by 3, then the new mean will also increase by 3 i.e., new arithmetic mean = 15 + 3 = 18

4. AM is affected with the change of origin and scale i.e., if the variable 'x' is changed to a new variable 'y' by a change of origin 'a' and scale 'b'

   $\Rightarrow y = a + bx$, then

   $AM_y = a + b \times AM_x$ or $\bar{y} = a + b\bar{x}$

   E.g.: If x and y are related by the equation $3x + y = 6$ and $\bar{x} = 1$ then

   $\bar{y} = 6 - 3\bar{x}$

   $\Rightarrow \bar{y} = 6 - 3(1) \Rightarrow \bar{y} = 3$

5. **Combined mean:** If the arithmetic means of two or more sets of data are known, then the arithmetic mean of the combined data can be obtained.

   If n_1 and n_2 are the number of observations and $\bar{x}_1$ and $\bar{x}_2$ are their respective means of two sets of data then the combined mean with observations is given by $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

Example 9. The sum of deviations of a certain number of observations measured from 4 is 72 and the sum of observations of the same value from 7 is -3. Find the number of observations and their mean.

(a) 24, 7.5 (b) 24, 6.77 (c) 25, 6.88 (d) 25, 6.5

Sol. (c) Given: Sum of deviations from 4 = 72

Sum of observations of the same value from 7 = –3

To find: Number of observations and Mean

Let n be the number of observations,

According to the question,

$(x_1 - 4) + (x_2 - 4) + ... + (x_n - 4) = 72$

and, $(x_1 - 7) + (x_2 - 7) + ... + (x_n - 7) = 3$

So, $(x_1 + x_2 + ... + x_n) - 4n = 72$

$\Rightarrow (x_1 + x_2 + ... + x_n) = 72 + 4n$

$\Rightarrow \Sigma x_1 = 72 + 4n$ ...(i)

Also, $(x_1 - 7) + (x_2 - 7) + ... + (x_n - 7) = -3$

$\Rightarrow (x_1 + x_2 + ... + x_n) - 7n = -3$

$\Rightarrow (x_1 + x_2 + ... + x_n) = -3 + 7n$

$\Rightarrow \Sigma x_i = 7n - 3$ ...(ii)

On equating (i) and (ii), we get

$\Rightarrow 72 + 4n = 7n - 3$

$\Rightarrow 7n - 4n = 72 + 3$

$\Rightarrow 3n = 75$

$\Rightarrow n = 25$

Now, Mean is given as,

$$\text{Mean} = \frac{\Sigma x_i}{\Sigma n}$$

Put n = 25 in order to find mean

$$= \frac{7n-3}{n} = \frac{7(25)-3}{25} = \frac{172}{25} = 6.88$$

Hence, the correct option is (c).

Example 10. The arithmetic mean of a set of 5 observations 5, 10, 15, 20 and 25 is 15. However, if each item is increased by 3, then the arithmetic mean will be

(a) 18 (b) 18.5 (c) 19 (d) 19.5

Sol. (a) **Detailed method:** According to the question,

Old Observations: 5, 10, 15, 20, 25

Mean = 15

If each observation is increased by 3, then

New observations: 8, 13, 18, 23, 28

Mean is given by the formula, Mean $= \frac{\sum x_i}{n}$

$\Rightarrow$ Mean $= \frac{8+13+18+23+28}{5}$

$\Rightarrow$ Mean $= \frac{90}{5}$ $\Rightarrow$ Mean = 18

Short-cut method: Since, the arithmetic mean of the given set of observations is 15. Now, each observation is increased by 3, thus the mean will also increase by 3.

Therefore, new mean = 15 + 3 = 18

Hence, the correct option is (a).

Example 11. The arithmetic mean of a set of 5 observations: 2, 4, 6, 8, 10 is 6. If each item is multiplied by 2, then the new arithmetic mean will be?

(a) 18 (b) 12 (c) 15 (d) 20

Sol. (b) **Detailed method:** According to the question,

Old Observations: 2, 4, 6, 8, 10

Mean = 6

If each observation is multiplied by 2, then

New observations: 4, 8, 12, 16, 20

Mean is given by the formula, Mean $= \frac{\sum x_i}{n}$

$\Rightarrow$ Mean $= \frac{4+8+12+16+20}{5}$

$\Rightarrow$ Mean $= \frac{60}{5}$

$\Rightarrow$ Mean = 12

Short-cut method: Since, the arithmetic mean of the given set of observations is 6.

Now, each observation gets multiplied by 2, thus the mean will also be multiplied by 2.

Therefore, new mean = 6 × 2 = 12

Hence, the correct option is (b).

Example 12. If it is known that 2 variables x and y are related by equation 2x + 3y = 5 and $\bar{x} = 1$, then $\bar{y}$ is

(a) 1 (b) 0 (c) 5 (d) 2

Sol. (a) Given equation: 2x + 3y = 5; $\bar{x} = 1$

Put $\bar{x} = 1$ in the above equation, we get

$2(1) + 3\bar{y} = 5$

$\Rightarrow 2 + 3\bar{y} = 5$

$\Rightarrow 3\bar{y} = 3 \Rightarrow \bar{y} = 3$

Hence, the correct option is (a).

Example 13. The mean salary for a group of 40 female workers is ₹52000 per month and that for a group of 50 male workers is ₹68000 per month. What is the combined mean salary?

(a) ₹65000 (b) ₹60888.88 (c) ₹51070 (d) ₹16650

Sol. (b) Given,

Number of Females $(n_1) = 40$

Mean salary of Females $(\bar{x}_1) =$ ₹52000

Number of Males $(n_2) = 50$

Mean Salary of Males $(\bar{x}_2) =$ ₹68000

We know, $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$\therefore$ Combined Salary $\bar{x} = \dfrac{(40)(52000) + (50)(68000)}{40 + 50} = \dfrac{(2080000) + (3400000)}{90}$

$= 60{,}888.88$

Hence, the correct option is (b).

Example 14. A distribution consists of three components with total frequencies of 200, 250 and 300 having means 25, 10 and 15 respectively. Find the mean of the combined distribution.

(a) 4 (b) 8 (c) 12 (d) 16

Sol. (d) Tabulate the data,

Let the frequencies be f_1 and the mean be x_i

f_i	x_i	$f_i \times x_i$
200	25	$200 \times 25 = 5000$
250	10	$250 \times 10 = 2500$
300	15	$300 \times 15 = 4500$
$\Sigma f_i = 750$		$\Sigma f_i \times x_i = 12000$

Now, Mean of Combined distribution will be given as,

$$= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{12000}{750} = 16$$

Hence, the correct option is (d).

Example 15. The mean weight of 150 students (boys and girls) in a class is 60 kg. The mean weight of boy students is 70 kg and that of girl students is 55 kg. The number of boys and girls respectively in the class are

(a) 50, 100 (b) 100, 50 (c) 120, 30 (d) 30, 120

Sol. (a) Let the number of boys be denoted as x and the number of girls be denoted as y.

As we know, Mean is given by the formula,

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Thus, Sum of all observations = Mean × Number of observations

Total number of students = x + y

$\therefore$ x + y = 150

$\Rightarrow$ x = 150 – y ...(i)

Now, Sum of weight of boys = '70x' kg and Sum of weight of girls = '55y' kg

Total weight of students (including boys and girls) = 150 × 60 = 9000 kg

$\Rightarrow$ 70x + 55y = 9000 ...(ii)

Now, on solving (i) and (ii), we get

70(150 – y) + 55y = 9000

$\Rightarrow$ 10500 – 70y + 55y = 9000

$\Rightarrow$ 15y = 1500

$\Rightarrow$ y = 100

Put y = 100 in (i), we get

$\Rightarrow$ x = 150 – 100

$\Rightarrow$ x = 50

So, number of Boys = 50 and number of girls = 100

Hence, the correct option is (a).

Example 16. Fifty students took up a test. The result of those who passed the test is given below:

Marks	4	5	6	7	8	9
Number of Students:	8	10	9	6	4	3

If the average for all 50 students was 5.16 marks, find the average of those who failed.

(a) 2.1 (b) 3.9 (c) 1.1 (d) 3.3

Sol. (a) According to the question,

Mean marks of 50 students = 5.16

Tabulate the data,

Let the marks be x_i and the number of students be f_i

x_i	f_i	$f_i \times x_i$
4	8	4 × 8 = 32
5	10	5 × 10 = 50
6	9	6 × 9 = 54
7	6	7 × 6 = 42
8	4	8 × 4 = 32
9	3	9 × 3 = 27

As per the question,

Let the average mark of failed student be y.

Let the number of failed students be F.

Now, number of failed students will be given by,

$\Rightarrow 50 - (8 + 10 + 9 + 6 + 4 + 3) = F$

$\Rightarrow F = 50 - 40$

$\Rightarrow F = 10$

So, According to the question,

$$\Rightarrow \frac{(32 + 50 + 54 + 42 + 32 + 27 + 10y)}{50} = 5.16$$

$\Rightarrow 237 + 10y = 258$

$\Rightarrow 10y = 21$

$\Rightarrow y = 2.1$

Hence, the correct option is (a).

CORRECTING INCORRECT MEAN

Example 17. The mean marks of 100 students were found to be 40. Later on, it was discovered that a score of 53 was misread as 83. Find the correct mean corresponding to the correct score.

(a) 67.5 (b) 39.7 (c) 51.70 (d) 66.5

Sol. (b) According to the question,

Initial mean (Incorrect mean) = 40

Number of observations (n) = 100

Wrong observation (x_w) = 83 and Right observation (x_r) = 53

Now, Incorrect mean, $\bar{x}_{in} = \frac{\sum x_w}{n}$

$$\Rightarrow 40 = \frac{\sum x_w}{100}$$

$$\Rightarrow \sum x_w = 4000 \quad \ldots(i)$$

Now, $\sum x_r = \sum x_w - x_w + x_r = 4000 - 83 + 53 = 3970$

Thus, Correct mean, $\bar{x}_c = \frac{\sum x_r}{n} = \frac{3970}{100} = 39.7$

Hence, the correct option is (b).

Note: We can use the direct formula for the above:

i.e., $\bar{x}_c = \bar{x}_{in} + \frac{x_c - x_{in}}{n}$,

where, $\bar{x}_c$ represents the correct mean, $\bar{x}_{in}$ is the incorrect mean, x_c is the correct observation and x_{in} is the incorrect observation.

Example 18. The mean salary paid to 1000 employees of an establishment was found to be ₹180.40. Later on, after disbursement of salary, it was discovered that the salary of the two employees was wrongly entered as ₹297 and ₹165. Their correct salaries were ₹197 and ₹185. Find the correct mean salary.

(a) 168.5 (b) 179.85 (c) 510.70 (d) 180.32

Sol. (d) According to the question,

Initial mean (Incorrect mean) = 180.40

Number of employees (n) = 1000

Wrong observations (x_w) = 297, 165 and Right observations (x_r) = 197, 185

Now, Incorrect mean, $\bar{x}_{in} = \frac{\sum x_w}{n}$

$\Rightarrow 180.40 = \frac{\sum x_w}{1000}$

$\Rightarrow \sum x_w = 180400$ …(i)

Now, $\sum x_r = \sum x_w - x_w + x_r = 180400 - 297 - 165 + 197 + 185 = 180320$

Thus, Correct mean, $\bar{x}_c = \frac{\sum x_r}{n} = \frac{180320}{1000} = 180.320$

Hence, the correct option is (d).

WEIGHTED ARITHMETIC MEAN

Let $w_1, w_2, \ldots\ldots w_n$ be the weights attached to n observations $x_1, x_2, \ldots x_n$ respectively then the weighted arithmetic mean, denoted by $\bar{x}_w$ is given by

Weighted mean $= \frac{\sum x_i w_i}{\sum w_i}$

Example 19. A candidate obtained the following percentage of marks in different subjects in the Half-yearly Examination: English 46%, Statistics 67%, Cost Accountancy 72%, Economics 58%, Income Tax 53%. It is agreed to give double weights to marks in English and Statistics as compared to other subjects. What is the simple and weighted arithmetic mean?

(a) 56.4, 59.42 (b) 58.42, 59.2 (c) 59.2, 58.42 (d) None of these

Sol. (a) Let N be the number of subjects i.e., 5.

Tabulate the data and solve,

According to the question, it is agreed to give double weights to marks in English and Statistics as compared to other subjects

So, weights of English and Statistics = 2 and weights of other subjects = 1

Subject	Marks (x)	Weight (w)	w × x
English	46	2	92
Statistics	67	2	134
Cost Accountancy	72	1	72
Economics	58	1	58
Income Tax	53	1	53
	$\Sigma x = 296$	$\Sigma w = 7$	$\Sigma wx = 409$

So, Simple Mean will be given as

$$\bar{x} = \frac{\sum x}{N} = \frac{296}{5} = 59.2$$

Now, weighted arithmetic mean will be given by the formula,

$$\overline{X_w} = \frac{\sum wx}{\sum w} = \frac{409}{7} = 58.42$$

Hence, the correct option is (c).

Example 20. Measures of central tendency for a given set of observations measures

(a) The scatterness of the observations (b) The central location of the observations

(c) Both (a) and (b) (d) One of these

Sol. (b) We know that,

Measures of central tendency for a given set of observations measures **the central location of the observations.**

Measures of central tendency is a mathematical concept that measures the central most value of a given series which can represent the whole set of the series. In the given series, the central location of the observation is measured through different tools in order to ascertain the most efficient value that can represent the series.

Hence, the correct option is (b).

Example 21. While computing the AM from a grouped frequency distribution, we assume that

(a) The classes are of equal length

(b) The classes have equal frequency

(c) All the values of a class are equal to the mid-value of that class

(d) None of these

Sol. (c) While computing the AM from a grouped frequency distribution, we assume that all the values of a class are equal to the mid-value of that class.

Hence, the correct option is (c).

Example 22. Which of the following statements is wrong?

(a) Mean is rigidly defined

(b) Mean is not affected due a sampling fluctuations

(c) Mean has some mathematical properties

(d) All of these

Sol. (b) Mean is not affected due to sampling fluctuations is a wrong statement.

The best measure of central tendency, usually, is the AM. It is rigidly defined, based on all the observations, easy to comprehend, simple to calculate and amenable to mathematical properties.

However, AM has one drawback in the sense that it is very much affected by sampling fluctuations.

Hence, the correct option is (b).

Example 23. If there are 3 observations 15, 20, 25 then the sum of deviation of the observations from their AM is

(a) 0 (b) 5 (c) -5 (d) None of these

Sol. (a) Given observations: 15, 20, 25

Firstly, we will find AM of the given data

$$AM = \frac{15 + 20 + 25}{3}$$

$$AM = \frac{60}{3}$$

AM = 20

Thus, the deviation of the observations from their AM is

15 – 20, 20 – 20, 25 – 20 i.e. –5, 0, 5

Therefore, the sum of deviation = –5 + 0 + 5 = 0

Hence, the correct option is (a).

Example 24. If there are two groups containing, 30 and 20 observations and having 50 and 60 as arithmetic means, then the combined arithmetic mean is

(a) 55 (b) 56 (c) 54 (d) 52

Sol. (c) Given: $\bar{x}_1 = 50$, $n_1 = 30$, $\bar{x}_2 = 60$ and $n_2 = 20$

The combined arithmetic mean is

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{30(50) + 20(60)}{30 + 20}$$

$$= \frac{1500 + 1200}{50} = \frac{2700}{50} = 54$$

Hence, the correct option is (c).

Example 25. The average salary of a group of unskilled workers is ₹10,000 and that of a group of skilled workers is ₹15,000. If the combined salary is ₹12,000, then what is the percentage of skilled workers?

(a) 40% (b) 50% (c) 60% (d) None of these

Sol. (a) Let the percentage of skilled worker be x, so unskilled worker will be (100 – x).

Therefore $n_1 = x, n_2 = 100 - x, \bar{x}_i = 12000$

$\bar{x}_1 = 15000$, $\bar{x}_2 = 10000$

$$\Rightarrow \bar{x}_i = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\Rightarrow 12000 = \frac{x(15000) + (100 - x)(10000)}{\{x + 100 - x\}}$$

$\Rightarrow$ 1200000 = 15000x + 1000000 – 10000x

$\Rightarrow$ 200000 = 5000x

$\Rightarrow$ x = 40%

Hence, the correct option is (a).

Example 26. If the relationship between two variables u and v are given by 2u + v + 7 = 0 and if the AM of u is 10, then the AM of v is **(ICAI)**

(a) 17 (b) –17 (c) –27 (d) 27

Sol. (c) Given,

Relationship between u and v is given by

$2u + v + 7 = 0$

AM of u = 10

$\Rightarrow v = -2u - 7$

Put u = 10 in the above equation to get AM of v, i.e.

$\bar{v} = -2(10) - 7$

$= -20 - 7 = -27$

Therefore, AM of v is –27.

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART A)

1. The weights of a group of individuals are recorded in kilograms: 65, 70, 75, 80, 85, 90, 95. Compute the mean weight.

(a) 75 kg (b) 80 kg (c) 85 kg (d) 90 kg

2. Following are the daily wages in thousands of a sample of workers: ₹58, ₹62, ₹48, ₹53, ₹70, ₹52, ₹60, ₹84, ₹75. Compute the mean wage.

(a) ₹60,000 (b) ₹61,740 (c) ₹62,440 (d) None of these

3. Find the mean salary of 60 workers in a factory from the following table.

Salary in ₹	Number of Workers
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1

(a) ₹5065.50 (b) ₹6000 (c) ₹5083.33 (d) None of the above

4. If the mean of the following distribution is 6, then the value of P is

X	2	4	6	10	P + 5
f	3	2	3	1	2

(a) 7 (b) 5 (c) 11 (d) 8

5. The mean of 6, 4, 1, 5, 6, 10 and 3 is 5. If each number is added with 2, then the new mean is _____.

(a) 7 (b) 5 (c) 6 (d) 10

6. The mean of a set of numbers is 8. If each number is multiplied by 3, then the new mean is _____.

(a) 24 (b) 8 (c) 3 (d) 12

7. What is the value of mean for the following data: **(ICAI)**

Marks	5 – 14	15 – 24	25 – 34	35 – 44	45 – 54	55 – 64
No. of Students	10	18	32	26	14	10

(a) 30 (b) 29 (c) 33.68 (d) 34.21

8. Calculate the arithmetic mean from the following frequency distribution:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	10	9	25	30	16	10

(a) 37 (b) 38.5 (c) 31.3 (d) 30

9. Given below is the distribution of marks obtained by 140 students in an examination:

Marks	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of Students	7	15	18	25	30	20	16	7	2

Find the mean of the distribution.

(a) 52.83 (b) 55.2 (c) 63.5 (d) None of these

10. Find the combined mean from the following data:

	Series X	Series Y
Arithmetic mean	12	20
Number of items	80	60

(a) 10.54 (b) 15.43 (c) 3.50 (d) None of these

11. The mean of 20 observations is 85, but it was later found that two of the observations were wrongly read as 75 and 70 instead of 57 and 60. Find the correct mean.

(a) 80.7 (b) 75.5 (c) 63.5 (d) 83.6

12. Calculate weighted mean from the following data:

Value	10	12	15	18	20
Weight	2	5	12	4	7

(a) 18.74 (b) 17.55 (c) 13.50 (d) 15.73

Answer Key

1. (b) **2.** (c) **3.** (c) **4.** (a) **5.** (a) **6.** (a) **7.** (c) **8.** (c) **9.** (a) **10.** (b)
11. (d) **12.** (d)

CRITERIA FOR AN IDEAL MEASURE OF CENTRAL TENDENCY

- It should be properly and unambiguously defined
- It should be easy to comprehend.
- It should be simple to compute.
- It should be based on all the observations
- It should have certain desirable mathematical properties.
- It should be least affected by the presence of extreme observations

MERITS AND DEMERITS OF ARITHMETIC MEAN

Merits

Arithmetic mean possesses the following merits

1. It is rigidly defined.
2. It is easy to calculate and simple to understand.
3. It is based on all the observations.
4. It is suitable for further mathematical treatment.
5. Of all the averages, arithmetic mean is affected least by fluctuations of sampling.

Demerits

Arithmetic mean has the following drawbacks:

1. It is very much affected by extreme values.
2. In a distribution with open-end classes the value of mean cannot be computed without making assumptions regarding the size of the class' interval of the open-end classes.
3. It can neither be determined by inspection nor can it be located graphically.
4. It cannot be computed for qualitative data such as honesty, beauty, intelligence etc.
5. It may lead to wrong conclusions if the details of the data from which it is obtained are not available.

MEDIAN

- Median, for a given set of observations, when arranged in an ascending order or a descending order of magnitude. It may be defined as the middle-most value.
- As distinct from the arithmetic mean, which is based on all the items of the distribution, the median is what is called a positional average.
- The position of the median in a distribution is such that the number of observations above it is equal to the number of observations below it.

CALCULATION OF MEDIAN: INDIVIDUAL OBSERVATIONS

For ungrouped data consisting of n observations, the calculation of median involves the following steps:

Step 1. Arrange the given set of observations in an ascending or descending order of magnitude.

Step 2. The median is given by

(a) When n is odd, the value is given by: $\left(\frac{n+1}{2}\right)^{th}$ observation

(b) When n is even, the value is given by: $\frac{\frac{n}{2}^{th} \text{ observation} + \left(\frac{n}{2}+1\right)^{th} \text{ observation}}{2}$

Example 27. What is the median for the following observations?

10, 2, 7, 9, 13

(a) 5 (b) 9 (c) 6 (d) 11

Sol. (b) Given data: 10, 2, 7, 9, 13

Arranging the data in increasing order, we get 2, 7, 9, 10, 13

Clearly, the number of observations is 5 i.e. odd

$\therefore$ Median $= \left(\frac{n+1}{2}\right)$ th term

$= \left(\frac{5+1}{2}\right)$

= 3rd term = 9

Therefore, the median is 9.

Hence, the correct option is (b) i.e., 9.

Example 28. The following number of goals were scored by a team in a series of 10 matches: 2, 3, 4, 5, 0, 1, 3, 3, 4, 3. Find the median of these scores.

(a) 5 (b) 6 (c) 3 (d) None of these

Sol. (c) Arranging the given data in ascending order: 0, 1, 2, 3, 3, 3, 3, 4, 4, 5

Here, number of observations (n) = 10, which is even

Thus, median $= \frac{\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)\text{th term}}{2}$

$= \frac{\frac{10}{2}\text{th} + \left(\frac{10}{2}+1\right)\text{th term}}{2}$

$= \frac{5\text{th term} + 6\text{th term}}{2}$

$= \frac{3+3}{2} = \frac{6}{2} = 3$

Therefore, the median of given observations is 3.

Hence, the correct option is (c) i.e. 3.

Example 29. The median of the observations 42, 72, 35, 92, 67, 85, 72, 81, 51, 56 is

(a) 69.5 (b) 72 (c) 64 (d) 61.5 **(Dec 2022)**

Sol. (a) Given observations: 42, 72, 35, 92, 67, 85, 72, 81, 51, 56

Arranging the observations in ascending order, we get

35, 42, 51, 56, 67, 72, 72, 81, 85, 92

Here, the number of observations (n) = 10, which is even.

Thus, Median = Average of two middle terms

$$\Rightarrow \text{Median} = \frac{67+72}{2} = \frac{139}{2} = 69.5$$

Therefore, the median of given observations is 69.5.

Example 30. The median of $x, \frac{x}{2}, \frac{x}{3}, \frac{x}{5}$ is 10. Find x, where x > 0.

(a) 24 (b) 32 (c) 8 (d) 16

Sol. (a) Given: Median = 10

Clearly, the given observations $x, \frac{x}{2}, \frac{x}{3}, \frac{x}{5}$ are even in number and in decreasing order, thus

$$\text{Median} = \frac{\frac{n}{2}^{\text{th}} \text{ term} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{ term}}{2}$$

$$\Rightarrow \text{Median} = \frac{2^{\text{nd}} \text{ term} + 3^{\text{rd}} \text{ term}}{2}$$

$$\Rightarrow 10 = \frac{1}{2}\left(\frac{x}{2}+\frac{x}{3}\right)$$

$$\Rightarrow 20 = \frac{5x}{6}$$

$$\Rightarrow 5x = 120$$

$$\Rightarrow x = 24$$

Therefore, the value of x is 24.

Hence, the correct option is (a) i.e. 24.

CALCULATION OF MEDIAN: DISCRETE SERIES

In the case of discrete series, where the variable takes the values $X_1, X_2, \ldots\ldots X_n$ with respective frequencies $f_1, f_2, \ldots f_n$ with $\Sigma f = N$, median is the size of $\left(\frac{N+1}{2}\right)$th observation. In this case, the calculation of median involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency (c. f) distribution.

Step 2: Find $\frac{N+1}{2}$

Step 3: See the c.f. just greater than or equal to $\frac{N+1}{2}$

Step 4: The value of the variable corresponding to the c.f. obtained in Step 3 gives the required median.

Example 31. For the distribution:

X	1	2	3	4	5	6
f	6	9	10	14	12	8

The value of median is

(a) 3.5 (b) 3 (c) 4 (d) 5

Sol. (c) Given data,

X	1	2	3	4	5	6
f	6	9	10	14	12	8
c.f.	6	15	25	39	51	59

Here, N = 59

We know that, median will be the observation having cumulative frequency just equal or greater than $\left(\frac{N+1}{2}\right)$th term.

Thus, median = $\left(\frac{59+1}{2}\right)$th term = 30th term

Thus, the cumulative frequency just greater than 30 is 39 which corresponds to 4.

Therefore, the median is 4.

Hence, the correct option is (c) i.e. 4.

Example 32. Calculate median from the following data

X	10	20	30	40	50	60	70
Y	1	5	12	20	19	9	4

(a) 50 (b) 70

(c) 40 (d) Cannot be determined

Sol. (c) Here,

X	F	Less than c.f.
10	1	1
20	5	6
30	12	18
40	20	38
50	19	57
60	9	66
70	4	70
	N = Σf = 70	

Thus, $\frac{N+1}{2} = \frac{71}{2} = 35.5$

Therefore, c.f just greater than 35.5 is 38 which corresponds to the value of 40.

i.e., median = 40

Hence, the correct option is (c).

CALCULATION OF MEDIAN - CONTINUOUS SERIES

In the case of continuous series, median is the size of $\frac{N}{2}$ th observation, where $N = \sum f$ is the total frequency. The calculation of median in this case involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency (c.f.) distribution.

Step 2: Find $\frac{N}{2}$

Step 3: See the c.f. just greater than or equal to $\frac{N}{2}$

Step 4: Find the class corresponding to the c.f. obtained in Step 3. This is called the median class.

Step 5: Apply the following interpolation formula for calculating the median:

$$\text{Median} = l + \frac{\frac{N}{2} - c.f}{f} \times h$$

where, l = lower limit of the median class, f = frequency of the median class, c.f. = cumulative frequency of the class preceding the median class, and h = size or width of the median class.

Example 33. The marks obtained by 100 students in a certain examination are given below:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of Students	10	9	25	30	16	10

Calculate the median marks.

(a) 32 (b) 50 (c) 30 (d) None

Sol. (a)

Marks	Number of Students (f)	Cumulative Frequency (c.f)
0 – 10	10	10
10 – 20	9	19
20 – 30	25	44
30 – 40	30	74
40 – 50	16	90
50 – 60	10	100
	N = 100	

Thus, c.f $> \frac{N}{2}$ i.e., c.f. > 50 is 74

Therefore, median class = 30 – 40

$\Rightarrow$ l = 30, f = 30, c.f = 44 and h = 10

Now, Median $= l + \frac{\frac{N}{2} - c.f}{f} \times h$

$= 30 + \frac{50 - 44}{30} \times 10 = 32$

Hence, the correct option is (a).

PROPERTIES OF MEDIAN

1. If x and y are two variables, to be related by y = a + bx for any two constants a and b, then the median of y is given by $y_{me} = a + bx_{me}$.
2. For a set of observations, the sum of absolute deviations is minimum when the deviations are taken from the median. This property states that $\Sigma |x - A|$ is minimum if we choose A as the median.

Example 34. Consider two variables, x and y, related by the equation y = 3x + 5. If the median of x is 10, what is the median of y?

(a) 20 (b) 40 (c) 37 (d) 35

Sol. (d) Given: y = 3x + 5 and the median of x is 10.

To find median of y, put x = 10 in the given equation

$\Rightarrow$ y = 3 × 10 + 5

$\Rightarrow$ y = 30 + 5

$\Rightarrow$ y = 35

Thus, the median of y is 35.

Hence, the correct option is (d) i.e. 35.

Example 35. Two variables x and y are given by y = 2x – 3. If the median of x is 20, what is the median of y?

(a) 20 (b) 40 (c) 37 (d) 35

Sol. (c) Given: y = 2x – 3 and the median of x is 20.

To find median of y, put x = 20 in the given equation

$\Rightarrow$ y = 2 × 20 – 3

$\Rightarrow$ y = 40 – 3

$\Rightarrow$ y = 37

Thus, the median of y is 37.

Hence, the correct option is (c).

Example 36. In case of an even number of observations, which of the following is median?

(a) Any of the two middle – most value

(b) The simple average of these two middle values

(c) The weighted average of these two middle values

(d) Any of these

Sol. (b) In case of an even number of observations then the simple average of these two middle values is the median of the even number of observations.

E.g.: 2, 3, 4, 7, 8, 10

Median of the even number of given observations $= \frac{4+7}{2} = \frac{11}{2} = 5.5$

Hence, the correct option is (b).

Example 37. What is the median for the following observations?

5, 8, 6, 9, 11, 4

(a) 6 (b) 7 (c) 8 (d) none of these

Sol. (b) Given data: 5, 8, 6, 9, 11, 4

Arranging the data in increasing order, we get

4, 5, 6, 8, 9, 11

Clearly, the number of observations is 6 i.e. even

$$\therefore \text{Median} = \frac{\frac{n}{2}^{th} + \left(\frac{n}{2}+1\right)^{th} \text{ term}}{2}$$

$$= \frac{\frac{6}{2}^{th} + \left(\frac{6}{2}+1\right)^{th} \text{ term}}{2} = \frac{3^{rd} \text{ term} + 4^{th} \text{ term}}{2}$$

$$= \frac{6+8}{2} = \frac{14}{2} = 7$$

Therefore, the median of given observations is 7.

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART B)

1. Find the median for the following data: 19 22 17 20 12

(a) 12 (b) 20 (c) 22 (d) 19

2. Find the median for the following data: 58 49 64 70 91 34

(a) 58 (b) 70 (c) 61 (d) None of these

3. The marks obtained by 9 students in a test are 25, 20, 15, 45, 18, 7, 10, 38, 12. Find the median marks.

(a) 20 (b) 18 (c) 10 (d) None of these

4. What is the value of median for the following data?

Marks	5-14	15-24	25-34	35-44	45-54	55-64
Number of Students	10	18	32	26	14	10

(a) 28 (b) 30 (c) 32.94 (d) 33.18

5. Two variables x and y are given by y - 5x - 5 = 0. If the median of x is 20, what is the median of y?

(a) 100 (b) 105 (c) 1105 (d) None of these

6. Find the median of the following data:

x	10	5	7	11	8
f	15	20	15	18	12

(a) 8 (b) 9 (c) 10 (d) 11

7. Find the missing frequency from the following data, given that the median mark is 23.

Mark	0–10	10–20	20–30	30–40	40–50
No. of Students	5	8	?	6	3

(a) 10 (b) 16 (c) 5 (d) 18

8. The median for the following data

Profit in '000₹	Below 5	Below 10	Below 15	Below 20	Below 25	Below 30
No. of Firms	10	25	45	55	62	65

(a) 11.60 (b) 11556 (c) 11875 (d) 11.50

9. Find the median of the following data:

Mark	0–10	10–20	20–30	30–40	40–50	50-60
No. of Students	4	6	20	10	7	3

(a) 15.6 (b) 25.8 (c) 11.5 (d) 27.5

10. Given below is the distribution of marks obtained by 140 students in an examination:

Marks	10–19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of Students	7	15	18	25	30	20	16	7	2

Find the median of the distribution.

(a) 55.60 (b) 65.75 (c) 51.17 (d) 61.50

Answer Key

1. (d) **2.** (c) **3.** (b) **4.** (c) **5.** (b) **6.** (a) **7.** (a) **8.** (c) **9.** (d) **10.** (c)

Merits of Median

Median possesses the following merits

1. It is rigidly defined.
2. It is easy to calculate and simple to understand.
3. It can be computed while dealing with a distribution with open end classes
4. Being a- positional average, it is not much affected by extreme observations.
5. it is the most appropriate average to be used while dealing with qualitative data.
6. It can sometimes be located by inspection and can also be determined graphically.

Demerits

Median has the following limitations:

1. Median, being a positional average, is not based on each and every item of the distribution.
2. It is not suitable for further mathematical treatment. For example, it is not possible to find the combined median of two or more groups.
3. It cannot be determined exactly for an ungrouped data consisting of an even number of observations. It is determined approximately as the mid-point of two middle 1 observations.
4. In comparison to arithmetic mean, it is much affected by sampling fluctuations.
5. For calculating the median, it is necessary to arrange the data in order of magnitude.

PARTITION VALUE OR QUARTILES OR DECILES OR PERCENTILES

Partition values, such as quartiles, deciles, and percentiles, are statistical measures that divide a given set of observations into equal parts.

- **Quartiles**: Quartiles divide the data into four equal parts, each containing 25% of the observations. The three quartiles are denoted as Q_1, Q_2 (which is the median), and Q_3. Q_1 represents the value below which 25% of the data falls, Q_2 is the median (50th percentile), and Q_3 represents the value below which 75% of the data falls.
- **Deciles:** Deciles divide the data into ten equal parts, each containing 10% of the observations. Deciles are often used to analyze income distributions or rank data. The first decile (D_1) represents the value below which 10% of the data falls, the second decile (D_2) represents the value below which 20% of the data falls, and so on until the ninth decile (D_9).
- **Percentiles:** The value which divides the data into hundred equal parts, each containing 1% of the observations. There are percentiles which are. Percentiles are used to understand the relative position of a particular value in a dataset.

  For example, the 75th percentile (P_{75}) represents the value below which 75% of the data falls, and the 90th percentile (P_{90}) represents the value below which 90% of the data falls. Thus, is the median.

CALCULATION OF QUARTILES - INDIVIDUAL OBSERVATIONS

For ungrouped data consisting of n observations (not necessarily all distinct), the calculation of kth quartile Q_k (k = 1, 2, 3,) involves the following steps:

Step 1: Arrange the given data in an ascending order of magnitude.

Step 2: The value of kth quartile Q_k is given by $\frac{k(n+1)}{4}$th observation.

Thus, $Q_1 = \frac{(n+1)}{4}$th observation

$Q_2 = \frac{2(n+1)}{4}$th observation

$Q_3 = \frac{3(n+1)}{4}$th observation

Example 38. Consider the following set of observations: 19, 12, 27, 14, 21, 18, 9, 15. What is the value of the first quartile?

(a) 11.5 (b) 12 (c) 12.5 (d) 13

Sol. (c) Given observations: 19, 12, 27, 14, 21, 18, 9, 15

Arranging the observations in increasing order, we get 9, 12, 14, 15, 18, 19, 21, 27

We know that,

First quartile,

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ value} = \left(\frac{8+1}{4}\right)^{th} \text{ value} = \left(\frac{9}{4}\right)^{th} \text{ value} = (2.25)^{th} \text{ value}$$

= 2nd value + 0.25 × Difference between 3rd and 2nd values

= 12 + 0.25(14-12) = 12 + 0.25(2) = 12 + 0.50 = 12.50

Therefore, the value of the first quartile is 12.50.

Hence, the correct option is (c) i.e. 12.50.

CALCULATION OF DECILES - INDIVIDUAL OBSERVATIONS

For ungrouped data consisting of n observations (not necessarily all distinct), the calculation of kth decile D_k(k = 1, 2, …, 9) involves the following steps:

Step 1: Arrange the given data in an ascending order of magnitude.

Step 2: The value of kth decile D_k is given by $\frac{k(n+1)}{10}$th observation.

Thus, $D_1 = \frac{(n+1)}{10}$ th observation

$D_2 = \frac{2(n+1)}{10}$th observation

$D_3 = \frac{3(n+1)}{10}$th observation and so on

CALCULATION OF PERCENTILES - INDIVIDUAL OBSERVATIONS

For ungrouped data consisting of n observations (not necessarily all distinct), the calculation of kth percentile P_k(k = 1, 2, …, 99) involves the following steps:

Step 1: Arrange the given data in an ascending order of magnitude.

Step 2: The value of kth percentile P_k is given by $\frac{k(n+1)}{100}$th observation.

Thus, $P_1 = \frac{(n+1)}{100}$ th observation

$P_2 = \frac{2(n+1)}{100}$th observation

$P_3 = \frac{3(n+1)}{100}$th observation and so on

Example 39. Consider the set of numbers: 14, 8, 19, 22, 17, 9, 13, 16. What is the value of the third decile?

(a) 11.8 (b) 13.4 (c) 13.5 (d) 14

Sol. (a) Given observations: 14, 8, 19, 22, 17, 9, 13, 16,

Arranging the observations in ascending order, we get 8, 9, 13, 14, 16, 17, 19, 22

We know that,

Third decile, $D_3 = \frac{3}{10} \times (n+1)^{th}$ value

$= \frac{3}{10} \times (8+1)^{th}$ value

$= \frac{3}{10} \times 9^{th}$ value $= 2.7^{th}$ value

$= 2^{nd}$ value + 0.7 × Difference between 3rd and 2nd values

$= 9 + 0.70(13 - 9) = 9 + 2.8 = 11.8$

Hence, the correct option is (a) i.e. 11.8.

Example 40. Consider the following set of observations: 19, 12, 27, 14, 21, 18, 9, 15. What is the value of the first quartile?

(a) 11.5 (b) 12 (c) 12.5 (d) 13

Sol. (c) Given observations: 19, 12, 27, 14, 21, 18, 9, 15

Arranging the observations in increasing order, we get

9, 12, 14, 15, 18, 19, 21, 27

We know that,

First quartile $Q_1 = \left(\frac{n+1}{4}\right)^{th}$ value $= \left(\frac{8+1}{4}\right)^{th}$ value $= \left(\frac{9}{4}\right)^{th}$ value

$= (2.25)^{th}$ value

= 2nd value + 0.25 × Difference between 3rd and 2nd values

$= 12 + 0.25(14 - 12)$

$= 12 + 0.25(2) = 12 + 0.50 = 12.50$

Therefore, the value of the first quartile is 12.50.

Hence, the correct option is (c).

Example 41. The marks obtained by 9 students in a test are 25, 20, 15, 45, 18, 7, 10, 38 and 12. Find the value of P_{70}.

(a) 25 (b) 7 (c) 10 (d) None of these

Sol. (a) Arranging the given data in ascending order, we get

7, 10, 12, 15, 18, 20, 25, 38, 45

Thus, $P_{70} = \frac{70(n+1)}{100} = \frac{70(9+1)}{100} = \frac{70(10)}{100}$ = 7th observation = 25

Therefore, the value of P_{70} is 25.

Hence, the correct option is (a).

CALCULATION OF QUARTILES - DISCRETE SERIES

In case of discrete frequency distribution where the variable X takes the values $X_1, X_2, \ldots X_n$ with respective frequencies $f_1, f_2, \ldots f_n$ with $\sum f = N$, the calculation of each quartile $Q_k (k = 1, 2, 3)$ involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{k(N+1)}{4}$

Step 3: See the c.f. just greater than or equal to $\frac{k(N+1)}{4}$

Step 4: The value of X corresponding to the c.f. obtained in Step 3 gives the required value of Q_k

CALCULATION OF DECILES - DISCRETE SERIES

In case of discrete frequency distribution where the variable X takes the values $X_1, X_2, \ldots X_n$ with respective frequencies $f_1, f_2, \ldots f_n$ with $\sum f = N$, the calculation of each decile D_k (k = 1, 2, 3,... 9) involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{k(N+1)}{10}$

Step 3: See the c.f. just greater than or equal to $\frac{k(N+1)}{10}$

Step 4: The value of X corresponding to the c.f. obtained in Step 3 gives the required value of D_k.

CALCULATION OF PERCENTILES - DISCRETE SERIES

In case of discrete frequency distribution where the variable X takes the values $X_1, X_2, \ldots X_n$ with respective frequencies $f_1, f_2, \ldots f_n$ with $\sum f = N$, the calculation of each percentile P_k (k = 1, 2, 3,... 99) involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{k(N+1)}{100}$

Step 3: See the c.f. just greater than or equal to $\frac{k(N+1)}{100}$

Step 4: The value of X corresponding to the c.f. obtained in Step 3 gives the required value of P_k.

Example 42. Calculate the value of Q_1 and P_{65} from the following data:

X	10	5	7	11	8
f	15	20	15	18	12

(a) 7, 10 (b) 35, 52.65 (c) 10, 7 (d) None of these

Sol. (a) Given data:

X	f	c.f. (less than)
5	20	20
7	15	35
8	12	47
10	15	62
11	18	80
	$N = \sum f = 80$	

For Q_1,

We have, $\frac{N+1}{4} = 20.25$

Thus, c.f. just greater than 20.25 is 35 which corresponds to X = 7.

Therefore, $Q_1 = 7$

For P_{65},

We have, $\frac{65(N+1)}{100} = 52.65$

Thus, c.f. just greater than 52.65 is 62 which corresponds to X = 10.

Therefore, $P_{65} = 10$

Hence, the correct option is (a).

CALCULATION OF QUARTILES - CONTINUOUS SERIES

In case of continuous frequency distribution, the calculation of Q_k (k = 1, 2, 3) involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{KN}{4}$, where $N = \sum f$ is the total frequency.

Step 3: See the c.f. just greater than or equal to $\frac{KN}{4}$

Step 4: Find the class, the class corresponding to c.f. obtained in Step 3.

Step 5: The value of Q_k is then obtained by using the following interpolation formula:

$$Q_k = l + \frac{\frac{kN}{4} - c}{f} \times h$$

where l = lower limit of Q_k class

C = c.f. of the class preceding the Q_k class

f = frequency of the Q_k class, and h = size or width of Q_k class.

CALCULATION OF DECILES - CONTINUOUS SERIES

In case of continuous frequency distribution, the calculation of D_k (k = 1, 2, …, 9) involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{KN}{4}$, where N = ∑f is the total frequency.

Step 3: See the c.f. just greater than or equal to $\frac{KN}{10}$

Step 4: Find the D_k class, the class corresponding to c.f. obtained in Step 3.

Step 5: The value of Q_k is then obtained by using the following interpolation formula:

$$Q_k = l + \frac{\frac{kN}{10} - c}{f} \times h$$

where l = lower limit of D_k class

C = c.f. of the class preceding the D_k class

f = frequency of the D_k class, and h = size or width of D_k class.

CALCULATION OF PERCENTILES - CONTINUOUS SERIES

In case of continuous frequency distribution, the calculation of P_k (k = 1, 2, …, 99) involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{kN}{100}$, where N = ∑f is the total frequency.

Step 3: See the c.f. just greater than or equal to $\frac{KN}{100}$

Step 4: Find the P_k class, the class corresponding to c.f. obtained in Step 3.

Step 5: The value of is then obtained by using the following interpolation formula:

$$P_k = l + \frac{\frac{kN}{100} - c}{f} \times h$$

where l = lower limit of P_k class

C = c.f. of the class preceding the P_k class

f = frequency of the P_k class, and h = size or width of P_k class.

Example 43. The third quartile and 65th percentile for the following data are:

Profit in '000₹	Less than 10	10-19	20-29	30-39	40-49	50-59
No. of Firms	5	18	38	20	9	2

(a) ₹33,500 and ₹29,184 (b) ₹33,000 and ₹28,680

(c) ₹33,600 and ₹29,000 (d) ₹33,250 and ₹29,250

Sol. (a) According to the given data,

Class Interval	Class- Boundaries	Frequency	Cumulative Frequency
Less than 10	Less than 9.5	5	5
10-19	9.5-19.5	18	23
20-29	19.5-29.5	38	61
30-39	29.5-39.5	20	81
40-49	39.5-49.5	9	90
50-59	49.5-59.5	2	92
		$n = \sum f_i = 92$	

Third quartile Q_3 = Value of $\left(\frac{3n}{4}\right)^{th}$ observation $= \left(\frac{3\times 92}{4}\right)^{th} = 69^{th}$ observation

69th observation lies in the class interval 29.5 - 39.5, so it is a quartile class.

Here, l = 29.5, cf = 61, f = 20, c = 39.5-29.5 = 10

We know that,

$$Q_3 = l + \frac{\frac{3n}{4} - cf}{f} \times c = 29.5 + \frac{69-61}{20} \times 10 = 33.5$$

$\Rightarrow Q_3 = 33.5 \times 1000$ ($\because$ Profits is given in thousands)

$= 33500$

Thus, third quartile is ₹33,500.

Now, 65th percentile P_{65} = Value of $\left(\frac{65n}{100}\right)^{th}$ observation

$= \left(\frac{65\times 92}{100}\right)^{th}$ = Value of 59.8th observation

Since, 59.8th observation lies in a class 19.5 - 29.5. So, it is 65th percentile class.

Here, l = 19.5, cf = 23, f = 38, c = 10

We know that,

$$P_{65} = l + \frac{\frac{65n}{100} - cf}{f} \times c = 19.5 + \frac{59.8-23}{38} \times 10 = 29.184$$

$P_{65} = 29.184 \times 1000$ ($\because$ Profits is given in thousands)

$P_{65} =$ ₹29184

Hence, the correct option is (a) i.e., ₹33,500 and ₹29,184.

PRACTICE QUESTIONS (PART C)

1. What is the value of the first quartile for observations 15, 18, 10, 20, 23, 28, 12, 16?

(a) 17 (b) 16 (c) 12.75 (d) 12

2. The third decile for the numbers 15, 10; 20, 25, 19, 11, 9, 12 is

(a) 13 (b) 10.70 (c) 11 (d) 11.50

3. Consider the set of numbers: 14, 8, 19, 22, 17, 9, 13, 16. What is the value of the third decile?

(a) 11.8 (b) 13.4 (c) 13.5 (d) 14

4. The fourth decile for the numbers 12, 15, 18, 20, 22, 25, 28, 30 is

(a) 12.50 (b) 19.20 (c) 28.0 (d) 30.0

5. Following are the wages of the laborer's:

₹82, ₹56, ₹90, ₹50, ₹120, ₹75, ₹75, ₹80, ₹130, ₹65

Find D_6 and P_{82}.

(a) 16.2 and 100.2 (b) 81.2 and 120.2

(c) 46.2 and 100.2 (d) None of these

6. Quartiles are the values dividing a given set of observations into

(a) Two equal parts (b) Four equal parts

(c) Five equal parts (d) None of these

7. The marks obtained by 9 students in a test are 25, 20, 15, 45, 18, 7, 10, 38 and 12. Find the values of Q_3 and D_2.

(a) 31.5 and 10 (b) 11 and 10

(c) 10 and 7 (d) None of these

8. From the following data, calculate the median and the first and third quartile wages.

Daily Wages (₹)	No. of Workers	Daily Wages (₹)	No. of Workers
30 – 32	2	40 – 42	62
32 – 34	9	42 – 44	39
34 – 36	25	44 – 46	20
36 – 38	30	46 – 48	11
38 - 40	49	48 - 50	3

(a) 37.7667 and 42.5385 (b) 30.5660 and 24.5385

(c) 37.7667 and 32.7380 (d) None of these

9. Find the 45th and 57th percentiles for the following data on marks obtained by 100 students:

Marks	20-25	25-30	30-35	35-40	40-45	45-50
No. of Students	10	20	20	15	15	20

(a) 30 and 37.34 (b) 33.35 and 37.34

(c) 40.5 and 44.6 (d) None of these

10. Calculate the values of Q_3, D_4 from the following data:

X	10	5	7	11	8
f	15	20	15	18	12

(a) 10, 7 (b) 20, 15 (c) 17, 7 (d) 10, 7

Answer Key

1. (c) **2.** (b) **3.** (a) **4.** (b) **5.** (b) **6.** (b) **7.** (a) **8.** (a) **9.** (b)
10. (d)

MODE

Mode is a measure of central tendency that represents the value or values in a dataset that occur most frequently. It is the data point with the highest frequency or the data points with equal highest frequencies.

E.g.: In dataset 1, 2, 3, 4, 4, 5, 2, 3, 4, the mode is 4 because it appears three times, which is more than any other value in the dataset.

Bimodal Distribution: If there are two modes in a distribution it is called Bimodal Distribution.

E.g.: 1, 2, 2, 2, 4, 5, 8, 6, 6, 6 then the modes are 2 and 6, as both values appear equally most frequently.

Multi-Modal Distribution

When we have multiple modes in a distribution it is called Multi- Modal Distribution.

E.g.: Dataset 1, 2, 2, 4, 4, 6, 8, 6 has three modes: 2, 4, and 6.

No Mode: When all observations have the same frequencies then the distribution has no mode.

E.g.: 1, 2, 3, 4, 5 has no mode as each value appears only once and has the same frequency.

CALCULATION OF MODE: DISCRETE SERIES

In discrete frequency distribution, mode can be determined just by inspection. It is the value of the variable corresponding to the maximum frequency

E.g.: Consider the following distribution:

X	1	2	3	4	5	6	7
f	2	3	12	5	2	6	3

Here, the highest frequency is 12 which corresponds to X = 3.

Therefore, Mode for the given distribution is 3.

Note: While determining mode by inspection in the case of discrete frequency distribution, an error of judgement is possible when the difference between the greatest frequency and the frequency preceding it or succeeding it is very small and the values are heavily concentrated on either side.

Example 44. What is the modal value for the numbers 5, 8, 6, 4, 10, 15, 18, 10?

(a) 18 (b) 10 (c) 14 (d) None of these

Sol. (b) Given data: 5, 8, 6, 4, 10, 15, 18, 10

We know that,

For a given set of observations, mode may be defined as the value that occurs the maximum number of times.

Here, occurs maximum number of times (2 times)).

Therefore, the modal value is 10.

Hence, the correct option is (b) i.e. 10.

Example 45. The heights (in centimeters) of a group of students in a class were recorded. The data set is as follows: 141, 150, 154, 152, 162, 162, 142, 155, 160, 167, 147, 152, 165. What is the mode(s) of the heights in the data set?

(a) 142 (b) 165

(c) 152 and 162 (d) No mode

Sol. (c) Given data: 141, 150, 154, 152, 162, 162, 142, 155, 160, 167, 147, 152, 165

We know that,

For a given set of observations, mode may be defined as the value that occurs the maximum number of times.

Here, we can see that the heights 152 and 162 both appear two times, which is more frequent than any other height in the data set.

Therefore, the modes of the heights in the data set are 152 and 162.

Hence, the correct option is (c).

Example 46. What is the mode for the following set of numbers? 7, 9, 12, 15, 18, 21, 24, 27

(a) 7 (b) 27

(c) 15, 18 (d) No mode

Sol. (d) We know that,

The mode represents the value(s) that occur most frequently in a set of numbers. In the given set, each number appears only once, and there are no repeated values. Therefore, there is no value that occurs more frequently than others, resulting in no mode for this set of numbers.

Hence, the correct option is (d).

Example 47. Modal group is:

Height in cms:	61 - 63	63 - 65	65 - 67	67 - 69	69 - 71
No.of students:	15	118	142	127	18

(a) 65 - 67 (b) 69 - 71 (c) 63 - 65 (d) None

Sol. (a) We know that,

For a given set of observations, mode may be defined as the value that occurs the maximum number of times.

Clearly, the highest frequency 142 is which lies in the class 65 - 67.

Therefore, the modal class is 65 – 67.

Hence, the correct option is (a) i.e., 65 – 67.

CALCULATION OF MODE - CONTINUOUS FREQUENCY DISTRIBUTION

The first step is to find the modal class, i.e., the class corresponding to the maximum frequency. The value of mode is then obtained by applying the following interpolation formula:

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

where, l = LCB of the modal class,

f_1 = frequency of modal class,

f_0 = frequency of pre-modal class,

f_2 = frequency of post-modal class,

c = class-length of the modal class

Example 48. The mode for the following frequency distribution.

Class Interval	350-369	370-389	390-409	410-429	430-449	450-469
Frequency	15	27	31	19	13	6

(a) 390 (b) 390.50 (c) 394.50 (d) 394

Sol. (c) According to the question,

Class	Frequency f	Class-Boundaries
350-369	15	349.5-369.5
370-389	27	369.5-389.5
390-409	31	389.5-409.5
410-429	19	409.5-429.5
430-449	13	429.5-449.5
450-469	6	449.5-469.5

Clearly, the maximum frequency is 31.

$\therefore$ The modal class is 389.5-409.5.

Here,

l = lower frequency point of modal class = 389.5

f_1 = frequency of the modal class = 31

f_0 = frequency of the preceding class = 27

f_2 = frequency of the succeeding class = 19

c = class length of the modal class = 20

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c = 389.5 + \left(\frac{31 - 27}{2(31) - 27 - 19}\right) \times 20 = 394.50$$

Hence, the correct option is (c).

Example 49. Following is an incomplete distribution having modal mark as 44.

Marks	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
No. of Students	8	18	?	12	5

What would be the mean marks? **(ICAI)**

(a) 45 (b) 46 (c) 47 (d) 48

Sol. (d) Let us assume that ? = x

Given, mode is 44.

$\Rightarrow$ Modal class is 40 - 60.

Thus, $l = 40$, $f_1 = x$, $f_0 = 18$, $f_2 = 12$ and $c = 20$

We know that,

$$\text{Mode } Z = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

$$\Rightarrow 44 = 40 + \frac{x - 18}{2x - 18 - 12} \times 20$$

$$\Rightarrow 44 - 40 = \frac{x - 18}{2x - 30} \times 20$$

$$\Rightarrow \frac{1}{5} = \frac{x - 18}{2x - 30}$$

On cross multiplication, we get

$2x - 30 = 5(x - 18)$

$\Rightarrow 2x - 30 = 5x - 90$

$\Rightarrow 3x = 60$

$$\Rightarrow x = \frac{60}{3}$$

$\Rightarrow x = 20$

$\Rightarrow ? = 20$

Now, to find mean:

Class	Frequency (f)	Mid Value (x)	$d = \frac{x - A}{h} = \frac{x - 50}{20}$, A = 50, h = 20	f × d
0 - 20	5	10	-2	-10
20 - 40	18	30	-1	-18
40 - 60	20	50 = A	0	0
60 - 80	12	70	1	12
80 - 100	5	90	2	10
	n = 60			$\sum f \cdot d = -6$

CA WALLAH

$$\text{Mean } \bar{x} = A + \frac{\sum f \cdot d}{n} \times h = 50 - \frac{6}{60} \times 20 = 48$$

Therefore, the mean marks is 48.

Hence, the correct option is (d) i.e. 48.

It may be remarked that the above formula for computing mode is based on the following assumptions:

1. The frequency distribution must be continuous with exclusive type classes without any gaps. If the data are not given in the form of continuous classes, it must first be converted into continuous classes before applying the above formula.
2. The class intervals must be uniform throughout, i.e., the size of all the class intervals must be the same. If they are unequal, they should first be made equal on the assumption that frequencies are uniformly distributed over all the classes.

PROPERTIES OF MODE

- If $y = a + bx$ then $y_{mode} = a + bx_{mode}$

EMPIRICAL RELATIONSHIP OF MEAN, MEDIAN AND MODE

For a grouped frequency distribution, we may consider the following empirical relationship between mean, median and mode:

Mean – Mode = 3(Mean – Median)

$\Rightarrow$ Mean – Mode = 3 Mean – 3 Median

$\Rightarrow$ 3 Median = 3 Mean – Mean + Mode

$\Rightarrow$ 3 Median = 2 Mean + Mode

Example 50. If mean $(\bar{X})$ is 10 and mode (Z) is 7, then find out the value of median (M).

(a) 9 (b) 17 (c) 3 (d) 4.33 **(Dec 2022)**

Sol. (a) Given: Mean (M) = 10, Mode (Z) = 7

To find: Median (M)

We know that,

3 Median = 2 Mean + Mode

$\Rightarrow$ 3M = 2(10) + 7

$\Rightarrow$ 3M = 27

$\Rightarrow$ M = 9

Therefore, the value of median (M) is 9.

Example 51. For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean mark and median mark were found to be 55.60 and 52.40. What is the modal mark?

(a) 46 (b) 16 (c) 51 (d) 66

Sol. (a) Given: Mean mark = 55.60 and Median mark = 52.40

As we know,

Mode = 3 × Median - 2 Mean = 3(52.40) - 2(55.60) = 46

Hence, the correct option is (a).

Example 52. If y = 2 + 1.50x and mode of x is 15, what is the mode of y?

(a) 7.5 (b) 24.5 (c) 10.7 (d) 17.5

Sol. (b) According to the question,

As we know, $y_{mo} = a + bx_{mo}$

Now, Put the values and compute,

$y_{mo} = 2 + 1.50(15) = 2 + 22.5 = 24.5$

Hence, the correct option is (b).

MERITS AND DEMERITS OF MODE

Merits

Mode possesses the following merits:

1. It is simple to understand and easy to calculate.
2. In some cases it can be located merely by inspection.
3. It can be determined graphically from a histogram.
4. It is not at all affected by extreme observations and can be calculated even if extreme values are not known.
5. It can be conveniently determined for distribution with open end classes.

Demerits

Mode has the following drawbacks:

1. It is not rigidly defined
2. It is not based on all the observations.
3. It is not suitable for further mathematical treatment.
4. As compared to mean, mode is affected to a greater extent by the fluctuations of sampling.
5. The value of mode cannot always be determined. In some cases, we may have a bi-modal distribution.

Example 53. What is the modal value for the numbers 5, 8, 6, 4, 10, 15, I8, 10?

(a) 18 (b) 10 (c) 14 (d) None the these

Sol. (b) Given data: 5, 8, 6, 4, 10, 15, I8, 10

We know that,

For a given set of observations, mode may be defined as the value that occurs the maximum number of times.

Here, 10 occurs maximum number of times (2 times).

Therefore, the modal value is 10.

Hence, the correct option is (b).

Example 54. If x and y are related by x – y - 10 = 0 and mode of x is known to be 23, then the mode of y is **(ICAI)**

(a) 20 (b) 13 (c) 3 (d) 23

Sol. (b) Given: x - y - 10 = 0

To find the mode of y, put x = 23 in the given equation.

$\Rightarrow$ 23 - y - 10 = 10

$\Rightarrow$ 13 - y = 0

$\Rightarrow$ y = 13

Therefore, mode of y is 13.

Hence, the correct option is (b) i.e. 13.

Example 55. Which of the following measure(s) satisfies (satisfy) a linear relationship between two variables?

(a) Mean (b) Median (c) Mode (d) All of these

Sol. (d) **GM:** Measure of the central tendency is difficult to compute.

Mean: For a given set of observations, the AM may be defined as the sum of all the observations divided by the number of observations.

GM: For a given set of n positive observations, the geometric mean is defined as the n-th root of the product of the observations.

Median: Median is a positional average which means that the value of the median is dependent upon the position of the given set of observations for which the median is wanted. Median, for a given set of observations, may be defined as the middle-most value when the observations are arranged either in an ascending order or a descending order of magnitude.

Mode: For a given set of observations, mode may be defined as the value that occurs the maximum number of times. Thus, mode is that value which has the maximum concentration of the observations around it.

Hence, the correct answer is option (d).

Example 56. Which of the following measures of central tendency is based on only fifty percent of the central values?

(a) Mean (b) Median (c) Mode (d) Both (a) and (b)

Sol. (b) **Median:** of central tendency is based on only fifty percent of the central values.

Mean: For a given set of observations, the AM may be defined as the sum of all the observations divided by the number of observations.

Median: Median is a positional average which means that the value of the median is dependent upon the position of the given set of observations for which the median is wanted. Median, for a given set of observations, may be defined as the middle-most value when the observations are arranged either in an ascending order or a descending order of magnitude.

Mode: For a given set of observations, mode may be defined as the value that occurs the maximum number of times. Thus, mode is that value which has the maximum concentration of the observations around it.

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART D)

1. The mode of the numbers 7, 7, 7, 9, 10, 11, 11, 11, 12 is

(a) 11 (b) 12 (c) 7 (d) 7 & 11

2. Mode is

Variable:	2	3	4	5	6	7
No. of Men:	5	6	8	13	7	4

(a) 6 (b) 4 (c) 5 (d) None of these

3. Calculate mode from the following data:

Height in Inches:	56	58	59	60	61	62	63	64	66	68
No. of Persons:	3	7	6	9	20	22	24	5	3	1

(a) 68 (b) 20 (c) 63 (d) 56

4. Compute mode for the distribution as described:

Weight in kgs.	40–45	46–50	51–55	56–60	61–65	66–70
No. of Friends:	4	3	9	8	7	2

(a) 67.55 (b) 54.29 (c) 70.65 (d) 66.50

5. Given below is the distribution of weights of a group of 60 students in a Class

Weights (in kg):	30–34	35–39	40-44	45-49	50–54	55–59	60–64
No. of Students	3	5	12	18	14	6	2

Find the mode of the distribution.

(a) 47.5 (b) 65.85 (c) 10.70 (d) 46.59

Answer Key

1. (d) **2.** (c) **3.** (c) **4.** (b) **5.** (a)

GEOMETRIC MEAN

The geometric mean, usually abbreviated as G.M., of a set of n observations x_1, x_2, ..., x_n is the nth root of their product. That is,

$$GM = (x_1 x_2 x_3 ..... x_n)^{\frac{1}{n}}$$

Note: It may be noted that if there are two observations, G.M. can be computed by taking the square root; if there are three observations, G.M. can be computed by taking the cube root of their product and so on. **E.g.:**,

G.M. of two numbers 4 and 9 is $\sqrt{4 \times 9} = 2 \times 3 = 6$

G.M. of three numbers 1, 4 and 128 is $\sqrt[3]{1 \times 4 \times 128} = 2^3 = 8$

Using Calculator tricks we can find Log and antilog for any number:

To find Logarithm	To find Anti-Logarithm
1. Type number and press root for 15 times	1. Multiply number by 0.000070274
2. Subtract 1 after step 1	2. Add 1 after step 1
3. Divide number by 0.000070274	3. Press multiply & = button for 15 times

Example 57. What is the geometric mean (GM) for the numbers 2, 8 and 32?

(a) 4 (b) 6 (c) 8 (d) 12

Sol. (c) Given: $x_1 = 2$, $x_2 = 8$, $x_3 = 32$ and $n = 3$

We know that,

$$GM = (x_1 x_2 x_3 \ldots\ldots\ldots\ldots x_n)^{\frac{1}{n}}$$

$$= (2 \times 8 \times 32)^{\frac{1}{3}}$$

$$= (2 \times 2 \times 4 \times 4 \times 4 \times 2)^{\frac{1}{3}}$$

$$= 2 \times 4$$

$$= 8$$

Therefore, the required geometric mean is 8.

Hence, the correct option is (c).

Example 58. What is the GM for the numbers 8, 24 and 40?

(a) 24 (b) 12 (c) $8\sqrt[3]{15}$ (d) 10

Sol. (c) Given: $x_1 = 8$, $x_2 = 24$, $x_3 = 40$ and $n = 3$

We know that,

$$GM = (x_1 x_2 x_3 \ldots\ldots\ldots\ldots x_n)^{\frac{1}{n}}$$

$$= (8 \times 24 \times 40)^{\frac{1}{3}}$$

$$= 8(3 \times 5)^{\frac{1}{3}}$$

$$= 8(15)^{\frac{1}{3}}$$

$$= 8\sqrt[3]{15}$$

Hence, the correct option is (c).

Example 59. The geometric mean of three numbers is 12 and two of the numbers are 4 and 16. What is the third number?

(a) 12 (b) 32 (c) 27 (d) 48

Sol. (c) Let the third number be x.

We know that, $G.M. = \sqrt[n]{a_1 \times a_2 \times a_3 \times ..... \times a_n}$

Since, Geometric mean of 4, 16 and x = 12

$\Rightarrow G.M. = \sqrt[3]{4 \times 16 \times x}$

$\Rightarrow 12 = \sqrt[3]{64 \times x}$

On cubing both the sides,

$\Rightarrow (12)^3 = 64 \times x$

$\Rightarrow x = \frac{(12)^3}{64}$

$\Rightarrow x = 27$

Hence, option (c) i.e. 27 is the correct answer.

CALCULATION OF GEOMETRIC MEAN - DISCRETE SERIES

The geometric mean of a set of n observations $X_1, X_2, ... X_n$ with their respective frequencies $f_1, f_2, ... f_n$ is given by:

$G.M. = \left(X_1^{f_1} \times X_2^{f_2} \times ... X_n^{f_n}\right)^{\frac{1}{n}}$

where $N = \Sigma f$ is the total frequency.

Taking logarithm of both sides, we obtain

$$\text{Log (G.M.)} = \frac{1}{N} \log (X_1^{f1} \times X_2^{f2} \times \ldots X_n^{fn})^{1/N} = \frac{1}{N} [\log (X_1^{f1} \times X_2^{f2} \times \ldots X_n^{fn})$$

$$= \frac{1}{N} [f_1 \log X_1 + f_2 \log X_2 + ... + f_n \log X_n] = \frac{1}{N} \Sigma f \log X$$

$$G.M. = AL \left[\frac{1}{N} \Sigma f \log X\right]$$

CALCULATION OF GEOMETRIC MEAN - CONTINUOUS SERIES

In the case of grouped or continuous frequency distribution, G.M. is given by

$$G.M. = AL \left[\frac{1}{N} \Sigma f \log X\right]$$

where $X_1, X_2, ..., X_n$ are the class marks (or mid-values) of a set of grouped data with corresponding class frequencies $f_1, f_2, ..., f_n$

Example 60. Find the geometric mean from the following data:

Diameter (mm)	130	135	140	145	143	148	149	150
No. of Screws	3	4	6	6	3	5	2	2

(a) 142 mm (b) 165 mm (c) 110 mm (d) 149 mm

Sol. (a) According to the given data,

Diameter (mm) (X)	f	log X	f log X
130	3	2.1139	6.3417
135	4	2.1303	8.5212
140	6	2.1461	12.8766
145	6	2.1614	12.9684
143	3	2.1553	6.4659
148	5	2.1703	10.8515
149	2	2.1732	4.3464
150	2	2.1761	4.3522
	$N = \sum f = 31$		$\sum f \log X = 66.7239$

$$\text{G.M.} = \text{AL}\left[\frac{1}{N}\sum f \log X\right] = \text{AL}\left[\frac{66.7239}{31}\right] = \text{AL}(2.1524) = 142.0 \text{ mm}$$

PROPERTIES OF GEOMETRIC MEAN

1. Logarithm of G for a set of observations is the AM of the logarithm of the observations i.e., $\log G = \frac{1}{r}\Sigma \log X$
2. If all the observations assumed by a variable are constants, (k), then the GM of the observations is also k.
3. GM of the product of two variables is the product of their GM's i.e., If z = xy then GM of z = GM of x × GM of y
4. GM of the ratio of two variables is the ratio of the GM's of two variables i.e., if $z = \frac{x}{y}$ then GM of $z = \frac{\text{GM of x}}{\text{GM of y}}$

USES OF GEOMETRIC MEAN

1. Geometric mean is used primarily to average data for which the ratio of consecutive terms remains approximately constant. This occurs, for example, with such data as rates of change, ratios, percent increase in sales, population sizes over consecutive time periods and the like.
2. It is the most appropriate average to be used in the construction of index numbers.
3. It is the most suitable average to be used when it is desired to give more weightage to smaller items and vice-versa.

MERITS AND DEMERITS OF GEOMETRIC MEAN

Merits

1. It is rigidly defined.
2. It is based on all the observations.

3. It is suitable for further mathematical treatment. For example, if the geometric means of two or more sets of data are known, then the geometric mean of the combined data can also be obtained. If G_1 and G_2 are the geometric means of two sets of data with number of observations n_1 and n_2 respectively, then the geometric mean G of the combined data with $n_1 + n_2$ observations is given by:

$$\log G = \frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}$$

4. It gives less weight to large items and more to small ones than does the arithmetic mean.
5. Unlike AM, geometric mean is affected to a lesser extent by extreme observations.
6. It is not affected much by fluctuations of sampling.

Demerits

Geometric mean has the following drawbacks.

1. It is difficult to understand.
2. It is not easy to calculate for a non-mathematical person.
3. If any of the observations is zero, the geometric mean becomes zero and if any one of the observations is negative, the value of GM cannot be calculated.

Example 61. Which of the following measures of the central tendency is difficult to compute?

(a) Mean (b) Median (c) Mode (d) GM

Sol. (d) **GM:** measure of the central tendency is difficult to compute.

Mean: For a given set of observations, the AM may be defined as the sum of all the observations divided by the number of observations.

GM: For a given set of n positive observations, the geometric mean is defined as the nth root of the product of the observations.

Median: Median is a positional average which means that the value of the median is dependent upon the position of the given set of observations for which the median is wanted. Median, for a given set of observations, may be defined as the middle-most value when the observations are arranged either in an ascending order or a descending order of magnitude.

Mode: For a given set of observations, mode may be defined as the value that occurs the maximum number of times. Thus, mode is that value which has the maximum concentration of the observations around it.

Hence, the correct answer is option (d).

Example 62. If GM of x is 10 and GM of y is 15, then the GM of xy is

(a) 150 (b) log 10 × log 15 (c) log 150 (d) None of these

Sol. (a) Given: GM of x = 10 and GM of y = 15

We know that,

GM of the product of two variables is the product of their GM's.

Thus, GM of xy = GM of x × GM of y = 10 × 15 = 150

Therefore, GM of xy is 150.

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART E)

1. Find the geometric mean of 2, 4, 8, 12, 16 and 24.

(a) 29.49 (b) 5.86 (c) 8.16 (d) 6.56

2. Find the geometric mean of 3, 6, 24 and 48.

(a) 9 (b) 12 (c) 16 (d) None of these

3. Find the GM for the following distribution:

x	2	4	8	16
f	2	3	3	2

(a) 2 (b) $4\sqrt{2}$ (c) 5 (d) $6\sqrt{3}$

4. The rates of returns from three different shares are 100%, 200% and 400% respectively, the average rate of return will be _____.

(a) 350% (b) 233.33% (c) 200% (d) 300%

5. Find the geometric mean for the following distribution:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	10	9	25	30	16	10

(a) 21.57 (b) 25.5 (c) 26.73 (d) None of these

6. The Geometric mean of 11, 2, 8, 9, 15, 24, 0, 18 is

(a) 0 (b) 25 (c) 47 (d) 90

7. The geometric mean of three numbers is 12 and two of the numbers are 4 and 16. What is the third number?

(a) 12 (b) 32 (c) 27 (d) 48

8. The geometric mean of three numbers 40, 50 and x is 10, the value of x is

(a) 5 (b) 4 (c) 2 (d) $\frac{1}{2}$

Answer Key

1. (c) **2.** (b) **3.** (b) **4.** (c) **5.** (c) **6.** (a) **7.** (c) **8.** (d)

HARMONIC MEAN

The harmonic mean, usually abbreviated as H.M., is defined as the reciprocal of the arithmetic mean of the reciprocals of the given set of observations.

Symbolically,

$$HM = \frac{n}{\Sigma \frac{1}{X}}$$

CALCULATION OF HARMONIC MEAN: INDIVIDUAL OBSERVATIONS

The harmonic mean of a set of n observations X_1, X_2, ……, X_n (not necessarily all distinct) is given by

$$HM = \frac{n}{\frac{1}{X_1}+\frac{1}{X_2}+\ldots\ldots\ldots\ldots+\frac{1}{X_n}} = \frac{n}{\Sigma\frac{1}{X}}$$

Example 63. Find the harmonic mean of 5 numbers 4, 5, 6, 10 and 12.

(a) 2.5 (b) 6.25 (c) 1.50 (d) None of these

Sol. (b) By definition of H.M.

$$\frac{n}{\sum\frac{1}{x}} = \frac{5}{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{10}+\frac{1}{12}} = \frac{5}{\frac{15+12+10+6+5}{60}} = \frac{5\times 60}{48} = \frac{25}{4} = 6.25$$

Therefore, the required harmonic mean of given numbers is 6.25.

Hence, the correct option is (b).

Example 64. The harmonic mean for the numbers 2, 3, 5 is

(a) 2.00 (b) 3.33 (c) 2.90 (d) $-\sqrt[3]{30}$

Sol. (c) Given: $x_1 = 2$, $x_2 = 3$, $x_3 = 5$ and $n = 3$

We know that,

$$HM = \frac{n}{\sum\frac{1}{x_i}} = \frac{3}{\frac{1}{2}+\frac{1}{3}+\frac{1}{5}} = \frac{3\times 30}{15+10+6} = \frac{90}{31} = 2.90$$

Therefore, the harmonic mean of the given numbers is 2.90.

Hence, the correct option is (c).

Example 65. What is the HM of $1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots\frac{1}{n}$?

(a) n (b) 2n (c) $\frac{2}{(n+1)}$ (d) $\frac{n(n+1)}{2}$

Sol. (c) We know that,

H.M of x_1, x_2, x_3,..... x_n (none of them is zero) is given by:

$$H.M = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)}$$

Thus, H.M of $1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots\frac{1}{n} = \frac{n}{1+2+3+\ldots\ldots n} = \frac{n}{\frac{n(n+1)}{2}} = \frac{2}{(n+1)}$

Hence, the correct option is (c).

CALCULATION OF HARMONIC MEAN: DISCRETE SERIES

The harmonic mean of a set of n observations X_1, X_2, ….., X_n with their respective frequencies f_1, f_2 ,……..., f_n is given by:

$$HM = \frac{n}{\Sigma\left[\frac{f}{X}\right]}$$

CALCULATION OF HARMONIC MEAN: CONTINUOUS SERIES

In the case of continuous series, we take X_1, X_2,... X_n are the class marks (or mid-values) of a set of grouped data with corresponding class frequencies f_1, f_2,... f_n, then harmonic mean is given by:

$$HM = \frac{n}{\Sigma\left[\frac{f}{X}\right]}$$

Example 66. Find the harmonic mean for the following distribution.

X	10	15	20	25	30
f	2	5	4	9	5

(a) 10.15 (b) 15.41 (c) 19.73 (d) None of these

Sol. (c) According to the given data,

X	f	$\frac{f}{X}$
10	2	0.2
15	5	0.34
20	4	0.2
25	9	0.36
30	5	0.167
	$N = \sum f = 25$	$\sum \frac{f}{X} = 1.267$

Thus, H.M $\frac{\sum f}{\sum \frac{f}{X}} = \frac{25}{1.267} = 19.73$

Hence, the correct option is (c).

USES OF HARMONIC MEAN

The harmonic mean is used

1. In averaging speeds when equal distances are covered with varying speed
2. In finding the average cost of some commodity when several different purchases are made by investing the same amount of money each time.

WEIGHTED HARMONIC MEAN

Let $w_1, w_2, \ldots, w_n$ be the weights attached to n observations $X_1, X_2, \ldots\ldots X_n$ respectively. Then the Weighted Harmonic Mean, denoted by H.M. $_{w,}$ is defined as

$$H{\cdot}M_{.w} = \frac{\Sigma w_i}{\Sigma\left(\frac{w_i}{x_i}\right)}$$

USE OF WEIGHTED HARMONIC MEAN

- In actual practice, the weighted harmonic mean is most frequently used in averaging speeds when different distances are covered with varying speeds.
- In finding the average cost of some commodity when several different purchases are made by putting in a different amount of money each time.

PROPERTIES

If all the observations taken by a variable are constants, say k, then the HM of the observations is also k.

If there are two groups with n_1 and n_2 observations and H_1 and H_2 as respective HM's then Combined HM is given by:

$$\frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$$

Example 67. The harmonic mean for 40 observations of group 1 data is 520 and for 50 observations of group 2 data is 680. What is the combined harmonic mean?

(a) 450.60 (b) 598.20 (c) 800.00 (d) None of these

Sol. (b) According to the given data,

$n_1 = 40$, $n_2 = 50$, $H_1 = 520$ and $H_2 = 680$

Thus, the combined HM

$$= \frac{40+50}{\frac{40}{520} + \frac{50}{680}} = \frac{90}{\frac{1}{13} + \frac{5}{68}} = 598.20$$

Therefore, the combined harmonic mean is 598.20.

Hence, the correct option is (b).

Example 68. If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observations, then the combined HM is given by

(a) 65 (b) 70.36 (c) 70 (d) 71

Sol. (c) We know that,

If there are two groups with n_1 and n_2 observations and H_1 and H_2 as respective HM's, then the combined HM is given by

$$\frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}.$$

Here, $n_1 = 15$, $n_2 = 13$, $H_1 = 75$ and $H_2 = 65$

Thus, the required combined harmonic mean

$$= \frac{15+13}{\frac{15}{75}+\frac{13}{65}} = \frac{28}{\frac{1}{5}+\frac{1}{5}} = \frac{28}{\frac{2}{5}} = 28 \times \frac{5}{2} = 14 \times 5 = 70$$

Therefore, the combined harmonic mean is 70.

Hence, the correct option is (c).

MERITS AND DEMERITS OF HARMONIC MEAN

Merits

Harmonic mean possess the following merits.

1. It is rigidly defined.
2. It is based on all the observations.
3. Since the reciprocals of the values of the observations are involved, It is not as much affected by one or two big observations.
4. It is not affected very much by fluctuations of sampling.

Demerits

Harmonic mean has the following drawbacks.

1. It is not easy to understand.
2. It is difficult to calculate.
3. Its value cannot be computed if any one of the observations is zero.
4. It gives the largest weight to the smallest item. This is generally not a desirable feature and as such this average is not very useful for the analysis of economic data.

Example 69. Which of the following measure(s) possesses (possess) mathematical properties?

(a) AM (b) GM (c) HM (d) All of these

Sol. (d) **AM:** For a given set of observations, the AM may be defined as the sum of all the observations divided by the number of observations.

GM: For a given set of n positive observations, the geometric mean is defined as the n-th root of the product of the observations.

HM: For a given set of non-zero observations, the harmonic mean is defined as the reciprocal of the AM of the reciprocals of the observation.

Hence, the correct option is (d).

PRACTICE QUESTIONS (PART F)

1. Find the Harmonic mean of 2, 4 and 6.

(a) 3.00 (b) 3.27 (c) 3.75 (d) 4.00

2. Calculate the Harmonic mean of $1, \frac{1}{3}, \frac{1}{6} \& \frac{1}{9}$

(a) 2.48 (b) 0.21 (c) 0.31 (d) 0.25

3. If there are two groups with 10 and 12 observations and harmonic mean of the two groups are 3 and 5 respectively, then the combined harmonic mean is

(a) 8.0 (b) 2.0 (c) 3.8 (d) 4.0

4. Find the HM for the following data;

x	2	4	8	16	20
f	2	3	3	2	1

(a) 3.75 (b) 4.78 (c) 5.15 (d) 5.86

5. A plane flies the first 500 miles of its trip at 400 mph and the next 1000 miles at 600 mph. What is the average speed of the plane for the entire journey?

(a) 428 mph (approx.) (b) 498 mph (approx.)

(c) 515 mph (approx.) (d) 568 mph (approx.)

6. An aeroplane flies from A to B at the rate of 500 km/hour and comes back from B to A at the rate of 700 km/hour. The average speed of the aeroplane is

(a) 100 km/hour (b) 583.33 km/hour

(c) $100\sqrt{35}$ km/hour (d) 620 km/hour

Answer Key

1. (b) **2.** (b) **3.** (c) **4.** (b) **5.** (c) **6.** (b)

RELATIONSHIP BETWEEN AM, GM AND HM

1. For any set of data, the values of A.M., G.M., and H.M. are connected by the following relation: AM ≥ GM ≥ HM

- If all the values in the dataset are equal then AM = GM = HM
- If all the values in the dataset are distinct then AM > GM > HM

2. For two numbers, we have $(G.M.)^2 = A.M. \times H.M.$

Example 70. The A.M. and G.M. of two numbers are 15 and 12 respectively. Find the H.M. of the numbers.

(a) 2 (b) 15 (c) 20 (d) 9.6

Sol. (d) We know that,

$AM \times HM = GM^2$

$\Rightarrow 15 \times HM = (12)^2$

$\Rightarrow HM = \frac{144}{15}$

$\Rightarrow HM = 9.6$

Therefore, the value of HM is 9.6.

Hence, the correct option is (d).

Example 71. If the AM and GM for 10 observations are both 15, then the value of HM is

(a) Less than 15 (b) More than 15

(c) 15 (d) Cannot be determined.

Sol. (c) Given: AM = GM = 15

We know that,

$AM \times HM = GM^2$

$\Rightarrow 15\ HM = (15)^2$

$\Rightarrow HM = \frac{(15)^2}{15}$

$\Rightarrow HM = 15$

Therefore, the value of HM is 15.

Hence, the correct option is (c).

Example 72. If the AM and GM for two numbers are 6.50 and 6 respectively, then the two numbers are

(a) 6 and 7 (b) 9 and 4 (c) 10 and 3 (d) 8 and 5

Sol. (b) If a and b are two positive observations such that a > b, then

According to the question

$\frac{a+b}{2} = 6.5\ (\therefore\ AM = 6.5)$

$\Rightarrow a + b = 13$ ...(i)

and $\sqrt{ab} = 6\ (\therefore\ GM = 6)$

$\Rightarrow ab = 36$ ...(ii)

We know, $(a - b)^2 = (a + b)^2 - 4ab$

$\Rightarrow (a - b)^2 = (13)^2 - 4 \times 36$

$\Rightarrow (a - b)^2 = 169 - 144$

$\Rightarrow (a - b)^2 = 25$

$\Rightarrow a - b = 5$ ...(iii)

Adding (i) and (iii), we get

$2a = 18$

$a = 9$

From (i), we get

$b = 13 - 9 = 4$

Therefore, the two numbers are 9 and 4.

Hence, the correct option is (b).

Example 73. If the AM and HM for two numbers are 5 and 3.2 respectively, then the GM will be

(a) 16.00 (b) 4.10 (c) 4.05 (d) 4.00

Sol. (d) Given: AM = 5 and GM = 3.2

We know that,

$AM \times HM = GM^2$

$\Rightarrow GM2 = 5\times 3.2$

$\Rightarrow GM^2 = 1.6$

$\Rightarrow GM = 4$

Therefore, GM will be 4.

Hence, the correct option is (d).

Example 74. Which of the following results hold for a set of distinct positive observations?

(a) $AM \geq GM \geq HM$ (b) $HM \geq GM \geq AM$

(c) $AM > GM > HM$ (d) $GM > AM > HM$

Sol. (c) AM > GM > HM true for a set of distinct positive observations.

AM: For a given set of observations, the AM may be defined as the sum of all the observations divided by the number of observations.

GM: For a given set of n positive observations, the geometric mean is defined as the n-th root of the product of the observations.

HM: For a given set of non-zero observations, the harmonic mean is defined as the reciprocal of the AM of the reciprocals of the observation.

Hence, the correct option is (c).

Example 75. For a moderately skewed distribution, which of the following relationships holds?

(a) Mean - Mode = 3 (Mean - Median) (b) Median - Mode 3 (Mean - Median)

(c) Mean Median = 3 (Mean Mode) (d) Mean - Median = 3 (Median - Mode)

Sol. (a) For a moderately skewed distribution, following relationship holds

Mean – Mode = 3 (Mean - Median)

Hence, the correct option is (a).

FULL CHAPTER PRACTICE QUESTIONS

1. Find the harmonic mean of the following numbers: 2, 3, 6, 8, 10

(a) 4.08 (b) 5.18 (c) 9.04 (4) None of these

2. Two values yielded an arithmetic mean of 24 and a harmonic mean of 6. The geometric mean of these values is _____.

(a) 8 (b) 12 (c) 14 (d) 16

3. The average of 5 quantities is 6 and the average of 3 is 8. What is the average of the remaining two?

(a) 4 (b) 5 (c) 3 (d) 3.5

4. From the record on sizes of shoes sold in a shop, one can compute the following to determine the most preferred shoe size.

(a) Mean (b) Median (c) Mode (d) Range

5. In a moderately skewed distribution, the mode and median are 20 and 24 respectively. The value of mean will be _____.

(a) 21 (b) 26 (c) 30 (d) None

6. The mean of 'n' observation is 'X'. If 'K' is added to each observation, then the new mean is _____.

(a) X (b) XK (c) X - K (d) X + K

7. If the AM and HM for two numbers are 5 and 3. 2 respectively then the GM will be

(a) 16.00 (b) 4.10 (c) 4.05 (d) 4.00

8. Mean of 7, 9, 12, x, 4, 11 & 5 is 9. Find the missing observation.

(a) 13 (b) 15 (c) 12 (d) None of these

9. The median of following numbers, which are given in ascending order is 25. Find the value of if data is 11, 13, 15, 19, (x + 2), (x + 4), 30, 35, 39, 46

(a) 22 (b) 20 (c) 15 (d) 30

10. For a distribution Mean, Median and Mode are 23, 24 and 25. 5 respectively, then it is most likely _____ skewed distribution.

(a) Positively (b) Symmetrical (c) Asymptotically (d) Negatively

11. There are n numbers. When 50 is subtracted from each of these number the sum of the numbers so obtained is –10. When 46 is subtracted from each of the original n numbers, then the sum of numbers so obtained is 70. What is the mean of the original n numbers?

(a) 56.8 (b) 25.7 (c) 49.5 (d) 53.8

12. Calculate the mode for the following data:

Monthly Wages (₹)	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of workers	4	6	20	12	33	17	8	2

(a) 259.50 (b) 350.78 (c) 400.42 (d) 428.38

13. The mean of 100 items was found to be 40. Later on it was discovered that two items of 45, 35 were wrongly taken as 35 and 25. Find the correct mean.

(a) 40.2 (b) 43.5 (c) 40 (d) None of these

14. An incomplete distribution is given below:

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Total Frequency	10	___	30	___	25	20	125

The missing frequencies if the median value is 33 are

(a) 10 and 15 (b) 15 and 20 (c) 15 and 25 (d) 20 and 25

15. A survey was conducted by a group of students as a part of their environmental awareness program, in which they collected the following data regarding the number of plants in 200 houses in a locality. Find the mean number of plants per house.

Number of Plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of Houses	1	2	1	5	6	2	3

(a) 162 (b) 20 (b) 15.5 (d) 8.1

Answer Key

1. (a) **2.** (b) **3.** (c) **4.** (c) **5.** (b) **6.** (d) **7.** (d) **8.** (b) **9.** (a) **10.** (d)
11. (c) **12.** (d) **13.** (a) **14.** (c) **15.** (d)

PREVIOUS YEAR QUESTION (PYQ'S)

1. The AM of 15 observations is 9 and the AM of first 9 observations is 11 and then AM of remaining observations is **(June 2019)**

(a) 11 (b) 6 (c) 5 (d) 9

2. In a moderately skewed distribution the values of mean & median are 12 & 18 respectively. The value of mode is **(June 2019)**

(a) 6 (b) 12 (c) 15 (d) 30

3. Which of the following is positional average?

(a) Median (b) GM (c) HM (d) AM

4. For the distribution **(June 2019)**

X	1	2	3	4	5	6
F	6	9	10	14	12	8

The value of median is

(a) 3.5 (b) 3 (c) 4 (d) 5

5. For a symmetric distribution

(a) Mean = Median = Mode (b) Mode = 3 Median - 2 Mean

(c) Mode $= \frac{1}{3}$ Median $= \frac{1}{2}$ (d) None **(June 2019)**

6. $\sum_{i=1}^{m}(x_i - \bar{x}) =$

(a) 1 (b) 0 (c) -1 (d) None of these

7. The median of the following frequency distribution is equal to **(Dec 2019)**

X	5	7	9	12	14	17	19	21
Y	6	5	3	6	5	3	2	4

(a) 6 (b) 12 (c) 13 (d) 14

8. Find median from the following data:

Marks	0-10	10-30	30-60	60-80	80-90
No. of Students	5	15	30	8	2

(a) 8 (b) 30 (c) 40 (d) 45

9. Find the mode from the following data: **(Dec 2019)**

Class	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	5	10	23	21	12	3

(a) 23 (b) 13.3 (c) 12.6 (d) 14.6

10. Find the mode of the following distribution? **(Dec 2019)**

Class	0-7	7-14	14-21	21-28	28-35	35-42	42-49
Frequency	19	25	36	72	51	43	28

(a) 24.3 (b) 25.4 (c) 72 (d) 21

11. The arithmetic mean of two numbers is 30 and geometric mean is 24 , find the two numbers. **(Dec 2019)**

(a) 12 and 48 (b) 14 and 46 (c) 10 and 50 (d) 16 and 44

12. Sum of the squares of deviations is minimum when deviations are taken from **(Dec 2019)**

(a) Mean (b) Median (c) Mode (d) An arbitrary value

13. Given the weights for the numbers 1, 2, 3,..., n are respectively 1^2, 2^2, 3^2,..., n^2 Then weighted HM is **(Dec 2020)**

(a) $\frac{2n+1}{2}$ (b) $\frac{2n+1}{6}$ (c) $\frac{2n+1}{3}$ (d) None

14. Which measure is suitable for open end classification? **(Dec 2020)**

(a) Median (b) Mean (c) Mode (d) GM

15. 50th percentile is equal to **(Dec 2020)**

(a) Median (b) Mode (c) Mean (d) None

16. For a distribution Mean, Median and Mode are 23, 24 and 25.5 respectively, then it is most likely _____ skewed distribution. **(Dec 2020)**

(a) Positively (b) Symmetrical (c) Asymptotically (d) Negatively

17. If any two numbers are in AP , then GM^2 = **(Dec 2020)**

(a) AM × HM (b) AM + HM (c) M × Z (d) AM × M

18. Two values yielded an arithmetic mean of 24 and a harmonic mean of 6. The geometric mean of these values is **(Dec 2020)**

(a) 8 (b) 12 (c) 14 (d) 16

19. The HM of A and B is $\frac{1}{3}$ and HM of C and D is $\frac{1}{5}$. Then HM of A, B, C and D is **(Dec 2020)**

(a) $\frac{8}{15}$ (b) $\frac{1}{4}$ (c) $\frac{15}{8}$ (d) $\frac{4}{15}$

20. Which one of these is least affected by extreme values? **(Dec 2020)**

(a) Mean (b) Median (c) Mode (d) None

21. A fire engine rushes to a place of fire accident with a speed of 110 kmph and after the completion of operation returned to the base at a speed of 35 kmph. The average speed per hour in per direction is obtained as _____ of those speeds. **(Dec 2020)**

(a) Speed Avg. of (b) HM of (c) GM of (d) Half of HM of

22. Ten matches data is given, then which of the following cannot be found? **(Dec 2020)**

(a) Least score (b) Highest score (c) Best score (d) Median score

23. If the AM and HM of two numbers are 6 and 9 respectively, then GM is **(Dec 2020)**

(a) 7.35 (b) 8.5 (c) 6.75 (d) None

24. From the record on sizes of shoes sold in a shop, one can compute the following to determine the most preferred shoe size. **(Jan 2021)**

(a) Mean (b) Median (c) Mode (d) Range

25. Which of the following measures does not possess mathematical properties? **(Jan 2021)**

(a) Arithmetic mean (b) Geometric mean

(c) Harmonic mean (d) Median

26. If $y = 3 + 4.5x$ and the mode of x-value is 20 , then the mode of y-value is **(Jan 2021)**

(a) 3.225 (b) 12 (c) 24.5 (d) 93

27. If there are two groups with n_1 and n_2 observations and H_1 and H_2 are respective harmonic means, then the harmonic mean of combined observation is **(Jan 2021)**

(a) $\frac{n_1H_1 + n_2H_2}{n_1 + n_2}$ (b) $\frac{n_1H_1 + n_2H_2}{H_1 + H_2}$ (c) $\frac{n_1 + n_2}{n_1H_1 + n_2H_2}$ (d) $\frac{(n_1 + n_2)H_1 + H_2}{n_1H_2 + n_2H_1}$

28. Expenditures of a company (in million rupees) per item in various years

Year	Item of Expenditures				
	Salary	Fuel and Transport	Bonus	Interest on Loans	Taxes
1998	288	98	3.00	23.4	83
1999	342	112	2.52	32.5	108
2000	324	101	3.84	41.6	74
2001	336	133	3.68	36.4	88
2002	420	142	3.96	49.4	98

What is the average amount of interest per year which the company had to pay during the period? **(July 2021)**

(a) 33.66 (b) 36.66 (c) 31.66 (d) 39.66

29. There are n numbers. When 50 is subtracted from each of these numbers, the sum of numbers so obtained is -10. When 46 is subtracted from each of the original n numbers, then the sum of numbers so obtained is 70. What is the mean of the original n numbers? **(July 2021)**

(a) 56.8 (b) 25.7 (c) 49.5 (d) 53.8

30. The mean of 'n' observation is 'X'. If K is added to each observation, then the new mean is **(July 2021)**

(a) X (b) XK (c) X – K (d) X + K

31. If y = 3 + 1.9x and mode of x is 15 , then the mode of y is **(July 2021)**

(a) 15.9 (b) 27.8 (c) 35.7 (d) 31.5

32. If there are 3 observations 15, 20, 25 then the sum of deviation of the observations from their AM is **(Dec 2021)**

(a) 0 (b) 5 (c) -5 (d) 10

33. If the AM and GM for 10 observations are both 15 , then the value of HM is **(Dec 2021)**

(a) Less than 15 (b) More than 15

(c) 15 (d) Cannot be determined

34. If the average mark for a group of 30 girls is 80 , a group of boys is 70 and combined average is 76 , then how many are in the boy's group? **(Dec 2021)**

(a) 21 (b) 20 (c) 22 (d) 19

35. If two variables a and b are related by c = ab, then GM of c is equal to **(Dec 2021)**

(a) GM of a + GM of b (b) GM of a × GM of b

(c) GM of a - GM of b (d) GM of a/GM of b

36. For a moderately skewed distribution, the median is twice the mean, then the mode is **(Dec 2021)**

(a) 3 (b) 2 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

37. The median value of the set of observations 48, 36, 72, 87, 19, 66, 56, 91 is **(Dec 2021)**

(a) 53 (b) 87 (c) 61 (d) 19

38. For a data having an odd number of values, the difference between the first and the middle value is equal to the difference between the last and the middle values. Similarly, the difference between the second and the middle values is equal to that of second last and middle value so on. Therefore, the middle value is equal to **(Dec 2021)**

(a) Half of the range (b) Half of standard deviation

(c) Mode (d) Mean

39. One hundred participants expressed their opinion on recommending a new product to their friends using the attributes: most unlikely, unlikely, not sure, likely, most likely. The appropriate measure of central tendency that can be used here is **(Dec 2021)**

(a) Mean (b) Mode

(c) Geometric mean (d) Harmonic mean

40. Ogive curves cannot be used to determine **(Dec 2021)**

(a) Mean (b) Median (c) Mode (d) Range

41. Along a road there are 5 buildings of apartments, marked as 1, 2, 3, 4,5. Number of people residing in each building is available. A bus stop is to be setup near one of the buildings so that the distance walked by the residents to the bus stop from their buildings must be kept minimum. One must consider involving _____ to find the position of the bus stop. **(Dec 2021)**

(a) Mean (b) Median (c) Mode (d) Weighted mean

42. Given that mean = 70.20 and mode = 70.50, the median is expected to be **(Dec 2021)**

(a) 70.15 (b) 70.20 (c) 7.30 (d) 70.35

43. _____ mean is calculated, when the values in series do not have equal importance. **(June 2022)**

(a) Arithmetic (b) Harmonic (c) Geometric (d) Weighted

44. A seller of pearls kept the pearls in seven boxes labeled from one to seven. At the end of a day, he found that J labeled box contained J pearls, the average number of pearls per box is **(June 2022)**

(a) 4 (b) 6.5 (c) 7.5 (d) 8

45. The mean of 100 students was 45. Later on, it was discovered that the marks of two students were misread as 85 and 54 instead of 58 and 45. Find out the correct mean. **(June 2022)**

(a) 68 (b) 36 (c) 44.64 (d) 52

46. Calculate the value of 3rd quartile from the following data; 40, 35, 51, 30, 21, 25, 16, 29, 27, 32

(a) 36.25 (b) 30.25 (c) 25 (d) 35

47. If mean $(\bar{x})$ is 10 and mode (Z) is 7 , then find out the median (M). **(Dec 2022)**

(a) 9 (b) 17 (c) 3 (d) 4.33

48. _____ is based on all the observations and _____ is based on the central fifty per cent of the observations. **(Dec 2022)**

(a) Mean deviation, Range

(b) Mean deviation, Quartile deviation

(c) Range, Standard deviation

(d) Quartile deviation, standard deviation

49. The relationship between two variables x and y is given by 4x – 10y = 20. If the median value of the variable x is 10 then the median value of variable y is **(Dec 2022)**

(a) 1.0 (b) 2.0 (c) 3.0 (d) 4.0

50. The median of the observations 42, 72, 35, 92, 67, 85, 72, 81, 51, 56 is **(Dec 2022)**

(a) 69.5 (b) 72 (c) 64 (d) 61.5

51. The mean of 50 observations is 36. If two observations 30 and 42 are to be excluded, then the mean of the remaining observations will be **(Dec 2022)**

(a) 36 (b) 38 (c) 48 (d) 50

52. If Arithmetic mean and geometric mean between two numbers are 5 and 4 respectively, then these numbers are **(Dec 2022)**

(a) 2 and 3 (b) 2 and 8 (c) 4 and 6 (d) 1 and 16

53. If Arithmetic between two numbers is 5 and Geometric mean is 4 , then what is the value of Harmonic mean? **(Dec 2022)**

(a) 3.2 (b) 3.4 (c) 3.5 (d) 3.6

54. The average age of 15 students in a class is 9 years. Out of them, the average age of 15 students is 13 years and that of 8 students is 5 years. What is the average of the remaining 2 students? **(Dec 2022)**

(a) 5 years (b) 9 years (c) 10 years (d) 15 years

55. A professor has given assignment to students in a Statistics class. A student Jagan computes the arithmetic mean and standard deviation for a set of 100 observations as 50 and 5 respectively. Later on, Sonali points out to Jagan that he has made of mistake in taking one observation as 100 instead of 50. What would be the correct mean if the wrong observation is corrected? **(June 2023)**

(a) 50.5 (b) 49.9 (c) 49.5 (d) 50.1

56. Find the mean of the following data **(June 2023)**

Class Interval	Frequency
10-20	9
20-30	13
30-40	6
40-50	4
50-60	6
60-70	2
70-80	3

(a) 23.7 (b) 35.7 (c) 39.7 (d) 43.7

57. The median of the following set of observations; 24, 18, 36, 42, 30, 28, 21, 29, 25, 33 is **(June 2023)**

(a) 26.5 (b) 27.5 (c) 28.5 (d) 29.5

58. Find the mode of the following data:

x	25-30	30-35	35-40	40-45	45-50	50-55
f(x)	20	53	42	42	41	43

(a) 31.75 (b) 30.75 (c) 33.75 (d) 35.75

59. For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean marks and median marks were found to be 55.60 and 52.40 respectively, what are the modal marks? **(June 2023)**

(a) 54.43 (b) 48 (c) 53.56 (d) 46

60. For a given data set; 5, 10, 3, 6, 4, 8, 9, 3, 15, 2, 9, 19, 11, 4 then what is the median? **(June 2023)**

(a) 8 (b) 6 (c) 4 (d) 9

61. If the mean of two numbers is 30 and geometric mean is 24 , then what will be the harmonic mean of two numbers?

(a) 19.2 (b) 21.8 (c) 22.3 (d) 18.4

62. The Geometric mean of 3, 7, 11, 15, 24, 28, 30, 0 is

(a) 6 (b) 0 (c) 9 (d) 12

63. The median of the following frequency distribution is; **(Dec 2023)**

x	0-10	10-20	20-30	30-40
f(x)	3	5	12	7

(a) 27.75 (b) 9.35 (c) 8.25 (d) 10.1

64. If the mode of a grouped data is 10 and the median is 6 , then what is the value of mean? **(Dec 2023)**

(a) 2 (b) 4 (c) 6 (d) 8

65. If mean and median of moderately asymmetrical series are 26.8 and 27.9 respectively then most probable value of mode is **(Dec 2023)**

(a) 35.6 (b) 30.1 (c) 34.3 (d) 70.8

66. The AM and HM of two numbers are 5 and 3.2 respectively, then GM will be **(Dec 2023)**

(a) 4.4 (b) 4.2 (c) 4.0 (d) 3.8

67. If AM and GM of two positive numbers 'a' and 'b' are 12 and 12 respectively, find the numbers. **(Dec 2023)**

(a) 18 and 6 (b) 15 and 9 (c) 16 and 8 (d) 12 and 12

68. Which of the following measure of central tendency will be unaffected if the lowest and highest observations are removed? **(June 2024)**

(a) Mean (b) Median (c) Mode (d) Range

69. If mean of a data set is 22 and median is 22.33, then the mode is **(June 2024)**

(a) 21 (b) 21.34 (c) 22.99 (d) 21.54

70. Which of the following measure of central tendency depends on the position of the observation? **(June 2024)**

(a) Mean (b) Median (c) Mode (d) Harmonic Mean

71. If there are two groups containing 40 and 30 observations and have arithmetic means as 50 and 60, then the combined arithmetic mean is **(June 2024)**

(a) 55.48 (b) 56.35 (c) 54.28 (d) 50.28

72. The mean of a set of 20 observations in 18.3. The mean is reduced by 0.6 when a new observation is added to the set. The new observation is **(June 2024)**

(a) 17.6 (b) 18.9 (c) 5.7 (d) 24.6

73. The mean of a group X is 70 and the mean of group Y is 85. If the number of observations in group Y is five times that of group X, then the combined mean of both the groups is:

(Sept 2024)

(a) 75 (b) 80 (c) 77.5 (d) 82.5

74. The Median of the following frequency distribution is: **(Sept 2024)**

x	0-10	10-20	20-30	30-40	40-50
f(x)	8	30	40	12	10

(a) 33 (b) 22.5 (c) 23 (d) 24

75. If the mean and median of a moderately asymmetrical series are 70.8 and 68.6 respectively, then the most probable mode is: **(Sept 2024)**

(a) 64.2 (b) 75.2 (c) 63.4 (d) 72.5

76. For a moderately-skewed distribution, which of the following relationship holds?

(Sept 2024)

(a) Mean - Mode = 3 (Mean - Median) (b) Median - Mode = 3 (Mean - Median)
(c) Mean - Median = 3 (Mean - Mode) (d) Mean - Median = 3 (Median - Mode)

77. The AM & GM for two observations are 8 and 2. Find the values of two observations.

(Jan 2025)

(a) 16, 1 (b) 15.75, 0.25 (c) 15, 1 (d) 14.75, 1.75

78. Find the Harmonic mean of 2, 4 and 6. **(Jan 2025)**

(a) 3.00 (b) 3.30 (c) 3.75 (d) 4.00

79. If the mode of the following data is 13, then the value of x in the dataset is **(Jan 2025)**

13, 8, 6, 3, 8, 13, 2x + 3, 8, 13, 3, 5, 7

(a) 5 (b) 6 (c) 7 (d) 8

80. The mean of three numbers is 135. Among the three numbers, the biggest number is 180. The difference between the remaining two numbers is 25, then the smallest number is

(Jan 2025)

(a) 125 (b) 130 (c) 120 (d) 100

81. The algebraic sum of deviations of set of observations from their arithmetic mean is

(Jan 2025)

(a) $\sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$ (b) $\frac{\sum x_i}{n}$ (c) $\frac{\sum x_i}{n-1}$ (d) Zero

82. The best measure of central tendency is **(Jan 2025)**

(a) Median (b) Mean (c) Mode (d) Range

83. Find out the mode from the following data; **(May 2025)**

100, 110, 125, 225, 325, 125, 90, 80, 455, 375, 125

(a) 325 (b) 110 (c) 455 (d) 125

84. If the AM and GM for two numbers are 6.50 and 6 respectively, then the two numbers are **(May 2025)**

(a) 6 and 7 (b) 9 and 4 (c) 10 and 3 (d) 8 and 5

85. An aeroplane flies from A to B at the rate of 500 km/hr and comes back from B to A at the rate of 700 km/hr. The average speed of the aeroplane is: **(May 2025)**

(a) 600 km/hr (b) 583.33 km/hr (c) $100\sqrt{35}$ km/hr (d) 620 km/hr

86. If Arithmetic Mean (A.M.) and Geometric Mean (G.M.) of two numbers are 6.50 and 6 respectively, then the two numbers are **(May 2025)**

(a) 6 and 7 (b) 9 and 4 (c) 10 and 3 (d) 8 and 5

87. If the mean of the following frequency distribution is 2.6, then the value of Y is **(May 2025)**

Mark (X)	1	2	3	4	5
No. of Students (f)	8	10	Y	2	4

(a) 16 (b) 6 (c) 26 (d) 12

88. Which one of the following measures of central tendency is based on only fifty percent (50%) of the central values? **(May 2025)**

(a) Geometric Mean (b) Harmonic Mean

(c) Median (d) Mode

89. The Arithmetic Mean (A.M.) and mode of the data are 32 and 26, respectively, then find the median of the data. **(May 2025)**

(a) 30 (b) 12 (c) 6 (d) 29

90. Find out the mode from the following data: **(May 2025)**

100, 110, 125, 225, 325, 125, 90, 80, 455, 375, 125

(a) 325 (b) 110 (c) 455 (d) 125

91. For a moderately skewed distribution of marks in English for a group of 145 students the mean marks and median marks were found to be 55.10 and 52.40 respectively. The modal marks are **(Sept 2025)**

(a) 47 (b) 42 (c) 53.1 (d) 52.2

92. If X and Y are related by 4X + 3Y + 5 – 0 and Mean of X is 10, then the Mean of Y is **(Sept 2025)**

(a) –23 (b) –15 (c) –20 (d) 23

93. Calculate the Harmonic mean of $1, \frac{1}{3}, \frac{1}{6} \& \frac{1}{9}$ **(Sept 2025)**

(a) 2.48 (b) 0.21 (c) 0.31 (d) 0.25

94. If there are two groups with 10 and 12 observations and harmonic mean of the two groups are 3 and 5 respectively, then the combined harmonic mean is **(Sept 2025)**

(a) 8.0 (b) 2.0 (c) 3.8 (d) 4.0

95. A set contains seven numbers whose average is 12. The average of four greater numbers is 18 and the average of four smaller numbers is 8. Which of the followings is the value of one of the numbers? **(Sept 2025)**

(a) 12 (b) 16 (c) 18 (d) 20

Answer Key

1. (b)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (b)
11. (a)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (d)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (c)
21. (b)	**22.** (c)	**23.** (d)	**24.** (c)	**25.** (d)	**26.** (d)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (d)
31. (d)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (b)	**37.** (c)	**38.** (d)	**39.** (b)	**40.** (d)
41. (b)	**42.** (c)	**43.** (d)	**44.** (a)	**45.** (c)	**46.** (a)	**47.** (a)	**48.** (b)	**49.** (b)	**50.** (a)
51. (a)	**52.** (b)	**53.** (a)	**54.** (d)	**55.** (c)	**56.** (b)	**57.** (c)	**58.** (c)	**59.** (d)	**60.** (b)
61. (a)	**62.** (b)	**63.** (a)	**64.** (b)	**65.** (b)	**66.** (c)	**67.** (d)	**68.** (b)	**69.** (c)	**70.** (b)
71. (c)	**72.** (c)	**73.** (d)	**74.** (c)	**75.** (a)	**76.** (a)	**77.** (b)	**78.** (b)	**79.** (a)	**80.** (d)
81. (d)	**82.** (b)	**83.** (d)	**84.** (b)	**85.** (b)	**86.** (b)	**87.** (a)	**88.** (c)	**89.** (a)	**90.** (d)
91. (a)	**92.** (b)	**93.** (b)	**94.** (c)	**95.** (d)					

CA WALLAH

Chapter 14 - Unit-2: Measure of Dispersion

WHAT IS DISPERSION?

The central tendency measures we've explored so far provide valuable insights into data, but they alone cannot offer a comprehensive description. Regardless of which measure we use, it doesn't reveal the distribution pattern of the data. We may encounter situations where different sets of observations share the same mean but exhibit substantial variations in their measurements around this average.

E.g.: consider the following three sets of observations, each containing 9 items:

Mean										Total	Mean
Set-A	20	20	20	20	20	20	20	20	20	180	20
Set-B	16	17	18	19	20	21	22	23	24	180	20
Set-C	12	14	16	18	20	22	24	26	28	180	20

All the three sets have the same mean i.e. 20 yet they are quite different.

Therefore, we can say that we need some more measures in addition to the central tendency to describe the data completely.

Definition of Dispersion: Dispersion in statistics is a way of describing how spread out a set of data is. It may be defined as the amount of deviation of the observations.

MEASURES OF DISPERSION

The degree to which the numerical data tends to deviate from the average value is called the variation or dispersion of the data.

The measure of dispersion can be classified as:

1. Absolute measures of dispersion
2. Relative measures of dispersion

ABSOLUTE MEASURES OF DISPERSION

- Absolute measures are dependent on the unit of the variable under consideration.
- Easy to comprehend and compute.

Different Measures ways:

- Range
- Mean Deviation
- Standard Deviation
- Quartile Deviation

RELATIVE MEASURES OF DISPERSION

- Relative measures of dispersion are unit free.
- For comparing two or more distributions, relative measures of dispersion are considered.

Different Measures ways:

- Coefficient of Range.
- Coefficient of Mean Deviation
- Coefficient of Variation
- Coefficient of Quartile Deviation.

Example 1. Dispersion measures **(ICAI)**

(a) The scatterness of a set of observations

(b) The concentration of a set of observations

(c) Both (a) and (b)

(d) Neither (a) and (b)

Sol. (a) We know that,

Dispersion measures scatterness of a set of observations.

Hence, the option (a) is correct.

Example 2. When it comes to comparing two or more distributions, we consider **(ICAI)**

(a) Absolute measures of dispersion (b) Relative measures of dispersion

(c) Both (a) and (b) (d) Either (a) or (b)

Sol. (c) For comparing two or more distributions we consider both Absolute measure of dispersion and Relative measure of dispersion.

Hence, the option (c) is correct.

RANGE

DEFINITION

The range of a set of data is defined as the difference between the largest and the smallest value in the set.

Range = Largest value - Smallest value

For a grouped frequency distribution, it is the difference between upper limit of the highest class and lower limit of the smaller class.

Range = Upper class boundary (U.C.B) – Lower class boundary (L.C.B)

Example 3. Following are the wages of 10 workers expressed in INR. 45, 72, 78, 90, 65, 20, 90, 65, 50, 70. Find the range

(a) 60 (b) 59 (c) 63.63 (d) None of the above

Sol. (d) Given: The wages of 10 workers in ₹ are 45, 72, 78, 90, 65, 20, 90, 65, 50, 70

Maximum wage (H) = 90

Minimum wage (L)= 20

So, the range = H – L = 90 – 20 = 70

Hence, the correct answer is option (d).

Example 4. The following data represents the heights (in centimeters) of a group of students in a Class

Height (cm)	Frequency
100 - 120	5
120 - 140	8

Height (cm)	Frequency
140 - 160	12
160 - 180	10
180 - 200	6

What is the range of heights for the given grouped frequency data?

(a) 100 cm (b) 120 cm (c) 160 cm (d) 200 cm

Sol. (a) We know that,

Range = Upper class boundary (U.C.B) - Lower class boundary (L.C.B)

According to the data given,

Minimum height (L.C.B) = 100 cm

Maximum height (U.C.B) = 200 cm

Therefore, the range of height = 200 cm – 100 cm = 100 cm

Hence, the correct option is (a) i.e., 100 cm.

COEFFICIENT OF RANGE

The range is an absolute measure of dispersion and is expressed in the unit of measurement of values of a distribution. Hence, it cannot be used to compare two distributions expressed in different units.

To overcome this difficulty, we need a relative measure which is independent of the units of measurement. This relative measure, called the coefficient of range, is defined as follows:

$$\text{Coefficient of Range} = \frac{\text{Range}}{\text{Sum of the largest and the smallest values}}$$

$$\text{i.e., Coefficient of range} = \frac{L - S}{L + S} \times 100$$

where, L is the largest value and S is the smallest value

Example 5. Following are the wages of 10 workers expressed in INR. 45, 72, 78, 90, 65, 20, 90, 65, 50, 70. Find the coefficient of range.

(a) 60 (b) 59 (c) 63.63 (d) None of these

Sol. (c) Given: The wages of 10 workers in ₹ are 45, 72, 78, 90, 65, 20, 90, 65, 50, 70

Maximum wage (L) = 90

Minimum wage (S) = 20

So, the range = L - S = 90 - 20 = 70

$$\text{Coefficient of range} = \frac{90-20}{90+20} \times 100$$

$$= \frac{70}{110} \times 100 = \frac{7}{11} \times 100 = 63.63$$

Hence, the correct answer is option (c).

Example 6. The following data represents the weights (in kilograms) of a group of individuals in a gym:

Weight (kg)	Frequency
41 – 50	6
51 – 60	12
61 – 70	15
71 – 80	8
81 – 90	5

Find the coefficient of range for the given grouped frequency data

(a) 1.13% (b) 3.82% (c) 4.29% (d) None of these

Sol. (b) According to the data given,

S = 41 - 0.5 = 40.5

L = 90 + 0.5 = 90.5

Thus, Coefficient of range = $\frac{L - S}{L + S} \times 100$

$= \frac{90.5 - 40.5}{90.5 + 40.5} \times 100 = \frac{50}{131} \times 100 = 3.82\%$

Hence, the correct option is (b) i.e., 3.82%.

Example 7. What is the coefficient of range for the following distribution?

Class Interval	10-19	20-29	30-39	40-49	50-59
Frequency	11	25	16	7	3

(a) 22 (b) 50 (c) 72.46 (d) 75.82

Sol. (c) The data is exclusive data,

First we find the boundary point for the first class & last class

The boundary points for the first class will be 9.5 – 19.5

So, the minimum value (S) = 9.5

The boundary points for last class will be 49.5 – 59.5

So, the maximum value (L) = 59.5

Thus, Coefficient of range = $\frac{L - S}{L + S} \times 100 = \frac{59.5 - 9.5}{59.5 + 9.5} \times 100 = \frac{50}{69} \times 100 = 72.46$

Hence, the correct answer is option (c).

PROPERTIES OF RANGE

Range remains unaffected due to a change of origin but affected in the same ratio due to a change in scale i.e., for any two constants a and b, two variables x and y are related by y = a + bx

Then, the range of y is given by $R_y = |b| \times R_x$

Example 8. If the relationship between x and y is given by 2x + 5y = 10 and the range of x is 5, what would be the range of y?

(a) 1 (b) 2 (c) 3 (d) 4

Sol. (b) As we know,

Range is given by the formula $R_y = |b| \times R_x$

As, 2x + 5y

$\Rightarrow 5y = 10 - 2x$

$\Rightarrow y = 2 - \frac{2x}{5}$

Now, $R_y = \frac{2}{5} \times 5$

$R_y = 2$

Hence, the correct option is (b).

Example 9. If R_x and R_y denote the range of x and y respectively where x and y are related by 3x + 2y + 10 = 0, What would be the relation between x and y?

(a) $R_x = R_y$ (b) $2R_x = 3R_y$

(c) $3R_x = 2R_y$ (d) $R_x = 2R_y$

Sol. (c) The relation of x and y is given by 3x + 2y + 10 = 0

$2y = -3x - 10$

$y = \frac{-3x-10}{2}$

So, the relation between their ranges will be

$R_y = \left|\frac{-3}{2}\right| R_x$

$3R_x = 2R_y$

Hence, the correct answer is option (c).

PRACTICE QUESTIONS (PART A)

1. The range of 15, 12, 10, 9, 17, 20 is

(a) 5 (b) 12 (c) 13 (d) 11

2. What is the coefficient of range for the wages of 8 workers?

₹80, ₹65, ₹90, ₹75, ₹70, ₹72, ₹85

(a) ₹ 30 (b) ₹ 20 (c) 30 (d) 20

3. Find the range of the daily wages of 10 persons given below:

₹240, ₹180, ₹250, ₹160, ₹200, ₹280, ₹220, ₹170, ₹210 and ₹270

(a) ₹130 (b) ₹140 (c) ₹120 (d) ₹150

4. If the range of x is 2, what would be the range of -3x + 50?

(a) 2 (b) 6 (c) -6 (d) 44

5. Find the range for the following frequency distribution:

x	3–5	6–8	9–11	12–14
f	3	2	2	3

(a) 12 (b) 12.5 (c) 11 (d) None of these

6. The following are the prices of shares of a company from Monday to Saturday:

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Price (in ₹)	55	54	52	53	56	58

Calculate the range and the coefficient of range.

(a) 5 and 0.064 (b) 6 and 0.054 (c) 8 and 0.034 (d) None of these

7. Find the coefficient of range for the following frequency distribution:

x	3–5	6–8	9–11	12–14
f	3	2	2	3

(a) 60.5% (b) 65.80% (c) 64.706% (d) None of these

Answer Key

1. (d) **2.** (d) **3.** (c) **4.** (b) **5.** (a) **6.** (b) **7.** (c)

MERITS

The range possesses the following merits

1. It is simple to understand and easy to calculate.
2. It requires minimum time to calculate the value of range.
3. It is useful in studying fluctuations in the share prices.

DEMERITS

The range has the following drawbacks:

1. It is not based on all the observations.
2. Range is a poor measure of variation. It considers only the extreme values and tells us nothing about the distribution of numbers in between.
3. It is very much affected by fluctuations of sampling. Its value varies widely from sample to sample
4. It cannot be calculated for grouped frequency distribution with open-end classes.
5. It is not suitable for further mathematical treatment.

CHARACTERISTICS FOR AN IDEAL MEASURE OF DISPERSION

An ideal measure of dispersion should be

- Properly defined

- Easy to comprehend
- Simple to compute
- Based on all the observations
- Unaffected by sampling fluctuations and amenable to some desirable mathematical treatment.

MEAN DEVIATION

As we saw, range is not based on all the observations. Moreover, it does not show any scatterness around an average. If we wish to measure variation in the sense of showing the scatter around an average, we must include the deviations of each and every item from an average.

- Mean deviation or the Average deviation helps us in achieving this goal. As the name suggests, this measure of dispersion is obtained by taking the average (arithmetic mean) of the deviations of the given values from a measure of central tendency.
- Mean Deviation (about an average A) = $\frac{1}{n}\Sigma|x_i - A|$ where, $|x_i - A|$ is the modulus value or the absolute value of the deviation from A, ignoring ± signs.

Usually, we obtain mean deviation about any one of the three averages Mean (M), Median (M_d) or Mode (M_0). As we know mode is generally ill-defined, in practice, mean deviation is computed about mean or median and if we calculate mean deviation about median, it will be much beneficial because the sum of the deviations of items from median is least when signs are ignored. But mostly, the mean is more frequently used in computing the average deviation and this is the reason why it is more commonly referred to as mean deviation.

PROCEDURE FOR COMPUTING THE MEAN DEVIATION

We now outline the procedure for computing the mean deviation:

Step 1. Calculate the average A about which mean deviation is to be computed, by the methods discussed earlier.

Step 2. Find the deviation of each observation X from A and denote it by D. That is, find D = X — A.

Step 3. Find the absolute value of the deviation of each observation from A ignoring the signs and denote it by |D|

Step 4. Find the sum of all absolute deviations obtained in Step 3 to get Σ |D|

Step 5. Divide the sum obtained in Step 4 by the number of observations to get the required mean deviation about the average A.

Example 10. What is the mean deviation for the following numbers?

15, 18, 20, 9, 12, 16

(a) 2 (b) 3 (c) 7 (d) 8

Sol. (b) Given numbers: 15, 18, 20, 9, 12, 16

$$\text{Mean } (\bar{x}) = \frac{15+18+20+9+12+16}{6} = 15$$

Now, mean deviation about mean

$$= \frac{|15-15|+|18-15|+|20-15|+|9-15|+|12-15|+|16-15|}{6}$$

$$= \frac{0+3+5+6+3+1}{6} = \frac{18}{6} = 3$$

Hence, the correct option is (b).

Example 11. What is the value of mean deviation about mean for the following numbers?

5, 8, 6, 3, 4 **(ICAI)**

(a) 5.20 (b) 7.20 (c) 1.44 (d) 2.23

Sol. (c) Given data: 5, 8, 6, 3, 4

Here, n = 5

Mean of the observations $(\bar{x}) = \frac{(5+8+6+3+4)}{5} = \frac{26}{5} = 5.2$

| Deviation from mean | = $|x_i - \bar{x}|$

= |5 – 5.2|, |8 – 5.2|, |6 – 5.2|, |3 – 5.2|, |4 – 5.2|

= 0.2, 2.8, 0.8, 2.2, 1.2

Thus, sum of deviations:

$\sum |x_i - \bar{x}|$ = 0.2 + 2.8 + 0.8 + 2.2 + 1.2 = 7.2

Mean deviation about mean = $\frac{\sum|x_i - \bar{x}|}{n} = \frac{7.2}{5} = 1.44$

Hence, the correct answer is option (c).

Example 12. Given the observations: 4, 9, 11, 14, 37. The mean deviation about the median is

(a) 11 (b) 8.5 (c) 7.6 (d) 7.45

Sol. (c) Given observations, X_i : 4, 9, 11, 14, 37

Here, n = 5

Since, n is odd

Thus, median of observation = $\frac{n+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3^{rd}$ Observation = 11

$\therefore$ |d| = $|X_i$ – Median| = $|X_i - 11|$

Xi	$\|d\| = \|X_i - 11\|$
4	7
9	2
11	0
14	3
37	26
	$\sum \|d\| = 38$

Mean deviation about the median = $\frac{\Sigma(xi - median)}{n} = \frac{\Sigma|d|}{n} = \frac{38}{5} = 7.6$

Hence, the correct option is (c) i.e.7.6.

Example 13. What is the value of mean deviation about mean for the following observation? 50, 60, 50, 50, 60, 60, 60, 50, 50, 50, 60, 60, 60, 50 **(ICAI)**

(a) 5 (b) 7 (c) 35 (d) 10

Sol. (a) Given data,

50, 60, 50, 50, 60, 60, 60, 50, 50, 50, 60, 60, 60, 50

The mean of the data

$= \frac{50+60+50+50+60+60+60+50+50+50+60+60+60+50}{14} = \frac{770}{14} = 55$

Now, the sum of deviation from mean

= 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 +5 + 5 +5 = 70

$\therefore$ Mean deviation about mean = $\frac{\text{Sum of deviation from mean}}{\text{Mean}} = \frac{70}{14} = 5$

Therefore, mean deviation about mean is 5.

Hence, the correct answer is option (a).

Example 14. The mean deviation about mode for the numbers $\frac{4}{11}, \frac{6}{11}, \frac{8}{11}, \frac{9}{11}, \frac{12}{11}, \frac{8}{11}$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{11}$ (c) $\frac{6}{11}$ (d) $\frac{5}{11}$ **(ICAI)**

Sol. (a) Given data (X_i): $\frac{4}{11}, \frac{6}{11}, \frac{8}{11}, \frac{9}{11}, \frac{12}{11}, \frac{8}{11}$

Since, $\frac{8}{11}$ is occurring most frequent (2 times), thus

Mode of the data = $\frac{8}{11}$

Now, the deviation from mode $|X_i - \text{Mode}| = |X_i - \frac{8}{11}| = \frac{4}{11}, \frac{2}{11}, 0, \frac{1}{11}, \frac{4}{11}, 0$

$\therefore$ Mean deviation about mode

$= \frac{\sum(X_i - \text{Mode})}{n} = \frac{\frac{4}{11} + \frac{2}{11} + 0 + \frac{1}{11} + \frac{4}{11} + 0}{6} = \frac{\frac{11}{11}}{6} = \frac{1}{6}$

Hence, the correct answer is option (a) i.e., $\frac{1}{6}$.

COMPUTATION OF MEAN DEVIATION - DISCRETE SERIES

- In case of discrete series where the variable X takes the values $X_1, X_2, \ldots, X_n$ with respective frequencies $f_1, f_2, \ldots, f_n$, the mean deviation about an average A is given by
- Mean Deviation (about an average A) = $\frac{\Sigma f_i |X_i - A|}{N}$

PROCEDURE FOR COMPUTING THE MEAN DEVIATION

Step 1. Calculate the average A about which mean deviation is to be computed.

Step 2. Take the deviation of each observation from A and denote it by D. That is, find $D = X - A$.

Step 3. Find the absolute value of the deviation of each observation from A ignoring ± signs and denote it by $|D|$.

Step 4. Multiply each absolute deviation $|D|$ by the corresponding frequency f_i to get $\Sigma |D|$.

Step 5. Add all the products obtained in Step 4 to get $\Sigma f_i |D|$.

Step 6. Divide the sum obtained in Step 5 by N, the total frequency, to get the required mean deviation.

Example 15. Calculate the mean deviation about the mean for the following data:

X	10	11	12	13	14	Total
f	3	12	18	12	3	48

(a) 12 (b) 0.75 (c) 15.5 (d) None of these

Sol. (b) According to the given data, we have

X	f	fX	$D = X - \bar{X}$	$\|D\|$	$f\|D\|$
10	3	30	-2	2	6
11	12	132	-1	1	12
12	18	216	0	0	0
13	12	156	1	1	12
14	3	42	2	2	6
	$N = \Sigma f = 48$	$\Sigma fX = 576$			$\Sigma f\|D\| = 36$

Mean $\bar{X} = \frac{\Sigma fX}{\Sigma f} = \frac{576}{48} = 12$

Thus, Mean Deviation about mean $= \frac{\Sigma f|D|}{N} = \frac{36}{48} = 0.75$

Hence, the correct option is (b).

COMPUTATION OF MEAN DEVIATION: CONTINUOUS SERIES

- The computation of the mean deviation in the case of continuous series is exactly the same as discussed above for discrete series.
- The only difference is that here we have to obtain the class marks (or mid-values) of the various classes and take absolute deviations of these values from the average A.
- Thus, if $x_1, x_2, \ldots\ldots, x_n$ are the class marks (or mid-values) of a set of grouped data with corresponding class frequencies $f_1, f_2, \ldots\ldots\ldots, f_n$, then the mean deviation about an average A is given by:

Mean Deviation (about an average A) $= \frac{\Sigma f_i |x_i - A|}{N}$

Example 16. Calculate mean deviation from the median for the following data:

Marks	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90
No. of Students	2	6	12	18	25	20	10	7

(a) 15.57 (b) 12.95 (c) 16.25 (d) None of these

Sol. (b) According to the given data, we have

Marks	Mid-Value X	No. of Students (f)	Less than cf	D	|D|	f|D|
10-20	15	2	2	-39.8	39.8	79.6
20-30	25	6	8	-29.8	29.8	178.8
30-40	35	12	20	-19.8	19.8	237.6
40-50	45	18	38	-9.8	9.8	176.4
50-60	55	25	63	0.2	0.2	5.0
60-70	65	20	83	10.2	10.2	204.0
70-80	75	10	93	20.2	20.2	202.0
80-90	85	7	100	30.2	30.2	211.4
		$N = \sum f = 100$				$\sum f \|D\| = 1294.8$

We have, $\frac{N}{2} = 50$

Thus, the c.f just greater than or equal to 50 is 63 which corresponds to class interval 50-60.

Therefore, Median $= 1 + \frac{\frac{N}{2} - c}{f} \times h = 50 + \frac{50 - 38}{25} \times 10 = 54.8$

Mean deviation $= \frac{\sum f|D|}{N} = \frac{1294.8}{100} = 12.9948 = 12.95$ (approx).

Hence, the correct option is (b).

Example 17. What is the coefficient of mean deviation for the following distribution of heights? Take deviation from AM. **(ICAI)**

Height in Inches	60-62	63-65	66-68	69-71	72-74
No. of Students	5	22	28	17	3

(a) 2.30 inches (b) 3.45 inches (c) 3.82 inches (d) 2.48 inches

Sol. (b) According to the given data,

C.I.	Class Boundary	Frequency (f_i)	x_i	$u_i = \frac{x_i - A}{h}$ A = 67, h = 3	$f_i u_i$
60-62	59.5 - 62.5	5	61	-2	-10
63-65	62.5 - 65.5	22	64	-1	-22
66-68	65.5 - 68.5	28	67 = A	0	0
69-71	68.5 - 71.5	17	70	1	17
72-74	71.5 - 74.5	3	73	2	6
		75			

Thus, Mean $(\bar{x}) = A + \frac{\sum f_i u_i}{n} \times h \Rightarrow \bar{x} = 67 + \frac{-9}{75} \times 3$

$\Rightarrow \bar{x} = 67 - 0.36 \Rightarrow \bar{x} = 66.64$

Now,

$d = \lvert x_i - \text{mean} \rvert$	$f_i.d$
5.64	28.20
2.64	58.08
0.36	10.08
3.36	57.12
6.36	19.08
	172.56

$$\text{Mean Deviation about mean} = \frac{\sum_{i=1}^{n} f_i \left|(x_i - \bar{x})\right|}{n} = \frac{172.56}{75} = 2.3008$$

$$\text{Coefficient of Mean Deviation} = \frac{\text{Mean Deviation}}{\bar{x}} \times 100$$

$$= \frac{2.3008}{66.64} \times 100 \approx 3.45 \text{ inches}$$

Hence, the correct option is (b).

COEFFICIENT OF MEAN DEVIATION

The relative measure corresponding to the mean deviation, called the coefficient of mean deviation, is given by

- Coefficient of M.D. = $\frac{1}{n}\Sigma|x_i - A|$
- Coefficient of M.D. about mean $= \frac{1}{n}\sum|x_i - \bar{x}|$ where, $\bar{x}$ is mean

- Coefficient of M.D. about median = $\frac{1}{n}\Sigma|x_i - \text{Median}|$

  Coefficient of mean deviation is a pure number independent of the units of measurement and can be used to compare two distributions expressed in different units.

PROPERTIES

- Mean deviation takes its minimum value when the deviations are taken from the median.
- Also mean deviation remains unchanged due to a change of origin but changes in the same ratio due to a change in scale i.e., if y = a + bx, a and b being constants, then
  MD of y = |b| × MD of x

Example 18. If the relation between x and y in 5y – 3x = 10 and the mean deviations about mean for x is 12, then the mean deviation of y about mean is

(a) 7.20 (b) 6.80 (c) 20 (d) 18.80

Sol. (a) Given, the relation is 5y – 3x = 10

$$\Rightarrow y = \frac{(10+3x)}{5}$$

$$\Rightarrow y = \frac{3}{5}x + 2$$

Mean deviation about mean of $y = \left|\frac{3}{5}\right| \times$ mean deviation about mean of x

$$= \left|\frac{3}{5}\right| \times 12 = \frac{3}{5} \times 12 = 7.2$$

Hence, the correct option is (a) i.e.,7.2.

MERITS AND DEMERITS OF MEAN DEVIATION

Merits

1. It is easy to understand and simple to calculate.
2. It is based on each and every item of the data.
3. It is rigidly defined.
4. As compared to standard deviation, it is less affected by extreme observations.
5. Since deviations are taken from a central value, comparison about formation of different distributions can easily be made.

Demerits

1. The major drawback of mean deviation is that algebraic signs are ignored while taking the deviations of the items.
2. It is not suitable for further mathematical treatment.
3. it cannot be computed for distribution with open-end classes.
4. It is rarely used in sociological studies.

Example 19. If two variables x and y are related by the equation 2x - 3y + 4 = 0 and the mean and mean deviation about mean of x are 4 and 0.6 respectively, then the coefficient of mean deviation of y about its mean is

(a) 5 (b) 8 (c) 10 (d) None of these

Sol. (c) Given,

The relation is $2x - 3y + 4 = 0$

$\Rightarrow \quad 3y = 2x + 4$

$\Rightarrow \quad y = \frac{2}{3}x + \frac{4}{3}$

Thus, the mean of $y = \frac{2}{3}(x) + \frac{4}{3}$

$\Rightarrow \quad y = \frac{2}{3}(4) + \frac{4}{3}$

$\Rightarrow \quad y = 4$

Mean deviation of $y = \left|\frac{2}{3}\right| \times$ mean deviation of x

$\Rightarrow \quad y = \frac{2}{3}(0.6)$

$\Rightarrow \quad y = 0.4$

Thus, Coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{mean}} \times 100$

$= \frac{0.4}{4} \times 100 = 10$

Hence, the correct option is (c) i.e., 10.

Example 20. The coefficient of mean deviation about mean for the first 9 natural number is

(a) $\frac{200}{9}$ (b) 80 (c) $\frac{400}{9}$ (d) 50 **(ICAI)**

Sol. (c) We know that,

First 9 natural numbers are 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Sum of first n natural numbers $= \frac{n(n+1)}{2}$

Then, sum of first 9 natural numbers $= \frac{9(9+1)}{2} = 45$

Mean of these numbers $= \frac{45}{9} = 5$

$\therefore \sum |\text{deviation from mean}| = 4 + 3 + 2 + 1 + 0 + 1 + 2 + 3 + 4 = 20$

The mean deviation about mean $= \frac{20}{9}$

$\therefore$ Coefficient of mean deviation $= \frac{\text{Mean deviation}}{\text{Mean}} \times 100$

$$= \frac{20}{9 \times 5} \times 100$$

$$= \frac{400}{9}$$

Hence, the correct answer is option (c).

Example 21. If two variables x and y are related by 2x + 3y - 7 = 0 and the mean and mean deviation about mean of x are 1 and 0.3 respectively, then the coefficient of mean deviation of y about its mean is **(ICAI)**

(a) –5 (b) 12 (c) 50 (d) 4.

Sol. (b) Given,

The relation is 2x + 3y - 7 = 0

$\Rightarrow 3y = 7 - 2x$

$\Rightarrow y = \frac{7}{3} - \frac{2}{3}x$

Thus, the mean of $y = \frac{7}{3} - \frac{2}{3}$ (mean of x)

$\Rightarrow y = \frac{7}{3} - \frac{2}{3}$ (1)

$\Rightarrow y = \frac{5}{3}$

Mean deviation of $y = \left|\frac{-2}{3}\right|$ mean deviation of x

$\Rightarrow y = \frac{2}{3}(0.3)$

$\Rightarrow y = 0.2$

Coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{mean}} \times 100$

$= \frac{0.2}{\frac{5}{3}} \times 100 = 12$

Hence, the correct answer is option (b).

PRACTICE QUESTIONS (PART B)

1. If x and y are related as 3x + 4y + 7 = 0 and mean deviation of x is 6.40, what is the mean deviation of y?

(a) 2.52 (b) 4.18 (c) 6.40 (d) None of these

2. Find the mean deviation about the median for the following data:

82, 56, 75, 70, 52, 80, 68

(a) 21.5 (b) 12.45 (c) 17.68 (d) None of these

3. Calculate mean deviation about the mean for the following data:

X	10	11	12	13	14	Total
Y	3	12	18	12	3	48

(a) 0.25 (b) 0.3 (c) 0.75 (d) 8.0

4. Compute the mean deviation about the arithmetic mean for the following data:

X	1	3	5	7	9
Y	5	8	9	2	1

(a) 1.72 (b) 1.38 (c) 1.55 (d) 1.80

5. The mean deviation of weights about median for the following data: **(ICAI)**

Weight (lb)	131-140	141-150	151-160	161-170	171-180	181-190
No. of persons	3	8	13	15	6	5

(a) 10.97 (b) 8.23 (c) 9.63 (d) 11.45

6. Calculate the mean deviation from the mean for the following data:

Marks	0-10	10–20	20–30	30–40	40–50	50–60	60-7
No. of Students	6	5	8	15	7	6	3

(a) 12.52 (b) 13.18 (c) 7.55 (d) 8.50

7. Calculate the coefficient of mean deviation for the following data:

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students	2	6	12	18	25	20	10	7

(a) 0.52 (b) 0.18 (c) 0.24 (d) 0.50

8. What is the mean deviation about median for the following data? **(ICAI)**

X	3	5	7	9	11	13	15
F	2	8	9	16	14	7	4

(a) 2.50 (b) 2.46 (c) 2.43 (d) 2.37

9. Calculate coefficient of mean deviation from the median for the following data:

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students	2	6	12	18	25	20	10	7

(a) 0.24 (b) 0.95 (c) 1.50 (d) None of these

Answer Key

1. (b) **2.** (d) **3.** (c) **4.** (a) **5.** (a) **6.** (b) **7.** (c) **8.** (d) **9.** (a)

QUARTILE DEVIATION OR SEMI INTER- QUARTILE RANGE

Inter-quartile Range is an absolute measure of dispersion defined by the formula:

Inter-quartile range = Q_3 - Q_1, where Q_1 and Q_3 are the first (lower) and the third (or upper) quartiles respectively.

Quartile deviation, also called semi inter-quartile range, is an absolute measure of dispersion defined by the formula:

$$\text{Quartile Deviation } (\text{Q.D.}) = \frac{Q_3 - Q_1}{2}$$

- In a symmetrical distribution, the two quartiles Q_1 and Q_3 are equidistant from the median, meaning their difference is equal.
- The quartile deviation provides a measure of the spread or dispersion of the data around the median.
- The quartile deviation can be used to estimate the spread of data and compare the variability between different datasets.
- The range of values from (median - Q.D.) to (median + Q.D.) covers exactly 50% of the data, making it useful for analysing the central half of the distribution.

COEFFICIENT OF QUARTILE DEVIATION

The coefficient of quartile deviation is a relative measure of dispersion that provides a standardized measure of the spread of data. It is defined by the formula:

$$\text{Coefficient of Quartile Deviation} = \frac{\text{Quartile Deviation}}{\text{Median}} \times 100 = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

The coefficient of quartile deviation is a pure number that is independent of the units of measurement used for the data. This makes it useful for comparing the variability of different distributions, even if they are expressed in different units.

Therefore, Quartile deviation can be computed from the distribution having open-end classes. It is affected considerably by the sampling fluctuations. Quartile deviation remains unaffected due to change of origin but is affected in the same ratio due to change in scale.

Example 22. When 1st quartile = 20, 3rd quartile = 30, the value of quartile deviation is

(a) 7 (b) 4 (c) -5 (d) 5 **(ICAI)**

Sol. (d) Given: $Q_1 = 20$ and $Q_3 = 30$

We know that,

$$\text{Quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{30 - 20}{2} = \frac{10}{2} = 5$$

Therefore, the value of quartile deviation is 5.

Hence, the correct answer is option (d) i.e., 5.

Example 23. The first quartile of a dataset is 25 and the third quartile is 75. What is the interquartile range of the dataset?

(a) 25 (b) 50 (c) 75 (d) 100

Sol. (b) To find the interquartile range, we need to calculate the difference between the third quartile and the first quartile.

Given:

$Q_1 = 25$

$Q_3 = 75$

Therefore, Interquartile range = 75 – 25 = 50

Therefore, the interquartile range of the dataset is 50.

Hence, the correct option is (b).

Example 24. Quartile Deviation for the data 1, 3, 4, 5, 6, 6, 10 is **(ICAI)**

(a) 3 (b) 1 (c) 6 (d) 1.5

Sol. (d) As we know that,

Quartile Deviation is given by $\frac{Q_3 - Q_1}{2}$

where, Q_3 = third quartile and Q_1 = first quartile

We have, 1, 3, 4, 5, 6, 6, 10

Given series of numbers is already arranged in ascending order and the number of observations is odd i.e. n = 7.

First Quartile $Q_1 = \left(\frac{n+1}{4}\right)^{th}$ observations

$= \left(\frac{7+1}{4}\right)^{th}$ observations = 2nd observation = 3

Third Quartile $Q_3 = \frac{3(n+1)^{th}}{4}$ observations

$= \left(\frac{24}{4}\right)^{th}$ observations = 6th observation = 6

Therefore, Quartile Deviation = $\frac{6-3}{2}$ = 1.5

Hence, the correct option is (d) i.e., 1.5.

Example 25. If median = 5, Quartile deviation = 1.5, then the coefficient of quartile deviation is

(a) 33 (b) 35 (c) 30 (d) 20 **(ICAI)**

Sol. (c) Given, Median = 5, Quartile deviation = 1.5

We know that,

Coefficient of Quartile Deviation = $\frac{\text{Quartile deviation}}{\text{median}} \times 100$

$= \frac{1.5 \times 100}{5} = 30$

Therefore, the coefficient of quartile deviation is 30.

Hence, the correct option is (c) i.e., 30.

Example 26. The quartile deviation for the data is:

x	2	3	4	5	6
y	3	4	8	4	1

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 0

Sol. (c) We know that,

Quartile deviation is given by Q. D. = $\frac{1}{2}(Q_3 - Q_1)$

where $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ observation and $Q_3 = \frac{3(N+1)^{th}}{4}$ observation

Calculating cumulative frequency, we get

x	2	3	4	5	6
f	3	4	8	4	1
cf	3	7	15	19	20

Here, $N = \Sigma x_i = 20$

So, $Q_1 = \frac{(20+1)^{th}}{4} = 5.25^{th}$observation = 3

Similarly, $Q_3 = \frac{3(20+1)^{th}}{4} = 15.75^{th}$ observation = 5

Therefore, Q.D. = $\frac{1}{2}(5 - 3) = 1$

Hence, the correct option (c) is i.e. 1.

Example 27. Following are the marks of the 10 students: 56, 48, 65, 35, 42, 75, 82, 60, 55, 50. Find the coefficient of quartile deviation.

(a) 16.50 (b) 18 (c) 18.42 (d) None of these

Sol. (c) On arranging the marks in ascending order, we get

35, 42, 48, 50, 55, 56, 60, 65, 75, 82

Now,

First Quartile $Q_1 = \left(\frac{10+1}{4}\right)^{th}$ observations

$= \left(\frac{11}{4}\right)^{th}$ observations = 2.75^{nd} observation

= 2nd observation + 0.75 × difference between 3rd and 2nd observation

= 42 + 0.75(48 – 42) = 46.50

Third Quartile $Q_3 = \frac{3(n+1)^{th}}{4}$ observations

$= \left(\frac{33}{4}\right)^{th}$ observations = 8.25^{th} observation

= 8th observation + 0.25 × difference between 9th and 8th observation

= 65 + 0.25(75 – 65) = 67.50

Thus, the coefficient of quartile deviation is given by $\frac{(Q_3 - Q_1)}{(Q_3 + Q_1)} \times 100$

$= \frac{67.50 - 46.50}{67.50 + 46.50} \times 100 = \frac{21}{114} \times 100 = 18.42$

Hence, the correct option is (c) i.e. 18.42.

Example 28. If x and y are related as 2x + 5y = 30 and the quartile deviation of x is 10, then the quartile deviation of y is:

(a) 2 (b) 4 (c) 5 (d) 6

Sol. (b) Given relation: 2x + 5y = 30

Also, quartile deviation of x = 10

Thus, 5y = 30 – 2x

$y = 6 - \frac{2}{5}x$

Therefore, Quartile deviation of y = $\left|-\frac{2}{5}\right|$ × Quartile deviation of x

Put the value of quartile deviation of x to find quartile deviation of y i.e.,

$y = \frac{2}{5}(10)$

y = 4

Therefore, the quartile deviation of y is 4.

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART C)

1. Following are the marks of the 10 students: 34, 28, 45, 26, 24, 47, 38, 36, 49, 50. Find the quartile deviation.

(a) 20 (b) 30
(c) 10 (d) None of these

2. The quartiles of a variable are 45, 52 and 65 respectively. Its quartile deviation is

(a) 10 (b) 20 (c) 25 (d) 8.30 **(ICAI)**

3. Find the interquartile range for the following dataset representing the scores of 10 students in a mathematics test: 35, 42, 48, 55, 60, 63, 68, 70, 72, 78

(a) 12 (b) 24
(c) 46.5 (d) None of these

4. If the first quartile is 56 and the third quartile is 77, then the coefficient of quartile deviation is

(a) 18.09 (b) 15.79 (c) 63.80 (d) 56.71

5. Find the value of the third quartile if the values of first quartile and quartile deviation are 90 and 20 respectively.

(a) 100 (b) 130 (c) 110 (d) 220

6. If the first quartile is 48 and quartile deviation is 6, find the median. (assuming the distribution to be symmetrical).

(a) 54 (b) 48 (c) 42 (d) 6

7. Find the interquartile range and the coefficient of quartile deviation from the following data:

Marks less than	10	20	30	40	50	60	70	80
No. of Students	4	16	40	76	96	112	120	125

(a) 50 and 25 (b) 20 and 25 (c) 50 and 30 (d) None

8. If the quartile deviation of x is 5 and $2x + 5y = 10$, what is the quartile deviation of y?

(a) 2 (b) 4 (c) 1 (d) cannot be determined

9. The quartiles of a variable are 45, 52 and 65 respectively. Its quartile deviation is

(a) 10 (b) 20 (c) 25 (d) 8.30.

10. If x and y are related as $3x + 4y = 20$ and the quartile deviation of x is 12, then the quartile deviation of y is

(a) 16 (b) 14 (c) 10 (d) 9

11. Calculate quartile deviation from the following distribution:

X	5-7	8-10	11-13	14-16	17-19
Frequency	14	24	38	20	4

(a) 1.55 (b) 2.27 (c) 3.05 (d) None of these

12. Find the value of the third quartile if the values of first quartile and quartile deviation are 104 and 18 respectively.

(a) 100 (b) 110 (c) 120 (d) 140

Answer Key

1. (c) **2.** (a) **3.** (b) **4.** (b) **5.** (b) **6.** (a) **7.** (b) **8.** (a) **9.** (a) **10.** (d)
11. (b) **12.** (d)

MERITS

- Quartile deviation provides the best measure of dispersion for open-end classification. In fact, it is the only measure of dispersion which can be obtained while dealing with a distribution having open-end classes.
- It is also less affected due to sampling fluctuations. It is not affected at all by extreme observations as it ignores 25% of the data from the beginning of the distribution and another 25% of the data from the top end.
- Quartile deviation is useful especially when it is desired to study variability in the central half part of the data.
- Like other measures of dispersion, quartile deviation remains unaffected due to a change of origin but is affected in the same ratio due to change in scale.

DEMERITS

1. Quartile deviation is not based on all the observations. In fact, it ignores 25% of the data at the lower end and 25% of the data at the upper end. Hence it cannot be considered as a good measure of dispersion.
2. Quartile deviation is not suitable for further mathematical treatment.
3. It is affected considerably by sampling fluctuations.

STANDARD DEVIATION

The standard deviation, abbreviated as S.D. of a given set of observations is defined as the positive square root of the arithmetic mean of the squares of deviations of the observations from their arithmetic mean. It is denoted by the Greek letter (read as sigma).

Thus, standard deviation of a set of n observations $X_1, X_2, \ldots, X_n$ is given by

$$\sigma = \sqrt{\frac{\sum (X - \bar{X})^2}{N}} \text{ where } \bar{X} = \frac{\sum X}{n}$$

If $X_1, X_2, \ldots\ldots, X_n$ are the class marks of a set of grouped data with class frequencies $f_1, f_2, \ldots f_n$, then the standard deviation is given by

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n}}$$

Example 29. The heights (in centimeters) of a group of students are as follows: 150, 160, 165, 155, 170, 155, 160, 155. What is the standard deviation of the heights?

(a) 4.16 (b) 5.20 (c) 6.62 (d) 7.07

Sol. (d) Given observations: 150, 160, 165, 155, 170, 155, 170, 155

$\therefore$ Sum of the observations =150 + 160 + 165 + 155 + 170 + 155 + 170 + 155 = 1280

Now, mean of the data $= \frac{1280}{8} = 160$

Thus, the sum of squares of the deviation from mean

$$\sum_{i=0}^{N}(x_i - \bar{x})^2 = 10^2 + 0^2 + 5^2 + 5^2 + 10^2 + 5^2 + 10^2 + 5^2$$

= 100 + 0 + 25 + 25 + 100 + 25 + 100 + 25 = 400

Therefore, $S.D = \sqrt{\frac{\sum_{i=0}^{N}(x_i - \bar{x})^2}{n}}$

$$S.D = \sqrt{\frac{400}{8}}$$

$$S.D = \sqrt{50} = 7.07$$

Hence, the correct option is (d).

Example 30. What is the standard deviation of 5, 5, 9, 9, 9, 10, 5, 10, 10? **(ICAI)**

(a) $\sqrt{14}$ (b) $\frac{\sqrt{42}}{3}$ (c) 4.50 (d) 8

Sol. (b) Given observations: 5, 5, 9, 9, 9, 10, 5, 10, 10

Sum of the observations = 5 + 5 + 5 + 10 + 10 + 10 + 9 + 9 = 72

Now, mean of the data

Thus, the sum of squares of the deviation from mean n

$$\sum_{i=0}^{N}(x_i - \bar{x})^2 = 3^2 + 3^2 + 3^2 + 2^2 + 2^2 + 2^2 + 1^2 + 1^2 + 1^2$$

= 9 + 9 + 9 + 1 + 1 + 1 + 4 + 4 + 4 = 42

Therefore, $S.D = \sqrt{\frac{\sum_{i=0}^{N}(x_i - \bar{x})^2}{n}}$

$$S.D = \sqrt{\frac{42}{9}}$$

$$S.D = \frac{\sqrt{42}}{3}$$

Hence, the correct answer is option (b) i.e., $\frac{\sqrt{42}}{3}$.

Example 31. Calculate the standard deviation for the following data:

X	20	30	40	50	60	70
Frequency	8	12	20	10	6	4

(a) 13.75 (b) 14.50 (c) 10.15 (d) None of these

Sol. (a) According to the given data,

X	f	fX	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
20	8	160	-21	441	3528
30	12	360	-11	121	1452
40	20	800	-1	1	20
50	10	500	9	81	810
60	6	360	19	361	2166
70	4	280	29	841	3364
	N = 60	$\sum fx = 2460$			$\sum f(x-\bar{x})^2 = 11340$

Therefore, $\bar{X} = \frac{\sum fx}{N} = \frac{2460}{60} = 41$

Now, Standard deviation $(\sigma) = \sqrt{\frac{\sum f(X-\bar{X})^2}{N}} = \sqrt{\frac{11340}{60}} = \sqrt{189} = 13.75$

Hence, the correct option is (a).

Example 32. Calculate the standard deviation from the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	10	15	25	25	10	10	5

(a) 11.65 (b) 13.39 (c) 14.40 (d) 15.94

Sol. (d) According to the given data,

Marks	Mid-value (X)	f	fX	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
0-10	5	10	50	-26	676	6760
10-20	15	15	225	-16	256	3840
20-30	25	25	625	-6	36	900
30-40	35	25	875	4	16	400
40-50	45	10	450	14	196	1960
50-60	55	10	550	24	576	5760
60-70	65	5	325	34	1156	5780
		N = 100	$\sum fx = 3100$			$\sum f(x-\bar{x})^2 = 25400$

Therefore, $\bar{X} = \frac{\sum fx}{N} = \frac{3100}{100} = 31$

Now, Standard deviation = $(\sigma) = \sqrt{\frac{\sum f(X-\bar{X})^2}{N}} = \sqrt{\frac{25400}{100}} = \sqrt{254} = 15.94$

Hence, the correct option is (d).

DIFFERENT METHODS OF CALCULATING STANDARD DEVIATION: UNGROUPED DATA

The standard deviation of an ungrouped data consisting of N observations $X_1, X_2, \ldots, X_n$ is given by

$$\sigma = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$$

The computation of standard deviation is very effective if $\bar{X}$ is an integer. However, if X comes out to be in fraction, its computation becomes very cumbersome and time-consuming. In that case we apply the following short-cut method which is very effective and reduces the numerical calculations to a great extent.

DIFFERENT METHODS OF CALCULATING STANDARD DEVIATION - GROUPED DATA

All the methods discussed earlier for calculating standard deviation in the case of ungrouped data can also be used in the case of grouped data. However, in practice it is the step deviation method that is mostly used.

Standard deviation is given by: $\sigma = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$

Example 33. From the following information, find standard deviations of X and Y variables:

$\sum X = 235, \sum X = 250, \sum X^2 = 6750, \sum Y^2 = 6840, N = 10$

(a) 11.08 and 7.68 (b) 12.55 and 8.06

(c) 29.50 and 16.76 (d) None of these

Sol. (a) Given: $\sum X = 235, \sum X = 250, \sum X^2 = 6750, \sum Y^2 = 6840, N = 10$

We know that, $\sigma_x = \sqrt{\frac{\sum X^2}{N} - \left(\frac{\sum X}{N}\right)^2} = \sqrt{\frac{6750}{10} - \left(\frac{235}{10}\right)^2} = \sqrt{675 - (23.5)^2}$

$= \sqrt{675 - 552.25} = \sqrt{122.75} = 11.08$

$$\sigma_Y = \sqrt{\frac{\sum Y^2}{N} - \left(\frac{\sum Y}{N}\right)^2} = \sqrt{\frac{6840}{10} - \left(\frac{250}{10}\right)^2} = \sqrt{684 - (25)^2} = \sqrt{684 - 625}$$

$= \sqrt{59} = 7.68$

Hence, the correct option is (a).

Example 34. From the following data, calculate the standard deviation:

X	10	11	12	13	14	15	16	17	18
Frequency	2	7	10	12	15	11	10	6	3

(a) 1.50 (b) 1.986 (c) 2.576 (d) None of these

Sol. (b) According to the given data, we have

X	f	d = X - 14	fd	fd^2
10	2	-4	-8	32
11	7	-3	-21	63
12	10	-2	-20	40
13	12	-1	-12	12
14	15	0	0	0
15	11	1	11	11
16	10	2	20	40
17	6	3	18	54
18	3	4	12	48
	N = 76		$\sum fd = 0$	$\sum fd^2 = 300$

Therefore, Standard deviation $= \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} = \sqrt{\frac{300}{76}} = 1.986$

Hence, the correct option is (b).

VARIANCE

The variance of a given set of observations is defined as the square of its standard deviation and is denoted by σ^2.

Thus,

For individual observations, $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ or $\sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2$

For Ungrouped/Grouped Observations, $\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{n}$

Example 35. $\sum x^2 = 3390$, n = 30, σ = 7, then $\bar{x}$ = _____

(a) 113 (b) 210 (c) 8 (d) None of these

Sol. (c) We know that, $\sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2$

Given that, $\sum x^2 = 3390$ n = 30, σ = 7

Putting the values, we get

$\sigma^2 = \frac{3390}{30} - (\bar{x})^2$

$\Rightarrow 49 = 113 - (\bar{x})^2$

$\Rightarrow (\bar{x})^2 = 64 \Rightarrow \bar{x} = 8$

Hence, the correct option is (c) i.e. 8.

Example 36. Find variance if $\sum D^2 = 150$ and N = 6 Deviations are taken from actual mean.

(a) 5 (b) 25 (c) 36 (d) None of these

Sol. (b) Given: $\sum D^2 = 150$ and N = 6

Thus, Variance $= \frac{\sum (x - \bar{x})^2}{N} = \frac{\sum D^2}{N} = \frac{150}{6} = 25$

Therefore, the variance is 25.

Hence, the correct option is (b).

Example 37. If the standard deviation for the marks obtained by a student in monthly test is 36, then the variance is

(a) 36 (b) 6 (c) 1296 (d) None of these

Sol. (c) We know that,

Variance is the square of standard deviation.

Since, SD (σ) = 36,

thus Variance = $\sigma^2 = (36)^2 = 1296$

Hence, option (c) is correct i.e. 1296.

CORRECTING INCORRECT VALUES OF MEAN AND STANDARD DEVIATION

Example 38. The mean and variance of 100 items were worked out as 40 and 25 respectively by a student. By mistake an item 50 was wrongly taken as 5 in calculating the above. You are required to find the correct mean and correct standard deviation.

(a) 2.54 (b) 3.05 (c) 3.68 (d) None of these

Sol. (c) Given,

Number of observations, n = 100,

Wrong mean, $\bar{x}_w = 40$

Variance, $\sigma^2 = 25$

Now, $\bar{x} = \dfrac{\sum x}{n}$

$\Rightarrow \bar{x}_w = n\bar{x} = 100\,(40) = 4000$

Now, $\sum x^2 = n(\sigma^2 + \bar{x}^2)$ $\qquad \left[\because \sigma^2 = \dfrac{\sum x^2}{n} - (\bar{x})^2\right]$

$= 100(25 + 1600) = 16250$

Since, wrong observation 5 is to be replaced by correct observation 50, then

Thus: Corrected $\sum x = 4000 - 5 + 50 = 4045$

Corrected $\sum x^2 = 162500 - 5^2 + (50)^2 = 164975$

$$\text{Corrected } \bar{x} = \frac{\text{corrected}\sum x}{n} = \frac{4045}{100} = 40.45$$

$$\text{Corrected } \sigma = \sqrt{\frac{\text{corrected}\sum x^2}{n} - (\text{corrected}\,\bar{x})^2} = \sqrt{\frac{164975}{100} - (40.45)^2}$$

$= \sqrt{13.55} = 3.68$

Hence, the correct option is (c).

COMBINED STANDARD DEVIATION

If two sets of data contain n_1 and n_2 observations having means $\bar{x}_1$ and $\bar{x}_2$, and standard deviations σ_1 and σ_2 respectively, then the standard deviation, σ, of the combined data with $n_1 + n_2$ observations is given by:

- $$\sigma = \sqrt{\frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}}$$
- $$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

The result can be generalized to more than two sets of data.

E.g.: if $\bar{X}_1, \bar{X}_2, \ldots \bar{X}_k$ be the means, σ_1, σ_2, ……. σ_k be the standard deviations and n_1, n_2, ……..., n_k be the number of observations in each set, then the standard deviation of the combined data with $n_1 + n_2 + \ldots\ldots + n_k$ observations is given by

- $$\sigma = \sqrt{\frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right) + \ldots\ldots + n_k\left(\sigma_k^2 + \sigma_k^2\right)}{n_1 + n_2 + \ldots\ldots + n_k}}$$

where $d_1 = \bar{X}_1 - \bar{X}, d_2 = \bar{X}_2 - \bar{X}, \ldots, d_k = \bar{X}_k - \bar{X}$

- $\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2 + ... + n_k\bar{x}_k}{n_1 + n_2 + ... + n_k}$ is the combined mean.

Example 39. If two samples of sizes 30 and 20 have means as 55 and 60, and variances as 16 and 25 respectively, then what would be the SD of the combined sample of size 50?

(a) 5.00 (b) 5.06 (c) 5.23 (d) 5.35

Sol. (b) According to the question, Variances, $\sigma_1^2 = 16$ and $\sigma_2^2 = 25$

$n_1 = 30$ and $n_2 = 20$

$\bar{x}_1 = 55$ and $\bar{x}_2 = 60$

Thus, combined mean $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \dfrac{32(55) + 20(60)}{30 + 20} = \dfrac{1650 + 1200}{50}$

$= \dfrac{2850}{50} = 57$

Thus, $d_1 = \bar{x}_1 - \bar{x} = 55 - 57 = -2$

$d_2 = \bar{x}_2 - \bar{x} = 60 - 57 = 3$

We know that,

Combined SD, $\sigma = \sqrt{\dfrac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}}$

$= \sqrt{\dfrac{30(16) + 20(25) + 30(4) + 20(9)}{30 + 20}}$

$= \sqrt{\dfrac{480 + 500 + 120 + 180}{50}} = \sqrt{\dfrac{1280}{50}} = 5.06$

Hence, the correct option is (b).

COEFFICIENT OF VARIATION

- The standard deviation is an absolute measure of dispersion, depending upon the units of measurement. It does not tell us much about the variability of a single set of data. The coefficient of standard deviation, based on standard deviation, is a relative measure of dispersion.
- This is a pure number independent of the units of measurement and hence can be used to compare the variability of two distributions expressed in different units.
- Perhaps a more appropriate measure is the coefficient of variation (C.V.), defined by

  Coefficient of Variation $= \dfrac{\text{S.D.}}{\bar{x}} \times 100$

  The above expression expresses the standard deviation as a percentage of the mean.
- Since C.V. is a measure of relative variation expressed as a percent, the coefficient of variation can be used to compare the variability of two or more sets of data even when observations are expressed in different units of measurement.
- A distribution for which the coefficient of variation is smaller is said to be less variable or more consistent, more uniform, more stable or more homogeneous.

- On the other hand, the distribution for which the coefficient of variation is greater is said to be more variable or less consistent, less uniform, less stable or less homogeneous.

Example 40. If Mean = 5, Standard deviation = 2.6, then the coefficient of variation is

(a) 49 (b) 51 (c) 50 (d) 52 **(ICAI)**

Sol. (d) We have, Mean = 5, Standard deviation = 2.6

We know that,

$$\text{Coefficient of variation} = \frac{\text{Standard deviation} \times 100}{\text{mean}}$$

$$= \frac{2.6 \times 100}{5} = 52$$

Hence, the correct option is (d) i.e.,52.

Example 41. If the coefficient of variation and standard deviation are 30 and 12 respectively, then the arithmetic mean of the distribution is **(Dec 2022)**

(a) 40 (b) 36 (c) 25 (d) 19

Sol. (a) **Given:** Coefficient of variation = 30, Standard deviation (σ) = 12

We know that,

$$\text{Coefficient of variation} = \frac{\sigma}{\bar{X}} \times 100$$

$$30 = \frac{12}{\bar{X}} \times 100$$

$$\Rightarrow \bar{X} = \frac{12}{30} \times 100 \Rightarrow \bar{X} = 40$$

Therefore, the arithmetic mean of the distribution is 40.

Example 42. What is the coefficient of variation of the following numbers?

(a) 8.09 (b) 18.08 (c) 20.23 (d) 20.45

Sol. (a) Given observations:

$\therefore$ Mean of the data

The sum of the squares of deviation from mean

The sum of the squares of deviation from mean $\sum_{i=0}^{N} (x_i - \bar{x})^2$

$= 5^2 + 6^2 + 3^2 + 2^2 + 6^2 = 25 + 36 + 9 + 4 + 36 = 110$

We know that.

$$\text{Standard deviation, S.D.} = \sqrt{\frac{\sum_{i=0}^{N} (x_i - \bar{x})^2}{n}}$$

$$\Rightarrow S.D = \sqrt{\frac{110}{5}} \Rightarrow S.D = \sqrt{22}$$

Coefficient of variation $= \frac{S.D}{\bar{x}} \times 100 = \frac{\sqrt{22}}{58} \times 100 = 8.09$

Hence, the correct answer is option (a) i.e., 8.09.

Example 43. The sum of squares of deviation from mean of 10 observations is 250. Mean of the datais 10. Find the co-efficient of variation.

(a) 10% (b) 25% (c) 50% (d) 0%

Sol. (c) Given, $N; N = 10; \sum (x - \bar{x})^2 = 250$ and Mean = 10

We know that,

Standard Deviation, $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}} = \sqrt{\frac{250}{10}} = 5$

Thus,

Coefficient of variance $= \frac{\sigma}{N} \times 100 = \frac{5}{10} \times 100 = 50\%$

Hence, the correct option is (c) i.e. 50%.

Example 44. Mean of a series is equal to 100, coefficient of variation is 45% then the S.D. is

(a) 45 (b) 0.45 (c) 4.5 (d) 40.5

Sol. (a) We know that, $(C.V.) = \frac{\sigma}{\bar{x}} \times 100$

$\bar{X} = 100, C.V = 45$

Given that $45 = \frac{\sigma}{100} \times 100$

Now, $\sigma = 45$

Hence, the correct option is (a) i.e., 45.

PROPERTIES FOR STANDARD DEVIATION

Standard deviation is independent of change of origin but not of scale, i.e., if there are two variables x and y related as y = a + bx for any two constants a and b, then SD of y is given by:

$$s_y = |b| \times s_x$$

Example 45. If x and y are related by 2x + 3y + 4 = 0 and SD of x is 6, then SD of y is

(a) 22 (b) 4 (c) 5 (d) 9

Sol. (b) Given relation. 2x + 3y + 4 = 0 and S.D. of x = 6

$\Rightarrow 3y = -2x - 4$

$$\Rightarrow y = \frac{-2x-4}{3}$$

$$\Rightarrow y = \frac{-2x}{3} - \frac{4}{3}$$

Thus, SD of $y = \left|\frac{-2}{3}\right| \times SD = \frac{2}{3} \times 6 = 4$

Hence, option (b) is the correct answer.

Example 46. If AM and C.V of a random variable x are 10 & 40 respectively, then the variance of $\left(-15 + \frac{3x}{2}\right)$

(a) 64 (b) 81 (c) 49 (d) 36

Sol. (d) We know that,

Coefficient of variance $(C.V) = \frac{\sigma}{\bar{x}} \times 100$

Given, $C.V = 40, \bar{X} = 10$

then, $40 = \frac{\sigma}{10} \times 100$

Thus, $\Rightarrow \sigma = \frac{400}{100} = 4$

Thus, $S.D\left(-15 + \frac{3x}{2}\right) = \frac{3}{2} \times SD(X)$

$\Rightarrow S.D\left(-15 + \frac{3X}{2}\right) = \frac{3}{2} \times 4 = 6$

Therefore, variance of $\left(-15 + \frac{3x}{2}\right) = 6^2 = 36$

Hence, option (d) is correct i.e. 36.

Note:

- Standard deviation is suitable for further mathematical treatment. For instance, if we know the sizes, means and standard deviations of two or more sets of data, then we can obtain the standard deviation of the combined data.
- The standard deviation of first natural numbers is $\sqrt{\frac{n^2-1}{12}}$.

- If all the observations assumed by a variable are constant i.e., equal, then the SD is zero. This means that if all the values taken by a variable x is k, say, then s = 0. This result applies to range as well as mean deviation.

Example 47. If the S.D. of the 1st n natural numbers is $\sqrt{30}$, then the value of n is

(a) 19 (b) 20 (c) 21 (d) None of these

Sol. (a) We know that, SD for first n natural numbers is given by

$$SD = \sqrt{\frac{n^2-1}{12}} = \sqrt{30} \Rightarrow \frac{n^2-1}{12} = 30$$

$\Rightarrow n^2 = 360 + 1$

$\Rightarrow n^2 = 361$

$\Rightarrow n = 19$

Therefore, the value of n is 19.

Hence, option (a) is correct i.e.19.

Example 48. The mean and SD for a, b and 2 are 3 and $\frac{2}{\sqrt{3}}$ respectively, The value of ab would be **(ICAI)**

(a) 5 (b) 6 (c) 11 (d) 3

Sol. (c) Given: Mean of a, b and 2 is 3.

$$\text{Thus, } \frac{a+b+2}{3} = 3$$

$\Rightarrow a + b + 2 = 9$

$\Rightarrow a + b = 7$

$$\text{Also, } S.D = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \text{Variance} = \frac{4}{3}$$

$$\text{We know that, Variance } (v(x)) = \frac{\sum_{i=1}^{n} X_i^2}{n} - \left(\frac{\sum_{i=1}^{n} X_i}{n}\right)^2$$

$$\Rightarrow \frac{4}{3} = \frac{\sum_{i=1}^{n} X_i^2}{n} - \left(\frac{\sum_{i=1}^{n} X_i}{n}\right)^2 \Rightarrow \frac{4}{3} = \frac{a^2+b^2+4}{3} - 9$$

$$\Rightarrow \frac{4}{3} = \frac{a^2+b^2+4-27}{3}$$

$\Rightarrow 4 = a^2 + b^2 - 23$

$\Rightarrow a^2 + b^2 = 27$

$\Rightarrow (a + b)^2 - 2ab = 27$ [$\because a^2 + b^2 + 2ab = (a + b)^2$]

$\Rightarrow (7)^2 - 2ab = 27$ [$\because$ From (i)]

$\Rightarrow 49 - 27 = 2ab$

$\Rightarrow 2ab = 22$

$\Rightarrow ab = 11$

Hence, the correct answer is option (c) i.e., 11.

Example 49. Which of the following companies A or B is more consistent so far as the payment of dividend is concerned? **(ICAI)**

Dividend paid byA	5	9	6	12	15	10	8	10
Dividend paid by B	4	8	7	15	18	9	6	6

(a) A (b) B (c) Both A & B (d) Neither A nor B

Sol. (a) We are given that

Dividend paid by A	5	9	6	12	15	10	8	10
Dividend paid by B	4	8	7	15	18	9	6	6

To check the consistency for the payment of dividend by A and B; we will find the coefficient of variation of A & B.

Let dividend paid by A be x, dividend paid by B be y

Then, $\Sigma X = 5 + 9 + 6 + 12 + 15 + 10 + 8 + 10 = 75$

$\Rightarrow \bar{X} = \frac{75}{8} = 9.375$

Also, $\Sigma X^2 = 5^2 + 9^2 + 6^2 + 12^2 + 15^2 + 10^2 + 8^2 + 10^2 = 775$

Now, SD is given by (σ_A)

$$= \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum X}{N}\right)^2}$$

$$\sqrt{\frac{775}{8} - \left(\frac{75}{8}\right)^2} = \sqrt{\frac{775}{8} - \frac{5625}{64}} = 2.99 \sim 3$$

Thus, $C.V_A = \frac{\sigma_A}{\bar{x}} \times 100 = \frac{3}{9.375} \times 100 = 32$

Similarly, $\sum Y = 73; \bar{Y} = \frac{73}{8} = 9.125, \sum Y^2 = 831$

$$\Rightarrow \sigma_B^2 = \frac{831}{8} - \left(\frac{73}{8}\right)^2 = \frac{831}{8} - \frac{5329}{64} = 20.61$$

Now, $C.V_B = \frac{4.54}{9.125} \times 100 = 49.75$

Since, $C.V_A < C.V_B$

Therefore, company A is more consistent than company B.

Hence, option (a) is correct.

PRACTICE QUESTIONS (PART D)

1. The variance of given data is 12 and their mean value is 40, what is the coefficient of variation (CV)?

(a) 5.66% (b) 6.66% (c) 7.50% (d) 8.65%

2. For any two numbers, SD is always **(ICAI)**

(a) Twice the range (b) Half of the range

(c) Square of the range (d) None of these

3. If all the observations are increased by 10, then **(ICAI)**

(a) SD would be increased by 10

(b) Mean deviation would be increased by 10

(c) Quartile deviation would increase by 10

(d) All these three remain unchanged.

4. If all the observations are multiplied by 2, then **(ICAI)**

(a) New SD would be also multiplied by 2

(b) New SD would be half of the previous SD

(c) New SD would be increased by 2

(d) New SD would be increased by 2

5. If the SD of x is 3, what is the variance of (5 - 2x)?

(a) 36 (b) 6 (c) 1 (d) 9

6. If x and y are related by y = 2x + 5 and the SD and AM of x are known to be 5 and 10 respectively, then the coefficient of variation is

(a) 25 (b) 30 (c) 40 (d) 20

7. Calculate the arithmetic mean and standard deviation from the following series:

Class Interval	5-15	15-25	25-35	35-45	45-50
Dividend paid by B	8	12	15	9	9

(a) 12.33 (b) 14.53 (c) 18.75 (d) None of these

8. The monthly test marks obtained by a student in Business Mathematics (out of 10) are 5, 9, 6, 12, 15, 10, 8, 10. The standard deviation for the student's marks is

(a) 5.05 (b) 1.80 (c) 2.99 (d) 3.51

9. The mean and standard deviation of 100 items is found to be 40 and 10. If at the time of calculations, two items are wrongly taken as 30 and 70 instead of 3 and 27, find the corrected mean and corrected standard deviation?

(a) 40 and 10 (b) 10.80 and 8.65

(c) 39.30 and 10.24 (d) 30.50 and 9.50

10. What is the standard deviation from the following data relating to the age distribution of 200 persons? **(ICAI)**

Age (year):	20	30	40	50	60	70	80
No. of People:	13	28	31	46	39	23	20

(a) 15.29 (b) 16.87 (c) 18.00 (d) 17.52

11. What is the coefficient of variation for the following distribution of wages? **(ICAI)**

Daily Wages (₹)	30-40	40-50	50-60	60-70	70-80	80-90
No of Workers	17	28	21	15	13	6

(a) ₹14.73 (b) 14.73 (c) 26.53 (d) 20.82

12. The mean and SD of a sample of 100 observations were calculated as 40 and 5.1 respectively by a CA student who took one of the observations as 50 instead of 40 by mistake. The current value of SD would be **(ICAI)**

(a) 4.90 (b) 5.00 (c) 4.88 (d) 4.85

Answer Key

1. (d) **2.** (b) **3.** (d) **4.** (a) **5.** (a) **6.** (c) **7.** (a) **8.** (c) **9.** (c) **10.** (b)
11. (c) **12.** (b)

FULL CHAPTER PRACTICE QUESTIONS

1. The sum of mean and SD of series is a + b, if we add 2 to each observation of the series then the sum of mean and SD is

(a) a + b + 2 (b) 6 – a + b (c) 4 + a – b (d) a + b +4

2. If the coefficient of quartile deviation is $\frac{1}{4}$, then $\frac{Q_3}{Q_1}$ is

(a) 5/3 (b) 4/3 (c) 3/4 (d) 3/5

3. For the distribution:

X	1	2	3	4	5	6
F	6	9	10	14	12	8

The value of median is

(a) 3.5 (b) 3 (c) 4 (d) 5

4. For a symmetric distribution

(a) Mean = Median = Mode (b) Mode = 3 median – 2 mean

(c) Mode $=\frac{1}{3}$ median $=\frac{1}{2}$ Mean (d) None of these

5. If the profits of a company remain some for the last ten months, then the S.D of profit of the company would be:

(a) Positive (b) Negative (c) Zero (d) (a) or (b)

6. S.D of first five consecutive natural numbers is

(a) $\sqrt{10}$ (b) $\sqrt{8}$ (c) $\sqrt{3}$ (d) $\sqrt{2}$

7. Which of the following is positional average?

(a) Median (b) GM (c) HM (d) AM

8. The Q.D of 6 numbers 15, 8, 36, 40, 38, 41 is equal to

(a) 12.4 (b) 25 (c) 13.5 (d) 37

9. Standard deviation is _____ times of $\sqrt{MD \times QD}$.

(a) $2\sqrt{3}$ (b) 4/5 (c) $\sqrt{\frac{15}{8}}$ (d) $\sqrt{\frac{8}{15}}$

10. If the points of inflexion of a normal curve are 40 and 60 respectively, then its mean deviation is

(a) 8 (b) 45 (c) 50 (d) 60

11. In a moderately skewed distribution the values of mean & median are 12 & 8 respectively. The value of mode is

(a) 0 (b) 12 (c) 15 (d) 30

12. The AM of 15 observations is 9 and the AM of first 9 observations is 11 and then AM of remaining observation is

(a) 11 (b) 6 (c) 5 (d) 9

13. If $\sigma^2 = 100$ and coefficient of variation = 20% then $\bar{X} =$

(a) 60 (b) 70 (c) 80 (d) 50

14. If the mean of the following distribution is 6 then the value of P is

X	2	4	6	10	P + 5
F	3	2	3	1	2

(a) 7 (b) 5 (c) 8 (d) 11

15. Find the interquartile range for the following dataset representing the scores of 10 students in a mathematics test:

35, 42, 48, 55, 60, 63, 68, 70, 72, 78

(a) 12 (b) 24 (c) 46.5 (d) None of these

16. If in a moderately skewed distribution the values of mode and mean are 32.1 and 35.4 respectively, then value of the median is

(a) 34.3 (b) 33.3 (c) 34 (d) 33

17. Standard deviation for the marks obtained by a student in monthly test in mathematics (out of 50) as 30, 35, 25, 20, and 15 is

(a) 25 (b) $\sqrt{50}$ (c) $\sqrt{30}$ (d) 50

18. If the variation of 5.7 and 11 is 4, then the coefficient of variation is :

(a) 15 (b) 25 (c) 17 (d) 19

19. If the range of a set of values is 65 and maximum values in the set is 83, then the minimum values in the set is

(a) 74 (b) 9 (c) 18 (d) None of these

20. Which one of the following is not a central tendency?

(a) Mean Deviation (b) Arithmetic mean

(c) Median (d) Mode

21. The Algebraic sum of the deviation of a set of values from their arithmetic mean is

(a) greater than 0 (b) equal to 0 (c) less than 0 (d) None of these

22. The Geometric mean of 3, 6, 24, and 48 is

(a) 8 (b) 12 (c) 24 (d) 20

23. The mean of 20 items of a data is 5 and If each item is multiplied by 3, then the new mean will be

(a) 5 (b) 10 (c) 15 (d) 20

24. The median of the data 5, 6, 7, 7, 8, 9, 10, 11, 11, 12, 15, 18, 18, and 19 is

(a) 10.5 (b) 10 (c) 11 (d) 11.5

25. The average of a series of overlapping averages, each of which is based on a certain number of items within a series is known as

(a) Moving average (b) Weighted average

(c) Simple average (d) None of these

26. For 899, 999, 391, 384, 390, 480, 485, 760, 111, 240 then rank of median is

(a) 2.7 (b) 5.5 (c) 8.25 (d) None

27. If each item is reduced by 15, then A.M is

(a) Reduced by 15 (b) Increased by 15 (c) Reduced by 10 (d) None

28. $\frac{(Q_3 - Q_1)}{(Q_3 + Q_1)}$ is known as

(a) Coefficient of Range. (b) Coefficient of Q.D.

(c) Coefficient of S.D (d) Coefficient of M. D.

29. Mean deviation is the least when deviation are taken from

(a) Mean (b) Median (c) Mode (d) Harmonic mean

30. If the mean value of seven numbers 7, 9, 12, x, 4, 11, and 5 is 9, then the missing number x will be:

(a) 13 (b) 14 (c) 15 (c) 8

Answer Key

1. (a)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (a)
11. (a)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (a)
21. (b)	**22.** (b)	**23.** (c)	**24.** (a)	**25.** (a)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (b)	**30.** (c)

PREVIOUS YEAR QUESTION (PYQ'S)

1. If $\sigma^2 = 100$ and coefficient of variation = 20% then $\bar{x}$ = **(June 2019)**

(a) 60 (b) 70 (c) 80 (d) 50

2. Coefficient of quartile deviation is 1/4 then Q_3/Q_1 is **(June 2019)**

(a) 5/3 (b) 4/3 (c) 3/4 (d) 3/5

3. The sum of mean and SD of a series is a + b, if we add 2 to each observation of the series then the sum of mean and SD is **(June 2019)**

(a) $a+b+2$ (b) ü+ + (c) ü+ − (d) $a+b+4$

4. Standard deviation is _____ times of $\sqrt{MD \times QD}$ **(June 2019)**

(a) $\frac{2}{3}$ (b) $\frac{4}{5}$ (c) $\sqrt{\frac{15}{8}}$ (d) $\sqrt{\frac{8}{15}}$

5. SD of first five consecutive natural numbers is **(June 2019)**

(a) $\sqrt{10}$ (b) $\sqrt{8}$ (c) $\sqrt{3}$ (d) $\sqrt{2}$

6. The Q.D. of 6 numbers 15, 8, 36, 40, 38, 41 is equal to **(June 2019)**

(a) 12.5 (b) 25 (c) 13.5 (d) 37

7. What will be the probable value of mean deviation when $Q_3 = 40$ and $Q_1 = 15$? **(Dec 2019, July 2021)**

(a) 17.50 (b) 18.75 (c) 15.00 (d) 16.00

8. Find the mean deviation about mean of 4, 5, 6, 8, 3. **(Dec 2019)**

(a) 5.20 (b) 7.20 (c) 1.44 (d) 2.33

9. The mean and coefficient of variance are 20 and 80 respectively. Find the value of variance. **(Dec 2019)**

(a) 16 (b) 256 (c) 36 (d) None

10. Find SD of 1, 2, 3, 4, 5, 6, 7, 8, 9 **(Dec 2019)**

(a) $\sqrt{\frac{20}{3}}$ (b) $\sqrt{\frac{81}{3}}$ (c) $\sqrt{\frac{20}{5}}$ (d) None of these

11. The standard deviation for the set of numbers 1, 4, 5, 7, 8 is 2.45 nearly. If 10 is added to each number then new standard deviation is **(Dec 2019)**

(a) 24.45 (b) 12.45 (c) 2.45 (d) 0.245

12. If every observation is increased by 5, then **(Dec 2019)**

(a) SD increased by 5 (b) MD increased by 5

(c) QD increased by 5 (d) Dispersion not affected

13. For given distribution, the arithmetic mean is 15 and the standard deviation is 9 then the coefficient of variation is **(Dec 2019)**

(a) $\frac{15}{9}\times 100$ (b) $\frac{15}{9}$ (c) $\frac{9}{15}$ (d) $\frac{9}{15}\times 100$

14. The mean of a distribution is 14 and the standard deviation is 5. What is the value of the coefficient of variation? **(Dec 2019)**

(a) 60.4% (b) 70% (c) 35.7% (d) 27.8%

15. Which of the following measures of dispersion is based on absolute deviations? **(Dec 2019)**

(a) Range (b) SD (c) Mean Deviation (d) Quartile Deviation

16. The best statistical measure used for comparing two series is **(Dec 2019)**

(a) Mean absolute deviation (b) Range

(c) Coefficient of variation (d) Standard deviation

17. The relationship between P-Series and Q series is given by $2P - 3Q = 10$. If the range of P Series is 18. What would be the range of Q ? **(Dec 2019)**

(a) 10 (b) 15 (c) 9 (d) 12

18. It is given that the mean $(\bar{X})$ is 10 and standard deviation (S.D) is 3.2. If the observations are increased by 4 , then the new mean and standard deviation are:

(a) $(\bar{X}) = 10, S.D = 7.2$ (b) $(\bar{X}) = 10, S.D = 3.2$

(c) $(\bar{X}) = 14, S.D = 3.2$ (d) $(\bar{X}) = 14, S.D = 7.2$

19. Which of the following is a relative measure of dispersion? **(Jan 2021)**

(a) Range (b) Mean deviation

(c) Standard deviation (d) Coefficient of quartile deviation

20. Find the coefficient of mean deviation about mean for the data: 5, 7, 8, 10, 11 13, 19 **(Jan 2021)**

(a) 17.28 (b) 28.57 (c) 32.11 (d) 18.56

21. The mean deviation of the numbers 3, 10, 6, 11, 14, 17, 9, 8, 12 about the mean is (correct to one decimal place) **(July 2021)**

(a) 8.7 (b) 4.2 (c) 3.1 (d) 9.8

22. The standard deviation of 1 to 9 natural numbers is _____. **(July 2021)**

(a) 6.65 (b) 2.58 (c) 6.75 (d) 5.62

23. If the numbers are 5, 1, 8, 7, 2 then the coefficient of variation is **(July 2021)**

(a) 56.13% (b) 59.13% (c) 48.13% (d) 44.13%

24. If every observation is increased by 7 then **(July 2021)**

(a) Standard deviation increases by 7 (b) Mean deviation increases by 7

(c) Quartile deviation increases by 7 (d) Not affected at all

25. If a school has 14 teachers, their heights (in cm) are: 172, 173, 164, 178, 168, 169, 173, 172, 173, 164, 178, 168, 169, 173 then average deviation of this data is **(July 2021)**

(a) 2.43 approx (b) 3.93 approx (c) 3.43 approx (d) 2.92 approx

26. If the relationship between x and y is given by $2x + 3y = 10$, and the range of y is 10 , then what is the range of x ? **(July 2021)**

(a) 10 (b) 18 (c) 8 (d) 15

27. The marks secured by 5 students in a subject are 82, 73, 69, 84, 66. What is the coefficient of Range? **(Dec 2021)**

(a) 0.12 (b) 12 (c) 120 (d) 0.012

28. Which one of the following is not a measure of central tendency? **(June 2022)**

(a) Median (b) Range

(c) Arithmetic mean (d) Harmonic mean

29. What is mean deviation about mean of the following numbers?

11, 8, 10, 10, 12, 9 **(June 2022)**

(a) 2 (b) 1 (c) 1.5 (d) 1.8

30. Following are the ages of 8 employees of a small old age home expressed in 96, 50, 67, 75, 71, 69, 64, 44. Find the range and its coefficient. **(June 2022)**

(a) 46, 31.51 respectively (b) 51, 37.67 respectively

(c) 43, 29.49 respectively (d) 49, 36.42 respectively

31. Find the standard deviation and coefficient of variation of 1, 6, 5, 9, 8. **(June 2022)**

(a) 2.78 and 40.83 respectively (b) 2.45 and 47.93 respectively

(c) 2.78 and 47.93 respectively (d) 2.87 and 49.37 respectively

32. The arithmetic mean and coefficient of variation of dataset x are respectively, 10 and 30. The variance of 30 - 2x is **(June 2022)**

(a) 28 (b) 32 (c) 34 (d) 36

33. Which measure of dispersion is based on the absolute deviation only? **(June 2022)**

(a) Range (b) Standard deviation

(c) Mean deviation (d) Quartile deviation

34. The coefficient of deviation based on 25 th and 75 th percentiles of 6, 9, 3, 8, 4, 5, 8 and 4 is **(June 2022)**

(a) 50 (b) $\frac{100}{3}$ (c) 30 (d) 25

35. If the coefficient of variation and standard deviation are 30 and 12 respectively, then the arithmetic mean of the distribution is **(Dec 2022)**

(a) 40 (b) 36 (c) 25 (d) 19

36. Which one of the following is not a method of measures of dispersion? **(Dec 2022)**

(a) Standard deviation (b) Mean deviation

(c) Range (d) Concurrent deviation method

37. Mean deviation is minimum when deviation are taken from **(Dec 2022)**

(a) Mean (b) Median (c) Mode (d) Range

38. If the first quartile is 56.50 and the third quartile is 77.50, then the coefficient of quartile deviation is **(Dec 2022)**

(a) 638.09 (b) 15.67 (c) 63.80 (d) 156.71

39. If the sum of squares of the values equals to 3390, number of observations are 30 and standard deviation is 7, what is the mean of the above observations? **(Dec 2022)**

(a) 14 (b) 11 (c) 8 (d) 5

40. If the variance of a random variable 'x' is 17, then what is the variance of y = 2x + 5? **(Dec 2022)**

(a) 34 (b) 39 (c) 68 (d) 78

41. If the variance of given data is 12 and their mean value is 40, what is the coefficient of variation (CV)? **(Dec 2022)**

(a) 5.66% (b) 6.66% (c) 7.50% (d) 8.65%

42. In a given set, if all data are of same value then variance would be **(Dec 2022)**

(a) 0 (b) 1 (c) -1 (d) 0.5

43. If x and y are related as 4x + 3y + 11 = 0 and mean deviation of y is 7.20, what is the mean deviation of x ? **(Dec 2022)**

(a) 2.70 (b) 7.20 (c) 4.50 (d) 5.40

44. The mean deviation about the mean for the data 12, 16, 24, 30, 35, 39, 40 is **(Dec 2022)**

(a) 9.14 (b) 9.41 (c) 8.91 (d) 9.81

45. If the standard deviation of data 2, 4, 5, 6, 8, 17 is 4.47, then standard deviation of the data 4, 8, 10, 12,16, 34 is

(a) 4.47 (b) 8.94 (c) 13.41 (d) 2.24

46. The mean and variance of a group of 100 observations are 8 and 9 respectively. Out of 100 observations, the mean and standard deviation of 60 observations are 10 and 2 respectively. Find the standard deviation of the remaining 40 observations?

(a) 4.5 (b) 3.5 (c) 2.5 (d) 1.5

47. For a given set of normally distributed data, the following statistical parameters are known, Mean = 6; Standard deviation = 2.6; Median = 5 and Quartile deviation = 1.5, then the coefficient of quartile deviation equals to

(a) 30 (b) 32 (c) 25 (d) 29

48. If the first quartile is 42.75 and the third quartile is 74.25, then the coefficient of quartile deviation is

(a) 29.62 (b) 15.75 (c) 17.57 (d) 29.62

49. If the range of a data is 20 and its smallest value is 5, then what is the largest value of the data? **(Dec 2023)**

(a) 20 (b) 25 (c) 5 (d) 30

50. If x and y are related as 3x – 4y = 20, and the quartile deviation of 'x' is 12, then the quartile deviation of 'y' is

(a) 9 (b) 8 (c) 7 (d) 6

51. If the quartile deviation is 12 and the first quartile is 25, then the value of the third quartile is **(Dec 2023)**

(a) 37 (b) 49 (c) 61 (d) 60

52. The coefficient of the range of the data: 7, 8, 4, 1, 9, 12, 18, 16, 94, 3, 5, -6 is **(June 2024)**

(a) 133.6 (b) 163.3 (c) 166.3 (d) 113.6

53. For the first 20 natural numbers, the standard deviation is **(June 2024)**

(a) 5.77 (b) 7.75 (c) 5.64 (d) 6.54

54. If Mean Deviation about Arithmetic Mean is 1.78 and Arithmetic Mean is 3.50 then coefficient of Mean Deviation about Arithmetic Mean is **(June 2024)**

(a) 50.85 (b) 52.65 (c) 51.85 (d) 53.65

55. If the Arithmetic Mean and coefficient of variation of y are 5 and 20 respectively, the variance of 12 – 3y **(June 2024)**

(a) 9 (b) 3 (c) 81 (d) 100

56. In a data set, 25 percent of values are smaller than 30 and one-fourth of values are larger than 70, then the coefficient of quartile deviation is _____ % **(June 2024)**

(a) 40 (b) 50 (c) 60 (d) 70

57. Consider the data sets: X = {–6, 2, –2, 6}, Y = {4, 8, 2, 6}, Z = {103, 100, 102, 101} Let S_x, S_y and S_z be the standard deviations of the sets X, Y and Z respectively. We have the relations, **(June 2024)**

(a) $S_x < S_y < S_z$ (b) $S_z < S_y < S_x$ (c) $S_z < S_x < S_y$ (d) $S_x < S_z < S_y$

58. What is the coefficient of range for the observations 20, 28, 32, 41, 48, 60? **(Sept 2024)**

(a) 50 (b) 20 (c) 40 (d) 200

59. In which of the following there is no impact of presence of extreme observations? **(Sept 2024)**

(a) Quartile deviation (b) Range
(c) Standard deviation (d) Variance

60. If each observation of a set is divided by 10, then the Standard Deviation of the new observation is: **(Sept 2024)**

(a) $\frac{1}{10}$ of Standard Deviation of original observation.

(b) $\frac{1}{100}$ th of Standard Deviation of original observation.

(c) 100 times of Standard Deviation of original observation.

(d) 10 times of Standard Deviation of original observation.

61. The Standard Deviation of the series 3, 6, 9, 12, 15 is: **(Sept 2024)**

(a) 4.24 (b) 6.36 (c) 4.12 (d) 3.28

62. The quartile deviation of the distribution of the following data is: **(Sept 2024)**

X	2	3	4	5	6
f(x)	2	4	8	4	1

(a) 0 (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

63. If x and y are related as $4x + 2y + 12 = 0$ and mean deviation of x is 4.5, then the mean deviation of y is **(Jan 2025)**

(a) 9 (b) –9 (c) 1.1 (d) 4.5

64. The standard deviation of the data 2, 4, 5, 6, 8, 17 is 23.33, then the standard deviation of the data 4, 8, 10, 12, 16, 34 is **(Jan 2025)**

(a) 46.66 (b) 23.33 (c) 12.23 (d) 0

65. The values of the first quartile and third quartile are 36.50 and 57.50. Then the semi-inter-quartile range is **(Jan 2025)**

(a) 12.50 (b) 47.50 (c) 10.50 (d) 11.50

66. For a distribution the mean is 30. The standard deviation is 2, then coefficient of variation? **(Jan 2025)**

(a) 9.45% (b) 6.67% (c) 7.5% (d) 2.5%

67. Which of the following is not a method of dispersion? **(Jan 2025)**

(a) Standard deviation (b) Mean deviation

(c) Range (d) Concurrent deviation method

68. Mean deviation is _____ when the deviations are taken from the median. **(Jan 2025)**

(a) minimum (b) maximum (c) zero (d) can't say

69. Find out co-efficient of variation, if N = 14, $\sum fx = 280$ and σ(S.D.) = 3. **(Jan 2025)**

(a) 20 (b) 15 (c) 4.67 (d) Zero

70. Which one of the following is the absolute measure of dispersion for open ended distribution? **(May 2025)**

(a) Range (b) Standard deviation

(c) Mean deviation (d) Quartile deviation

71. The monthly profit/loss for six months of the firm is as under; **(May 2025)**

Months	January	February	March	April	May	June
Profit/Loss (in)	1000	900	0	-200	-400	2000

The coefficient of Range of the above data is

(a) 122 (b) 150 (c) 33.33 (d) 55.55

72. Best measure of Dispersion for open-ended classification is _____ , which does not change with the change of _____. **(Sept 2025)**

(a) Quartile Deviation, Scale (b) Standard Deviation, Scale

(c) Quartile Deviation, Origin (d) Standard Deviation, Origin

73. Coefficient of range of 84, 93, 53, 70, 82, 65 is **(Sept 2025)**

(a) 28.38 (b) 27.39 (c) 26.75 (d) 29.31

74. Find the coefficient of variation for the following numbers; **(Sept 2025)**

7, 5, 9, 3, 6

(a) 33.33 (b) 66.66 (c) 3 (d) 300

75. A data set has first eleven positive multiples of 6. The semi inter-quartile range is _____. **(Sept 2025)**

(a) 12 (b) 24 (c) 18 (d) 36

76. If the sum of ten values is 20 and sum of squares of these values is 80, then the standard deviation is _____. **(Sept 2025)**

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 4

Answer Key

1. (d)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (a)
11. (c)	**12.** (d)	**13.** (d)	**14.** (c)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (d)	**20.** (c)
21. (c)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (c)	**26.** (d)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (a)
31. (c)	**32.** (d)	**33.** (c)	**34.** (b)	**35.** (a)	**36.** (d)	**37.** (b)	**38.** (b)	**39.** (c)	**40.** (c)
41. (d)	**42.** (a)	**43.** (d)	**44.** (a)	**45.** (b)	**46.** (d)	**47.** (a)	**48.** (d)	**49.** (b)	**50.** (a)
51. (b)	**52.** (d)	**53.** (a)	**54.** (a)	**55.** (a)	**56.** (a)	**57.** (b)	**58.** (a)	**59.** (a)	**60.** (a)
61. (a)	**62.** (b)	**63.** (a)	**64.** (a)	**65.** (c)	**66.** (b)	**67.** (d)	**68.** (a)	**69.** (b)	**70.** (d)
71. (b)	**72.** (c)	**73.** (b)	**74.** (a)	**75.** (c)	**76.** (b)				

SUMMARY

MEASURES OF CENTRAL TENDENCY

- **Arithmetic mean:** Sum of the observations divided by the number of observations

$$\text{Arithmetic Mean (Mean)} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

Methods to find mean:

(i) Direct method: $\text{Mean} = \frac{f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \ldots + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$

(ii) Step-deviation method: Mean $= a + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h$

- **Geometric mean:**

(i) For individual series: GM = $(x_1 x_2 x_3 \ldots\ldots\ldots\ldots x_n)^{1/n}$

(ii) For discrete series: G.M. = AL $\left[\frac{1}{N}\sum f \log X\right]$

(iii) For continuous series: G.M. = AL $\left[\frac{1}{N}\sum f \log X\right]$

- **Harmonic mean:**

(i) For individual series: H.M. $= \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\ldots+\frac{1}{x_n}}$

(ii) For discrete series: HM $= \frac{n}{\sum\left[\frac{f}{X}\right]}$

(iii) For continuous series: HM $= \frac{n}{\sum\left[\frac{f}{X}\right]}$

- **Mode:** The value that occurs the maximum number of times.

For continuous frequency distribution: Mode = $l + \frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times c$

- **Median:**

(i) **For individual series:** Arrange the dataset in ascending order or descending order, then

(a) If n is odd, the value is given by: $\left(\frac{n+1}{2}\right)$th observation

(b) If n is even, the value is given by: $\frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$

(ii) **For discrete series:**

Median is the size of $\left(\frac{N+1}{2}\right)$th observation

(iii) **For continuous series:**

Median $= l + \frac{\frac{N}{2} - c.f}{f} \times h$

- **Quartiles** divide the entire dataset into four equal parts. So: there are three quartiles; first second and third represented by Q_i, Q_2 and Q_3 respectively. Q_1 is the lower quartile and median of the lower half of the data set. Q_2 is the median.
- **Deciles** divide the entire dataset into ten equal parts. There are 9 deciles
- **Percentiles** divide the entire dataset into hundred equal parts. There are 99 percentiles.

TABLE FOR PARTITION VALUE

Calculation for	Quartiles	Deciles	Percentiles
Individual Observations	Q_k = size of $\frac{k(n+1)}{4}$ th observation	$D_k = \frac{k(n+1)}{10}$ th observation	$P_k = \frac{k(n+1)}{100}$ th observation
Discrete Series	The value of X corresponding to the c.f. just greater than or equal to $\frac{k(n+1)}{4}$	The value of X corresponding to the c.f. just greater than or equal to $\frac{k(n+1)}{10}$	The value of X corresponding to the c.f. just greater than or equal to $\frac{k(n+1)}{100}$
Continuous Series	$Q_k = l + \frac{\frac{KN}{4} - C}{f} \times h$ l = lower limit of Q_k class C = c.f. of the class preceding the Q_k class f = frequency of the Q_k class, h = size or width of Q_k class.	$D_k = l + \frac{\frac{KN}{10} - C}{f} \times h$	$P_k = l + \frac{\frac{KN}{100} - C}{f} \times h$

- Algebraic sum of deviations of a set of observations from their AM is zero i.e. $\sum(x_i - \bar{x}) = 0$
- AM is affected due to a change of origin and/or scale which implies that if the original variable is changed to another variable by effecting a change of origin,
- For given two positive numbers, A.M. × H.M.= $(G.M)^2$
- AM ≥ GM ≥ HM The equality sign occurs, as we have already seen, when all the observations are equal.
- Mode = 3 median - 2 mean

MEASURES OF DISPERSION

Range, interquartile range, and standard deviation are the three commonly used measures of dispersion.

- **Range** is the difference between the largest and the smallest observation in the data.
  - (i) For individual series: Range = Largest value - Smallest value
  - (ii) For grouped frequency distribution: Range = Upper class boundary (U.C.B) - Lower class boundary (L.C.B)
- **Coefficient of Range**:

$$\text{Coefficient of Range} = \frac{\text{Range}}{\text{Sum of the largest and the lowest values}}$$

$$\text{i.e., Coefficient of range} = \frac{L-S}{L+S} \times 100$$

- **Quartile Deviation or Semi Inter-Quartile Range:**
  - **(i) Interquartile range** is defined as the difference between the 25th and 75th percentile (also called the first and third quartile). Hence the interquartile range describes the middle 50% of observations. If the interquartile range is large it means that the middle 50% of observations are spaced wide apart.

    Inter-quartile range = $Q_3 - Q_1$
  - **(ii) Quartile deviation**, also called semi inter-quartile range, is an absolute measure of dispersion defined by the formula:

$$\text{Quartile Deviation (Q.D.)} = \frac{Q_3 - Q_1}{2}$$

  - **(iii) Coefficient of Quartile Deviation**

$$\text{Coefficient of Quartile Deviation} = \frac{\text{Quartile Deviation}}{\text{Median}} \times 100 = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

- **Mean Deviation:**
  - **(i) Discrete Series:** Mean Deviation (about an average A) $= \frac{\sum f_i |x_i - A|}{N}$
  - **(ii) Continuous Series:** Mean Deviation (about an average A) $= \frac{\sum f_i |x_i - A|}{N}$ where x_1, x_2, ..., x_n are the class marks (or mid-values) of a set of grouped data

Coefficient of Mean Deviation

The relative measure corresponding to the mean deviation, called the coefficient of mean deviation, is given by

1. Coefficient of M.D. = $\frac{1}{n}\sum |x_i - A|$

2. Coefficient of M.D. about mean $\frac{1}{n}\sum|x_i - \bar{x}|$ where, $\bar{x}$ is mean

3. Coefficient of M.D. about median = $\frac{1}{n}\sum|x_i - \text{Median}|$

- **Standard Deviation** (SD) is the most commonly used measure of dispersion. It is a measure of spread of data about the mean. SD is the square root of the sum of squared deviation from the mean divided by the number of observations.

  (i) For individual series: $\sigma = \sqrt{\frac{\sum(X-\bar{X})^2}{N}}$ where $\bar{X} = \frac{\sum X}{n}$

  (ii) For grouped data: $\sigma = \sqrt{\frac{\sum f_i(x_i - \bar{x})^2}{n}}$

  (iii) For ungrouped data: $\sigma = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$

  Standard deviation of first natural numbers $= \sqrt{\frac{n^2-1}{12}}$

- **Variance:** It is the square of the standard deviation.

  (i) For individual observations, $\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}$ or $\sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2$

  (ii) For Ungrouped/Grouped Observations, $\sigma^2 = \frac{\sum f_i(x_i - \bar{x})^2}{n}$

- **Coefficient of Variation:**

  Coefficient of Variation $= \frac{\text{S.D.}}{\bar{x}} \times 100$

❑❑❑

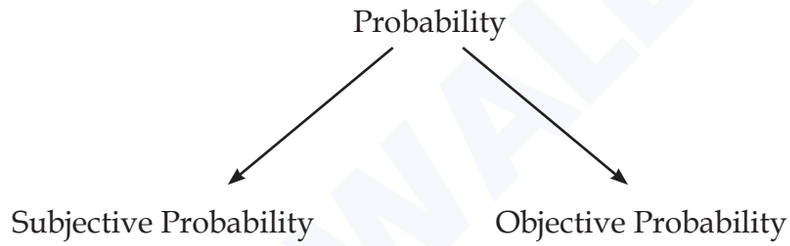
CHAPTER

15 Probability

If we were to consider an India vs. Pakistan cricket match today, predicting the outcome involves probability.

PROBABILITY

- The terms 'Probably' 'in all likelihood', 'chance', 'odds in favor', 'odds against' are too familiar nowadays and they have their origin in a branch of Mathematics, known as Probability.
- Thus, it is a branch of mathematics that deals with quantifying uncertainty and analyzing the likelihood of events occurring.



SUBJECTIVE PROBABILITY

- Subjective probability is based on an individual's personal beliefs, judgments, and opinions about the likelihood of an event.
- It is often used when there is limited or no historical data available to estimate probabilities objectively.
- Subjective probabilities are influenced by personal experiences, biases, and perceptions.

OBJECTIVE PROBABILITY

- Objective probability, also known as frequentist probability, is based on observed or historical data and the relative frequency of an event occurring in a large number of trials or observations.
- It is considered a more objective and data-driven approach to probability, as it relies on empirical evidence.
- Objective probability can be expressed as a ratio of the number of favorable outcomes to the total number of possible outcomes.

  In order to develop a sound knowledge about probability, it is necessary to get ourselves familiar with a few terms.
- **EXPERIMENT:** An experiment refers to the performance of certain tasks to produce certain results.

- **RANDOM EXPERIMENT**: A random experiment is one in which the results depend solely on chance and cannot be predicted with certainty.
- **SAMPLE SPACE:** The set of all possible distinct outcomes of a random experiment is known as sample space. It is denoted by 'S'.

  E.g.: When a coin is tossed, then S = {H, T}
- **EVENTS:** Events are the results or outcomes of a random experiment. Sometimes, it gives a combination of outcomes.

  For example, {HH} represents the event of getting 2 heads.

TYPES OF EVENTS

1. **Simple Event:** An event consisting of a single outcome.
   - **Example:** Rolling a 4 on a die (E = {4}).
2. **Compound Event:** An event consisting of multiple outcomes.
   - **Example:** Rolling an even number on a die (E = {2, 4, 6}).
3. **Certain Event:** An event that is guaranteed to occur.
   - **Example:** Rolling a number less than 7 on a die (E = S = {1, 2, 3, 4, 5, 6}).
4. **Impossible Event:** An event that cannot occur.
   - **Example:** Rolling a 7 on a standard die (E = φ).

MUTUALLY EXCLUSIVE EVENTS OR INCOMPATIBLE EVENTS

These are events that cannot occur simultaneously.

E.g.: Consider rolling a six-sided die. The events "rolling a 1" and "rolling a 2" are mutually exclusive because it is impossible for both events to occur simultaneously.

- Two events A and B are mutually exclusive if $P(A \cap B) = 0$, meaning there are no common outcomes between A and B.
- Similarly, three events A, B, and C are mutually exclusive if $P(A \cap B \cap C) = 0$, meaning there are no common outcomes among all three events.

EXHAUSTIVE EVENTS

Exhaustive events are events that together cover all possible outcomes of an experiment.

E.g.:

1. In a coin toss, there are two exhaustive events: "Getting heads" or "Getting tails".
2. In throwing a die, there are 6 exhaustive events: {1, 2, 3, 4, 5, 6}.
   (a) Two events A and B are exhaustive if $P(A \cup B) = 1$, meaning all possible outcomes are covered by A and B.
   (b) Similarly, three events A, B, and C are exhaustive if $P(A \cup B \cup C) = 1$, meaning all possible outcomes are covered by A, B, and C.

EQUALLY LIKELY EVENTS or Mutually Symmetric Events or Equi-Probable Events: Equally likely events are events that have the same probability of occurring.

E.g.: In a fair coin toss, getting heads or tails is an example of equally likely events.

- Three events A, B, and C are equally likely if P(A) = P(B) = P(C), meaning the probabilities of each event are the same.

ABOUT PLAYING CARDS

TOTAL CARDS = 52

- Diamond (13)
- Heart (13)
- Club (13)
- Spade (13)

1. Total playing cards: 52
2. Total number of suits: 4
3. Number of cards in a suit: 13
4. Number of Ace cards: 4
5. Number of King cards: 4
6. Number of Queen cards: 4
7. Number of Jack cards: 4
8. Number of face cards: 12

CLASSICAL DEFINITION OF PROBABILITY OR A PRIOR DEFINITION

FOR FINITE ELEMENTARY EVENTS

Let's consider a random experiment that results in n finite elementary events, which are assumed to be equally likely. If n_A ($\leq n$) events are favorable to an event A, then the probability of occurrence of event A is defined as the ratio of the number of events favorable to A to the total number of events. This can be expressed as:

$$P(A) = \frac{n_A}{n} = \frac{\text{Number of events favorable to A}}{\text{Total number of events}}$$

FOR COMPOSITE EVENTS

In the case of composite events that are mutually exclusive, exhaustive, and equally likely, we can consider m ($\leq n$) such events. If m_A ($\leq n_A$) represents the number of mutually exclusive, exhaustive, and equally likely events favourable to A, then the probability of event A is given by:

$$P(A) = \frac{m_A}{m} = \frac{\text{Number of mutually exclusive, exhaustive, and equally likely events favorable to A}}{\text{Total number of mutually exclusive, exhaustive, and equally likely events}}$$

E.g.: Consider the rolling of a dice once. The sample space S is given by S = {1, 2, 3, 4, 5, 6}.

We define events A, B and C as follows:

A: The event of getting an even number: A = {2, 4, 6}

B: The event of getting an odd number: B = {1, 3, 5}

C: The event of getting a multiple of 3: C = {3, 6}

Calculation of Probabilities

Now, let's apply the classical definition of probability to the given example:

- P(A) is the probability of getting an even number. Since there are 3 even numbers in S and a total of 6 sample points, we have $P(A) = \frac{3}{6} = \frac{1}{2}$
- P(B) is the probability of getting an odd number. There are 3 odd numbers in S, so $P(B) = \frac{3}{6} = \frac{1}{2}$

- P(C) is the probability of getting a multiple of 3. Among the 6 sample points, 2 of them are multiples of 3. Therefore, $P(C) = \frac{2}{6} = \frac{1}{3}$
- Also, since P (A ∩ B) = 0 since A ∩ B = {2, 4, 6} ∩ {1, 3, 5} = ϕ
  Thus, A and B are mutually exclusive events.

Example 1. A die is thrown, then the probability of getting a prime number is

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) None of these

Sol. (b) When a die is thrown, then the sample space will be:

Total outcomes = 6

Now, event of getting a prime number is: {2, 3, 5}

i.e., Number of favourable outcomes = 3

Thus, the required probability = $\frac{3}{6} = \frac{1}{2}$

Hence, the correct option is (b).

Example 2. The probability that exactly one head appears in a single throw of two fair coins is **(ICAI)**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) None of these

Sol. (b) The sample space of throwing two coins in a single time is given by: {(H, H); (H, T); (T, H); (T, T)}

⇒ Total no. of possible outcomes = 4

Now, the outcomes of appearing exactly one head is (H, T); (T, H)

⇒ Total no. of favorable outcomes = 2

We know that,

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Hence, the correct option is (b) i.e. $\frac{1}{2}$.

Example 3. A bag contains 15 one rupee coins, 25 two rupee coins and 10 five rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one rupee coin is **(ICAI)**

(a) 0.30 (b) 0.70 (c) 0.25 (d) 0.20

Sol. (b) We know that,

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total outcomes}}$$

Number of favourable outcomes = 25 + 10 = 35

Total outcomes = 25 + 10 + 15 = 50

Therefore, the required probability $= \frac{35}{50} = \frac{7}{10} = 0.7$

Hence, the correct option is (b) i.e., 0.7

Example 4. Three coins are tossed together. The probability of getting three tails is **(ICAI)**

(a) 5/8 (b) 3/8 (c) 1/8 (d) None of these

Sol. (c) The sample space of tossing 3 coins is given by,

S = { HHH, TTT, HTT, THT, TTH, THH, HTH, HHT}

where, { H → Heads, T → Tails}

Here, total outcomes = 8

Favorable outcomes = { TTT} = 1

We know that,

$$\text{Probability (P)} = \frac{\text{Favorable Outcomes}}{\text{Total Number of Outcomes}} = \frac{1}{8}.$$

Hence, the correct answer is option (c) i.e., $\frac{1}{8}$.

Example 5. A coin is tossed three times. What is the probability of getting:

(I) 2 tails (II) at least 2 tails

(a) $\frac{3}{8}, \frac{5}{8}$ (b) $\frac{3}{8}, \frac{3}{8}$ (c) $\frac{3}{8}, \frac{1}{8}$ (d) None of these

Sol. (d) Given: A coin is tossed three times

Thus, the events are: {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)}

(I) Events of 2 tails: {(HTT), (THT), (TTH)}

Therefore, the required probability $= \frac{3}{8}$

(II) Events of at least 2 tails: {(HTT), (THT), (TTH), (TTT)}

Therefore, the required probability $= \frac{4}{8} = \frac{1}{2}$

Hence, the correct option is (d).

Some key points related to the classical definition of probability:

1. The probability of an event lies between 0 and 1, inclusive. It cannot be negative or greater than 1.
2. The non-occurrence of event A is denoted by A' or A^c or $\overline{A}$ and is known as the complementary event of A. The event A and its complementary event A' form a set of mutually exclusive and exhaustive events.
3. The ratio of the number of favourable events to the number of unfavourable events is known as the odds in favour of event A. Its inverse ratio is known as the odds against event A. i.e. If 'p' be the number of favourable outcomes of an event and 'q' be the number of unfavourable outcomes of the event, then

Probability $= \frac{p}{p+q}$ where $\frac{p}{q}$ is the odds in favour of event and $\frac{q}{p}$ is the odds against the event.

Example 6. If $P(A) = \frac{5}{9}$, then the odds against the event A is **(ICAI)**

(a) 5 : 9 (b) 5 : 4 (c) 4 : 5 (d) 5 : 14

Sol. (c) We know that,

Probability of the event is given by $\frac{p}{p+q}$

where $\frac{p}{q}$ are the odds in favor of an event and $\frac{q}{p}$ are the odds against an event.

Given, $P(A) = \frac{5}{9}$

$\Rightarrow \frac{p}{p+q} = \frac{5}{9}$

$\Rightarrow 9p = 5p + 5q$

$\Rightarrow 4p = 5q$

$\Rightarrow \frac{q}{p} = \frac{4}{5}$

Hence, the correct option is (c).

Example 7. Three unbiased coins are tossed simultaneously, then the probability of getting at least 2 heads is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$

Sol. (b) Here, the sample space

= {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

Now, the number of favourable outcomes ={HHT, HTH, THH, HHH}

Thus, the required probability $= \frac{4}{8} = \frac{1}{2}$

Hence, the correct option is (b).

Example 8. A dice is rolled twice. What is the probability of getting a difference of 5 points?

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{9}$ (d) None of these

Sol. (a) A dice is rolled twice

Then the total number of outcomes,

n(S) = {(1, 1); (1, 2); (1, 3); (1, 4); (1, 5); (1, 6)

(2, 1); (2, 2); (2, 3); (2, 4); (2, 5); (2, 6)

(3, 1); (3, 2); (3, 3); (3, 4); (3, 5); (3, 6)
(4, 1); (4, 2); (4, 3); (4, 4); (4, 5); (4, 6)
(5, 1); (5, 2); (5, 3); (5, 4); (5, 5); (5, 6)
(6, 1); (6, 2); (6, 3); (6, 4); (6, 5); (6, 6)}

The favourable outcomes of getting a difference of 5 points are {(1, 6); (6,1)}

We know that,

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Then, p (getting difference of 5 points) = $\frac{n(A)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

Hence, the correct option is (a).

Example 9. Find the probability that a four digit number comprising the digits 2, 5, 6 and 7 would be divisible by 4.

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{24}$ (d) None of these

Sol. (b) Given digits: 2, 5, 6, 7

Here, these 4 numbers can be form a 4 digit number in 4! = 24 ways

For a four digit number to be divisible by 4, its last two digits should also be divisible by 4.

So, the possible last digits: 52, 56, 72, 76.

Thus, if we have the last two digits as 52, so the 1st two places of the four digit number can be filled up using the remaining 2 digits i.e., in 2! or 2 ways.

i.e., 6752 and 7652

Thus there are 2 four digit numbers that end with 52.

Similarly, the number of four digit numbers that are divisible by 4 is 4 × 2 = 8.

We know that,

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Thus, the required probability = $\frac{8}{24} = \frac{1}{3}$

Hence, the correct option is (b).

Example 10. A committee of 6 members is to be formed from a group comprising 6 gentlemen and 4 ladies. What is the probability that the committee would comprise:

(I) 3 ladies (II) at least 3 ladies

(a) $\frac{8}{21}, \frac{19}{42}$ (b) $\frac{24}{110}, \frac{19}{42}$ (c) $\frac{1}{21}, \frac{5}{21}$ (d) None of these

Sol. (a) Given that,

Number of gentlemen = 6

Number of ladies = 4

Total members = 6 + 4 = 10

(I) The committee has 3 ladies:

Total ways of selection of 6 members

$= {}^{10}C_6 = \frac{10!}{6! \times 4!} = \frac{10 \times 9 \times 8 \times 7}{4!} = 10 \times 3 \times 7 = 210$

Since, the committee should have 3 ladies thus the number of gentlemen would be 3.

Thus, the possible ways $= {}^{6}C_3 \times {}^{4}C_3 = \frac{6!}{3! \times 3!} \times \frac{4!}{3! \times 1!} = 5 \times 4 \times 4 = 80$

The required probability $= \frac{80}{210} = \frac{8}{21}$

(II) At least 3 ladies:

Total ways of selection of 3 or more ladies = "3 ladies and 3 gentlemen" or "4 ladies and 2 gentlemen"

$= ({}^{6}C_3 \times {}^{4}C_3) + ({}^{6}C_2 \times {}^{4}C_4) = (20 \times 4) + (15 \times 1) = 80 + 15 = 95$

The required probability $= \frac{95}{210} = \frac{19}{42}$

Hence the correct answer is option (a).

PRACTICE QUESTIONS (PART A)

1. A coin is tossed twice, what is the probability that at least one tail occurs?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) 1

2. Two broad divisions of probability are **(ICAI)**

(a) Subjective probability and objective probability

(b) Deductive probability and non-deductive probability

(c) Statistical probability and Mathematical probability

(d) None of these

3. A die is thrown, then the probability of getting a number greater than or equal to 3 is

(a) $\frac{4}{5}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

4. A bag contain 6 white and 5 black balls. One ball is drawn. The probability that it is white is

(a) $\frac{5}{11}$ (b) 1 (c) $\frac{6}{11}$ (d) $\frac{1}{11}$

5. Two dice are thrown simultaneously, then the probability that sum on the faces is exactly 5 is

(a) $\frac{1}{9}$ (b) $\frac{1}{36}$ (c) $\frac{1}{4}$ (d) None of these

6. A bag contains 5 white, 6 red and 7 green balls. 3 balls are drawn at random. Find the probability that balls are 2 white and 1 green.

(a) $\frac{35}{136}$ (b) $\frac{1}{6}$ (c) $\frac{35}{408}$ (d) None of these

7. Three unbiased coins are tossed simultaneously, then the probability of getting at most 2 tails is

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{3}{4}$

8. A committee of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise 2 ladies?

(a) $\frac{37}{210}$ (b) $\frac{24}{110}$ (c) $\frac{140}{429}$ (d) None of these

9. Three events A, B and C are mutually exclusive, exhaustive and equally likely. What is the probability of the complementary event of (A' – B) ?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) None of these

10 Two dice are thrown simultaneously. What is the probability that the sum of the numbers rolled is a prime number?

(a) $\frac{5}{12}$ (b) $\frac{1}{4}$ (c) $\frac{7}{12}$ (d) 1

11. 20 Books are placed at random in a shelf. Find the probability that a particular pair of books is always together.

(a) 5 (b) $\frac{1}{10}$ (c) $\frac{1}{10!}$ (d) None of these

12. Two cards are drawn at random from a pack of 52 playing cards. The probability of getting 2 diamond cards is

(a) $\frac{1}{17}$ (b) $\frac{2}{17}$ (c) $\frac{2}{13}$ (d) None of these

Answer Key

1. (c) **2.** (a) **3.** (d) **4.** (c) **5.** (a) **6.** (c) **7.** (c) **8.** (c) **9.** (b) **10.** (a)
11. (b) **12.** (b)

THIS CLASSICAL DEFINITION OF PROBABILITY HAS THE FOLLOWING DEMERITS OR LIMITATIONS

1. It is applicable only when the total number of events is finite

2. It can be used only when the events are equally likely or equi-probable. This assumption is made well before the experiment is performed.

3. This definition has only a limited field of application like coin tossing, dice throwing, drawing cards etc. where the possible events are known well in advance. In the field of uncertainty or where no prior knowledge is provided, this definition is inapplicable.

RELATIVE FREQUENCY DEFINITION OF PROBABILITY

KEY POINTERS TO STUDY

- Concept of Relative frequency was first developed by the British mathematicians in connection with the survival probability of a group of people.

- If a random experiment is performed by repeating "n times" (under an identical set of condition) then the probability of an event A can be defined as limiting value of the ratio of event occurrence (f_A) to number of times experiment is being repeated (n)

  i.e. $P(A) = \lim_{n\to\infty} \frac{f_A}{n}$

  This statistical definition is applicable if the above limit exists and tends to a finite value.

Example 11. The following data relate to the distribution of salary of a group of employee:

Salary (Thousand ₹)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Workers	12	23	24	32	19	11	8

If an employee is selected at random from the entire group of employees, what is the probability that

(I) His Salary would be less than ₹30,000?

(a) 0 (b) 0.5 (c) 3 (d) 0.7

(II) His Salary would be less than ₹60,000?

(a) 0.3 (b) 0.457 (c) 0.8 (d) 0.44

(III) His Salary would be more than ₹100,000?

(a) 0 (b) 0.4 (c) 0.2 (d) 0.55

(IV) His salary would be between ₹40,000 and ₹80,000?

(a) 0.66 (b) 0.759 (c) 0.99 (d) 0.1

Sol. (I) (a) According to the given data there are no workers with a salary less than ₹30,000, so the probability is 0.

Hence, the correct answer is option (a).

(II) (b) Total number of employees = (12 + 23 + 24 + 32 + 19 + 11 + 8) = 129

There are (12 + 23 + 24) workers whose salary is less than ₹60,000.

Thus, Probability $= \frac{12+23+24}{129} = \frac{59}{129} = 0.457$

Hence, the correct answer is option (b).

(III) (a) According to the given data,

There are no workers with salaries more than ₹1,00,000, so the probability is 0.

Hence, the correct answer is option (a).

(IV) (b) There are (23 + 24 + 32 + 19) workers whose salary is between ₹40,000 and ₹80,000.

Thus, Probability $= \frac{23+24+32+19}{129} = \frac{98}{129} = 0.759$

Hence, the correct answer is option (b).

AXIOMATIC OR MODERN DEFINITION OF PROBABILITY

The axiomatic or modern definition of probability states that for a sample space S and an event A defined on S, the probability of A, denoted as P(A), is determined by the following axioms:

1. The probability of any event A is always greater than or equal to zero, i.e., $P(A) \geq 0$ for every $A \subseteq S$ (subset).
2. The probability of the entire sample space S is equal to 1, i.e., P(S) = 1.
3. For any sequence of mutually exclusive events A_1, A_2, A_3, ..., the probability of their union is equal to the sum of their individual probabilities, i.e.,

   $P(A_1 \cup A_2 \cup A_3 \cup ...) = P(A_1) + P(A_2) + P(A_3) + ...$

ADDITION THEOREMS OR THEOREMS ON TOTAL PROBABILITY

- **THEOREM 1:** For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B. i.e. $P(A \cup B)$ or P(A + B) = P(A) + P(B) ..... or P(A or B) whenever A and B are mutually exclusive.

  E.g.: Let's consider the events A and B representing the outcomes of rolling a fair six-sided die. If A is the event of getting an even number (2, 4 or 6), and B is the event of getting an odd number (1, 3 or 5), then A and B are mutually exclusive. The probability of either getting an even number or an odd number is given by:

  $$P(A \cup B) = P(A) + P(B) = \frac{3}{6} + \frac{3}{6} = 1$$

  This means that the probability of getting either an even number or an odd number when rolling the die is 1, which is the total probability of the entire sample space.

Example 12. If A and B are two mutually exclusive events such that $P(A \cup B) = \frac{2}{3}$, $P(A) = \frac{2}{5}$, then P(B) =

(a) $\frac{4}{15}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{15}$

Sol. (a) Given: $P(A \cup B) = \frac{2}{3}$ and $P(A) = \frac{2}{5}$

Since, A and B are two mutually exclusive events thus

$P(A \cup B) = P(A) + P(B)$

$\Rightarrow \frac{2}{3} = \frac{2}{5} + P(B)$

$\Rightarrow P(B) = \frac{2}{3} - \frac{2}{5}$

$\Rightarrow P(B) = \frac{10-6}{15} = \frac{4}{15}$

Hence, the correct option is (a) i.e. $\frac{4}{15}$.

Example 13. A number is selected from the first 20 natural numbers. What is the probability that it would be divisible by 3 or 8?

(a) 0 (b) 0.4 (c) 0.33 (d) 0.75

Sol. (b) Total observations: 1, 2, 3, …… 20

Now, numbers divisible by 3 = 3, 6, 9, 12, 15, 18

Numbers divisible by 8 = 8, 16

Since, the events that the numbers would be divisible by 3 or 8 is a mutually exclusive events, thus probability that the numbers would be divisible by 3 or 8 is given by:

P(3 or 8) = P(3) + P(8)

$= \frac{6}{20} + \frac{2}{20} = \frac{8}{20}$

= 0.4

Hence, the correct answer is option (b).

- **THEOREM 2**: For any k (≥ 2) mutually exclusive events A_1, A_2, A_3 ..., A_k then probability that at least one of them occurs is given by the sum of the individual probabilities of the K events.
  i.e. $P(A_1 \cup A_2 \cup A_3 \cup .... \cup A_k) = P(A_1) + P(A_2) + ... + P(A_k)$
- **THEOREM 3:** For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.
  i.e., $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example 14. A number is selected at random from the first 1000 natural numbers. What is the probability that it would be a multiple of 3 or 7?

(a) 0 (b) 0.426 (c) 0.33 (d) 0.75

Sol. (b) Given, A number is selected at random from the first 1000 natural numbers.

According to the question, n(S) = 1000

Number of multiples of 7 in first 1000 natural numbers,

$$P(A) = \frac{1000}{7} = 142\,(\text{approx})$$

Number of multiples of 3 in first 1000 natural numbers,

$$P(B) = \frac{1000}{3} = 333\,(\text{approx})$$

And, numbers divisible by 7 and 3 are

$$P(A \cap B) = \frac{1000}{(3 \times 7)} = 47\,(\text{approx})$$

Therefore, the probability that the number so selected would be a multiple of 7 or 11

$= P(A) + P(B) - P(A \cap B)$

$= \frac{142}{1000} + \frac{333}{1000} - \frac{47}{1000} = \frac{142 + 333 - 47}{1000} = 0.426$

Hence, the correct answer is option (b).

- **THEOREM 4:** For any three events A, B and C, the probability that at least one of the events occurs is given by: $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

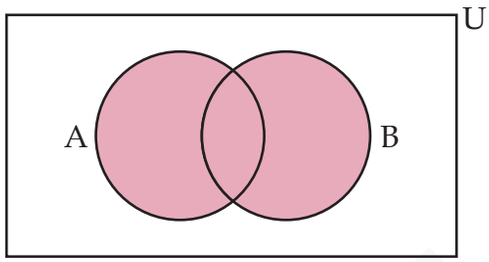
VENN DIAGRAM REPRESENTATION OF PROBABILITY RULES

- P (Atleast one set)

  P (A or B)

  $= P(A \cup B)$

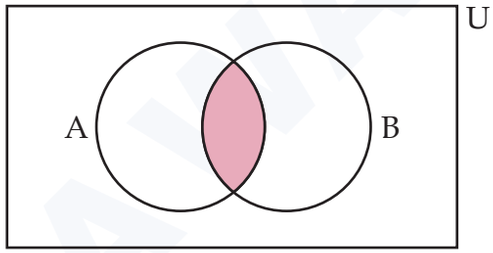
  $= P(A) + P(B) - P(A \cap B)$



- P (A & B both)

  $= P(A \cap B)$

  $= P(A) + P(B) - P(A \cup B)$



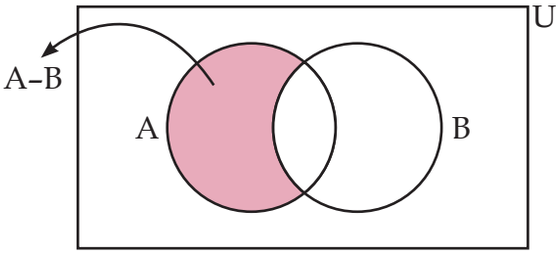
- P (Only A)

  = P(A but not B)

  = P(A – B)

  $= P(A \cap B')$

  $= P(A) - P(A \cap B)$



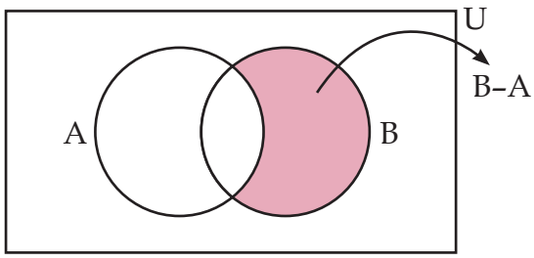
- P(Only B)

  = P(B but not A)

  = P(B – A)

$= P(B \cap A')$

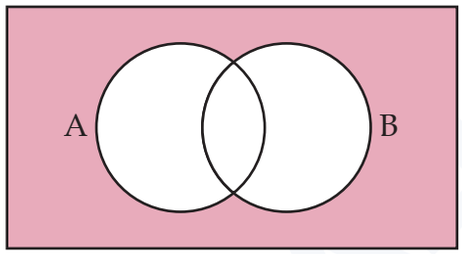
$= P(B) - P(A \cap B)$



- P(Neither A nor B)

$= P\left(\overline{A} \cap \overline{B}\right)$

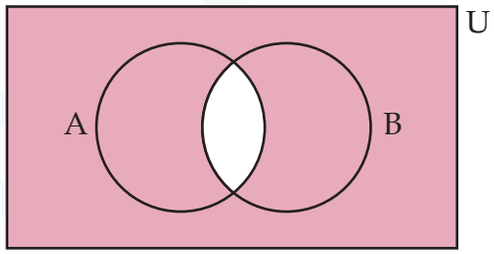
$= 1 - P(A \cup B)$



- P(Not A or Not B)

$= P\left(\overline{A} \cup \overline{B}\right)$

$= 1 - P(A \cap B)$



Example 15. There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

(a) 1 (b) 0.9 (c) 0.5 (d) None of these

Sol. (b) According to the given information, we have

$P(A) = 0.80, P(B) = 0.60, P(C) = 0.50,$

$P(A \cap B) = 0.46, P(B \cap C) = 0.32, P(A \cap C) = 0.48$ and

$P(A \cap B \cap C) = 0.26$

Now, the probability that atleast one of them survives another 5 years is given by $P(A \cup B \cup C)$

On substituting the values, we get

$P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$= 0.80 + 0.60 + 0.50 - 0.46 - 0.32 - 0.48 + 0.26$

$= 0.90$

Therefore, the required probability is 0.9.

Hence, the correct option is (b).

Example 16. Which of the following pairs of events are mutually exclusive?

(a) A: The team wins the football match.
B: The team lost the football match.

(b) A: The card drawn is a heart.
B: The card drawn is a red card.

(c) A: Anita is 20 years old.
B: She is a great dancer.

(d) A: The dice shows an even number.
B: The dice shows a prime number.

Sol. (a) We know that,

Mutually exclusive events are those events that do not occur at the same time.

We know that,

Mutually exclusive events are those events that do not occur at the same time.

Clearly, option (a) i.e., the team wins the football match and the team loses the football match.

Hence, the correct answer is option (a).

Example 17. If two events A and B, $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{2}{3}$ then find $P(A \cap B)$

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

Sol. (b) Given: $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{2}{3}$

We know that,

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{2}{3}$

$\Rightarrow P(A \cap B) = \frac{3+2-4}{6}$

$\Rightarrow P(A \cap B) = \frac{1}{6}$

Hence, the correct option is (b) i.e. $\frac{1}{6}$.

Example 18. If A and B are mutually exclusive events, then **(ICAI)**

(a) P(A) = P(A – B) (b) P(B) = P(A – B)

(c) P(A) = P(A ∩ B) (d) P(B) = P(A ∩ B)

Sol. (a) Given, A and B are mutually exclusive events.

Thus, A ∩ B = 0

We know that,

P(A – B) = P(A) – P(A ∩ B)

$\Rightarrow$ P(A – B) = P(A) [$\because$ P(A ∩ B) = 0]

Hence, the correct option is (a) i.e. P(A) = P(A – B).

Example 19. A coin is tossed thrice. What is the probability of getting 2 or more tails?

(a) 0 (b) 0.4 (c) 0.5 (d) 0.75

Sol. (c) Total cases: {TTT, HHH, HTT, THT, TTH, HHT, HTH, THH} = 8

Cases of getting 2 or more tails i.e., 2 tails or 3 tails

For 2 tails, favourable outcomes = {HTT, THT, TTH}

for 3 tails, favourable outcomes = {TTT}

Therefore, the required Probability = P(2 tails) + P(3 tails)

$\frac{3}{8} + \frac{1}{8} = \frac{4}{8} = 0.5$

Hence, the correct option is (c).

Example 20. A certain problem has odds of 5 to 2 against A solving it, and odds of 3 to 1 in favor of B solving it. What is the probability of the problem being solved if both A and B attempt it?

(a) $\frac{13}{14}$ (b) $\frac{15}{28}$ (c) $\frac{9}{14}$ (d) None of these

Sol. (a) Given, Odds against A solving a certain problem are 5 to 2 and Odds in favour of B solving the problem are 3 to 1.

Then, $P(A) = \frac{5}{5+2} = \frac{5}{7}$ (Probability to solve the problem)

$P(A)' = 1 - P(A) = 1 - \frac{5}{7} = \frac{2}{7}$

$P(B) = \frac{3}{3+1} = \frac{3}{4}$ (Probability to solve the problem)

$P(B)' = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$

Required Probability will be given as,

$\Rightarrow 1 - [P(A)' \times P(B)']$

$$= 1 - \left[\frac{2}{7} \times \frac{1}{4}\right] = 1 - \left(\frac{2}{28}\right) = \frac{28-2}{28} = \frac{26}{28} = \frac{13}{14}$$

Hence, the correct answer is option (a) i.e., $\frac{13}{14}$.

PRACTICE QUESTIONS (PART B)

1. A class consists of 10 boys and 20 girls of which half of boys and half the girls have blue eyes. Find the probability that a student chosen at random is a boy and has blue eyes.

(a) $\frac{1}{6}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) None of these

2. An event that can be split into further events is known as **(ICAI)**

(a) Complex event (b) Mixed event

(c) Simple event (d) Composite event

3. Which of the following pairs of events are mutually exclusive? **(ICAI)**

(a) A: The student reads in a school. B: He studies Philosophy.

(b) A: Raju was born in India. B: He is a fine Engineer.

(c) A: Ruma is 16 years old. B: She is a good singer.

(d) A: Peter is under 15 years of age. B: Peter is a voter of Kolkata.

4. If an unbiased coin is tossed once, then the two events Head and Tail are

(a) Mutually exclusive (b) Exhaustive

(c) Equally likely (d) All these (a), (b) and (c)

5. If $P(A) = \frac{1}{3}$, then the odds against the event A is

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 2 : 3

6. If a number is selected at random from the first 50 natural numbers, what will be the probability that the selected number is a multiple of 3 and 4? **(Dec 2022)**

(a) 5/50 (b) 2/25 (c) 3/50 (d) 4/25

7. If A and B are events such that $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, then $P(A \cup B) =$

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$ (c) $\frac{1}{2}$ (d) None of these

8. The probability that an applicant for an Accountant's job has a B.Com. degree is 0.75. The probability that they have knowledge of Tally is 0.40. The probability that they have a B.Com. degree or knowledge of Tally is 0.30. Out of 1000 applicants, how many would be B.Com. experts in Tally?

(a) 0 (b) 850 (c) 1150 (d) 750

9. A, B, C are three mutually independent with probabilities 0.3, 0.2 and 0.4 respectively. What is $P(A \cap B \cap C)$? **(ICAI)**

(a) 0.400 (b) 0.240 (c) 0.024 (d) 0.500

10. There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years is 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

(a) 1.00 (b) 0.28 (c) 0.45 (d) 0.90

11. For any two events A and B,

(a) $P(A) + P(B) > P(A \cap B)$ (b) $P(A) + P(B) < P(A \cap B)$

(c) $P(A) + P(B) \geq P(A \cap B)$ (d) $P(A) \times P(B) \leq P(A \cap B)$

12. A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6? **(ICAI)**

(a) 0.30 (b) 0.25 (c) 0.20 (d) 13

13. The probability of two events A and B are 0.25 and 0.50 respectively. The probability of their simultaneous occurrence is 0.14. Find the probability that neither A nor B occurs.

(a) 0.39 (b) 0.61 (c) 0.75 (d) None

14. Let A and B be two possible outcomes of a random experiment and $P(A) = \frac{1}{3}$, $P(A \cup B) = \frac{1}{2}$ and $P(B) = x$. For what value of x are A and B mutually exclusive events?

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{8}$

15. If $P(A) = 0.65$ and $P(B) = 0.15$, then $P(\overline{A}) + P(\overline{B})$ is:

(a) 1.2 (b) 1.5 (c) 0.8 (d) 0.35

Answer Key

1. (a) **2.** (d) **3.** (b) **4.** (d) **5.** (a) **6.** (b) **7.** (b) **8.** (b) **9.** (c) **10.** (d)
11. (c) **12.** (d) **13.** (a) **14.** (b) **15.** (a)

CONDITIONAL PROBABILITY AND COMPOUND THEOREM OF PROBABILITY

COMPOUND PROBABILITY OR JOINT PROBABILITY

The probability of an event, discussed so far, is technically known as unconditional or marginal probability. But if there are two or more events occurring simultaneously, how to calculate the probability.

The probability of occurrence of two events A and B simultaneously is known as the Compound Probability or Joint Probability of the events A and B and is denoted by $P(A \cup B)$.

In a similar manner, the probability of simultaneous occurrence of k events A_1, A_2, A_3 ..., A_k, is denoted by $P(A_1 \cup A_2 \cup A_3 \cup ... \cup A_k)$

CONDITIONAL PROBABILITY

Let A and B be two events and S be the sample space, then the probability of event B given that event A has already occurred is called the conditional probability of B given A.

It is denoted be $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

E.g.: Let two unbiased coins be tossed then Sample space, S = {HH, HT, TH, TT}

Now,

Let A be the event of getting atleast one head = {HH, HT, TH}

B be the event of getting both head = {HH}

then, $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{4}$

If it is known that A has already happened, then it is sure that TT cannot occur.

Thus, $P\left(\frac{B}{A}\right) = \frac{1}{3}$

Also, $\frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

Similarly, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

Example 21. If $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{6}$, then $P\left(\frac{A}{B}\right)$ is

(a) $\frac{1}{6}$ (b) $\frac{2}{9}$ (d) $\frac{1}{2}$ (d) $\frac{1}{8}$

Sol. (b) Given, $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{6}$

Thus, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{4}{18} = \frac{2}{9}$

Hence, the correct option is (b).

Example 22. If $P(A)=\frac{1}{3}$, $P(B)=\frac{3}{4}$ and $P(A\cup B)=\frac{11}{12}$, then $P\left(\frac{B}{A}\right)$ is **(Dec 2022)**

(a) $\frac{1}{6}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

Sol. (c) Given, $P(A)=\frac{1}{3}, P(B)=\frac{3}{4}$ and $P(A\cup B)=\frac{11}{12}$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{11}{12}=\frac{1}{3}+\frac{3}{4}-P(A\cap B)$

$\Rightarrow P(A\cap B)=\frac{1}{3}+\frac{3}{4}-\frac{11}{12}$

$\Rightarrow P(A\cap B)=\frac{4+9-11}{12}$

$\Rightarrow P(A\cap B)=\frac{2}{12}=\frac{1}{6}$

Thus, $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{6}}{\frac{1}{3}}=\frac{1}{2}$

Hence, the correct option is (c).

Example 23. If $P(\bar{A}\cup\bar{B})=\frac{5}{6}, P(A)=\frac{1}{2}$ and $P(B)=\frac{2}{3}$, what is $P(A \cup B)$? **(ICAI)**

(a) 1 (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

Sol. (a) Given, $P(\bar{A}\cup\bar{B})=\frac{5}{6}, P(A)=\frac{1}{2}$ and $P(B)=\frac{2}{3}$

We know that,

$P(\bar{A}\cup\bar{B})=P(\overline{A\cap B})=1-P(A\cap B)$

$\Rightarrow \frac{5}{6}=1-P(A\cap B)$

$\Rightarrow P(A\cap B)=1-\frac{5}{6}=\frac{1}{6}$

We also know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$=\frac{1}{2}+\frac{2}{3}-\frac{1}{6}=\frac{7}{6}-\frac{1}{6}=1$

Hence, the correct option is (a) i.e. 1.

INDEPENDENT EVENTS

Independent events are events in which the occurrence or outcome of one event does not affect the probability of the other event. The probability of the second event happening is the same whether the first event has occurred or not.

E.g.: Coin Flips Suppose you flip a fair coin twice. The outcome of the first coin flip (e.g., getting heads) has no influence on the outcome of the second coin flip. The probability of getting heads on the second flip remains $\frac{1}{2}$, regardless of whether you got heads or tails on the first flip. These events are independent.

Thus, if A and B are two independent events, then

$$P\left(\frac{A}{B}\right) = P(A) \text{ and } P\left(\frac{B}{A}\right) = P(B)$$

If A and B are independent events, then

$P(A \cap B) = P(A) \times P(B)$

Similarly,

- $P(A \cap C) = P(A) \times P(C)$
- $P(B \cap C) = P(B) \times P(C)$
- $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

  If events A and B are independent, the pairs A and B′, A′ and B, and A′ and B′ are also independent, where A′ represents the complement of event A and B′ represents the complement of event B.

Example 24. For any two events A_1, A_2 : let $P(A_1) = \frac{2}{3}, P(A_2) = \frac{3}{8}, P(A_1 \cap A_2) = \frac{1}{4}$, then A_1, A_2 are

(a) Mutually Exclusive but not independent events

(b) Mutually Exclusive and independent events

(c) Independent but not mutually exclusive

(d) None of these

Sol. (c) Given, $P(A_1) = \frac{2}{3}, P(A_2) = \frac{3}{8}$ and $P(A_1 \cap A_2) = \frac{1}{4}$

As we know, for mutually exclusive events

$P(A_1 \cap A_2) = 0$

On the contrary, given in the question,

$$P(A_1 \cap A_2) = \frac{1}{4} \neq 0$$

Thus, they are not mutually exclusive events.

Now, for independent events: $P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)$

Put the values and check for independent events,

$$\Rightarrow \frac{1}{4} = \frac{2}{3} \times \frac{3}{8}$$

$= \frac{6}{24} \Rightarrow \frac{1}{4} = \frac{1}{4}$ (verified)

Thus, they are independent events however they are not mutually exclusive events.

Hence, the correct answer is option (c) i.e., Independent but not mutually exclusive.

Example 25. If events A and B are given to be independent such that P(A) = 0.2, P(A ∪ B) = 0.6, then P(B) is

(a) 0.4 (b) 0.5 (c) 0.7 (d) None of these

Sol. (b) Given, A and B are independent events such that P(A) = 0.2, P(A ∪ B) = 0.6

We know,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$\Rightarrow 0.6 = 0.2 + P(B) - 0.2 \times P(B)$

$\Rightarrow 0.4 = 0.8 \times P(B)$

$\Rightarrow P(B) = \frac{0.4}{0.8} = \frac{1}{2} = 0.5$

Hence, the correct option is (b).

Example 26. A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that the problem will be solved?

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{5}$ (d) None of these

Sol. (b) Let A, B and C be the events such that $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$

Clearly, A, B and c are independent events.

Thus, the probability that the problem will be solved = 1 – P(problem is not solved)

= 1 – P(A')P(B')P(C')

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$

Hence, the correct option is (b).

Example 27. Ronaldo is known as one of the football players to hit 4 goals out of 10 shots whereas Messi is known to hit 5 goals out of 11 shots. What is the probability that the target would be hit once they both have hit for the penalty shootout?

(a) 0 (b) 0.67 (c) 0.33 (d) 0.75

Sol. (b) Probability of Ronaldo hitting the goal, P(A) = 0.4

Probability of Messi hitting the goal, $P(B) = \frac{5}{11} = 0.45$

Thus, $P(A') = 1 - P(A)$ and $P(B') = 1 - P(B)$

Probability that the target would be hit once they both have hit for the penalty shootout

$= P(A)\,P(B') + P(A') \cdot P(B) + P(A) \cdot P(B)$

$= 0.4(1 - 0.45) + (1 - 0.4)0.45 + (0.4)(0.45)$

$= 0.22 + 0.27 + 0.18 = 0.67$

Hence, the correct answer is option (b).

DEPENDENT EVENTS

Dependent events are events in which the outcome of one event does affect the probability of the other event. The probability of the second event happening is influenced by the outcome of the first event.

E.g.: Drawing Cards

Suppose you have a deck of 52 playing cards. If you draw a card and do not replace it before drawing the next card, these events are dependent. For example, if you draw an Ace of Spades as the first card, there are now only 51 cards left in the deck, and the probability of drawing another Ace of Spades as the second card is $\frac{1}{51}$, not $\frac{1}{52}$ as it was initially. The probabilities change because the events are dependent on each other.

THEOREMS OF COMPOUND PROBABILITY

- **Theorem 5:** For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred i.e.,

  $$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right), \text{ provided } P(A) > 0$$

- **Theorem 6:** For any three events A, B and C, the probability that they occur jointly is given by

  $$P(A \cap B \cap C) = P(A) \times P\left(\frac{B}{A}\right) \times P\left(\frac{C}{A \cap B}\right), \text{ provided } P(A \cap B) > 0$$

Example 28. If for two events A and B, $P(A \cap B) \neq P(A) \times P(B)$, then the two events A and B are

(a) Independent (b) Dependent

(c) Not equally likely (d) Not exhaustive

Sol. (b) We know that,

For two events A and B to be dependent, happening of one affect the happening of other i.e., $P(A \cap B) \neq P(A) \times P(B)$

Hence, the correct option is (b).

Example 29. In a group of 15 males and 10 females, 5 males and 7 females are service holders. What is the probability that a person selected at random from the group is not a service holder given that the selected person is a female?

(a) 0 (b) 0.5 (c) 0.3 (d) 0.75

Sol. (c) Given, Total number of males = 15

Total number of females = 10

Number of female service holders = 7

Number of male service holders = 5

Let A be the event that a person selected is female and B be the event that a person is not a service holder.

To find: $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}$

Probability that the person is not a service holder and a female:

$$P(B\cap A)=\frac{3}{15+10}=\frac{3}{25}=0.12$$

Probability that the person is female:

$$P(A)=\frac{10}{25}=0.4$$

Therefore, the required probability $=\frac{0.12}{0.4}=0.3.$

Hence, the correct answer is option (c).

Example 30. A card is drawn from a pack of 52 cards, the card drawn is a red card. What is the probability of its being a card of diamond?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{5}$

Sol. (b) Let A be the event of drawing a red card and B be the event of drawing a diamond.

Thus, $P(A)=\frac{26}{52}$

Now, probability that the card is red and card of diamond is: $P(A\cap B)=\frac{13}{52}$

Now, probability of its being a card of diamond given that it is red card is:

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$$

$$\Rightarrow P\left(\frac{B}{A}\right)=\frac{\frac{13}{52}}{\frac{26}{52}}=\frac{13}{52}\times\frac{52}{26}=\frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

Hence, the correct answer is option (b).

Example 31. A pair of dice is thrown together and the sum of points of the two dice is noted to be 9. What is the probability that one of the two dice has shown the point 4?

(a) 0 (b) 0.5 (c) 0.33 (d) 0.75

Sol. (b) Since, sum of points of the two dice is 9

Here, Possible ways to get 9 as a sum = (4, 5); (5, 4); (3, 6); (6, 3) i.e., total outcomes = 4

Number of favourable outcomes (one of the two dice has shown the point 4) = 2

Therefore, probability $= \frac{2}{4} = 0.5$

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART C)

1. If A and B are two events such that P(A) = 0.6, P(B) = 0.3 and P(A ∩ B) = 0.2, then $P\left(\frac{A}{B}\right) =$

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) None of these

2. If P(B) = 0.5 and P(A ∩ B) = 0.32, then $P\left(\frac{A}{B}\right) =$

(a) $\frac{25}{16}$ (b) $\frac{16}{25}$ (c) $\frac{7}{16}$ (d) None of these

3. Events A and B are given to be independent such that P(A) = 0.40 and P(A ∪ B) = 0.70, then P(B) =

(a) 0.2 (b) 0.4 (c) 0.5 (d) 1

4. In connection with a random experiment, it is found that

$P(A) = \frac{1}{3}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{4}$

Evaluate the following probabilities: **(ICAI)**

(I) P(A/B) (II) P(B/A) (III) P(A′/ B)

(IV) P(A/ B′) (V) P(A′/ B′)

5. The probability of the occurrence of a number greater than 2 in a throw of a die if it is known that only even numbers can occur is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) None of these

6. Given that for two events A and B, $P(A) = \frac{3}{5}, P(B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{5}$, what is $P\left(\frac{A}{B}\right)$?

(a) 0.655 (b) $\frac{13}{60}$ (c) $\frac{60}{31}$ (d) 0.775

Answer Key

1. (a) **2.** (b) **3.** (c) **4.** (I) 13/18 (II) 13/20 (III) 5/18 (IV) 7/12 (V) 5/12
5. (c) **6.** (d)

Example 32. If $P(A \cap B) = 0$, then the two events A and B are

(a) Mutually exclusive (b) Exhaustive

(c) Equally likely (d) Independent

Sol. (a) If $P(A \cap B) = 0$, it means the probability of the intersection of events A and B is 0, which implies that events A and B have no common outcomes.

In such a case, the events A and B are considered mutually exclusive.

Hence, the correct option is (a).

Example 33. If A, B and C are mutually exclusive, independent and exhaustive events then what is the probability that they occur simultaneously?

(a) 1 (b) 0.50

(c) 0 (d) any value between 0 and 1

Sol. (c) If events A, B, and C are mutually exclusive, it means that they cannot occur simultaneously. This implies that the probability of all three events occurring simultaneously is 0.

Hence, the correct option is (c).

Example 34. If for two independent events A and B, $P(A \cup B) = \frac{2}{3}$ and $P(A) = \frac{2}{5}$, what is P(B)?

(a) $\frac{4}{15}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{15}$

Sol. (a) We know that,

$P(B) = P(A \cup B) + P(A) + P(A \cap B)$

Here, $P(A \cap B)$ is 0 as they are independent events.

Thus, $P(B) = \frac{2}{3} - \frac{2}{5} + 0$

Hence, the correct answer is option (a).

Example 35. For three events A, B and C, the probability that only A occur is

(a) P(A) (b) $P(A \cup B \cup C)$

(c) $P(A' \cap B \cap C)$ (d) $P(A \cap B' \cap C')$

Sol. (d) We know that,

For sets A, B and C the probability of occurrence of event A only is given by:

$P(A \cap B' \cap C')$

Hence, the correct option is (d).

Example 36. The probability that is leap year has 53 Monday is: **(Dec 2022)**

(a) $\frac{1}{7}$ (b) $\frac{2}{3}$ (c) $\frac{2}{7}$ (d) $\frac{3}{5}$

Sol. (c) We know that,

In a year there are 52 weeks, thus in a leap year there are 52 weeks and 2 odd days.

The two odd days can be:

(Saturday, Sunday),
(Sunday, Monday),
(Monday, Tuesday),
(Tuesday, Wednesday),
(Wednesday, Thursday),
(Thursday, Friday),
(Friday, Saturday)

So, there are 7 possibilities out of which 2 have Monday.

Therefore, Probability of 2 Monday $= \frac{2}{7}$

Hence, the correct answer is option (c).

Example 37. There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls

Box II: 4 Red + 8 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

If one ball is drawn at random, then what is the probability that they would be of the same colour?

(a) $\frac{89}{729}$ (b) $\frac{97}{729}$ (c) $\frac{82}{729}$ (d) $\frac{23}{32}$

Sol. (a) Given, There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls

Box II: 4 Red + 4 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

Here,

The probability of selection of red ball from box 1 is $\frac{5}{5+7+6} = \frac{5}{18}$

The probability of selection of red ball from box 2 is $\frac{4}{4+8+6} = \frac{4}{18}$

The probability of selection of red ball from box 3 is $\frac{3}{3+4+2} = \frac{3}{9}$

Now,

The probability of selection of white ball from box $\frac{7}{18}$

The probability of selection of white ball from box 2 is $\frac{8}{18}$

The probability of selection of white ball from box 3 is $\frac{4}{9}$

Now,

The probability of selection of blue ball from box 1 is $\frac{6}{18}$

The probability of selection of blue ball from box 2 is $\frac{6}{18}$

The probability of selection of blue ball from box 3 is $\frac{2}{9}$

Now,

By using multiplication theorem of probability we get, the required probability as

$\Rightarrow \frac{5}{18}\times\frac{4}{18}\times\frac{3}{9}+\frac{7}{18}\times\frac{8}{18}\times\frac{4}{9}+\frac{6}{18}\times\frac{6}{18}\times\frac{2}{9}$

$\Rightarrow \frac{89}{729}$

Hence, the final option is (a).

Example 38. A problem in probability was given to three CA students A, B and C whose chances of solving it are $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{2}$ respectively. What is the probability that the problem would be solved?

(a) $\frac{4}{15}$ (b) $\frac{7}{8}$ (c) $\frac{8}{15}$ (d) $\frac{11}{15}$

Sol. (d) Given: Probability was given to three CA students A, B and C whose chances of solving it are $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{2}$ respectively.

i.e., $P(A)=\frac{1}{3}, P(B)=\frac{1}{5}$ and $P(C)=\frac{1}{2}$

Thus,

$P(\bar{A})=1-\frac{1}{3}=\frac{2}{3}$

$P(\bar{B})=1-\frac{1}{5}=\frac{4}{5}$

$P(\bar{C})=1-\frac{1}{2}=\frac{1}{2}$

Thus, the probability that the problem would not be solved if none of them solved the problem is given by $P(\bar{A}\cap\bar{B}\cap\bar{C})$

Since, they are independent events thus $P(\bar{A}\cap\bar{B}\cap\bar{C})=P(\bar{A})\times P(\bar{B})\times P(\bar{C})$

$P(\bar{A}\cap\bar{B}\cap\bar{C})=\frac{2}{3}\times\frac{4}{5}\times\frac{1}{2}=\frac{4}{15}$

Therefore, the probability that the problem would be solved

=1 – Probability (problem not solved)

$=1-\frac{4}{15}=\frac{11}{15}$

Hence, the correct answer is option (d).

Example 39. For a group of subjects 30%, 40% and 50% failed in Physics, Chemistry and at least one of the two subjects respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry? **(ICAI)**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

Sol. (a) Let A be the event that a student fails in Physics.

Let B be the event that a student fails in Chemistry.

According to the question,

Then, P(A) = 0.30, P(B) = 0.40

Also, the probability of failing in at least one of the subjects is 0.50

$\Rightarrow P(A \cup B) = 0.50$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) - P(A \cap B) = 0.50$

$\Rightarrow 0.30 + 0.40 - P(A \cap B) = 0.50$

$\Rightarrow P(A \cap B) = 0.70 - 0.50$

$\Rightarrow P(A \cap B) = 0.20$

Now, the probability that if student pass in Physics then he will fail in chemistry is denoted by $P\left(\frac{\bar{A}}{B}\right)$

We know that,

$$P\left(\frac{\bar{A}}{B}\right) = \frac{P(\bar{A} \cap B)}{P(B)}$$

Here, $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= 0.40 - 0.20 = 0.20$

$$\Rightarrow P\left(\frac{\bar{A}}{B}\right) = \frac{0.20}{0.40} = \frac{1}{2}$$

Hence, the correct answer is option (a) i.e., $\frac{1}{2}$.

Example 40. An article consists of two parts A and B. The manufacturing process of each part is such that probability of defect in A is 0.08 and that B is 0.05. What is the probability that the assembled product will not have any defect?

(a) 0.934 (b) 0.864 (c) 0.85 (d) 0.874

Sol. (d) Given, Probability of defect in A, P(A) = 0.08

Probability of defect in B, P(B) = 0.05

Now, Probability that part A will have no defect,

P(A)' = 1 – 0.08 = 0.92

Now, Probability that part B will have no defect,

$P(B)' = 1 - 0.05 = 0.95$

Now, Products are non defective will be given as,

$P(A' \cap B') = P(A)' \times P(B)'$

$\Rightarrow 0.92 \times 0.95$

$\Rightarrow 0.874$

Hence, the correct answer is option (d) i.e., 0.874.

PRACTICE QUESTIONS (PART D)

1. If for two events A and B, $P(A \cap B) \neq P(A) \times P(B)$, then the two events A and B are

(a) Independent (b) Dependent

(c) Not equally likely (d) Not exhaustive

2. If two events A and B are independent, then

(a) They can be mutually exclusive

(b) They cannot be mutually exclusive

(c) They cannot be exhaustive

(d) Both (b) and (c)

3. If $P\left(\frac{A}{B}\right) = P(A)$, then

(a) A is independent of B (b) B is independent of A

(c) B is dependent of A (d) Both (a) and (b).

4. If $P(A - B) = P(B - A)$, then the two events A and B satisfy the condition

(a) $P(A) = P(B)$ (b) $P(A) + P(B) = 1$

(c) $P(A \cap B) = 0$ (d) $P(A \cup B) = 1$

5. If $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$, then $P\left(\frac{B}{A}\right)$ is **(Dec 2022)**

(a) $\frac{1}{6}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

6. If $P(\overline{A} \cup \overline{B}) = \frac{5}{6}, P(A) = \frac{1}{2}$ and $P(\overline{B}) = \frac{2}{3}$, what is $P(A \cup B)$?

(a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

7. If two unbiased dice are rolled, what is the probability of getting points neither 6 nor 9?

(a) 0.25 (b) 0.50 (c) 0.75 (d) 0.80

8. A machine is made of two parts A and B. The manufacturing process of each part is such that probability of defective in part A is 0.08 and that B is 0.05. What is the probability that the assembled part will not have any defect? **(Dec 2022)**

(a) 0.934 (b) 0.864 (c) 0.85 (d) 0.874

9. If $P(\bar{A} \cup \bar{B}) = \frac{5}{6}, P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$, what is $P(A \cup B)$?

(a) 1 (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

10. The probability of winning of a person is $\frac{6}{11}$ and at a result he gets ₹77. The expectation of this person is

(a) ₹35 (b) ₹42 (c) ₹58 (d) None of these

11. In a class 40% students read Mathematics, 25% Biology and 15% both Mathematics and Biology. One student is selected at random. The probability that he reads Mathematics if it is known that he reads Biology is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) None of these

12. Two cards are drawn from a pack of 52 cards. The probability that one is a spade and one is a heart; is

(a) $\frac{29}{34}$ (b) $\frac{3}{20}$ (c) $\frac{47}{100}$ (d) $\frac{13}{102}$

13. A father had three sons namely, Kailash3, Harish and Prakash. All are above 65 years in age. Prakash happens to be the eldest while Kailash as youngest. As per the health history, it is estimated that the probability tha Kailash survives another 5 years is $\frac{4}{5}$, Harish survives another 5 years is $\frac{3}{5}$ and Prakash survives another 5 years is $\frac{1}{2}$. The probabilities that Kailash and Harish survive another 5 years is 0.46, Harish and Prakash survive another 5 years is 0.32 and Kailash and Prakash survive another 5 years is 0.48. The probability that all three sons survive another 5 years is 0.26. What shall be the probability that at least one of them survives another 5 years?

(a) 0.72 (b) 0.78 (c) $\frac{7}{10}$ (d) $\frac{9}{10}$

Answer Key

1. (b) **2.** (b) **3.** (a) **4.** (a) **5.** (c) **6.** (c) **7.** (c) **8.** (d) **9.** (a) **10.** (b)
11. (b) **12.** (d) **13.** (d)

RANDOM VARIABLE - PROBABILITY DISTRIBUTION

A random variable or stochastic variable is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment.

A random variable is denoted by a capital letter.

E.g.: Consider the experiment of tossing a coin three times. Let X represent the number of heads obtained. In this case, the sample space is given by:

{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

The different values of X would be:

X = 0 (no heads)

X = 1 (one head)

X = 2 (two heads)

X = 3 (three heads)

TYPE OF RANDOM VARIABLE

Discrete Random Variable	Continuous Random Variable
A random variable from the discrete sample space is called a discrete random variable. For example, consider the experiment of tossing a coin three times. Let X represent the number of heads obtained. In this case, the sample space is given by: {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} The different values of X would be:	A random variable from the continuous sample space is called a discrete random variable. For example, consider the experiment of measuring the height of a randomly selected person. Let X represent the height in centimeters. The values of X would be any real number within a certain range, such as 150 cm to 200 cm.

Discrete Random Variable	Continuous Random Variable
X = 0 (no heads) X = 1 (one head) X = 2 (two heads) X = 3 (three heads)	Since the height can take on any value within the interval, X is a continuous random variable

PROBABILITY DISTRIBUTION OF A RANDOM VARIABLE

It may be defined as a statement where we take different values for random variables with their corresponding probabilities. Writing it mathematically, if a random variable X assumes n finite values $X_1, X_2, X_3, ..., X_n$ with corresponding probabilities $P_1, P_2, P_3, ..., P_n$ such that

1. $P_i \geq 0$ (for every i)
2. $\sum P_i = 1$ (over all i)

then the probability distribution of the random variable X is given by

X:	X_1	X_2	X_3	...	X_n	Total
P:	P_1	P_2	P_3	...	P_n	1

For example, if an unbiased coin is tossed three times and if X denotes the number of heads then, as we have already discussed, X is a random variable and its probability distribution is given by:

Probability Distribution of X (Number of heads when a coin is tossed thrice)

X	0	1	2	3	Total
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

PROBABILITY MASS FUNCTION (PMF) OF X

If a function f(X) exists which defines the probability (P) as a function of X, where X is discrete random variable, where f(X) satisfies the below given condition:

1. $f(X) \geq 0$ for every X
2. $\sum f(X) = 1$

where, f(X) is given by $f(X) = P(X = x)$

Example 41. Which of the following set of function define a probability space on S = {a1, a2, a3}?

(a) $P(a_1) = \frac{1}{3}, P(a_2) = \frac{1}{2}, P(a_3) = \frac{1}{4}$

(b) $P(a_1) = \frac{1}{3}, P(a_2) = \frac{1}{6}, P(a_3) = \frac{1}{6}$

(c) $P(a_1) = \frac{2}{3}, P(a_2) = \frac{2}{3}, P(a_3) = \frac{1}{4}$

(d) None of these

Sol. (b) Given, $S = \{a_1, a_2, a_3\}$

As we know that, the sum of all the probabilities is equal to 1.

For option (a):

$$\Rightarrow \frac{1}{3} + \frac{1}{2} + \frac{1}{4} = \frac{13}{12} \neq 1$$

Here, the sum of probabilities is not equal to 1.

For option (b):

$$\Rightarrow \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = \frac{6}{6} = 1$$

Therefore, the sum of probabilities is equal to 1.

For option (c):

$$\Rightarrow \frac{2}{3} + \frac{2}{3} + \frac{1}{4} = \frac{19}{12} \neq 1$$

Here, the sum of probabilities is not equal to 1.

Hence, the correct option is (b).

Example 42. Let P be a probability function on $S = \{X_1, X_2, X_3\}$ if $P(X_1) = \frac{1}{4}, P(X_3) = \frac{1}{3}$ then $P(X_2)$ is

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{3}{4}$ (d) None of these

Sol. (a) Given, $P(X_1) = \frac{1}{4}$ and $P(X_3) = \frac{1}{3}$

We know that, the sum of all probabilities of all the elements of the sample space is 1.

Thus, $P(X_1) + P(X_2) + P(X_3) = 1$

$\Rightarrow \frac{1}{4} + P(X_2) + \frac{1}{3} = 1$

$\Rightarrow P(X_2) = 1 - \frac{7}{12} = \frac{5}{12}$

$\Rightarrow P(X_2) = \frac{5}{12}$

Hence, the correct option is (a) i.e. $\frac{5}{12}$.

Example 43. The probability distribution of a random variable is as follows:

x	1	2	3	4	5
P	3k	2k	3k	k	k

Find the value of k and P ($x \leq 3$).

(a) $\frac{1}{10}, 0.2$ (b) $\frac{1}{10}, 0.5$ (c) $\frac{1}{5}, 1.5$ (d) None of these

Sol. (b) As it is given as probability distribution function,

$\Rightarrow \Sigma P = 1$

$\Rightarrow 10k = 1$

$\Rightarrow k = \frac{1}{10}$

Now, $P(x < 3) = P(x = 1) + P(x = 2)$

$= 3k + 2k = 5k$

$= 5\left(\frac{1}{10}\right) = \frac{1}{2} = 0.5$

Hence, the correct option is (a) i.e., 0.5.

Example 44. A random variable X taking values 0, 1, 2 has the following probability distribution for some number k.

$P(X) = k$ if $X = 0$

$= 2k$ if $X = 1$

$= 3k$ if $X = 2$

Find the value of k.

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) None of these

Sol. (c) As it is given as probability distribution function,

$\Rightarrow \Sigma P(X) = 1$

$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) = 1$

$\Rightarrow k + 2k + 3k = 1$

$\Rightarrow 6k = 1$

$\Rightarrow k = \frac{1}{6}$

Therefore, the value of k is $\frac{1}{6}$.

Hence, the correct option is (c).

Example 45. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	2k	3k	k	2k	k^2	$7k^2$	$2k^2 + k$

Find the value of k.

(a) 10 (b) –1 (c) $\frac{1}{10}$ (d) None of these

Sol. (c) We know that,

$\Sigma P(X) = 1$

$\Rightarrow 0 + 2k + 3k + k + 2k + k^2 + 7k^2 + 2k^2 + k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow 10k^2 + 10k - k - 1 = 0$

$\Rightarrow 10k(k + 1) - (k + 1) = 0$

$\Rightarrow (10k - 1)(k + 1) = 0$

$\Rightarrow 10k - 1 = 0, k + 1 = 0$

$\Rightarrow k = \frac{1}{10}, -1$

Since, k cannot be negative.

Therefore, the value of k is $\frac{1}{10}$.

Hence, the correct option is (c).

PROBABILITY DENSITY FUNCTION (PDF)

When x is a continuous random variable defined over an interval $[\alpha, \beta]$, where $\beta > \alpha$, then x can assume an infinite number of values from its interval. And in such cases we assign intervals of values to probability rather than assigning individual probability to every mass point x.

Then, if a function f(x) exists such that it defines the probability. It will be called as probability density function if it satisfies the below given condition:

1. $f(x) \geq 0$ for $x \in [\alpha, \beta]$
2. and if the probability that x lies between two specified values a and b, where $\alpha \leq a < b \leq \beta$ is given by $\int_a^b f(x)dx$

Example 46. If $f(x)=\begin{Bmatrix} cx^2, 0<x<1 \\ 0, \text{elsewhere} \end{Bmatrix}$, elsewhere is the probability density function of a continuous random variable X, find the value of c.

(a) 3 (b) 2 (c) 1 (d) None of these

Sol. (a) Given function, $f(x)=\begin{Bmatrix} cx^2, 0<x<1 \\ 0, \text{elsewhere} \end{Bmatrix}$ Since, f(x) represents probability density function of a random variable X, then

$$\int_0^1 f(x)=1 \Rightarrow \int_0^1 cx^2=1$$

$$\Rightarrow \left[c\frac{x^3}{3}\right]_0^1=1$$

$$\Rightarrow \frac{c}{3}[(1)^3-(0)^3]=1$$

$$\Rightarrow \frac{c}{3}[1-0]=1 \Rightarrow c=3$$

Therefore, the value of x is 3.

Hence, the correct option is (a).

EXPECTED VALUE OF A RANDOM VARIABLE

Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

Note: Expected value gives the mean of all values.

Hence, if a random variable x assumes n values $x_1, x_2, x_3, ..., x_n$ with corresponding probabilities $p_1, p_2, p_3, ..., p_n$, where p_i satisfies

1. $p_i(x_i) \geq 0$ for every X
2. $\sum p_i x_i = 1$

then the expected value of x is given by $\mu = E(x) = \sum p_i x_i$

- Expected Value of x^2 is given by $E(x^2) = \sum p_i x_i^2$
  In particular expected value of a monotonic function g(x) is given by $E[g(x)] = \sum p_i E(x_i)$
- Variance of x, to be denoted by, σ^2 is given by
  $V(x) = \sigma^2 = E(x-\mu)^2 = E(x^2) - \mu^2$

The positive square root of variance is known as standard deviation and is denoted by σ.

- If y = a + bx, for two random variables x and y and for a pair of constants a and b, then the mean i.e., expected value of y is given by:
  $\mu_y = a + b\mu_x$
  and the standard deviation of y is given by:
  $\sigma_y = |b| \times \sigma_x$

Now, when x is discrete random variable and f(x) is the probability mass function, then the expected value (i.e. Mean) is given by

$$\mu = E(x) = \sum_x xf(x)$$

Its variance is given by $\sigma^2 = E(x^2) - \mu^2$, where, $E(x^2) = \sum_x x^2 f(x)$

If x is continuous random variable defined in $(-\infty, \infty)$, then the expected value is given by

$$E(x) = \int_{-\infty}^{\infty} xf(x)dx \text{ and } \sigma^2 = E(x^2) - \mu^2$$

where, $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

PROPERTIES OF EXPECTED VALUES

1. Expectation of a constant k is k i.e., if all values of x is equal to k, then the expected value will be equal to E(k) = k for any constant k.
2. Expectation of sum of two random variables is the sum of their expectations
   i.e., E(x + y) = E(x) + E(y) for any two random variables x and y.
3. Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.
   i.e., E(kx) = k.E(x) for any constant k.
4. Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.
   i.e., E(xy) = E(x) × E(y) where x and y are independent.

Example 47. If x and y are random variables having expected values as 4.5 and 2.5 respectively, then the expected value of (x – y) is **(ICAI)**

(a) 2 (b) 7 (c) 6 (d) 0

Sol. (a) Given, x and y are random variables having expected values as 4.5 and 2.5

$\Rightarrow$ E(x) = 4.5 and E(y) = 2.5

$\therefore$ Expected value of (x – y)

= E(x – y) = E(x) – E(y)

= 4.5 – 2.5 = 2

Therefore, the required expected value is 2.

Hence, the correct option is (a) i.e. 2.

Example 48. The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively. If A, B and C prepare 60, 70 and 90 such statements, then the expected number of correct statement is **(ICAI)**

(a) 170 (b) 176 (c) 178 (d) 180

Sol. (c) Given, The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively.

$P(\bar{A}) = 0.2, P(\bar{B}) = 0.3, P(\bar{C}) = 0.1$

So, probability that there is no error will be

$P(A) = 1 - P(\bar{A}) = 1 - 0.2 = 0.8$

$P(B) = 1 - P(\bar{B}) = 1 - 0.3 = 0.7$

$P(C) = 1 - P(\bar{C}) = 1 - 0.1 = 0.9$

If A, B and C prepare 60, 70 and 90 such statements, so the expected correct statement is given as follows:

X	60	70	90
P(X)	0.8	0.7	0.9

Thus, $E(X) = \Sigma XP(X)$

Expected value = $60 \times 0.8 + 70 \times 0.7 + 90 \times 0.9 = 48 + 49 + 81 = 178$

Here, the correct answer is option (c) i.e., 178.

Example 49. A wholesaler can make a profit of ₹12,000 or incur a loss of ₹8,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.8 and 0.2 respectively. What is his expected profit?

(a) ₹8,000 (b) ₹12,000 (c) ₹4,000 (d) None of these

Sol. (a) According to the given information,

X	12,000	8,000
P	0.8	0.2

Thus, the expected profit is given by:

$E(X) = 12,000(0.8) + (-8,000)(0.2)$

$E(X) = 9,600 - 1,600$

$E(X) = 8,000$

Therefore, his expected profit is ₹8,000.

Hence, the correct option is (a).

Example 50. A number is selected at random from a set containing the first 50 natural numbers and another number is selected at random from another set containing the first 100 natural numbers. What is the expected value of

(I) the sum (II) the product

(a) $76, \frac{5151}{2}$ (b) $66, \frac{8151}{2}$ (c) $50, \frac{9158}{2}$ (d) $45, \frac{5190}{2}$

Sol. (a) According to the given information,

X	1	2	3	4	....	50
P_i	$\frac{1}{50}$	$\frac{1}{50}$	$\frac{1}{50}$	$\frac{1}{50}$	....	$\frac{1}{50}$
P_iX_i	$\frac{1}{50}$	$\frac{2}{50}$	$\frac{3}{50}$	$\frac{4}{50}$	....	$\frac{50}{50}=1$

Y	1	2	3	4	....	100
P_i	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	....	$\frac{1}{100}$
P_iY_i	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{3}{100}$	$\frac{4}{100}$	....	$\frac{100}{100}=1$

(I) Sum $= E(X+Y) = \sum P_iX_i + \sum P_iY_i$

$$= \left\{\frac{1}{50}+\frac{2}{50}+\frac{3}{50}+\ldots\ldots\ldots+\frac{50}{50}\right\}+\left\{\frac{1}{100}+\frac{2}{100}+\frac{3}{100}+\ldots\ldots\ldots\frac{100}{100}\right\}$$

$$= \frac{1+2+3\ldots\ldots+50}{50}+\frac{1+2+3+\ldots\ldots\ldots+100}{100}$$

$$= \frac{\frac{50(50+1)}{2}}{50}+\frac{\frac{100(100+1)}{2}}{100}$$

$$= \frac{51}{2}+\frac{101}{2}=\frac{152}{2}=76$$

(II) Product $= E(XY) = \left(\sum P_iX_i\right)\left(\sum P_iY_i\right)$

$$= \frac{51}{2}\times\frac{101}{2}=\frac{5151}{2}$$

Hence, the correct answer is option (a).

PRACTICE QUESTIONS (PART E)

1. A random variable has the following probability distribution:

X	4	5	7	8	10
P	0.15	0.20	0.40	0.15	0.10

Find $E[x - E(x)]^2$. Also obtain v(3x – 4).

(a) 3.04, 17.36 (b) 3.04, 27.36

(c) 4.15, 37.65 (d) None of these

2. A box contains 10 tubelights of which 3 are defective. A man selects four tube lights at random. What is the expected number of defective tube lights in his selection?

(a) 1.21 (b) 1.69 (c) 1.71 (d) 1.20

3. Raju draws 2 balls from a bag containing 3 white and 5 Red balls. He gets ₹500 if he draws a white ball and ₹200 if he draws a red ball. What is his expectation? But here are the twists, for each game he has to pay ₹400 for participating in the game, would he consider it a fair game and participate?

(a) Yes, the game is fair and should participate

(b) No, the game is not fair

(c) Cannot be determined

(d) None of these

4. A dice is thrown repeatedly till a '1' appears. Write down the sample space. Also find the expected number of throws.

(a) 0 (b) 1 (c) 6 (d) None of these

5. A random variable x has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	2k	3k	k	2k	k^2	$7k^2$	$2k^2 + k$

Find:

(I) $P(x \leq 2)$ (II) $P(x > 3)$ (III) $P(3 \leq x < 6)$

(a) 0.50, 0.40, 0.30 (b) 0.40, 0.40, 0.30

(c) 0.50, 0.40, 0.30 (d) 0.50, 0.40, 0.30

6. If all the values taken by a random variable are equal then

(a) its expected value is zero

(b) its standard deviation is zero

(c) its standard deviation is positive

(d) its standard deviation is a real number

7. If x and y are independent, then

(a) E(xy) = E(x) x E(y) (b) E(xy) = E(x) + E(y)

(c) E(x – y) = E(x) + E(y) (d) E(x – y) = E(x) + x E(y)

8. If two random variables x and y are related by y = 2 – 3x, then the SD of y is given by

(a) –3 × SD of x (b) 3 × SD of x **(ICAI)**

(c) 9 × SD of x (d) 2 × SD of x

9. If an unbiased die is rolled once, the odds in favour of getting a point which is a multiple of 3 is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

10. If a random variable x assumes the values 0, 1 and 2 with probabilities 0.30, 0.50 and 0.20, then its expected value is **(ICAI)**

(a) 1.50 (b) 3 (c) 0.90 (d) 1

Answer Key

1. (b) **2.** (d) **3.** (a) **4.** (c) **5.** (d) **6.** (b) **7.** (a) **8.** (b) **9.** (a) **10.** (c)

FULL CHAPTER PRACTICE QUESTIONS

1. When 2 – dice are thrown simultaneously then the probability of getting at least one 5 is

(a) $\frac{11}{36}$ (b) $\frac{5}{36}$ (c) $\frac{8}{15}$ (d) $\frac{1}{7}$

2. If a coin is tossed 5 times then the probability of getting tail and head occurs alternatively is

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{1}{64}$

3. Two event A and B are such that do not occurs simultaneously then they are called _____ events

(a) Mutually exhaustive (b) Mutually exclusive

(c) Mutually independent (d) Equally likely

4. If $Y \geq X$ then mathematical expectation is

(a) $E(X) > E(Y)$ (b) $E(X) \leq E(Y)$ (c) $E(X) = E(Y)$ (d) $E(X) \cdot E(Y) = 1$

5. If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$, then $P(\bar{A}) + P(\bar{B})$

(a) 0.3 (b) 0.5 (c) 0.7 (d) 0.9

6. Two different dice are thrown simultaneously, then the probability that the sum of the numbers appearing on the top of dice ids 9 is

(a) $\frac{8}{9}$ (b) $\frac{1}{9}$ (c) $\frac{7}{9}$ (d) None of these

7. Ram is known to hit a target in 2 out of 3 shots whereas Shyam is known to hit the same target in 5 out of 11 shots. What is the probability that the target would be hit if they both try?

(a) $\frac{9}{11}$ (b) $\frac{3}{11}$ (c) $\frac{10}{33}$ (d) $\frac{6}{11}$

8. A coin is tossed six times, then the probability of obtaining heads and tails alternatively is

(a) $\frac{1}{2}$ (b) $\frac{1}{64}$ (c) $\frac{1}{32}$ (d) $\frac{1}{16}$

9. The Probability that a non-leap year has 53 Wednesday is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{2}{3}$ (d) $\frac{1}{7}$

10. If $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$, and $P(A \cap B) = \frac{1}{4}$, then $P(A \cup B)$ is equal to

(a) $\frac{11}{12}$ (b) $\frac{10}{12}$ (c) $\frac{7}{12}$ (d) $\frac{1}{6}$

11. Sum of all probability mutually exclusive and exhaustive events is equal to

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 1

12. What is the probability of having at least one 'six' from 3 throws of a perfect die?

(a) $\frac{5}{6}$ (b) $\left(\frac{5}{6}\right)^3$ (c) $1-\left(\frac{1}{6}\right)^3$ (d) $1-\left(\frac{5}{6}\right)^3$

13. If two random variables x and y are related by y = 2 - 3x, then the SD of y is given by

(a) –3 × SD of x (b) 3 × SD of x

(c) 9 × SD of x (d) 2 × SD of x

14. Variance a random variable x is given by

(a) $E(X-\mu)^2$ (b) $E[X-E(X)]^2$ (c) $E(X^2-\mu)$ (d) (a) or (b)

15. The theorem of compound probability states that for any two events A and B

(a) $P(A \cap B) = P(A) \times P(B/A)$ (b) $P(A \cup B) = P(A) \times P(B/A)$

(c) $P(A \cap B) = P(A) \times P(B)$ (d) $P(A \cup B) = P(A) \times P(B) - P(A \cap B)$

16. The term "chance" and probability are synonyms **(ICAI)**

(a) True (b) False (c) Both (d) None of these

17. Two broad divisions of probability are **(ICAI)**

(a) Subjective probability and objective probability

(b) Deductive probability and mathematical Probability

(c) Statistical probability and mathematical probability

(d) None of these

18. Given $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(A \cap B)=\frac{1}{4}$, the value of P(A/B) is

(a) 1/2 (b) 1/6 (c) 2/3 (d) 3/4

19. The Probability distribution of the demand for a commodity is given below:

Demands (x)	5	6	7	8	9	10
Probability [P(x)]	0.05	0.10	0.30	0.40	0.10	0.05

The expected value of demands will be

(a) 7.55 (b) 7.85 (c) 1.25 (d) 8.35

20. If for two mutually exclusive events A and B, $P(A \cup B)=\frac{2}{3}$ and $P(A)=\frac{2}{5}$, then what is the value of P(B)?

(a) $\frac{4}{15}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{15}$

21. Let A and B are two events with $P(A)=\frac{2}{3}, P(B)=\frac{1}{4}$ and $P(A \cap B)=\frac{1}{12}$, then P(B/A) will be

(a) 7/8 (b) 1/3 (c) 1/8 (d) 8/7

22. If a coin is tossed 10 times, what is the probability of getting at least 7 tails?

(a) 0.1719 (b) 0.3438 (d) 0.5000 (d) 0.8281

23. For two events A,B let $P(A)=\frac{2}{3}, P(B)=\frac{3}{8}$ and $P(A\cap B)=\frac{1}{4}$, then A and B are

(a) Mutually exclusive but not independent

(b) Independence but not mutually exclusive

(c) Mutually exclusive and independent

(d) None of these

24. A bag contains 6 white and 5 red balls. One ball is drawn. The probability that it is red is

(a) $\frac{5}{11}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

25. If two unbiased dice are rolled, what is the probability of getting points neither 3 nor 6?

(a) 0.25 (b) 0.50 (c) 0.75 (d) 0.80

26. Three coins are tossed together, the probability of getting exactly two head is:

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{8}$ (d) None of these

27. A bag contain 15 one rupee coins, 25 two rupees coins and 10 five rupees coins, if a coin is selected at random then probability for not selecting a one rupee coin is:

(a) 0.30 (b) 0.20 (c) 0.25 (d) 0.70

28. If a random variable x assumes the values x_1, x_2, x_3, x_4 with corresponding probabilities p_1, p_2, p_3, p_4; then the expected value of x is **(ICAI)**

(a) $p_1 + p_2 + p_3 + p_4$ (b) $x_1p_1 + x_2p_3 + x_3p_2 + x_4p_4$

(c) $p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$ (d) None of these

29. When two fair dice are thrown, what is the probability of getting a sum that is a multiple of 7?

(a) $\frac{1}{12}$ (b) $\frac{1}{36}$ (c) $\frac{1}{6}$ (d) $\frac{1}{18}$

30. If there are 20 cars, 14 of them are SUVs and 6 of them are sedans, then the probability of randomly selecting 4 cars that include 2 SUVs and 2 sedans is

(a) 0.04 (b) 0.17 (c) 0.28 (d) 0.23

31. Let X be a random variable having following Probability distribution:

x	–3	6	9
P(x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find E(X) and $E(X^2)$.

(a) $\frac{11}{2}, \frac{93}{2}$ (b) $\frac{17}{2}, \frac{75}{2}$ (c) $\frac{11}{3}, \frac{97}{3}$ (d) None of these

Answer Key

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(d)	**6.**	(b)	**7.**	(a)	**8.**	(c)	**9.**	(d)	**10.**	(c)
11.	(d)	**12.**	(d)	**13.**	(b)	**14.**	(d)	**15.**	(a)	**16.**	(a)	**17.**	(a)	**18.**	(d)	**19.**	(a)	**20.**	(a)
21.	(c)	**22.**	(a)	**23.**	(b)	**24.**	(a)	**25.**	(d)	**26.**	(b)	**27.**	(d)	**28.**	(c)	**29.**	(c)	**30.**	(c)
31.	(a)																		

PREVIOUS YEAR QUESTION (PYQ'S)

1. If $y \geq x$, then mathematical expectation is **(June 2019)**

(a) $E(x) > E(y)$ (b) $E(x) \leq E(y)$ (c) $E(x) = E(y)$ (d) $E(x).E(y) = 1$

2. Two events A and B are such that they do not occurs simultaneously then they are called _____ events. **(June 2019)**

(a) Mutually exhaustive (b) Mutually exclusive

(c) Mutually independent (d) Equally likely

3. If a coin is tossed 5 times, then the probability of getting tail and head occurs alternatively is **(June 2019)**

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{1}{64}$

4. When 2-dice are thrown simultaneously then the probability of getting at least one 5 is **(June 2019)**

(a) $\frac{11}{36}$ (b) $\frac{5}{36}$ (c) $\frac{8}{15}$ (d) $\frac{1}{7}$

5. If two letters are taken at random from the word HOME, what is the probability that none of the letters would be vowels ? **(Dec 2019)**

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

6. The chance of getting 7 or 11 in a throw of 2 dice is **(Dec 2019)**

(a) $\frac{7}{9}$ (b) $\frac{5}{9}$ (c) $\frac{2}{9}$ (d) None of these

7. A bag contains 15 one rupee coins, 25 two rupee coins and 10 five rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one rupee coins is **(Dec 2019)**

(a) 0.30 (b) 0.70 (c) 0.25 (d) 0.20

8. When 2 fair dice are thrown what is the probability of getting the sum which is a multiple of 3? **(Dec 2020)**

(a) $\frac{4}{36}$ (b) $\frac{8}{36}$ (c) $\frac{2}{36}$ (d) $\frac{12}{36}$

9. When two coins are tossed simultaneously, then the probability of getting atleast one tail is **(Dec 2020)**

(a) 1 (b) 0.75 (c) 0.5 (d) 0.25

10. When 3 dice are rolled simultaneously, the probability on the third die is greater than the sum of the numbers on two dice is **(Dec 2020)**

(a) $\frac{12}{216}$ (b) $\frac{36}{216}$ (c) $\frac{48}{216}$ (d) $\frac{60}{216}$

11. If A speaks 75% of truth and B speaks 80% of truth. In what percentage both of them likely contradict each other in narrating the same questions? **(Dec 2020)**

(a) 0.60 (b) 0.45 (c) 0.65 (d) 0.35

12. An event that can be sub-divided into further events is called as **(Jan 2021)**

(a) A composite event (b) A complex event

(c) A mixed event (d) A simple event

13. Three identical and balanced dice are rolled. The probability that the same number will appear on each of them is **(Jan 2021)**

(a) $\frac{1}{6}$ (b) $\frac{1}{18}$ (c) $\frac{1}{36}$ (d) $\frac{1}{24}$

14. A basket contains 15 white balls, 25 red balls and 10 blue balls. If a ball is selected at random, the probability of selecting not a white ball is **(Jan 2021)**

(a) 0.20 (b) 0.25 (c) 0.60 (d) 0.70

15. Two dice are thrown simultaneously. The probability of a total score of 5 from the outcomes of dice is **(Jan 2021)**

(a) $\frac{1}{18}$ (b) $\frac{1}{12}$ (c) $\frac{1}{9}$ (d) $\frac{2}{5}$

16. Two dice are thrown simultaneously. The probability of a total score of 5 from the outcomes of dice is **(July 2021)**

(a) $\frac{1}{18}$ (b) $\frac{1}{12}$ (c) $\frac{1}{9}$ (d) $\frac{2}{5}$

17. A biased coin is such that the probability of getting a head is thrice the probability of getting a tail. If the coin is tossed 4 times, what is the probability of getting a head all the times? **(July 2021)**

(a) $\frac{2}{5}$ (b) $\frac{81}{128}$ (c) $\frac{81}{256}$ (d) $\frac{81}{64}$

18. If there are 16 phones, 10 of them are Android and 6 of them are of Apple, then the probability of 4 randomly selected phones to include 2 Android and 2 Apple phone is **(July 2021)**

(a) 0.47 (b) 0.51 (c) 0.37 (d) 0.27

19. If there are 48 marbles marked with numbers 1 to 48, then the probability of selecting a marble having the number divisible by 4 is **(July 2021)**

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

20. If in a class, 60% of the student study Mathematics and Science and 90% of the student study Science, then the probability of a student Mathematics given that he/she is already studying Science is **(July 2021)**

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{1}{2}$

21. A bag contains 7 blue and 5 green balls. One ball is drawn at random. The probability of getting a blue ball is **(July 2021)**

(a) $\frac{5}{12}$ (b) $\frac{12}{35}$ (c) $\frac{7}{12}$ (d)0

22. The probability that a football team losing a match at Kolkata is $\frac{3}{5}$ and winning a match at Bengaluru is $\frac{6}{7}$, then the probability of the team winning at least one match is **(July 2021)**

(a) $\frac{3}{35}$ (b) $\frac{18}{35}$ (c) $\frac{32}{35}$ (d) $\frac{17}{35}$

23. For any two dependent events A and B, $P(A) = \frac{5}{9}$ and $P(B) = \frac{6}{11}$ and $P(A \cap B) = \frac{10}{33}$. What are the values of $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$? **(Dec 2021)**

(a) $\frac{5}{9}, \frac{6}{11}$ (b) $\frac{5}{6}, \frac{6}{11}$ (c) $\frac{1}{9}, \frac{2}{9}$ (d) $\frac{2}{9}, \frac{4}{9}$

24. Which of the following pair of events E and F are mutually exclusive? **(Dec 2021)**

(a) E = {Ram's age is 13} and F = {Ram is studying in a college}

(b) E = {Sita studies in a school} and F = {Sita is a playback singer}

(c) E = {Raju is an elder brother in a family} and F = {Raju's father has more than one son}

(d) E = {Banu studied B.A. English literature} and F = {Banu can read English novels}

25. Four unbiased coins are tossed simultaneously. The expected number of heads is **(Dec 2021)**

(a) 1 (b) 2 (c) 3 (d) 4

26. Assume that the probability for rain on a day is 0.4. An umbrella salesman can earn Rs 400 per day in case of rain on the day and will lose Rs 100 per day if there is no rain. The expected earnings (in Rs.) per day of the salesman is **(Dec 2021)**

(a) 400 (b) 200 (c) 100 (d) 0

27. The probability distribution of a random variable x is given below: **(Dec 2021)**

x:	1	2	4	5	6
P:	0.15	0.25	0.2	0.3	0.1

What is the standard deviation of x?

(a) 1.49 (b) 1.56 (c) 1.69 (d) 1.72

28. In a group of 20 males and 15 females, 12 males and 8 females are service holders. What is the probability that a person selected at random from the group is a service holder given that the selected person is a male? **(Dec 2021)**

(a) 0.40 (b) 0.60 (c) 0.45 (d) 0.55

29. There are 3 boxes with the following composition:

Box I: 7 Red + 5 White + 4 Blue balls

Box II: 5 Red + 6 White + 3 Blue balls

Box III: 4 Red + 3 White + 2 Blue balls

One of the boxes is selected at random and a ball is drawn from it. What is the probability the drawn ball is red? **(Dec 2021)**

(a) $\frac{1249}{3024}$ (b) $\frac{1247}{3004}$ (d) $\frac{1147}{3024}$ (d) $\frac{1}{2}$

30. For a probability distribution, probability is given by $P(X) = \frac{X_i}{k}; X_i = 1,2, ...,9$. The value of k is **(Dec 2021)**

(a) 55 (b) 9 (c) 45 (d) 81

31. If P(A) = 0.3, P(B) = 0.8 and $P\left(\frac{B}{A}\right) = 0.5$. Find P (A ∪ B). **(June 2022)**

(a) 0.7 (b) 0.95 (c) 0.60 (d) 0.59

32. What is the chance that a leap year selected at random will contain 53 Fridays? **(June 2022)**

(a) $\frac{3}{7}$ (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) $\frac{4}{7}$

33. Two balanced dice are rolled. The probability of getting 1 in at least one dice is $\frac{x}{36}$, where x is **(June 2022)**

(a) 12 (b) 1 (c) 11 (d) 2

34. Thirty balls are serially numbered and placed in a bag. Find the chance that the first ball drawn is a multiple of 3 or 5. **(June 2022)**

(a) $\frac{8}{15}$ (b) $\frac{2}{15}$ (c) $\frac{1}{2}$ (d) $\frac{7}{15}$

35. The odds in favour of an event A is 2 : 3 and odds against an event B is 6:4, the probability that only one of A and B occurs is $\frac{y}{25}$, where y is **(June 2022)**

(a) 12 (b) 15 (c) 18 (d) 9

36. The odds in favour of event A in a trial is 3 : 1. In a three independent trials, the probability of no occurence of the event A is **(June 2022)**

(a) $\frac{1}{64}$ (b) $\frac{1}{32}$ (c) $\frac{1}{27}$ (d) $\frac{1}{8}$

37. A machine is made of two parts A and B. The manufacturing process of each part is such that probability of defective in part A is 0.08 and that B is 0.05. What is the probability that the assembled part will not have any defect? **(Dec 2022)**

(a) 0.934 (b) 0.864 (c) 0.85 (d) 0.874

38. If $P(A)=\frac{1}{3}, P(B)=\frac{3}{4}$ and $P(A\cup B)=\frac{11}{12}$ then $P\left(\frac{B}{A}\right)$ is **(Dec 2022)**

(a) $\frac{1}{6}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

39. The probability that a leap year has 53 Monday is **(Dec 2022)**

(a) $\frac{1}{7}$ (b) $\frac{2}{3}$ (c) $\frac{2}{7}$ (d) $\frac{3}{5}$

40. Suppose A and B are two independent events that probabilities P (A) ≠ 0 and P (B) ≠ 0. Let A' and B' be their complements. Which one of the following statements is false? **(Dec 2022)**

(a) $P(A\cap B)=P(A).P(B)$ (b) $P\left(\frac{A}{B}\right)=P(A)$

(c) $P(A\cup B)=P(A)+P(B)$ (d) $P(A'\cap B')=P(A').P(B')$

41. The theorem of Compound Probability states that for any events A and B is **(Dec 2022)**

(a) $P(A\cap B)=P(A)\times P\left(\frac{B}{A}\right)$ (b) $P(A\cup B)=P(A)\times P\left(\frac{B}{A}\right)$

(c) $P(A\cap B)=P(A)\times P(B)$ (d) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

42. If a number is selected at random from the first 50 natural numbers, what will be the probability that the selected is a multiple of 3 or 4? **(Dec 2022)**

(a) $\frac{5}{50}$ (b) $\frac{2}{25}$ (c) $\frac{3}{50}$ (d) $\frac{4}{25}$

43. If three coins are tossed simultaneously, what is the probability of getting two heads together? **(Dec 2022)**

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{5}{8}$ (d) $\frac{3}{8}$

44. Company 'A' produces 10% defective products, company 'B' produces 20% defective products and company 'C' produces 5% defective products. If choosing a company is an equally likely event, what is the probability that the product chosen is free from defect? **(June 2023)**

(a) 0.88 (b) 0.80 (c) 0.79 (d) 0.78

45. For any two events, 'A' and 'B' it is known that $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{8}$ and $P(A \cap B) = \frac{1}{4}$, then the events A and B are; **(June 2023)**

(a) Mutually exclusive and independent

(b) Mutually not exclusive and independent

(c) Mutually exclusive but not independent

(d) Neither independent nor mutually exclusive

46. The probability that a four digit number comprising the digits 2, 5, 6, and 7, without repetition of digits, would be divisible by 4 is: **(June 2023)**

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

47. Four persons are chosen at random from a group of 3 men, 2 women and 4 children. The probability that exactly 2 of them are children is: **(June 2023)**

(a) $\frac{10}{21}$ (b) $\frac{1}{12}$ (c) $\frac{1}{5}$ (d) $\frac{1}{9}$

48. If $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}, P\left(\frac{A}{B}\right)=\frac{1}{6}$, the probability $P\left(\frac{B}{A}\right)$ is: **(June 2023)**

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$

49. A number is selected from the first 30 natural numbers. What is the probability that it would be divisible by 3 or 8? **(Dec 2023)**

(a) 0.2 (b) 0.4 (c) 0.6 (d) 0.8

50. If $P(A \cap B)=\frac{1}{3}, P(A \cup B)=\frac{5}{6}, P(\overline{B})=\frac{1}{2}$, then $P(\overline{A})=?$ **(Dec 2023)**

(a) 0.2 (b) 0.4 (c) 0.6 (d) 0.8

51. If a card is drawn at random from a pack of 52 cards, what is the chance of getting a club or king? **(Dec 2023)**

(a) $\frac{13}{52}$ (b) $\frac{4}{52}$ (c) $\frac{17}{52}$ (d) $\frac{16}{52}$

52. If $P(A)=\frac{1}{2}, P(B)=\frac{1}{4}, P(A \cup B)=\frac{1}{6}$, then find $P(A \cap B)$? **(Dec 2023)**

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$

53. A box contains 20 electrical bulbs out of which 4 are defective. Two bulbs are chosen at random from this box. The probability that at least one of them is defective is **(Dec 2023)**

(a) $\frac{7}{19}$ (b) $\frac{8}{20}$ (c) $\frac{3}{16}$ (d) $\frac{4}{19}$

54. A number is selected at random from the first 100 natural numbers. What is the probability that it would be a multiple of 3 or 7? **(Dec 2023)**

(a) $\frac{33}{100}$ (b) $\frac{4}{100}$ (c) $\frac{21}{100}$ (d) $\frac{43}{100}$

55. A random variable has the following probability distribution: **(June 2024)**

X	2	3	5
P	K	2K	2K

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{2}{3}$

56. A number is selected at random from the set {1, 2, ..., 99}. The probability that it is divisible by 9 or 11 is **(June 2024)**

(a) $\frac{19}{100}$ (b) $\frac{19}{99}$ (c) $\frac{10}{100}$ (d) $\frac{10}{99}$

57. The following table gives the probability function of X: **(June 2024)**

X	0	1	2	3	4	5
f(x)	$\frac{6}{30}$	$\frac{5}{30}$	$\frac{13}{30}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{1}{30}$

The expectation of X is

(a) 1.8 (b) 1.7 (c) 1.5 (d) 1.6

58. From a bag containing 4 red, 5 blue and 6 white caps, two caps are drawn without replacement. What is the probability that the caps are of different colours? **(June 2024)**

(a) $\frac{74}{105}$ (b) $\frac{37}{105}$ (c) $\frac{94}{105}$ (d) $\frac{31}{105}$

59. A question in statistics is given to three students A, B and C. Their chances of solving the question are $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{7}$ respectively. The probability that the question would be solved is: **(June 2024)**

(a) $\frac{19}{35}$ (b) $\frac{1}{105}$ (c) $\frac{16}{35}$ (d) $\frac{104}{105}$

60. A company produces two types of products A and B. The probability of a defective product in type A is 0.05 and in type B is 0.03. If the company produces 60% type A and 40% type B, what is the probability of a randomly selected product being defective? **(June 2024)**

(a) 0.042 (b) 0.048 (c) 0.050 (d) 0.052

61. Which of the following pairs of events are mutually exclusive? **(Sept 2024)**

(a) A: The student studies in a school.
B: He studies Geography.

(b) A: Archna was born in India.
B: She is a fine lawyer.

(c) A: Sita is 16 years old.
B: She is a good folk dancer.

(d) A: Imran is under 15 years of age.
B: He is a voter of Delhi.

62. Which one holds correct for any two events A and B ? **(Sept 2024)**

(a) $P(A - B) = P(A) - P(B)$ (b) $P(A - B) = P(A) - P(A \cap B)$

(c) $P(A - B) = P(B) - P(A \cap B)$ (d) $P(A - B) = P(B) + P(A \cap B)$

63. Eight labourers are working at a construction site with the following wages for each day of working (in Rs):

500, 620, 400, 700, 450, 560, 320, 450

If one of the workers is selected at random, what is the probability that his wage would be less than the average wage? **(Sept 2024)**

(a) 0.625 (b) 0.375 (c) 0.500 (d) 0.450

64. A box contains shoe pairs of same pattern of different sizes numbered from 1 to 12. If a shoe pair is selected at random, what is the probability that the number on the shoe pair will be a multiple of 5 or 6 ? **(Sept 2024)**

(a) 0.33 (b) 0.25 (c) 0.20 (d) 0.375

65. Two cards are drawn at random from a pack of 52 cards. The probability of getting either both the red cards or both Kings cards is: **(Sept 2024)**

(a) 0.2488 (b) 0.4288 (c) 0.8248 (d) 0.8428

66. The probability of success of three students in CA Foundation examination are $\frac{1}{5}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. Find the probability that at least two students will get success. **(Sept 2024)**

(a) $\frac{2}{5}$ (b) $\frac{3}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{5}$

67. If $P(A) = 0.65$ and $P(B) = 0.15$, then $P(\overline{A}) + P(\overline{B})$ is: **(Sept 2024)**

(a) 1.5 (b) 1.2 (c) 0.8 (d) 0.35

68. Let $P(A) = \frac{1}{5}$ and $P(\overline{B}) = \frac{3}{5}$. If A and B are mutually exclusive events then $P(A \cup B)$ is **(Jan 2025)**

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $\frac{4}{5}$

69. Let A and B be two possible outcomes of a random experiment and $P(A) = \frac{1}{3}$, $P(A \cup B) = \frac{1}{2}$ and P(B) = x. For what value of x are A and B mutually exclusive events? **(Jan 2025)**

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{8}$

70. A random variable X has the following probability density function; $f(x) = 6x(1 - x)$, $0 \le x \le 1$, then the mean is **(Jan 2025)**

(a) $\frac{1}{3}$ (b) $\frac{1}{12}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

71. A random variable has the following probability distribution: **(Jan 2025)**

X	0	1	2	3
P	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

Find the expected value of X.

(a) 1.20 (b) 1.43 (c) 1.80 (d) 2.00

72. A number is selected from the first 20 natural numbers. Find the probability that it would be divisible by 3 or 7. **(May 2025)**

(a) $\frac{7}{20}$ (b) $\frac{12}{37}$ (c) $\frac{24}{67}$ (d) $\frac{8}{20}$

73. A father had three sons namely, Kailash, Harish and Prakash. All are above 65 years in age. Prakash happens to be the eldest while Kailash as youngest. As per the health history, it is estimated that the probability that Kailash survives another 5 years is $\frac{4}{5}$. Harish survives another 5 years is $\frac{3}{5}$ and Prakash survives another 5 years is $\frac{1}{2}$. The probabilities that Kailash and Harish survive another 5 years is 0.46, Harish and Prakash survive another 5 years is 0.32 and Kailash and Prakash survive another 5 years is 0.48. The probability that all three sons survive another 5 years is 0.26. What shall be the probability that at least one of them survives another 5 years? **(May 2025)**

(a) 0.78 (b) 0.72 (c) $\frac{7}{10}$ (d) $\frac{9}{10}$

74. Two dice are thrown simultaneously. Find the probability that the sum of digits on the two dice would be 8 or more. **(May 2025)**

(a) $\frac{5}{18}$ (b) $\frac{5}{13}$ (c) $\frac{5}{36}$ (d) $\frac{7}{12}$

75. Two cards are drawn from a pack of 52 cards. The probability that one is a spade and one is a heart; is **(May 2025)**

(a) $\frac{3}{20}$ (b) $\frac{29}{34}$ (c) $\frac{47}{100}$ (d) $\frac{13}{102}$

76. A problem is given to 5 students P, Q, R, S and T. If the probability of solving the problem individually is $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{5}$ and $\frac{1}{6}$ respectively then find the probability that the problem is solved. **(May 2025)**

(a) 0.47 (b) 0.93 (c) 0.57 (d) 0.27

77. In a leap year, what is the probability that there will be 53 Sundays? **(May 2025)**

(a) $\frac{53}{365}$ (b) $\frac{1}{7}$ (c) $\frac{3}{7}$ (d) $\frac{2}{7}$

78. In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that 1 girl and 2 boys are selected is **(May 2025)**

(a) $\frac{21}{46}$ (b) $\frac{25}{17}$ (c) $\frac{1}{50}$ (d) $\frac{3}{25}$

79. Two-person X and Y appear in an interview for two vacancies for the same post. The probability of X's selection is $\frac{1}{5}$ and that of Y's selection is $\frac{1}{3}$. The probability that none of them will be selected is **(Sept 2025)**

(a) $\frac{7}{15}$ (b) $\frac{8}{15}$ (c) $\frac{9}{15}$ (d) $\frac{10}{15}$

80. Three components A, B and C are manufactured separately and then assembled into a finished product. While producing the three components, it is found that 5 percent of component A, 4 percent of component B and 1 percent of component C are defective. What is the probability that the assembled product is free from defects? **(Sept 2025)**

(a) 0.75 (b) 0.8 (c) 0.85 (d) 0.9

81. Two persons are playing a set of matches. The winner of 4 matches is declared as the winner. Any player has 50% chance to win a match. The probability that the game comes to an end at the fourth match is _____. **(Sept 2025)**

(a) $\frac{5}{8}$ (b) $\frac{4}{8}$ (c) $\frac{3}{8}$ (d) $\frac{1}{8}$

82. A number is selected at random from the first 50 natural numbers. What is the probability that it would be either a two-digit prime number or a composite number lying between 5 and 40? **(Sept 2025)**

(a) 0.54 (b) 0.48 (c) 0.64 (d) 0.72

83. Some dice with six faces have numbers written from Four to Nine. Two such dice are thrown simultaneously. Find the probability that the sum of numbers on the two dice would be 14 or less. **(Sept 2025)**

(a) $\frac{11}{18}$ (b) $\frac{13}{18}$ (c) $\frac{1}{6}$ (d) $\frac{2}{9}$

84. The probability mass function of a distribution is given below in a tabular form: **(Sept 2025)**

X	0	1	2	3	4
P(x)	k	$2k + k^2$	3k	$2k + k^2$	k

Where k is a non-negative constant. The median of the distribution is

(a) 3k (b) 2 (c) 2k (d) 3

Answer Key

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(a)	**6.**	(c)	**7.**	(b)	**8.**	(d)	**9.**	(b)	**10.**	(d)
11.	(d)	**12.**	(a)	**13.**	(c)	**14.**	(d)	**15.**	(c)	**16.**	(c)	**17.**	(c)	**18.**	(c)	**19.**	(d)	**20.**	(b)
21.	(c)	**22.**	(c)	**23.**	(a)	**24.**	(a)	**25.**	(b)	**26.**	(c)	**27.**	(c)	**28.**	(b)	**29.**	(a)	**30.**	(b)
31.	(b)	**32.**	(c)	**33.**	(c)	**34.**	(d)	**35.**	(a)	**36.**	(a)	**37.**	(d)	**38.**	(c)	**39.**	(c)	**40.**	(c)
41.	(a)	**42.**	(b)	**43.**	(d)	**44.**	(a)	**45.**	(b)	**46.**	(d)	**47.**	(a)	**48.**	(a)	**49.**	(b)	**50.**	(b)
51.	(d)	**52.**	(c)	**53.**	(a)	**54.**	(d)	**55.**	(c)	**56.**	(b)	**57.**	(a)	**58.**	(a)	**59.**	(a)	**60.**	(a)
61.	(d)	**62.**	(b)	**63.**	(c)	**64.**	(a)	**65.**	(a)	**66.**	(c)	**67.**	(b)	**68.**	(a)	**69.**	(a)	**70.**	(d)
71.	(b)	**72.**	(d)	**73.**	(d)	**74.**	(b)	**75.**	(d)	**76.**	(b)	**77.**	(d)	**78.**	(a)	**79.**	(b)	**80.**	(d)
81.	(d)	**82.**	(d)	**83.**	(b)	**84.**	(b)												

SUMMARY

- **Experiment:** An experiment may be described as a performance that produces certain results.
- **Random Experiment:** An experiment is defined to be random if the results of the experiment depend on chance only.
- **Events:** The results or outcomes of a random experiment are known as events. Sometimes events may be a combination of outcomes.
- **The events are of two types:**
  (i) Simple or Elementary,
  (ii) Composite or Compound.
- **Mutually Exclusive Events or Incompatible Events:** A set of events A_1, A_2, A_3, ... is known to be mutually exclusive if not more than one of them can occur simultaneously.
- **Exhaustive Events:** The events A_1, A_2, A_3, ... are known to form an exhaustive set if one of these events must necessarily occur.
- **Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events:** The events of a random experiment are known to be equally likely when all necessary evidence are taken into account, no event is expected to occur more frequently as compared to the other events of the set of events.
- The probability of occurrence of the event is defined as the ratio of the number of events favorable to A to the total number of events. Denoting this by P(A), we have

$$P(A) = \frac{\text{No. of equally likely favorable events}}{\text{Total no. of equally likely events}}$$

- The probability of an event lies between 0 and 1, both inclusive i.e., $0 \le P(A) \le 1$.
- When $P(A) = 0$, A is known to be an impossible event and when $P(A) = 1$, A is known to be a sure event.
- Non-occurrence of event A is denoted by A′ or A^c or $\bar{A}$ and it is known as complimentary event of A. The event A along with its complementary A′ forms a set of mutually exclusive and exhaustive events.
- The ratio of no. of favorable events to the no. of unfavorable events is known as odds in favor of the event and its inverse ratio is known as odds against event A.

  i.e. odds in favor of $A = m_A : (m - m_A)$

  and odds in against $A = (m - m_A) : m_A$
- For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B.

  i.e. or $P(A + B) = P(A) + P(B)$
- For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.

  i.e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  Conditional Probability: $P(B/A) = \frac{P(A \cap B)}{P(A)}$
- For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred i.e., Compound Probability or Joint Probability Provided.
- A random variable or stochastic variable is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment.
- Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

❑❑❑

CHAPTER

16 Theoretical Distributions

THEORETICAL DISTRIBUTION OF PROBABILITY

A probability distribution is a way of describing the probabilities of different outcomes in a random experiment or event. It is similar to a frequency distribution, where we distribute different frequencies over class intervals, but in a probability distribution, we distribute different probabilities.

1. In the case of a **Discrete random variable**, which takes on a finite or countable number of values, the probability distribution is typically represented by a probability mass function (PMF). The PMF gives the probability of each possible outcome.

   E.g.: Binomial Distribution and the Poisson Distribution

2. In the case of a **Continuous random variable**, which can take on any value within a certain range, the probability distribution is typically represented by a probability density function (PDF). The PDF gives the probability density at each point. The area under the PDF represents the probability of an event occurring within a certain range.

   E.g.: Normal Distribution

As a probability distribution exists only in theory, it is called the theoretical distribution of Probability.

WHY DO WE NEED TO STUDY?

- We generally do sampling by frequency distribution, and based on the same samples if we need to find some results. We need to learn probability distribution.

  For example: Finding the age of different tube lights
- Theoretical probability distribution will help you create future projections.

  For Example: Finding the projection of profitability of tube lights in next 10 years
- Statistical analysis is possible only on the basis of theoretical probability distribution.
- A probability distribution also possesses all the characteristics of an observed distribution.

Like finding mean (μ), median (μ), mode(μ_0), standard deviation (σ) etc. exactly the same way we have done earlier.

THINGS TO STUDY

- Discrete Probability Distributions:
  - Binomial Distribution
  - Poisson Distribution
- Continuous Probability Distributions:
  - Normal Distribution

BINOMIAL DISTRIBUTION

- It is derived from a particular type of random experiment known as Bernoulli process named after the famous mathematician Bernoulli.
- When 'trial' is attempted to produce a particular outcome which is neither certain nor impossible.
- The characteristics of Bernoulli trials are stated below:
  (a) Trial is associated with two mutually exclusive and exhaustive outcomes
  (b) The occurrence of one of which is known as a 'success' and as such its non occurrence as a 'failure'.
  As an example, flipping a coin multiple times and counting the number of heads.
- The trial is independent.
- The probability of a success, usually denoted by p, and hence that of a failure, usually denoted by q = 1 – p, remain unchanged throughout the process
- The number of trials is a finite positive integer.

Under such conditions, A discrete random variable x is defined to follow binomial distribution with parameters n and p, to be denoted by x ~ B (n, p), if the probability mass function of x is given by

$f(x) = P(X = x) = {}^{n}C_{x}\,(p)^{x}\,(q)^{n-x}$ or x = 0, 1, 2, ..., n

= 0 otherwise

Example 1. What is the probability of getting 3 heads if 6 unbiased coins are tossed simultaneously? **(ICAI)**

(a) 0.50 (b) 0.25 (c) 0.3125 (d) 0.6875

Sol. (c) According to the question,

Number of trials: n = 6,

Specific outcome (getting 3 heads): x = 3

Probability of success: $p = \frac{1}{2}$,

Probability of failure: $q = 1 - \frac{1}{2} = \frac{1}{2}$

Binomial distribution is given by the formula,

$$P(X = x) = {}^{n}C_{x}(p)^{x}(q)^{n-x}$$

$$P(X = 3) = {}^{6}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{6-3} \quad \left[\because {}^{n}C_{r} = \frac{n!}{r! \times (n-r)}\right]$$

$$= \frac{6!}{3! \times (6-3)!} \times \left(\frac{1}{8}\right)\left(\frac{1}{8}\right) = \frac{20}{64} = 0.3125$$

Therefore, the required probability is 0.3125.

Hence, the correct answer is option (c).

Example 2. What is the probability of making 3 correct guesses in 5 True – False answer type questions? **(ICAI)**

(a) 0.3125 (b) 0.5676 (c) 0.6875 (d) 0.4325

Sol. (a) According to the question,

Number of trials: n = 5,

Specific outcome (making 3 correct guesses): x = 3

Probability of success: $p = \frac{1}{2}$

Probability of failure: $q = 1 - \frac{1}{2} = \frac{1}{2}$

Binomial distribution is given by the formula,

$P(X = x) = {}^{n}C_{x}(p)^{x}(q)^{n-x}$

$P(X = 3) = {}^{5}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{5-3} = \frac{5!}{2! \times 3!}\left(\frac{1}{8}\right)\left(\frac{1}{4}\right)$

$= \frac{10}{32} = \frac{5}{16} = 0.3125$

Hence, the correct answer is option (a).

IMPORTANT POINTS IN CONNECTION WITH BINOMIAL DISTRIBUTION

1. As $n > 0$, $p, q, \geq 0$, it follows that $f(x) \geq 0$ for every x. Also, $\Sigma f(x) = f(0) + f(a) + f(b) + ... + f(n) = 1$.
2. Binomial distribution is known as bi-parametric distribution as it is characterized by two parameters n and p. This means that if the values of n and p are known, then the distribution is known completely.
3. The mean of the binomial distribution is given by $\mu = np$.
4. Depending on the values of the two parameters, binomial distribution may be unimodal or bi- modal μ_0, the mode of binomial distribution, is given by:

   μ_0 = the largest integer contained in:

   (a) $(n + 1)p$ if $(n + 1)p$ is a non-integer.

   (b) $(n + 1)p$ and $(n + 1)p - 1$ if $(n + 1)p$ is an integer.
5. The variance of the binomial distribution is given by $\sigma 2 = npq$

   $\Rightarrow$ Variance of a binomial variable is always less than its mean. Since p and q are numerically less than or equal to 1 and Variance is maximum at $p = q = 0.5$ and the maximum value is $\frac{n}{4}$
6. Additive property of binomial distribution. If X and Y are two independent variables such that $X \sim B(n_1, P)$ and $Y \sim B(n_2, P)$ then $(X + Y) \sim B(n_1 + n_2, P)$.

APPLICATIONS OF BINOMIAL DISTRIBUTION

- Binomial distribution is applicable when the trials are independent and each trial has just two outcomes: success and failure. It is applied in coin tossing experiments, sampling inspection plans, genetic experiments and so on.

Example 3. If in Binomial distribution, np = 9 and npq = 2.25 then q is equal to **(ICAI)**

(a) 0.25 (b) 0.75 (c) 1 (d) None

Sol. (a) Given: np = 9 and npq = 2.25

$$\Rightarrow \frac{npq}{np} = \frac{2.25}{9}$$

$\Rightarrow q = 0.25$

Therefore, the value of q is 0.25.

Hence, the correct answer is option (a) i.e. 0.25.

Example 4. X is a binomial variable with n = 20. What is the mean of X if it is known that X is symmetric? **(ICAI)**

(a) 5 (b) 10 (c) 2 (d) 8

Sol. (b) Given: Number of trials (n) = 20

We know that,

If X is symmetric then occurrence of success is equal to failure.

p = q = 0.5 where p is success, q is failure

Mean is given by the formula,

$\Rightarrow \overline{X} = np$

= 20(0.5)

= 10

Hence, the correct answer is option (b).

Example 5. What is the number of trials of a binomial distribution having mean and SD as 3 and 1.5 respectively? **(ICAI)**

(a) 2 (b) 4 (c) 8 (d) 12

Sol. (d) Given: Mean (X) = 3 Standard Deviation (σ) = 1.5

Mean is given by the formula,

$\overline{X} = np$

Standard Deviation is given by the formula,

$S.D(\sigma) = \sqrt{Var(X)} = \sqrt{npq}$

$\Rightarrow np = 3$ and $\sqrt{npq} = 1.5$

Put the value of $np = 3$ in $\sqrt{npq} = 1.5$

$\Rightarrow \sqrt{3q} = 1.5$

$\Rightarrow 3q = 2.25$

$\Rightarrow q = \frac{2.25}{3}$

$\Rightarrow q = 0.75$

As we know, $p = 1 - q$

$\Rightarrow p = 1 - 0.75$

$\Rightarrow p = 0.25$

Put $p = 0.25$ in $np = 3$,

$\Rightarrow n(0.25) = 3$

$\Rightarrow n = \frac{3}{0.25}$

$\Rightarrow n = 12$

Hence, the correct answer is option (d).

Example 6. If X is a binomial variate with parameter 15 and $\frac{1}{3}$, what is the value of mode of the distribution? **(ICAI)**

(a) 5 and 6 (b) 5 (c) 5.50 (d) 6

Sol. (b) Given: Number of trials (n) = 15

Success $(p) = \frac{1}{3}$

Here, $(n+1)p = \frac{16}{3}$, which is a non-integer

We know that,

Mode = the largest integer contained in $(n + 1)p$ if $(n + 1)p$ is a non-integer.

Here, $(n+1)p = \frac{16}{3} = 5\frac{1}{3}$

Thus, the largest integer is 5.

Therefore, the mode is 5.

Hence, the correct answer is option (b).

Example 7. If it is known that the probability of a missile hitting a target is $\frac{1}{8}$, what is the probability that out of 10 missiles fired, at least 2 will hit the target? **(ICAI)**

(a) 0.4258 (b) 0.3968 (c) 0.5238 (d) 0.3611

Sol. (d) Given: Number of trials (n) = 10

Success $(p) = \frac{1}{8}$

$$\text{Failure}(q) = 1 - p = 1 - \frac{1}{8} = \frac{7}{8}$$

The probability that at least 2 will hit the target will be given by,

= 1 - [Probability that none will hit the target + Probability that one will hit the target]

= 1 - [P (X = 0) + P(X = 1)]

Binomial distribution is given by the formula,

$$P(X = x) = {}^{n}C_{x}(p)^{x}(q)^{n-x} \qquad \left[\because {}^{n}C_{r} = \frac{n!}{r! \times (n-r)!}\right]$$

$$\text{Required Probability} = 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - {}^{10}C_{0}\left(\frac{1}{8}\right)^{0}\left(\frac{7}{8}\right)^{10-0} + {}^{10}C_{1}\left(\frac{1}{8}\right)^{1}\left(\frac{7}{8}\right)^{10-1}$$

$$= 1 - \frac{10!}{0! \times 10!}(1)\left(\frac{7}{8}\right)^{10} + \frac{10!}{1! \times 9!}\left(\frac{1}{8}\right)^{1}\left(\frac{7}{8}\right)^{9}$$

$$= 1 - \left(\frac{7}{8}\right)^{9}\left(\frac{7}{8} + \frac{10}{8}\right) = 1 - \left[\left(\frac{7}{8}\right)^{9}\left(\frac{17}{8}\right)\right]$$

= 1 - 0.6389 = 0.3611

Hence, the correct answer is option (d).

Example 8. Assuming that one-third of the population is tea drinkers and each of 1000 enumerators takes a sample of 8 individuals to find out whether they are tea drinkers or not, how many enumerators are expected to report that five or more people are tea drinkers?

(a) 100 (b) 95 (c) 88 (d) 90 **(ICAI)**

Sol. (c) Given: 1000 enumerators i.e., N = 1000

Number of trials (n) = 8

$$\text{Success}(p) = \frac{1}{3}$$

$$\text{Failure}(q) = 1 - \frac{1}{3} = \frac{2}{3}$$

To find: Enumerators are expected to report that five or more people are tea drinkers i.e., $P(X \geq 5)$

Binomial distribution is given by the formula,

$$P(X = x) = {}^{n}C_{x}(p)^{x}(q)^{n-x} \qquad \left[\because {}^{n}C_{r} = \frac{n!}{r! \times (n-r)!}\right]$$

Thus, $P(X \geq 5) = P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$

$$= {}^{8}C_{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{8-5} + {}^{8}C_{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{8-6} + {}^{8}C_{7}\left(\frac{1}{3}\right)^{7}\left(\frac{2}{3}\right)^{8-7} + {}^{8}C_{8}\left(\frac{1}{3}\right)^{8}\left(\frac{2}{3}\right)^{8-8}$$

$$= 56\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^3 + 28\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^2 + 8\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right)^1 + \left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^0$$

$$= \left(\frac{1}{3}\right)^5\left[56\left(\frac{2}{3}\right)^3 + 28\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + 8\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^3\right]$$

$$= \left(\frac{1}{3}\right)^5\left[\frac{448+112+16+1}{3^3}\right]$$

$$= \frac{1}{3^8}(577) = 0.08794$$

As N = 1000,

Thus, the required enumerators (the expected frequency)

$N \times P(X = x) = 0.08794 \times 1000 = 87.94 \approx 88$

Hence, the correct answer is option (c).

Example 9. In 10 independent rollings of a biased die, the probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. What is the probability that an even number will appear twice when the die is rolled 8 times? **(ICAI)**

(a) 0.0304 (b) 0.1243 (c) 0.2315 (d) 0.1926

Sol. (a) Given: Number of trials (n) = 10

Success (p) = Even number appear on dice

To find: Probability that an even number will appear twice when the die is rolled 8 times

Binomial distribution is given by the formula,

$$P(X = x) = {}^nC_x(p)^x(q)^{n-x} \qquad \left[{}^nC_r = \frac{n!}{r! \times (n-r)!}\right]$$

$n \rightarrow$ Number of trials

$x \rightarrow$ number of times for a specific outcome within n trials

$p \rightarrow$ Success

$q \rightarrow$ Failure

According to the question,

$\Rightarrow P(X = 5) = 2P(X = 4)$

$$\Rightarrow {}^{10}C_5(p)^5(q)^{10-5} = 2\left[10_{C_4}(p)^4(q)^{10-4}\right]$$

$$\Rightarrow {}^{10}C_5(p)^5(q)^5 = 2\left[10_{C_4}(p)^4(q)^6\right]$$

$$\Rightarrow \frac{p}{5} = \frac{2q}{6}$$

$\Rightarrow p = \frac{5q}{3}$

Success and failures adds up to 1,

So, p + q = 1

Put the value of $p = \frac{5q}{3}$ in p + q = 1

$\Rightarrow \frac{5q}{3} + q = 1$

$\Rightarrow \frac{5q + 3q}{3} = 1$

$\Rightarrow 8q = 3$

$\Rightarrow q = \frac{3}{8}$

$\Rightarrow p = 1 - q = 1 - \frac{3}{8} = \frac{5}{8}$

Now, $P(X = 2) = 8_{C_2}\left(\frac{5}{8}\right)^2\left(\frac{3}{8}\right)^{8-2}$

$\Rightarrow \frac{8!}{2! \times 6!}\left(\frac{5}{8}\right)^2\left(\frac{3}{8}\right)^6$

$\Rightarrow 28 \times \frac{5^2 \times 3^6}{8^8}$

$\Rightarrow 0.03041 \approx 0.0304$

Hence, the correct answer is option (a).

PRACTICE QUESTIONS (PART A)

1. What are the parameters of binomial distribution?

(a) n (b) p (c) Both n and p (d) None of these

2. The Standard Deviation of Binomial distribution is **(Dec 2022)**

(a) npq (b) $\sqrt{npq}$ (c) np (d) $\sqrt{np}$

3. If X is a binomial variate with $p = \frac{1}{3}$ for the experiment of 90 trials, them the standard deviation is equal to **(July 2021)**

(a) $-\sqrt{5}$ (b) $\sqrt{5}$ (c) $2\sqrt{5}$ (d) $\sqrt{15}$

4. There is a fire in a house, chances of catching fire to fireman is 25%. What is the probability that out of 5 workmen, 4 or more will be in the fire?

(a) $\frac{15}{4^5}$ (b) $\frac{1}{64}$ **(c)** $\frac{1}{256}$ (d) None of these

5. An unbiased die is tossed 500 times. The mean of the number of 'sixes' in these 500 tosses is **(ICAI)**

(a) $\frac{50}{6}$ (b) $\frac{500}{6}$ (c) $\frac{5}{6}$ (d) None of these

6. If in a Binomial distribution if mean = 20, S.D. = 4, then n is equal to **(ICAI)**

(a) 80 (b) 100 (c) 90 (d) None of these

7. In Binomial distribution if n = 4 and p = $\frac{1}{3}$ then the value of variance is **(ICAI)**

(a) $\frac{8}{3}$ (b) $\frac{8}{9}$ (c) $\frac{4}{3}$ (d) None of these

8. For binomial distribution E(x) = 2, V(x) = $\frac{4}{3}$. Find the value of n.

(a) 3 (b) 4 (c) 5 (d) 6

9. If 15 dates are selected at random, then the probability of getting two sundays is :

(a) 0.29 (b) 0.99 (c) 0.49 (d) 0.39

10. An experiment succeeds twice as often as it fails. What is the probability that in next five trials there will be at least three successes?

(a) $\frac{33}{81}$ (b) $\frac{46}{81}$ (c) $\frac{64}{81}$ (d) $e^{\frac{6}{7}}$

11. Find the probability of success in a binomial distribution for which mean and standard deviation are 6 and 2 respectively.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

12. Find the probability of a success for the binomial distribution satisfying the following relation 4P (x = 4) = P (x = 2) and having the parameter n as six.

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$

13. The probability of a man hitting the target is $\frac{1}{4}$. If he fires 7 times, the probability of hitting the target at least twice is

(a) $1-\left(\frac{5}{2}\right)\left(\frac{3}{4}\right)^6$ (b) $1-\frac{15}{2}\left(\frac{3}{4}\right)^6$ (c) $1-\left(\frac{5}{6}\right).3^5$ (d) $1-\left(\frac{3}{4}\right)^6$

14. If X ~ B (n, p), what would be the greatest value of the variance of x when n =16?

(a) 2 (b) 4 (c) 8 (d) $\sqrt{2}$

15. For a Binomial distribution B (6, p), P(x = 2) = 9P(x = 4) , then P is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{10}{13}$ (d) $\frac{1}{4}$

Answer Key

1. (c)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (a)	**10.** (c)
11. (a)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (d)					

POISSON DISTRIBUTION

- Poisson distribution is a theoretical discrete probability distribution which can describe many processes. Simon Denis Poisson of France introduced this distribution way back in the year 1837.

  Let us think of a random experiment under the following conditions:
- The probability of finding success in a very small time interval (t, t + dt) is kt, where k (> 0) is a constant.
- The probability of having more than one success in this time interval is very low.
- The probability of having success in this time interval is independent of t as well as earlier successes.
- The above model is known as the Poisson Model. The probability of getting x successes in a relatively long time interval T containing m small time intervals t i.e. T = kt is given by $\frac{e^{-kt}}{x!}(kt)^x$ for $x = 0, 1, 2, \ldots$

  Taking kt = m, the above form is reduced to $\frac{e^{-m}m^x}{x!}$ for $x = 0, 1, 2, \ldots$

DEFINITION OF POISSON DISTRIBUTION

- A random variable X is defined to follow Poisson distribution with parameter m, to be denoted by X ~ P (m) if the probability mass function of x is given by

$$f(x) = P(X = x) = \frac{e^{-m}m^x}{x!} \text{ for } x = 0, 1, 2, \ldots\ldots\ldots$$

Here, e is a transcendental quantity with an approximate value as 2.71828.

Example 10. If 1.5 percent of items produced by a manufacturing units are known to be defective, what is the probability that a sample of 200 items would contain no defective item? **(ICAI)**

(a) 0.05 (b) 0.15 (c) 0.20 (d) 0.22

Sol. (a) Given: 1.5% defective items i.e. $(p) = \frac{1.5}{100} = 0.015$

Number of units (n) = 200

In Poisson Distribution,

$m = np = 200(0.015) \Rightarrow m = 3$

Probability mass function will be given as,

$$P(X=x)=\frac{e^{-m}m^x}{x!}$$

As we want 0 defective unit according to the question, then x = 0

$$\Rightarrow P(X=0)=\frac{e^{-3}3^0}{0!}$$

$$=\frac{1}{e^3}=\frac{1}{(2.71828)^3}=0.04978\approx 0.05$$

Hence, the correct answer is option (a).

Example 11. X is a Poisson variate satisfying the following condition 9 P(X = 4) + 90 P(X = 6) = P(X = 2). What is the value of P(X ≤ 1)? **(ICAI)**

(a) 0.5655 (b) 0.6559 (c) 0.7358 (d) 0.8201

Sol. (c) As we know, For the poisson distribution with mean m

$$P(X=x)=\frac{e^{-m}m^x}{x!}$$

According to the question,

$$9P(X=4)+90P(X=6)=P(X=2)$$

Take e^{-m} and m^2 across the equation,

$$\Rightarrow 9\frac{m^2}{24}+90\frac{m^4}{720}=\frac{1}{2}$$

$$\Rightarrow \frac{3m^2}{8}+\frac{m^4}{8}=\frac{1}{2}$$

$\Rightarrow m^4 + 3m^4 - 4 = 0$

$\Rightarrow m^4 + 4m^2 - m^2 - 4 = 0$

$\Rightarrow m^2 (m^2 + 4) - 1 (m^2 + 4) = 0$

$\Rightarrow (m^2 - 1) (m^2 + 4) = 0$

$\Rightarrow m^2 - 1 = 0$ and $m^2 + 4 = 0$

$\Rightarrow m = \pm 1$ and

$m^2 = -4$ which is not possible.

So, m = 1, as mean cannot be negative

Now, P(X ≤ 1) will be given as,

P(X ≤ 1) = P(X = 0) + P(X = 1) when m = 1

$$=\frac{e^{-1}1^0}{0!}+\frac{e^{-1}1^1}{1!}=\frac{1}{e}+\frac{1}{e}$$

$$= \frac{2}{e} = \frac{2}{2.71828}$$

$= 0.73579 \approx 0.7358$

Hence, the correct answer is option (c) i.e., 0.7358.

Example 12. If 1 percent of an airline's flights suffer a minor equipment failure in an aircraft, what is the probability that there will be exactly two such failures in the next 100 such flights? **(ICAI)**

(a) 0.50 (b) 0.184 (c) 0.265 (d) 0.256

Sol. (b) Given: Probability that an airline's flights suffer a minor equipment failure (p) = 1%

$$= \frac{1}{100} = 0.01$$

Number of flights (n) = 100

Thus, m = np = 100(0.01)

$\Rightarrow$ m =1

Probability mass function will be given as,

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

According to the question,

For exactly 2 equipment failure i.e. x = 2

$$\Rightarrow P(X = 2) = \frac{e^{-1} 1^2}{2!} = \frac{1}{2e}$$

$= 0.183932 \approx 0.184$

Hence, the correct answer is option (b).

It is wiser to remember the following important points in connection with Poisson distribution:

1. Since $e^{-m} = \frac{1}{e^m} > 0$, whatever may be the value of m, m > 0, it follows that f(x) ≥ 0 for every x. Also it can be established that f(x)=1

   i.e., f(0) + f(a) + f(b) + ... = 1
2. Poisson distribution is known as a uniparametric distribution as it is characterized by only one parameter m.
3. The mean of Poisson distribution is given by m i.e. μ = m
4. The variance of Poisson distribution is given by σ^2 = m
5. Like binomial distribution, Poisson distribution could be also unimodal or bimodal depending upon the value of the parameter m.

   We have μ_0 =

   (a) The largest integer contained in m if m is a non-integer

   (b) m and m - 1 if m is an integer

6. Poisson approximation to Binomial distribution:

The Poisson distribution can be used as an approximation to the binomial distribution under certain conditions. When the number of trials in a binomial distribution (n) is large and the probability of success (p) is small, the binomial distribution becomes computationally challenging. In such cases, the Poisson distribution provides a simpler and more convenient approximation.

(a) The conditions for using the Poisson approximation to the binomial distribution are as follows:

(b) The number of trials (n) is large, typically greater than 20.

(c) The probability of success (p) is small, typically less than or equal to 0.05.

(d) The events occur independently of each other.

Then, B(n, p) ϑ P(m)

7. Additive property of Poisson distribution:

The Poisson distribution has an additive property, which means that if we have two or more independent Poisson-distributed random variables, the sum of these variables will also follow a Poisson distribution.

Mathematically, if X and Y are two independent Poisson-distributed random variables with parameters m_1 and m_2 respectively, then the sum Z = X + Y will be a Poisson-distributed random variable with parameter $m = m_1 + m_2$.

i.e., If $X \sim P(m_1)$ and $Y \sim P(m_2)$ and X and Y are independent then

$Z = X + Y \sim P(m_1 + m_2)$

Application of Poisson distribution:

Poisson distribution is applied when the total number of events is pretty large but the probability of occurrence is very small. Thus we can apply Poisson distribution, rather profitably, for the following cases:

1. The distribution of the number of printing mistakes per page of a large book.
2. The distribution of the number of road accidents on a busy road per minute.
3. The distribution of purchases of number of shoe laces per minute at shoe store
4. The distribution of the number of demands per minute for health centers and so on.

FITTING A POISSON DISTRIBUTION

As explained earlier, we can apply the method of moments to fit a Poisson distribution to an observed frequency distribution. Since Poisson distribution is uniparametric, we equate m, the parameter of Poisson distribution, to the arithmetic mean of the observed distribution and get the estimate of m.

i.e $\hat{m} = \bar{x}$

The fitted Poisson distribution is then given by $\hat{f}(x) = \frac{e^{-m}(\hat{m})^x}{x!}$ for $x = 0, 1, 2, \ldots$

Example 13. If the standard deviation of a Poisson variate X is 2, what is P(1.5 < X < 2.9)?

(a) 0.231 (b) 0.158 (c) 0.15 (d) 0.146 **(ICAI)**

Sol. (d) Given : Standard Deviation (m) of a Poisson variate X is 2.

To find : P(1.5 < X < 2.9)

In Poisson Distribution, Mean $\left(\overline{X}\right)$ = Variance (γ) = m

As we know, Standard deviation $(\sigma) = \sqrt{m}$

$\Rightarrow 2 = \sqrt{m}$

$\Rightarrow m = 4$

As X only takes discrete values, thus

For, 1.5 < X < 2.9

$\Rightarrow X = 2$

Now, $P(X = x) = \frac{e^{-m} \times m^x}{x!}$

$P(X = 2) = \frac{e^{-4} \times 4^2}{2!}$

$= e^{-4} \times 8 = 0.146$

Hence, the correct answer is option (d) i.e., 0.146.

Example 14. For a Poisson variate X, P (X = 1) = P (X = 2). What is the mean of X? **(ICAI)**

(a) 1.00 (b) 1.50 (c) 2.00 (d) 2.50

Sol. (c) Given: P (X = 1) = P (X = 2)

Probability mass function will be given as,

$P(X = x) = \frac{e^{-m} m^x}{x!}$

$\Rightarrow \frac{e^{-m} m^1}{1!} = \frac{e^{-m} m^2}{2!} \Rightarrow 1 = \frac{m}{2}$

$\Rightarrow m = 2$

Therefore, the mean of X is 2.

Hence, the correct answer is option (c).

Example 15. If X is a Poisson variate such that, P (X = 1) = 0.7, P (X = 2) = 0.3, then P (X = 0) = **(July 2021)**

(a) $e^{\frac{6}{7}}$ (b) $e^{-\frac{6}{7}}$ (c) $e^{-\frac{2}{3}}$ (d) $e^{-\frac{1}{3}}$

Sol. (b) We know that,

Formula for poisson distribution for random variable $X, P(X) = \frac{e^{-m} m^x}{x!}$

Given P(x = 1) = 0.7, P(x = 2) = 0.3

So, $\frac{e^{-m}m^1}{1!} = me^{-m} = 0.7$

Also $\frac{e^{-m}m^2}{2!} = \frac{m^2e^{-m}}{2} = 0.3$

Dividing them, we get

Now, $\frac{m^2e^{-m}}{2me^{-m}} = \frac{0.3}{0.7}$

$\Rightarrow m = \frac{6}{7}$

Now, $P(x = 0) = \frac{e^{-m}m^0}{0!} = e^{-6/7}$

Hence, option (b) is correct.

Example 16. If for a Poisson variable X, f(2) = 3f(4), what is the variance of X ? **(ICAI)**

(a) 2 (b) 2 (c) $\sqrt{2}$ (d) 3

Sol. (a) Given: f(2) = 3f(4)

Probability mass function will be given as,

$P(X = x) = \frac{e^{-m}m^x}{x!}$

In Poisson Distribution, Mean = Variance = m

According to the question,

$\Rightarrow P(X = 2) = 3P(X = 4)$

$\Rightarrow \frac{e^{-m}m^2}{2!} = 3\left(\frac{e^{-m}m^4}{4!}\right)$

$\Rightarrow m^2 = 4 \Rightarrow m = \sqrt{4}$

$\Rightarrow m = 2$

Hence, the correct answer is option (a).

Example 17. A Company has two cars which it hires out during the day. The number of Cars demanded with mean 1.5. Then percentage of days on which only one car was in demand is equal to

(a) 23.26 (b) 33.47 (c) 44.62 (d) 46.40

Sol. (b) Let X denote the number of cars which are hired out per day.

For Poisson distribution, mean = m = 1.5

We know that, $P(x) = \frac{e^{-m} \cdot m^x}{x!}$

So, $P(X=1)=\frac{e^{-1.5}\cdot(1.5)^1}{1!}=\frac{0.2231\times1.5}{1}$ = 0.33465 = 33.47%

Hence, the correct option is (b) i.e. 33.47%

Example 18. If the parameter of poisson distribution is m and mean + S.D. $=\frac{6}{25}$ then find m.

(a) $\frac{3}{25}$ (b) $\frac{1}{25}$ (c) $\frac{4}{25}$ (d) $\frac{3}{5}$ **(Dec 2020)**

Sol. (b) We know that for poisson distribution the mean and variance is same.

Given, Mean + SD = $\frac{6}{25}$

Let mean = Vairance = m

Then, $SD=\sqrt{m}$

$\Rightarrow m+\sqrt{m}=\frac{6}{25}$

For the given options,

Option **(a)**: $\frac{3}{25}+\sqrt{\frac{3}{25}}\neq\frac{6}{25}$

Option **(b)**: $\frac{1}{25}+\sqrt{\frac{1}{25}}=\frac{1}{25}+\frac{1}{5}=\frac{6}{25}$ which is true

Option **(c)**: $\frac{4}{35}+\sqrt{\frac{4}{25}}=\frac{4}{25}+\frac{2}{5}=\frac{14}{25}\neq\frac{6}{25'}$

Option **(d)**: $\frac{3}{5}+\sqrt{\frac{3}{5}}\neq\frac{6}{25'}$

Hence, $m=\frac{1}{25}$, option (b) is correct.

Example 19. In a certain manufacturing process, 5% of the tools produced turn out to be defective. Find the probability that in a sample of tools, at most 2 will be defective. Given e^{-2} = 0.135

(a) 0.555 (b) 0.932 (c) 0.785 (d) 0.675

Sol. (d) Given: 5% defective items i.e. $(p)=\frac{5}{100}=0.05$

Number of units (n) = 40

In Poisson Distribution,

m = np = 40(0.05)

$\Rightarrow$ m = 2

Probability mass function will be given as,

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

Now, P(at most 2 defective tools) is given by

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}$$

$= e^{-2}(1 + 2 + 2) = 0.135 \times 5 = 0.675$

Hence, the correct answer is option (d) i.e., 0.675.

Example 20. The number of accidents in a year attributed to taxi drivers in a locality follows Poisson distribution with an average 2. Out of 500 taxi drivers of that area, what is the number of drivers with at least 3 accidents in a year? **(ICAI)**

(a) 162 (b) 180 (c) 201 (d) 190

Sol. (a) Given,

Number of units (n) = 40

Total no. of taxi drivers (N) = 500

Mean (m) = 2

The probability of at least 3 accidents is given by $P(X \ge 3) = 1 - P(X < 3)$

We know that, $P(X = x) = \frac{e^{-m} m^x}{x!}$

$\Rightarrow P(X \ge 3) = 1 - P(X < 3)$

$\Rightarrow P(X \ge 3) = 1 - [P(0) + P(1) + P(2)]$

$$\Rightarrow P(X \ge 3) = 1 - \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}$$

$$\Rightarrow P(X \ge 3) = 1 - \frac{1}{(2.71828)^2}[1 + 2 + 2]$$

$$\Rightarrow (PX \ge 3) = 1 - \frac{1}{(2.71828)^2}[5] \Rightarrow P(X \ge 3) = 1 - 0.06767$$

$\Rightarrow P(X \ge 3) = 0.3233$

Now, the number of drivers with at least 3 accidents in a year = $N \times P(X \ge 3)$

$= 500 \times 0.3233$

$= 161.65 \approx 162$

Therefore, the number of drivers with at least 3 accidents in a year are 162.

Hence, the correct answer is option (a)

Example 21. A renowned hospital usually admits 200 patients every day. One percent of patients, on an average, require special room facilities. On one particular morning, it was found that only one special room was available. What is the probability that more than 3 patients would require special room facilities? **(ICAI)**

(a) 0.1428 (b) 0.1732 (c) 0.2235 (d) 0.3450

Sol. (a) Given: Number of patients (n) = 200

Success (p) = 1% = 1/100 = 0.01

Thus, it follows Poisson Distribution.

As we know, for the poisson distribution with mean m,

Mean = Variance = m

As, m = np

m = 200(0.1)

m = 2

According to the question,

$P(X > 3) = 1 - P(X \leq 3)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)]$

As we know that,

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

$$P(X > 3) = 1 - \left[\frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!} + \frac{e^{-2}2^3}{3!}\right]$$

$$= 1 - \left(\frac{1}{(2.71828)^2} \times \frac{19}{2}\right) = 1 - 0.85712$$

$= 0.14288$

Hence, the correct answer is option (a).

Example 22. A random variable X follows Poisson distribution and its coefficient of variation is 50. What is the value of $P\left(\frac{x > 1}{x > 0}\right)$? **(ICAI)**

(a) 0.1876 (b) 0.2341 (c) 0.9254 (d) 0.8756

Sol. (c) Given: Coefficient of Variation (C.V) = 50

As we know, for the Poisson distribution with mean m

Mean = Variance = m and Standard Deviation (S.D) = $\sqrt{m}$

Coefficient of Variation (C.V) is given by the formula,

$$C.V. = \frac{S.D.}{m} \times 100$$

$$50 = \frac{\sqrt{m}}{m} \times 100$$

$$\sqrt{m} = 2$$

Squaring both sides,

m = 4

As we know that,

$$P(X=x)=\frac{e^{-m}m^{x}}{x!}$$

$$P\left(\frac{X>1}{X>0}\right)=\frac{1-[P(X=0)+P(X=1)]}{1-[P(X=0)]}$$

Now, P(X > 1),

$$=1-\left[\frac{e^{-m}m^{x}}{x!}+\frac{e^{-m}m^{x}}{x!}\right]$$

$$=1-\left[\frac{e^{-4}4^{0}}{0!}+\frac{e^{-4}4^{1}}{1!}\right]=1-\frac{5}{(2.71828)^{4}}$$

= 0.9085

Now, P(X > 0),

$$=1-\left[\frac{e^{-4}4^{0}}{0!}\right]$$

$$=1-\frac{1}{(2.71828)^{4}}$$

= 0.981685

Now, $P\left(\frac{x>1}{x>0}\right)=\frac{0.9085}{0.981685}$

= 0.92544

Hence, the correct answer is option **(c)**.

Example 23. A car hire firm has 2 cars which are hired out everyday. The number of demands per day for a car follows Poisson distribution with mean 1.20. What is the proportion of days on which some demand is refused? (Given $e^{1.20} = 3.32$) **(ICAI)**

(a) 0.25 (b) 0.3012 (c) 0.12 (d) 0.03

Sol. (d) According to the question,

As we know, For the poisson distribution with mean m

Mean = Variance = m = 1.20 (given)

As we know that,

$$P(X=x)=\frac{e^{-m}m^{x}}{x!}$$

The service will be refused if P(X > 3)

P(X > 3) will be given as,

P(X > 3) = 1 – P(X ≤ 3)

= 1 – [P(X = 0) + P (X = 1) + P (X = 2) + P (X = 3)]

$$= 1 - \left[\frac{e^{-m}m^x}{x!} + \frac{e^{-m}m^x}{x!} + \frac{e^{-m}m^x}{x!} + \frac{e^{-m}m^x}{x!}\right]$$

$$= 1 - \left[\frac{e^{-1.20}1.20^0}{0!} + \frac{e^{-1.20}1.20^1}{1!} + \frac{e^{-1.20}1.20^2}{2!} + \frac{e^{-1.20}1.20^3}{3!}\right]$$

$$= 1 - \frac{1}{e^{12}}(1 + 1.2 + 0.72 + 0.288) = 1 - 0.9663$$

$= 0.0337 \sim 0.03$

Hence, the correct answer is option (d).

PRACTICE QUESTIONS (PART B)

1. In Poisson distribution, which of the following is same ? **(Dec 2019)**

(a) Mean and variance (b) Mean and SD

(c) Both (d) None of these

2. Let X be a Poisson random variable with parameter λ. Then p(X) is equal to

(a) $\frac{e^{\lambda} - e^{-\lambda}}{2}$ (b) $\frac{e^{\lambda} + e^{-\lambda}}{2}$ (c) $\frac{e^{2\lambda} - 1}{2}$ (d) $\frac{\lambda^x \cdot e^{-\lambda}}{x!}$ **(Dec 2019)**

3. Which one of the following has Poisson distribution? **(Dec 2020)**

(a) The number of days to get a complete cure.

(b) The number of defects per meter on long roll of coated polythene sheet.

(c) The errors obtained in repeated measuring of the length of a rod.

(d) The number of claims rejected by an insurance agency.

4. Mean of poisson distribution is 6 then variance is _____ .

(a) 6 (b) $\sqrt{6}$ (c) 4 (d) 3

5. If the overall percentage of success in an exam is 60, what is the probability that out of a group of 4 students, at least one has passed? **(ICAI)**

(a) 0.6525 (b) 0.9744 (c) 0.8704 (d) 0.0256

6. If 5% of the families in Kolkata do not use gas as a fuel, what will be the probability of selecting 10 families in a random sample of 100 families who do not use gas as a fuel.

(Given $e^{-5} = 0.0067$)

(a) 0.038 (b) 0.028 (c) 0.048 (d) 0.018

7. The probability that a random variable x following Poisson distribution would assume a positive value is $(1-e^{-2.7})$. What is the mode of the distribution?

(a) 0 (b) 1 (c) 2 (d) None of these

8. Find the mean and standard deviation of x, where x is a Poisson variate satisfying the condition P(x = 3) = P(x = 4).

(a) 4, 2 (b) 4, 4 (c) 2, 2 (d) None of these

9. For a Poisson distributed variable, we have, P (X = 7) = 8P (X = 9) the mean of the distribution is **(Dec 2020)**

(a) 4 (b) 3 (c) 7 (d) 9

10. What is the standard deviation of the number of recoveries among 48 patients when the probability of recovering is 0.75? **(ICAI)**

(a) 36 (b) 81 (c) 9 (d) 3

11. The mean of Poisson distribution is 4. The probability of two successes is:

(a) $\frac{8}{e^4}$ (b) $\frac{4}{e^4}$ (c) $\frac{16}{e^4}$ (d) $\frac{8}{e^2}$

12. If Standard deviation of a poisson distribution is 2, then its

(a) Mode is 2 (b) Mode is 4

(c) Modes are 3 and 4 (d) Modes are 4 and 5

13. A manufacture, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contains at least two defectives:

(a) 7 (b) 13 (c) 9 (d) 11

14. The probability than a man aged 45 years will die within a year is 0.012. What is the probability that of 10 men, at least 9 will reach their 46th birthday? [Given: $e^{-0.12} = 0.88692$]

(a) 0.0935 (b) 0.9934 (c) 0.9335 (d) 0.9555

15. If 5% of the families in large population city do not use gas as a fuel, what will be the probability of selecting 10 families in a random sample of 100 families who do not use gas as a fuel? [Given that $e^{-5} = 0.0067$]

(a) 0.038 (b) Zero (c) 0.018 (d) 0.048

Answer Key

1. (a) **2.** (d) **3.** (b) **4.** (a) **5.** (b) **6.** (d) **7.** (c) **8.** (a) **9.** (b) **10.** (d)
11. (a) **12.** (c) **13.** (c) **14.** (c) **15.** (c)

NORMAL OR GAUSSIAN DISTRIBUTION

- The two distributions discussed so far, namely binomial and Poisson, are applicable when the random variable is discrete.
- In the case of a continuous random variable like height or weight, it is impossible to distribute the total probability among different mass points because between any two unequal values, there remains an infinite number of values.

- Thus, a continuous random variable is defined in term of its probability density function f (x), provided, of course, such a function really exists, f (x) satisfies the following condition: $f(x) \geq 0$ or for $x \in (-\infty, \infty)$ and $\int_{-\infty}^{\infty} f(x)dx = 1$
- It is also known as Gaussian Distribution as Karl Gauss was instrumental for deriving normal distribution and as such normal distribution but the contribution of De-Moivre, Laplace was significant,
- A continuous random variable x is defined to follow normal distribution with parameters μ and σ^2, to be denoted by $X \sim N(\mu, \sigma^2)$
- If the probability density function of the random variable x is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \text{ for } -\infty < x < \infty \text{ where } \mu \text{ and } \sigma \text{ are constants and } \sigma > 0.$$

Example 24. The mean and the variance of a random variable X having the probability density function $P(X = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-4)^2}, -\infty < x < \infty$ is

(a) $4, \frac{1}{2}$ (b) $4, \frac{1}{\sqrt{2}}$ (c) 2, 2 (d) $2, \frac{1}{2}$

Sol. (a) Normal distribution density function is :

$$P(X = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty$$

Given equation:

$$P(X = x) = \frac{1}{\sqrt{\pi}} e^{-(x-4)^2} = \frac{1}{\frac{1}{\sqrt{2}}\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-4}{\frac{1}{\sqrt{2}}}\right)^2}$$

Comparing given function with the standard form, we get

Mean (μ) = 4

S. D. $(\sigma) = \frac{1}{\sqrt{2}}$

Variance $(\sigma^2) = \frac{1}{2}$

Hence, the correct option is (a) i.e. $4, \frac{1}{2}$

Example 25. What is the coefficient of variation of x, characterized by the following probability density function: $f(x) = \frac{1}{4\sqrt{2\pi}} e^{\frac{-(x-10)^2}{32}}$ for $-\infty < x < \infty$ **(ICAI)**

(a) 50 (b) 60 (c) 40 (d) 30

Sol. (c) General form of probability density function is given as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Given in the question, $f(x) = \frac{1}{4\sqrt{2\pi}} e^{\frac{-(x-10)^2}{32}}$

where, σ is the standard deviation, μ is the mean

Comparing the coefficients,

σ = 4, μ = 10

Coefficient of Variation is given by the formula,

$$C.V. = \frac{\sigma}{\mu} \times 100$$

$$C.V. = \frac{4}{10} \times 100$$

$\Rightarrow$ C. V. = 40

Hence, the correct answer is option (c) i.e., 40.

Example 26. What is the first quartile of X having the following probability density function?

$$f(x) = \frac{1}{\sqrt{72\pi}} e^{\frac{-(x-10)^2}{72}} \text{ for } -\infty < x < \infty$$ **(ICAI)**

(a) 4 (b) 5 (c) 5.95 (d) 6.75

Sol. (c) General form of probability density function is given as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(x-\mu)^2}{\sigma^2}\right)}$$

Given in the question,

$$f(x) = \frac{1}{\sqrt{72\pi}} e^{\frac{-(x-10)^2}{72}}$$

where, σ is the Standard deviation, μ is the mean

Comparing the coefficients,

$\Rightarrow 2\sigma^2 = 72$ and $\mu = 10$

$\Rightarrow \sigma = 6$ and $\mu = 10$

First Quartile is given as,

$Q_1 = \mu - 0.675(\sigma)$

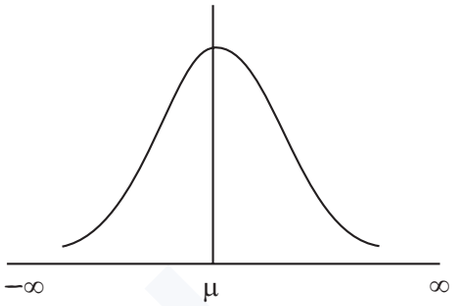
$= 10 - 0.675(6)$

$= 10 - 4.05 = 5.95$

Hence, the correct answer is option (c).

SOME IMPORTANT POINTS RELATING TO NORMAL DISTRIBUTION ARE LISTED BELOW

1. The name Normal Distribution has its origin some two hundred years back as the then mathematicians were in search for a normal model that can describe the probability distribution of most of the continuous random variables.



2. If we plot the probability function y = f (x), then the curve, known as probability curve, takes the following shape:
   - **Curve is symmetric:** If you look above curve is the symmetrical curve which is bell shaped and has one peak representing the mode of normal distribution.
   - **The line of symmetry is x = μ** which is similar to binomial distribution, which is symmetric about p = 0.5

   Tails of the Curve: If we look at the two tails of the normal curve, it extends indefinitely on both sides of the curve and both the left and right tails never touch the horizontal axis. The total area of the normal curve or for that any probability curve is taken to be unity i.e. one. Since the vertical line drawn through x = divides the curve into two equal halves, it automatically follows that The area between $-\infty$ to μ = the area between μ to ∞ = 0.5

When the mean is zero, we have

Area between $-\infty$ to 0 = Area between 0 to ∞ = 0.5

$\Rightarrow$ If we take μ = 0 and σ = 1, we have

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \text{ for } -\infty < z < \infty$$

The random variable z is known as **Standard normal variate** (or variable) or standard normal deviate.

The probability that a standard normal variate X would take a value less than or equal to a particular value say X = x is given by

$\phi(x) = P(X \le x)$

$\phi(x)$ is known as the cumulative distribution function.

We also have $\phi(0) = P(X \le 0)$

= Area of the standard normal curve between $-\infty$ and 0

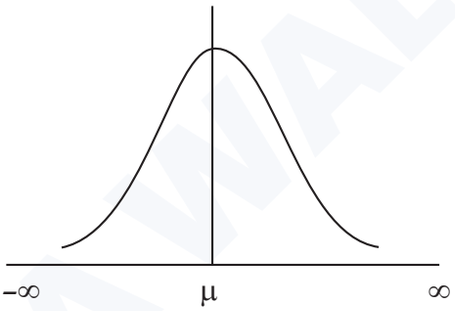
= 0.5

The normal distribution is known as biparametric distribution as it is characterized by two parameters μ and σ^2. Once the two parameters are known, the normal distribution is completely specified.

PROPERTIES OF NORMAL DISTRIBUTION

- Since $\pi = \frac{22}{7}, e^{-\theta} = \frac{1}{e^{\theta}} > 0$ whatever maybe, it follows that $f(x) \ge 0$ for every x.

- It can be shown that $\int_{-\infty}^{\infty} f(x) = 1$
- The mean of the normal distribution is given by μ . Further, since the distribution is symmetrical about $x = \mu$, it follows that the mean, median and mode of a normal distribution coincide, all being equal to μ.
- The standard deviation of the normal distribution is given by σ.
- Mean deviation of normal distribution is $\sigma\sqrt{\frac{2}{\pi}} = 0.8\sigma$
- The first and third quartiles are given by
  $Q_1 = m - 0.675\,\sigma$ and $Q_3 = m + 0.675\,\sigma$
  So, Quartile Deviation = 0.675σ
- The normal distribution is symmetrical about $x = \mu$. As such, its skewness is zero i.e. the normal curve is neither inclined to move towards the right (negatively skewed) nor towards the left (positively skewed).
- The normal curve $y = f(x)$ has two points of inflexion to be given by $x = \mu - \sigma$ and $x = \mu + \sigma$ i.e. at these two points, the normal curve changes its curvature from concave to convex and from convex to concave.



- If $x \sim N(\mu,\sigma^2)$ then $z = \frac{(x-\mu)}{\sigma} \sim N(0, 1)$, $z \sim N(0, 1)$, z is known as standardized normal variate or normal deviate.
- We also have $P(z \le k) = \phi(k)$ which is known as the cumulative distribution function.
- The values of $\phi(k)$ for different k are given in a table known as "Biometrika." Because of symmetry, we have $\phi(-k) = 1 - \phi(k)$
  We can evaluate the different probabilities in the following manner:
- $P(x < a) = P\left[\frac{x-u}{\sigma} < \frac{a-\mu}{\sigma}\right] = P(z < k)$ where, $k = \frac{a-\mu}{\sigma} = \phi(k)$
- Also $P(x \le a) = P(x < a)$ as x is continuous.
- $P(x > b) = 1 - P(x < b) = 1 - \phi\left(\frac{b-\mu}{\sigma}\right)$
- $P(a < x < b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$ ordinate at $x = a$ is given by $\left(\frac{1}{\sigma}\right)\phi\left(\frac{a-\mu}{\sigma}\right)$
- Also, $\phi(-k) = \phi(k)$

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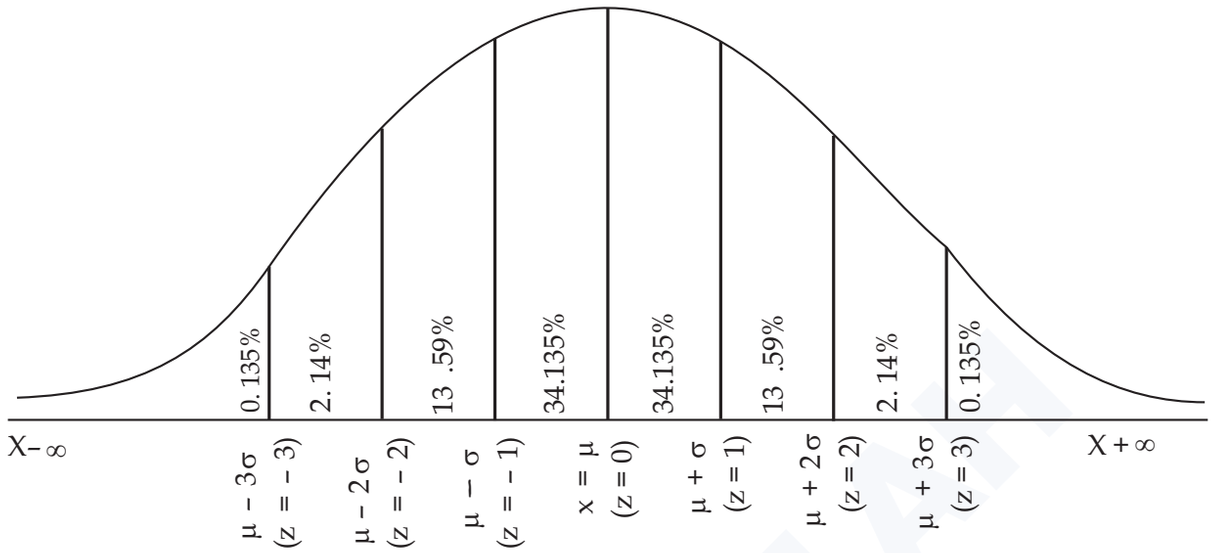
- The value of $\phi(k)$ for different k are also provided in the Biometrika Table.
- Area under the normal curve is shown in the following figure :



- If x and y are independent normal variables with means and standard deviations as μ_1 and μ_2 and σ_1 and σ_2 respectively, then z = x + y also follows normal distribution with mean ($\mu_1 + \mu_2$) and $SD = \sqrt{\sigma_1^2 + \sigma_2^2}$ respectively.

SOME IMPORTANT PROPERTIES OF STANDARD NORMAL VARIATE

If we take $\mu = 0$ and $\sigma = 1$,we have $f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ for $-\infty < z < \infty$

1. z has mean, median and mode all equal to zero.
2. The standard deviation of z is 1. Also the approximate values of mean deviation and quartile deviation are 0.8 and 0.675 respectively.
3. The standard normal distribution is symmetrical about z=0.
4. The two points of inflexion of the probability curve of the standard normal distribution are –1 and 1.
5. The two tails of the standard normal curve never touch the horizontal axis.
6. The upper and lower p per cent points of the standard normal variable z are given by $P(Z > z_p) = p$ and $P(Z < z_1 - p) = p$ i.e., $P(Z < z_p) = p$ respectively since for a standard normal distribution

   $(z_1 - p = - z_p)$

   Selecting P = 0.005, 0.025, 0.01 and 0.05 respectively, We have

   $z_{0.005} = 2.58$

   $z_{0.025} = 1.96$

   $z_{0.01} = 2.33$

   $z_{0.05} = 1.645$

APPLICATION OF NORMAL DISTRIBUTION

The applications of normal distribution are not restricted to statistics only. Many science subjects, social science subjects, management, commerce etc. find many applications of normal distributions. Most of the continuous variables like height, weight, wage, profit etc. follow normal distribution. If the variable under study does not follow normal distribution, a simple transformation of the variable, in many cases, would lead to the normal distribution of the changed variable.

Example 27. If the two quartiles of N (μ,σ^2) are 14.6 and 25.4 respectively, what is the standard deviation of the distribution? **(ICAI)**

(a) 9 (b) 6 (c) 10 (d) 8

Sol. (d) Given: First Quartile $(Q_1) = 14.6$, Third Quartile $(Q_3) = 25.4$

To find: Standard Deviation (σ)

As we know,

Quartile deviation is given as, $Q_D = \frac{Q_3 - Q_1}{2}$

$$Q_D = \frac{25.4 - 14.6}{2}$$

$$Q = 5.4$$

Standard deviation is given as, S.D $(\sigma) = \frac{3 \times Q_D}{2}$

$$\sigma = \frac{3(5.4)}{2}$$

$$\sigma = \frac{16.2}{2}$$

$$\sigma = 8.1 \approx 8$$

Hence, the correct answer is option (d).

Example 28. If the mean deviation of a normal variable is 16, what is its quartile deviation?

(a) 10.00 (b) 13.50 (c) 15.00 (d) 12.05 **(ICAI)**

Sol. (b) Given: Mean Deviation = 16

To find: Quartile Deviation (Q_D)

Mean deviation is given by the formula,

Mean Deviation = $0.8 \times \sigma$ [$\sigma \rightarrow$ Standard Deviation]

$16 = 0.8\ \sigma$

$$\sigma = \frac{16}{0.8}$$

$\sigma = 20$

Quartile deviation is given as,

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$Q_D = 0.675 \times \sigma$

$= 0.675(20)$

$= 13.50$

Hence, the correct answer is option (b) i.e., 13.50.

Example 29. If the quartile deviation of a normal curve is 4.05, then its mean deviation is

(a) 5.26 (b) 6.24 (c) 4.24 (d) 4.80 **(ICAI)**

Sol. (d) Given: Quartile Deviation (Q_D) = 4.05

We know that,

Quartile deviation is given as,

$\Rightarrow Q_D = 0.675 \times \sigma$

$\Rightarrow 4.05 = 0.675 \times \sigma$

$\Rightarrow \sigma = \frac{4.05}{0.675}$

$\Rightarrow \sigma = 6$

Now, Mean Deviation will be given as,

Mean Deviation = $0.8 \times \sigma$ [$\sigma \rightarrow$ Standard Deviation]

$= 0.8 \times 6 = 4.8$

Hence, the correct answer is option (d).

Example 30. If the First quartile and mean deviation about median of a normal distribution are 13.25 and 8 respectively, then the mode of the distribution is **(ICAI)**

(a) 20 (b) 10 (c) 15 (d) 12

Sol. (a) Given: First Quartile (Q_1) = 13.25

Mean Deviation = 8

Mean deviation is given by the formula,

Mean Deviation = $0.8 \times \sigma$ [$\sigma \rightarrow$ Standard Deviation]

$\Rightarrow 8 = 0.8 \times \sigma$

$\Rightarrow \sigma = \frac{8}{0.8}$

$\Rightarrow \sigma = 10$

First Quartile is given as,

$\Rightarrow Q_1 = \mu - 0.675(\sigma)$ [$\mu \rightarrow$ Mean]

$\Rightarrow 13.25 = \mu - 0.675(10)$

$\Rightarrow \mu = 13.25 + 6.75$

$\Rightarrow \mu = 20$

As we know, in Normal Distribution,

Mean = Median = Mode

So, Mode = 20

Hence, the correct answer is option (a).

Example 31. If the inflexion points of a normal distribution are 6 and 14..Find its standard deviation?

(a) 4 (b) 6 (c) 10 (d) 12

Sol. (a) Points of inflexion of a normal curve = Mean ± Standard Deviation

i.e. $x = \mu \pm \sigma$

Here, x values are 6 and 14

So, $\mu + \sigma = 14$ ...(i)

$\mu - \sigma = 6$ ...(ii)

Add equation (i) and (ii),

Substitute in eq. (i)

$\Rightarrow 10 + \sigma = 14$

$\Rightarrow \sigma = 14 - 10$

$\Rightarrow \sigma = 4$

Hence, the correct option is (a) i.e. 4.

Exampl 32. If the area of standard normal curve between z = 0 to z = 1 is 0.3413, then the value of $\varphi(a)$ is **(ICAI)**

(a) 0.5000 (b) 0.8413 (c) -0.5000 (d) 1

Sol. (b) Given:

Area of standard normal curve between z = 0 to z = 1 is 0.3413

To find: $\varphi(1)$

$\varphi(1)$ will be given as, the area of standard normal curve between $-\infty$ to 1.

This will divided into 2 segments, i.e., ($-\infty$ to 0) and (0 to 1)

We know that,

Area between $z = -\infty$ to z = 0 is 0.5.

Now, area between $z = -\infty$ to z = 1

= 0.5 + 0.3413

= 0.8413

Therefore, $\varphi(1) = 0.8413$

Hence, the correct answer is option **(b)**.

Example 33. Area of the normal curve **(ICAI)**

(a) between $-\infty$ to μ is 0.50 (b) between μ to $-\infty$ is 0.50

(c) between $-\infty$ to ∞ is 0.50 (d) both (a) and (b)

Sol. (d) Normal Distribution is also called Gaussian or bell curve. It is defined as probability distribution where the values of a random variable are distributed symmetrically. These values are equally distributed on the left and the right side of the central tendency.

It is formulated as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for} -\infty < x < \infty$$

As we can see in the formula x lies in between -∞ to ∞.

So by definition, Area of normal curve is between - ∞ to μ is 0.50 and between μ to - ∞ is 0.50.

Hence, the correct answer is option (d).

Example 34. The cumulative distribution function of a random variable X is given by **(ICAI)**

(a) $F(x) = P(X \le x)$ (b) $F(x) = P(X < x)$

(c) $F(x) = P(X \ge x)$ (d) $F(x) = P(X = x)$

Sol. (a) The cumulative distribution function (CDF) calculates the cumulative probability for a given x-value.

CDF is used to determine the likelihood that a random observation taken from the population will be less than or equal to a particular value.

It is defined as $F(x) = P(X \le x)$.

Hence, the correct answer is option (a).

Example 35. If X follows normal distribution with μ = 50 and σ = 10, what is the value of $P\left(\frac{x \le 60}{x > 50}\right)$? **(ICAI)**

(a) 0.8413 (b) 0.6828 (c) 0.1587 (d) 0.7256

Sol. (b) Given: X follows normal distribution where μ = 50 and σ = 10

To find: $P\left(\frac{x \le 60}{x > 50}\right)$

z is given by the formula, $z = \frac{x-\mu}{\sigma}$

$P\left(\frac{X \le 60}{X > 50}\right)$ this will be given as, $P\left(\frac{50 < X \le 60}{X > 50}\right)$

When $P(50 < X \le 60)$,

$$P\left(\frac{50-50}{10} < \frac{x-\mu}{\sigma} \le \frac{60-50}{10}\right)$$

$P(0 < z \le 1) = 0.3414$ (Using z score table)

When $P(X > 50)$

$$P\left(\frac{x-\mu}{\sigma} > \frac{50-50}{10}\right)$$

$P(z > 0) = 0.500$ (Using z score table)

Now, put the value of

$P(50 < X \le 60)$ and $P(X > 50)$ in $P\left(\frac{50 < x \le 60}{x > 50}\right)$, we get

$= \frac{0.3414}{0.500} = 0.6828$

Hence, the correct answer is option (b).

Example 36. For a certain type of mobiles, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. A person owns one of those mobiles and wants to know the probability that length of time will be between 50 & 70 hours is

(Given: ϕ (1.33) = 0.9082, ϕ (0) = 0.5) **(July 2021)**

(a) -0.4082 (b) 0.5 (c) 0.4082 (d) -0.5

Sol. (c) We know that $z = \frac{x - \mu}{\sigma}$

Given that μ = 50 hours, S. D. = σ = 15

So, $P(50 \le X \le 70) = P\left(\frac{50-50}{15} \le z \le \frac{70-50}{15}\right)$

$P(50 \le X \le 70) = P(0 \le z \le 1.33)$

$= \phi(1.33) - \phi(0)$

$= 0.9082 - 0.5$

$= 0.4082$

Hence, option (c) is correct.

PRACTICE QUESTIONS (PART C)

1. The probability density function of a normal variable is given by:

(a) $f(x) = \sigma\frac{1}{\sqrt{2\pi\sigma}} \cdot e^{\frac{(x-\lambda^2)}{2\sigma^2}}$ for $0 < x < \infty$

(b) $f(x) = \frac{1}{\sqrt{2\pi\sigma}} \cdot e^{\frac{(x-\mu^2)}{2\sigma^2}}$ for $-\infty < x < \infty$

(c) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$

(d) None of these

2. X and Y are two independent Normal variables, then the distribution of X + Y is _____

(a) Normal Distribution (b) t-distribution

(c) Chi-Square distribution (d) F-distribution

3. The Normal Curve is

(a) Positively Skewed (b) Negatively Skewed

(c) Symmetrical (d) All of these

4. If the two quartiles of N (μ,σ^2) are 47.30 and 52.70 respectively, what is the mean deviation about the median of the distribution?

(a) 9.0 (b) 6.5 (c) 3.20 (d) 8.45

5. Shape of Normal Distribution Curve

(a) Depends on its parameters (b) Does not depend on its parameters

(c) Either (a) or (b) (d) Neither (a) nor (b)

6. In Normal distribution 95% observation lies between _____ & _____ .

(a) $(\mu - 2\sigma, \mu + 2\sigma)$ (b) $(\mu - 3\sigma, \mu + 3\sigma)$

(c) $(\mu - 1.96\sigma, \mu + 1.96\sigma)$ (d) $(\mu - 2.58\sigma, \mu + 2.58\sigma)$

7. If x is a poisson variate with mean m then $z = \frac{x-m}{\sqrt{m}}$ follows _____ distribution.

(a) Normal (b) Binomial (c) Bernoulli (d) None of the above

8. The points of inflexion of the normal curve $f(t) = \frac{1}{4\sqrt{2\pi}} e^{\frac{-(t-10)^2}{32}}$ are

(a) 6, 14 (b) 5, 15 (c) 4, 16 (d) None of these

9. X follows normal distribution with mean as 50 and variance as 100. What is $P(x \geq 60)$? [Given$\varphi(1) = 0.8413$]

(a) 0.20 (b) 0.40 (c) 0.16 (d) 0.30

10. If the points of inflexion of a normal curve are 40 and 60 respectively, then its mean is

(a) 40 (b) 45 (c) 50 (d) 60

Answer Key

1. (c) **2.** (a) **3.** (c) **4.** (c) **5.** (a) **6.** (c) **7.** (a) **8.** (a) **9.** (c) **10.** (c)

Example 37. The mean of a binomial distribution with parameter n and p is **(ICAI)**

(a) n (1 – p) (b) np (1-p) (c) np (d) $\sqrt{np(1-p)}$

Sol. (c) Binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^nC_x\, p^x q^{n-x}$

Where, n → Number of Trials

x → 0, 1, 2, 3,....

p → Success

q → Failure

The mean or expected value (E) is given as multiplying the number of trials (n) and the probability of success (p).

$\Rightarrow (E) = np$

Hence, the correct answer is option (c).

Example 38. The variance of a binomial distribution with parameters n and p is **(ICAI)**

(a) $np^2 (1 - p)$ (b) $\sqrt{np(1-p)}$ (c) $nq (1 - q)$ (d) $n^2p^2 (1 - p)^2$

Sol. (c) Binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^nC_x\, p^x q^{n-x}$

Where, $n \rightarrow$ Number of Trials

$x \rightarrow 0, 1, 2, 3, ...$

$p \rightarrow$ Success

$q \rightarrow$ Failure

Variance of the binomial distribution is a measure of the dispersion of the probabilities with respect to the mean value.

It is formulated by, $\sigma = npq$ where, $p = 1 - q$

Thus, $\sigma = nq(1-q)$

Hence, the correct answer is option (c).

Example 39. The maximum value of the variance of a binomial distribution with parameters n and p is **(ICAI)**

(a) $\frac{n}{2}$ (b) $\frac{n}{4}$ (c) $np(1 - p)$ (d) $2n$

Sol. (b) Binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^nC_x\, p^x q^{n-x}$

Where, $n \rightarrow$ Number of Trials

$x \rightarrow 0, 1, 2, 3, ...$

$p \rightarrow$ Success

$q \rightarrow$ Failure

Variance of the binomial distribution is a measure of the dispersion of the probabilities with respect to the mean value.

It is formulated by, $\sigma = npq$

$\Rightarrow np(1 - p)$

When, $p = q = \frac{1}{2}$ (i.e., when the occurrence of success and failure is equal)

$$\Rightarrow n \times \frac{1}{2} \times \left(1 - \frac{1}{2}\right)$$

$$\Rightarrow n \times \frac{1}{2} \times \frac{1}{2}$$

$$\Rightarrow \frac{n}{4}$$

Hence, the correct answer is option (b).

Example 40. The method usually applied for fitting a binomial distribution is known as

(a) Method of least square **(ICAI)**

(b) Method of moments

(c) Method of probability distribution

(d) Method of deviations

Sol. (b) Binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

Where, $n \rightarrow$ Number of Trials

$x \rightarrow 0,1,2,3, \ldots.$

$p \rightarrow$ Success

$q \rightarrow$ Failure

The method usually applied for fitting a binomial distribution is known as the method of moments.

Hence, the correct answer is option (b).

Example 41. The normal curve is **(ICAI)**

(a) Positively skewed (b) Negatively skewed

(c) Symmetrical (d) All of these

Sol. (c) Normal Distribution is also called Gaussian or bell curve. It is defined as probability distribution where the values of a random variable are distributed symmetrically. These values are equally distributed on the left and the right side of the central tendency.

It is formulated as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for } -\infty < x < \infty$$

The key points associated with Normal curve are listed below:

- It is symmetrical about the ordinate at the central point of the curve.
- The skewness for a normal distribution is zero and the data is symmetric.

From the above points we can clearly see that the normal curve is symmetric.

Hence, the correct answer is option (c).

Example 42. The mean and mode of a normal distribution **(ICAI)**

(a) May be equal (b) May be different

(c) Are always equal (d) (a) or (b)

Sol. (c) Normal Distribution is also called Gaussian or bell curve. It is defined as probability distribution where the values of a random variable are distributed symmetrically. These values are equally distributed on the left and the right side of the central tendency.

It is formulated as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \text{ for } -\infty < x < \infty$$

As normal distribution is symmetrical, i.e., mean, median and mode are all equal.

Hence, the correct answer is option (c).

Example 43. The interval $(\mu - 3\sigma, \mu + 3\sigma)$ covers **(ICAI)**

(a) 95% area of a normal distribution

(b) 96% area of a normal distribution

(c) 99% area of a normal distribution

(d) all but 0.27% area of a normal distribution

Sol. (d) Probability density function defines the density of the probability that a continuous random variable will lie within a particular range of values.

It is formulated as, $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$

According to the question, the interval $(\mu - 3\sigma, \mu + 3\sigma)$ will cover all but 0.27% area of a normal distribution.

Hence, the correct answer is option (d).

Example 44. The probability mass function of binomial distribution is given by **(ICAI)**

(a) $f(x) = p^x q^{n-x}$ (b) $f(x) = {}^nC_x p^x q^{n-x}$

(c) $f(x) = {}^nC_x q^x p^{n-x}$ (d) $f(x) = {}^nC_x p^{n-x} q^x$

Sol. (b) Probability mass function of binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^nC_x\ p^x q^{n-x}$

Where, $n \rightarrow$ Number of Trials

$p \rightarrow$ Probability of getting success

$q \rightarrow$ Probability of getting failure

Hence, the correct answer is option (b) i.e., $f(x) = {}^nC_x\ p^x q^{n-x}$.

Example 45. If X ~ P (m) and its coefficient of variation is 50, what is the probability that X would assume only non-zero values? **(ICAI)**

(a) 0.018 (b) 0.982 (c) 0.989 (d) 0.976

Sol. (b) Given: Coefficient of variation (C.V.) = 50

Coefficient of Variation is given by the formula,

$$C.V = \frac{\sigma}{\bar{X}} \times 100 \quad [\sigma \rightarrow \text{Standard deviation}, \ \bar{X} \rightarrow \text{Mean}]$$

In Poisson Distribution, Mean $(\bar{X})$ = Variance $(\gamma) = m$

As we know, Standard Deviation $(\sigma) = \sqrt{\gamma} = \sqrt{m}$

Put the respective values,

$$50 = \frac{\sqrt{m}}{m} \times 100$$

$$\Rightarrow \sqrt{m} = 2$$

$$\Rightarrow m = 4$$

Probability mass function will be given as,

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

$$P(X = 0) = \frac{e^{-4} 4^0}{0!}$$

$$= \frac{1}{e^4} = \frac{1}{(2.71282)^4}$$

= 0.01846

Thus, the probability that X would assume only non-zero values i.e.

P (X ≥ 1) = 1 – P (X = 0)

= 1 – 0.01846

= 0.98154 ≈ 0.982

Hence, the correct answer is option (b).

Example 46. If the points of inflexion of a normal curve are 40 and 60 respectively, then its mean deviation is

(a) 8 (b) 5 (c) 10 (d) 6

Sol. (a) Given: Points of inflexion of a normal curve are 40 and 60

Let σ be the standard deviation and μ be the mean, then

According to the question,

μ – σ = 40 ...(i)

μ + σ = 60 ...(ii)

Adding (i) and (ii)

$2\mu = 100$

$\Rightarrow \mu = \frac{100}{2}$

$\Rightarrow \mu = 50$

Put $\mu = 50$ in (i)

$\Rightarrow 50 - \sigma = 40$

$\Rightarrow \sigma = 50 - 40$

$\Rightarrow \sigma = 10$

Since, the distribution is symmetric about mean value

Thus, mean, median and mode will coincide.

Mean deviation will be given as,

Mean Deviation = $0.8\ \sigma = 0.8(10) = 8$

Hence, the correct answer is option (a).

Example 47. The quartile deviation of a normal distribution with mean 10 and standard deviation 4 is **(Dec 2020)**

(a) 54.24 (b) 23.20 (c) 0.275 (d) 2.70

Sol. (d) Given,

Mean = 10, Standard deviation = 4

We know that,

Quartile deviation is given as,

$\Rightarrow Q_D = 0.675 \times \sigma$

$= 0.675(4) = 2.70$

Hence, the correct answer is option (d).

Example 48. The salary of workers of a factory is known to follow normal distribution with an average salary of ₹ 10,000 and standard deviation of salary as ₹ 2,000. If 50 workers receive salary more than ₹ 14,000, then the total number of workers in the factory is

(a) 2193 (b) 2000 (c) 2200 (d) 2500 **(ICAI)**

Sol. (a) Given: $\mu = 10,000$ and $\sigma = 2,000$

Now, $P(x > 14,000)$

$= P\left(\frac{x - 10000}{2000} > \frac{14000 - 10000}{2000}\right)$

$= P(Z > 2)$

$= 0.5 - P(0 \le Z \le 2)$

$= 0.5 - 0.4772$

$= 0.0228$

Number of workers having salary more than 14,000 = 50

Therefore, $N \times P(X > 14000) = 50$

$\Rightarrow N \times 0.0228 = 50$

$\Rightarrow N = \frac{50}{0.0228}$

$\Rightarrow N = 2192.98$

$\Rightarrow N = 2193$

Therefore, the total number of workers in the factory is 2,193.

Hence, the correct answer is option (a).

Example 49. 50 percent of a certain product has weight 60 kg or more whereas 10 per cent have weight 55 kg or less. On the assumption of normality, what is the variance of weight? Given $\varphi(1.28) = 0.90$ **(ICAI)**

(a) 15.21 (b) 9.00 (c) 16.00 (d) 22.68

Sol. (a) Given,

$P(X > 60) = 0.5$

$$\Rightarrow P\left(\frac{X-\mu}{\sigma} > \frac{60-\mu}{\sigma}\right) = 0.5$$

$$\Rightarrow P(Z > 0) = 0.5$$

$$\Rightarrow \frac{60-\mu}{\sigma} = 0$$

$\Rightarrow \mu = 0$

Also, $\phi(1.28) = 0.90$

$$\Rightarrow P(-\infty < Z < 1.28) = 0.90$$

$$\Rightarrow P(0 < Z < 1.28) = 0.90 - 0.50$$

$$\Rightarrow P(0 < Z < 1.28) = 0.4$$

$P(X < 55) = 0.10$

$$\Rightarrow P\left(\frac{X-60}{\sigma} < \frac{55-60}{\sigma}\right) = 0.10$$

$$\Rightarrow P\left(Z < -\frac{5}{\sigma}\right) = 0.1$$

$$\Rightarrow P\left(-\frac{5}{\sigma} < 0\right) = 0.4$$

$$\Rightarrow -\frac{5}{\sigma} = -1.28$$

$$\Rightarrow \sigma = \frac{-5}{-1.28}$$

$$\Rightarrow \sigma^2 = \left(\frac{5}{1.28}\right)^2 = 15.258$$

Therefore, the variance of weight is 15.21 (approx.).

Hence, the correct answer is option (a).

Example 50. If the weekly wages of 5000 workers in a factory follows normal distribution with mean and standard deviation as ₹ 700 and ₹ 50 respectively, what is the expected number of workers with wages between ₹ 660 and ₹ 720? **(ICAI)**

Given, $\Phi(0.8) = 0.2881$, $\Phi(0.4) = 0.1554$

(a) 2,050 (b) 2,200 (c) 2,218 (d) 2,300

Sol. (c) Given,

X follows normal distribution with $\mu = 700$, $\sigma = 50$ and $N = 5000$

Number of workers with wages between ₹ 660 and ₹ 720 =

$N \times P(660 < X < 720)$

Now, $P(660 < X < 720)$

$$= P\left(\frac{660-700}{50} < \frac{X-700}{50} < \frac{720-700}{50}\right)$$

$= P(-0.8 < Z < 0.4)$

$= P(-0.8 < Z < 0) + P(0 < Z < 0.4)$

$= P(o < Z < 0.8) + P(0 < Z < 0.4)$

$= 0.2881 + 0.1554$

$= 0.4435$

Number of workers with wages between ₹ 660 and ₹ 720

$\Rightarrow N \times P(660 < X < 720)$

$= 5000 \times 0.4435$

$= 2217.5 \approx 2218$

Hence, the correct answer is option (c).

Example 51. In a sample of 800 students, the mean weight and standard deviation of weight are found to be 50 kg and 20 kg respectively. On the assumption of normality, what is the number of students weighing between 46 kg and 62 kg ?

Given area of the standard normal curve between z = 0 to z = 0.20 = 0.0793 and area between z = 0 to z = 0.60 = 0.2257. **(ICAI)**

(a) 250 (b) 244 (c) 240 (d) 260

Sol. (b) Given, The weight of the student follows normal distribution with $\mu = 50$ and $\sigma = 20$.

Number of students having weights between 46 and 62

$= N \times P(46 \le X \le 62)$

Now, P (46 < X < 62)

$\Rightarrow P\left(\frac{46-50}{20} < \frac{X-50}{20} < \frac{62-50}{20}\right)$

$\Rightarrow$ P (–0.2 < Z < 0.6)

$\Rightarrow$ P (-0.2 < Z < 0) + P (0 < Z < 0.6)

$\Rightarrow$ P (0 < Z < 0.2) + P (0 < Z < 0.6)

$\Rightarrow$ 0.0793 + 0.2257

$\Rightarrow$ 0.3050

Number of students having weighing between 46 and 62

= N × P (46 ≤ X ≤ 62)

$\Rightarrow$ 800 × 0.3050

$\Rightarrow$ 244

Hence, the correct answer is option (b).

FULL CHAPTER PRACTICE QUESTIONS

1. Standard deviation of binomial distribution is: **(ICAI)**

(a) $(npq)^2$ (b) $\sqrt{npq}$ (c) $(np)^2$ (d) $\sqrt{np}$

2. The wages of workers of factory follows: **(June 2016)**

(a) Binomial distribution (b) Poisson distribution

(c) Normal distribution (d) Chi-square distribution

3. The normal curve is:

(a) Positively skewed (b) Negatively skewed

(c) Symmetrical (d) All of these

4. If for a Binomial distribution B(n, p); n = 4 and also P(x = 2) = P(x = 3) then the value of p is **(Dec 2019)**

(a) 9/11 (b) 1 (c) 1/3 (d) 1/9

5. In a discrete random variable X follows uniform distribution and assumes only the values 8, 9, 11, 15, 18, 20. Then P(X ≤ 15) is _____

(a) 1/2 (b) 1/3 (c) 2/3 (d) 2/5

6. If x and y are independent normal variates with Mean and Standard Deviation as μ_1 and μ_2 and σ_1 and $\sigma_2$respectively, then z = x + y also follows normal distribution with

(a) Mean = $\mu_1 + \mu_2$ and S.D. = 0

(b) Mean = 0 and S.D. = $\sigma_1^2 + \sigma_2^2$

(c) Mean = $\mu_1 + \mu_2$ and S.D. = $\sqrt{\sigma^2_1 + \sigma_2^2}$

(d) None of these.

7. In Poisson distribution $\mu_4 = 2$, its variance i.e. μ_2 is

(a) 2/3 (b) 1/2 (c) 1/3 (d) 3/2

8. Name the distribution which has Mean = Variance.

(a) Binomial distribution (b) Poisson distribution

(c) Normal distribution (d) Chi-square

9. An example of a bi-parametric discrete probability distribution is: **(Dec 2017)**

(a) Binomial distribution (b) Poisson distribution

(c) Normal distribution (d) Both (a) and (b)

10. If X ~ N (50, 16), then which of the following is not possible? **(June 2017)**

(a) P(X > 60) = 0.30 (b) P(X < 50) = 0.50

(c) P(X < 60) = 0.40 (d) P(X > 50) = 0.50

11. If for a distribution mean = variance, then the distribution is said to be:

(a) Normal (b) Binomial

(c) Poisson (d) None of these

12. For a Binomial distribution if variance = $(\text{Mean})^2$, then the values of n and p will be:

(a) 1 and 1/2 (b) 2 and 1/2 (c) 3 and 1/2 (d) 1 and 1

13. In a normal distribution about 95 per cent of the observations lie between _____and_____.

(a) $\mu - 2\sigma, \mu + 2\sigma$ (b) $\mu - 3\sigma, \mu + 3\sigma$

(c) $\mu - 1.96\sigma, \mu + 1.96\sigma$ (d) $\mu - 2.58\sigma, \mu + 2.58\sigma$

14. An example of a bi-parametric discrete probability distribution is

(a) Binomial distribution (b) Poisson distribution

(c) Normal distribution (d) Both (a) and (b)

15. The variance of a binomial distribution with parameters n and p is:

(a) $np^2(1- p)$ (b) $np(1- p)$ (c) $nq(1- q)$ (d) $n^2p^2 (1 - p)^2$

16. X is a Poisson variate satisfying the following condition:

$9\ P(X = 4) + 90(X = 6) = P\ (X = 2)$.What is the value of $P(X \leq 1)$?

(a) 0.5655 (b) 0.6559 (c) 0.7358 (d) 0.8201

17. Probability distribution may be

(a) discrete (b) continuous (c) infinite (d) (a) or (b)

18. For a Poisson variate X, $P(X = 2) = 3P\ (X = 4)$, then the standard deviation of X is

(a) $\sqrt{2}$ (b) 3 (c) 4 (d) 5 **(Nov 2018)**

19. The mean of the Binomial distribution $B(4, \frac{1}{3})$ is equal to **(Nov 2018)**

(a) 3/5 (b) 4/3 (c) 8/3 (d) 3/4

20. If for a normal distribution $Q_1 = 54.52$ and $Q_3 = 78.86$, then the median of the distribution is **(Nov 2018)**

(a) 12.17 (b) 39.43 (c) 66.69 (d) None of these

21. What is the mean of X having the following density function?

$$f(x) = \frac{1}{4\sqrt{2\pi}} e^{-\frac{(x-10)^2}{32}} \text{ for } -\infty < x < \infty$$ **(Nov 2018)**

(a) 4 (b) 10 (c) 40 (d) None of the above

22. The probability that a student is not a swimmer is 1/5, then the Probability that out of five students four are swimmer is

(a) $(4/5)^4(1/5)$ (b) ${}^5C_1\,(1/5)^4(4/5)$

(c) ${}^5C_4\,(4/5)^1\,(1/5)^4$ (d) None of these

Answer Key

1. (c)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (c)
11. (c)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (b)	**20.** (c)
21. (a)	**22.** (c)								

PREVIOUS YEAR QUESTION (PYQ'S)

1. 4 coins are tossed 1600 times. What is the probability that all 4 coins do not turn head upward at a time? **(June 2019)**

(a) $1600e^{-100}$ (b) $1000e^{-100}$ (c) $100e^{-1600}$ (d) e^{-1600}

2. If mean and variance are 5 and 3 respectively, then relation between p and q is: **(June 2019)**

(a) $p > q$ (b) $p < q$ (c) $p = q$ (d) p is symmetric

3. In a Poisson distribution, if $P(x=4) = P(x=5)$, then the parameter of Poisson distribution is: **(June 2019)**

(a) $\frac{4}{5}$ (b) $\frac{5}{4}$ (c) 4 (d) 5

4. Area between -1.96 to $+1.96$ in a normal distribution is: **(June 2019)**

(a) 95.45% (b) 95% (c) 96% (d) 99%

5. In normal distribution, what is the ratio of QD: MD : SD. **(June 2019)**

(a) 12 : 10 : 15 (b) 15 : 10 : 12 (c) 10 : 15 : 12 (d) 10 : 12 : 15

6. For a normal distribution, $\sqrt{\frac{2}{\pi}} e^{-2(x-3)^2}$, mean and standard deviation will be **(Dec 2019)**

(a) $3, \frac{1}{2}$ (b) $3, \frac{1}{\sqrt{2}}$ (c) $3, \sqrt{2}$ (d) None of these

7. Area covered in a normal curve by $(\mu \pm 3\sigma)$ is **(Dec 2019)**

(a) 68.28% (b) 95.96% (c) 99.73% (d) 99.23%

8. If x is binomial variate with parameter 15 and $\frac{1}{3}$, then what is the value of mode of the distribution? **(Dec 2019)**

(a) 5 and 6 (b) 5.5 (c) 5 (d) 6

9. In Poisson distribution, which of the following is same? **(Dec 2019)**

(a) Mean and Variance (b) Mean and SD

(c) Both (d) None of these

10. If for a Binomial distribution, B(n, p); n = 4 and also P(x = 2) = 3 P(x = 3), then the value of P is equal to **(Dec 2019)**

(a) $\frac{9}{11}$ (b) 1 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

11. Let x be a Poisson random variate with parameter λ. Then p(x) is equal to **(Dec 2019)**

(a) $\frac{e^{\lambda}-e^{-\lambda}}{2}$ (b) $\frac{e^{\lambda}+e^{-\lambda}}{2}$ (c) $\frac{e^{2\lambda}-1}{2}$ (d) $\frac{\lambda^x e^{-\lambda}}{x!}$

12. Which of the following is a uni-parametric distribution? **(Dec 2020)**

(a) Normal (b) Poisson (c) Binomial (d) Hyper geometric

13. If the probability of success in a binomial distribution is less than one-half, then the binomial distribution _____ **(Dec 2020)**

(a) Is skewed to the left (b) Is skewed to the right

(c) Has two modes (d) Has median at a point > mean + $\frac{1}{2}$

14. If we change the parameter(s) of a _____ distribution, the shape of the probability curve does not change. **(Dec 2020)**

(a) Binomial (b) Normal (c) Poisson (d) Non-Gaussian

15. Which one of the following has Poisson distribution? **(Dec 2020)**

(a) The number of days to get a complete cure.

(b) The number of defects per meter on a long roll of coated polythene sheet.

(c) The errors obtained in repeated measuring of the length of a rod.

(d) The number of claims rejected by an insurance agency.

16. For a Poisson distributed variable X, we have P(X = 7) = 8. P(X = 9), the mean of the distribution is **(Dec 2020)**

(a) 4 (b) 3 (c) 7 (d) 9

17. The quartile deviation of a normal distribution with mean 10 and standard deviation 4 is **(Dec 2020)**

(a) 54.24 (b) 23.20 (c) 0.275 (d) 2.70

18. If the parameter of Poisson distribution is m and mean + S.D $= \frac{6}{25}$, then find m.

(Dec 2020)

(a) $\frac{3}{25}$ (b) $\frac{1}{25}$ (c) $\frac{4}{25}$ (d) $\frac{3}{5}$

19. If an unbiased coin is tossed twice, then the probability of obtaining at least on tails is

(Jan 2021)

(a) 1 (b) 0.5 (c) 0.75 (d) 0.25

20. If an unbiased coin is tossed three times, what is the probability of getting more than one head? **(Jan 2021)**

(a) $\frac{1}{2}$ (b) $\frac{3}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{3}$

21. A coin with probability for head as $\frac{1}{5}$ as tossed 100 times. The standard deviation of the number of head 5 turned up is **(Jan 2021)**

(a) 3 (b) 2 (c) 4 (d) 6

22. If x is a Poisson variable and $P(x=1)=P(x=2)$, then $P(x=4)$ is **(Jan 2021)**

(a) $\frac{2}{3}e^{-2}$ (b) $\frac{2}{3}e^{4}$ (d) $\frac{3}{2}e^{-2}$ (e) $\frac{3}{2}e^{4}$

23. Which one of the following is an uni-parametric distribution? **(Jan 2021)**

(a) Poisson (b) Normal (c) Binomial (d) Hyper geometric'

24. For a normal distribution, the value of third moment about mean is **(Jan 2021)**

(a) 0 (b) 1 (c) 2 (d) 3

25. The value of K for the probability density function of a variate X is equal to

(July 2021)

X	0	1	2	3	4	5	6
P(X)	5K	3K	4K	6K	7K	9K	11K

(a) 39 (b) $\frac{1}{40}$ (c) $\frac{1}{49}$ (d) $\frac{1}{45}$

26. In normal distribution, Mean, Median and Mode are **(July 2021)**

(a) Zero (b) Not equal (c) Equal (d) Null

27. If X is a Poisson variate such that $P(x=1)=0.7, P(x=2)=0.3$, then $P(x=0)=$ **(July 2021)**

(a) $e^{\frac{6}{7}}$ (b) $e^{-\frac{6}{7}}$ (c) $e^{-\frac{2}{3}}$ (d) $e^{-\frac{1}{3}}$

28. If X is a binomial variate with $p = \frac{1}{3}$, for the experiment of 90 trials, then the standard deviation is equal to **(July 2021)**

(a) $-\sqrt{5}$ (b) $\sqrt{5}$ (c) $\sqrt[2]{5}$ (d) $\sqrt{15}$

29. For a certain type of mobiles, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. A person owns one of these mobiles and wants to know the probability that the length of time will be between 50 and 70 hours is **(July 2021)**

(Given $\phi(1.33) = 0.9082, \phi(0) = 0.5$)

(a) -0.4082 (b) 0.5 (c) 0.4082 (d) -0.5

30. The average number of advertisements per page appearing in a newspaper is 3. What is the probability that in a particular page zero number of advertisements are there? **(Dec 2021)**

(a) e^{-3} (b) e^{0} (c) e^{+3} (d) e^{-1}

31. If for a Poisson distributed random variable X, the probability for X taking value 2 is 3 times the probability for X taking value 4, then the variance of X is **(Dec 2021)**

(a) 4 (b) 3 (c) 2 (d) 5

32. Let X be a normal distribution with mean 2.5 and variance 1. If $P(a < X < 2.5) = 0.4772$ and that the cumulative normal probability value at 2 is 0.9772, then $a = ?$ **(Dec 2021)**

(a) 0.5 (b) 3 (c) -3.5 (d) -4.5

33. The manufacturer of a certain electronic component is certain that 2% of his product is defective. He sells the components in boxes of 120 and guarantees that not more than 2% in any box will be defective.

Find the probability that a box selected at random would fail to meet the guarantee?

(Given that $e^{-2.4} = 0.0907$) **(Dec 2021)**

(a) 0.49 (b) 0.39 (c) 0.37 (d) 0.43

34. A renowned hospital usually admits 200 patients everyday. One percent of patients, on an average, require special room facilities. On one particular morning, it was found that only one special room was available. What is the probability that more than 3 patients would require special room facilities? **(Dec 2021)**

(a) 0.1428 (b) 0.1732 (c) 0.2235 (d) 0.3450

35. The binomial distribution, having mean and standard deviation as 3 and 1.5, has number of trials equal to **(June 2022)**

(a) 3 (b) 6 (c) 8 (d) 12

36. The mean of binomial distribution is **(June 2022)**

(a) Always less than its variance (b) Always more than its variance

(b) Always equal than its variance (d) Always equal to its standard deviation

37. The variance of a normal distribution is given to be 16. The mean deviation about mode is **(June 2022)**

(a) 3.2 (b) 8 (c) 12.8 (d) 12

38. The standard deviation of a Poisson variate X is 1.732. The $P(-2.9 < X < 3.54)$ is **(June 2022)**

(a) $13e^{-3}$ (b) $9e^{-3}$ (c) $4e^{-2}$ (d) e^{-6}

39. For a normal distribution, the first and third quartiles are given to be 37 and 49, the mode of the distribution is **(June 2022)**

(a) 37 (b) 49 (c) 43 (d) 45

40. Skewness of normal distribution is **(Dec 2022)**

(a) Negative (b) Positive (c) Zero (d) Undefined

41. If a Poisson distribution is such that $P(X=2) = P(X=3)$, then the variance of the distribution is **(Dec 2022)**

(a) $\sqrt{3}$ (b) 3 (c) 6 (d) 9

42. The standard deviation of Binomial distribution is **(Dec 2022)**

(a) npq (b) $\sqrt{npq}$ (c) np (d) $\sqrt{np}$

43. The speed of a number of bikes follows a normal distribution model with a mean of 83 km/hr and a standard deviation of 9.4 km/hr. Find the probability that a bike picked at random is traveling at more than 95 km/hr. **(Dec 2022)**

(a) 0.102 (b) 0.38 (c) 0.49 (d) 0.278

44. Between 9 am and 10 am, the average number of phone calls per minute coming into the switchboard number of a company is 4. Find the probability that during the particular minute, there will be either 2 phone calls or no phone calls. (Given that $e^{-4} = 0.018316$) **(Dec 2022)**

(a) 0.156 (b) 0.165 (c) 0.149 (d) 0.194

45. The probability distribution of x is given below:

Value of x	1	0	Total
Probability	p	1 – p	1

Mean is equally to: **(Dec 2022)**

(a) p (b) 1 – p (c) 0 (d) 1

46. If a Poisson distribution is such that $P(x=2) = \frac{1}{3}P(x=3)$, then the standard deviation of the distribution is

(a) $\sqrt{3}$ (b) 3 (c) 2 (d) 1

47. If a random variable X has the following probability distribution, then the expected value of X is

X	-1	-2	0	1	2
f(x)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{3}$

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

48. On a commodity exchange when booking trades with provision for stop-losses, a trader can make a profit of ₹50,000 or incur a loss of ₹20,000. The probabilities of making a profit and incurring loss, from the past experience are known to be 0.75 and 0.25 respectively. The expected profit to be made by trader should be:

(a) ₹32,500 (b) ₹35,000 (c) ₹30,000 (d) ₹40,000

49. The incidence of skin disease in a chemical plant occurs in such a way that the workers have 20% chance of suffering from it. What is the probability that out of 6 workers 4 or more will have skin diseases?

(a) 0.1696 (b) 0.01696 (c) 0.1643 (d) 0.01643

50. For a binomial distribution, the mean and SD are 10 and 3 respectively. Find the value of n. **(Dec 2023)**

(a) 30 (b) 9 (c) 90 (d) 100

51. If six coins are tossed simultaneously. The probability of obtaining exactly two heads are **(Dec 2023)**

(a) 0.2343 (b) 0.9841 (c) 0.1268 (d) 0.0156

52. By plotting cumulative frequency against the respective class boundary, we get **(Dec 2023)**

(a) Frequency curve (b) Ogives
(c) Frequency polygon (d) Histogram

53. In a cumulative frequency curve, what is represented on the y-axis? **(Dec 2023)**

(a) Class interval (b) Cumulative frequency
(c) Frequency density (d) Relative frequency

54. A perpendicular line drawn from the point of intersection of two ogives on the horizontal axis gives the value of **(Dec 2023)**

(a) 2nd quartile (b) 3rd quartile (c) Mode (d) 1st quartile

55. If mean and variance of random variable which follows the binomial distribution are 7 and 6 respectively, then the probability of success is **(Dec 2023)**

(a) $\frac{6}{7}$ (b) $\frac{36}{49}$ (c) $\frac{1}{7}$ (d) $\frac{1}{49}$

56. In a standard normal distribution, the value of the mean (μ) and the standard deviation (σ) is **(Dec 2023)**

(a) $\mu = 0, \sigma = 0$ (b) $\mu = 0, \sigma = 1$ (c) $\mu = 1, \sigma = 0$ (d) $\mu = 1, \sigma = 1$

57. If x and y are two independent normal variables with means and SDs as μ_1, μ_2 and σ_1, σ_2 respectively, then $z = x + y$ follows normal distribution with mean and SD as: **(Dec 2023)**

(a) Mean $= \mu_1 + \mu_2$, SD $= \sqrt{{\sigma_1}^2 + {\sigma_2}^2}$ (b) Mean $= \frac{\mu_1 + \mu_2}{2}$, SD $= \sqrt{\frac{{\sigma_1}^2 + {\sigma_2}^2}{2}}$

(c) Mean $= \mu_1 + \mu_2$, SD $= \sqrt{{\sigma_1}^2 - {\sigma_2}^2}$ (d) Mean $= \frac{\mu_1 + \mu_2}{2}$, SD $= \sqrt{\frac{{\sigma_1}^2 - {\sigma_2}^2}{2}}$

58. The mean of Poisson distribution is 4. The probability of two successes is **(June 2024)**

(a) $\frac{8}{e^4}$ (b) $\frac{4}{e^4}$ (c) $\frac{16}{e^4}$ (d) $\frac{8}{e^2}$

59. For a normal distribution, the ratio of mean deviation to the standard deviation is **(June 2024)**

(a) 0.4 (b) 0.6 (c) 0.8 (d) 1.0

60. For a binomial distribution the variance is 0.2 and the mean is 0.6. The probability of getting 3 successes out of a trial of 5 is **(June 2024)**

(a) $\frac{80}{3^5}$ (b) $\frac{20}{3^5}$ (c) $\frac{40}{3^5}$ (d) $\frac{160}{3^5}$

61. A company produces 5 defective items out of 300 items. The probability distribution follows a: **(June 2024)**

(a) Binomial distribution (b) Normal distribution

(c) Poisson distribution (d) Standard normal distribution

62. The quartile deviation of a normal distribution with Mean of 10 and Standard Deviation of 4 is: **(Sept 2024)**

(a) 2.70 (b) 3.20 (c) 0.675 (d) 6.75

63. If X and Y are 2 independent normal variables with mean as 10 and 12 and Standard Deviation (S.D.) as 3 and 4 respectively, then (X + Y) is normally distributed with: **(Sept 2024)**

(a) Mean = 22 and S.D. = 7 (b) Mean = 22 and S.D. = 25

(c) Mean = 22 and S.D. = 5 (d) Mean = 22 and S.D. = 49

64. The number of accidents in a year attributed to taxi drivers in a locality follows Poisson distribution with average 2. Out of 500 taxi drivers of that area, what is the number of drivers with at least 3 accidents in a year? (Given that e = 2.718) **(Sept 2024)**

(a) 162 (b) 180 (c) 201 (d) 190

65. In a class of 100 students, the mean marks was 50 with standard deviation 14.9. Assuming the distribution of marks to be normal, find the number of students who obtained more than 70% marks [at Z = 1.34, area = 0.4099]. **(Sept 2024)**

(a) 10 (b) 9 (c) 8 (d) 7

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67. If a random variable X follows Poisson distribution such that P(X = 1) = P(X = 2), then the mean of the distribution is: **(Sept 2024)**

(a) 2 (b) 1 (c) 0 (d) $\frac{1}{2}$

66. If 3 percent of ceramic cup manufactured by a company are known to be defective. What is the probability that a sample of 100 cups are taken from the production process, of that company would contain exactly one defective cup? **(Jan 2025)**

(a) 0.03 (b) 0.15 (c) 0.09 (d) 0.30

67. What will be the mode of the Binomial distribution in which mean is 20 & standard deviation is $\sqrt{10}$? **(Jan 2025)**

(a) 21 (b) 20.5 (c) 20 (d) 41

68. If X is a Poisson variable such that P(X = 1) = P(X = 2) then the variance is **(Jan 2025)**

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) 3

69. The mean deviation of normal distribution is approximately equal to **(Jan 2025)**

(a) 314 σ (b) 0.5 σ (c) 1.14 σ (d) 0.8 σ

70. If 3 percent of ceramic cup manufactured by a company are known to be defective. What is the probability that a sample of 100 cups are taken from the production process, of that company would contain exactly one defective cup? **(Jan 2025)**

(a) 0.03 (b) 0.15 (c) 0.09 (d) 0.30

71. Poisson probability distribution is appropriately applied in **(May 2025)**

(a) The height of students in the university.

(b) The distribution of passing of students in university examinations.

(c) Tossing of a coin hundred times.

(d) Number of deaths by a rare disease.

72. What is the probability of making 3 corrected guesses in 5 True-False answer type questions? **(May 2025)**

(a) 0.3125 (b) 0.4156 (c) 1.3888 (d) 0.5235

73. If 5% of the families in large population city do not use gas as a fuel, what will be the probability of selecting 10 families in a random sample of 100 families who do not use gas as a fuel? [Given that $e^{-5} = 0.0067$] **(May 2025)**

(a) 0.038 (b) Zero (c) 0.018 (d) 0.048

74. If the points of inflexion of a normal curve are 6 and 14, then standard deviation of the distribution is **(May 2025)**

(a) 4 (b) 8 (c) 9.17 (d) 32

75. The Mode of binomial distribution B(7, $\frac{1}{3}$) is **(Sept 2025)**

(a) 3 (b) 2 (c) $\frac{7}{3}$ (d) $\frac{8}{3}$

76. It is given that X has normal distribution with mean zero and standard deviation one. Also given that P[–2 < X < 2] = 0.95, P[–2 < X < –1.5] = 0.045. Find the probability for P[0 < X < 1.5] **(Sept 2025)**

(a) 0.63 (b) 0.53 (c) 0.33 (d) 0.43

77. An emergency room receives an average of 3 patients per hour. What is the probability that exactly 2 patients arrive in an hour? **(Sept 2025)**

(Given: $e^0 = 1$, $e^{-1} = 0.367$, $e^{-2} = 0.135$, $e^{-3} = 0.049$, $e^{-4} = 0.018$, $e^{-5} = 0.0067$)

(a) 0.22 (b) 0.3 (c) 0.27 (d) 0.25

78. If a binomial distribution has n = 20 and p = 0.3, what is its variance? **(Sept 2025)**

(a) 4.2 (b) 5.6 (c) 3.4 (d) 2.9

Answer Key

1. (d)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (c)
11. (d)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (a)
21. (c)	**22.** (a)	**23.** (a)	**24.** (a)	**25.** (d)	**26.** (c)	**27.** (b)	**28.** (c)	**29.** (c)	**30** (a)
31. (c)	**32.** (a)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (b)	**37.** (a)	**38.** (a)	**39.** (c)	**40.** (c)
41. (b)	**42.** (b)	**43.** (a)	**44.** (b)	**45.** (a)	**46.** (b)	**47.** (c)	**48.** (a)	**49.** (b)	**50.** (d)
51. (a)	**52.** (b)	**53.** (b)	**54.** (a)	**55.** (c)	**56.** (b)	**57.** (a)	**58.** (a)	**59.** (c)	**60.** (a)
61. (c)	**62.** (a)	**63.** (c)	**64.** (a)	**65.** (b)	**66.** (a)	**67.** (c)	**68.** (b)	**69.** (d)	**70.** (b)
71. (d)	**72.** (a)	**73.** (c)	**74.** (a)	**75.** (b)	**76.** (d)	**77.** (a)	**78.** (a)		

SUMMARY

- **Binomial distribution:** Trials are independent and each trail has only two outcomes Success & failure. $P(X = x) = {}^nC_x\, p^x q^{n-x}$

  Denoted by X ~ B (n, p)

  mean : $\mu = np$

  variance : $\sigma^2 = npq$

  $p + q = 1$
- **Poisson distribution:** Trials are independent and probability of occurrence is very small in given time. $P(X = x) = \frac{e^{-\lambda}\, \lambda^x}{x!}$

  Denoted by X ~ P(m)

  mean : $\mu = m$

  variance: $\sigma^2 = m$

- **Normal distribution:** When distribution is symmetric $f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

  Mean = Median = Mode = μ

  Variance = σ^2
- We note that 99.73 percent of the values of a normal variable lies between $\mu - 3\sigma$ & $\mu + 3\sigma$.
- If x and y are independent normal variables with means and standard deviations as μ_1 μ_2 and μ_2 and σ_1, and σ_2 respectively, then z = x + y also follows normal distribution with mean $\mu_1 + \mu_2$ and SD $=\sqrt{\sigma_1^2+\sigma_2^2}$ respectively.
- If a continuous random variable z follows standard normal distribution, to be denoted by z ~ N (0, 1), then the probability density function of z is given by

  $f(z)=\frac{1}{\sqrt{2\pi}}e^{-\pi^2/2}$ for $-\infty < z < \infty$
- Some important properties of z are listed below :

  (i) z has mean, median and mode all equal to zero.

  (ii) The standard deviation of z is 1 . Also the approximate values of mean deviation and quartile deviation are 0.8 and 0.675 respectively.

  (iii) The standard normal distribution is symmetrical about z = 0.

  (iv) The two points of inflexion of the probability curve of the standard normal distribution are -1 and 1.

❑❑❑

CA MALLAH

CHAPTER

17 Correlation and Regression

CORRELATION

Correlation is a fundamental concept in statistics that allows us to measure the statistical relationship between two variables. It helps us understand the extent to which changes in one variable are associated with changes in another. Imagine it as a way to quantify how two things interact or influence each other.

BIVARIATE DATA

In real life, we generally deal with more than one variable.

E.g.:

	Math-Enthusiast	Math-Neutral	Math-Disliker	Marginal (English)
English-Lover	10	5	3	18
English-Neutral	3	8	2	13
English-Disliker	1	2	7	10
Marginal (Math)	14	15	12	41

- The above type of data is called a Bivariable data.
- When we involve more than two variables in our study, that type of data is called Multivariate data.

However, in real-life situations, we often deal with more than just two variables, leading to what we call multivariate data. Multivariate data involves the measurement or observation of multiple variables for each individual or object in a study.

Thus, in the given scenario, the data of the singing competition involves two variables which represent bivariate data. If we further consider additional variables such as the age, gender, or singing experience of the participants, it would be an example of multivariate data.

MARGINAL DISTRIBUTION

The marginal distribution of a variable in a bivariate frequency distribution is the distribution of that variable alone, without considering the other variable. It is obtained by summing (or adding) the frequencies along one of the margins (rows or columns) of the distribution table.

E.g.: Consider a bivariate frequency distribution table representing the preferences of students regarding two subjects, Math and English. The table might look like this:

	Math-Enthusiast	Math-Neutral	Math-Disliker	Marginal (English)
English-Lover	10	5	3	18
English-Neutral	3	8	2	13
English-Disliker	1	2	7	10
Marginal (Math)	14	15	12	41

1. The marginal distribution of the variable "English preference" is obtained by summing the frequencies along the "Marginal (English)" column.
2. The marginal distribution of the variable "Math preference" is obtained by summing the frequencies along the "Marginal (Math)" row.

CONDITIONAL DISTRIBUTION

The conditional distribution of a variable in a bivariate frequency distribution is the distribution of that variable under the condition that a specific value of the other variable is known. It is calculated by dividing the joint frequency by the corresponding marginal frequency.

To find the conditional distribution of "Math preference" given that a student is an "English Lover," divide each frequency in the "English Lover" row by the marginal frequency in the "Marginal (English)" column.

	Math-Enthusiast	Math-Neutral	Math-Disliker	Marginal (English)
English-Lover	$\frac{10}{18}$	$\frac{5}{18}$	$\frac{3}{18}$	18

CORRELATION

Correlation refers to the statistical relationship between two variables. It measures the extent to which the change in one variable is accompanied by a change in the other. Correlation helps us understand the association or connection between different variables and how they interact with each other.

E.g.: If we consider the height of students as variable X and their weight as variable Y, we would expect that large values of X (tall students) correspond to large values of Y (higher weight), while small values of X (short students) correspond to small values of Y (lower weight). This indicates a correlation between height and weight, as they tend to change together.

Let's see one more example to understand correlation. We might find a high degree of relationship between the price of a product and consumer demand for that product. As the price increases, the demand may decrease, and vice versa. This demonstrates the correlation between price and demand.

POSITIVE AND NEGATIVE CORRELATION

- **Positive Correlation:** It indicates that as one variable increases (or decreases), the other variable also increases (or decreases). For instance, in a business context, if the amount spent on digital advertising (X) by a firm increases, there is a corresponding increase in total annual sales (Y). Conversely, a reduction in advertising expenditure is associated with a decrease in total sales.

- **Negative Correlation:** It indicates that as one variable increases (or decreases), the other variable decreases (or increases). Consider the price of a product (X) and its demand (Y). If there is a negative correlation, an increase in the product's price leads to a decrease in demand. Conversely, a decrease in price tends to result in an increase in demand for the product.
- **Zero Correlation:** No relationship between the variables.

For example, the number of hours studied and the color of your shoes.

Interpretation:

Close to +1: Strong positive relationship.

Close to -1: Strong negative relationship.

Close to 0: Weak or no relationship.

Example 1. The correlation between shoe-size and intelligence is

(a) Zero (b) Positive (c) Negative (d) None of these.

Sol: (a) We know that two variables are said to be correlated if with the increase in one variable there is either increase or decrease in the value of the other variable.

We can observe that with the increase in shoe-size there is neither increase nor decrease in intelligence.

Therefore, the correlation between shoe size and intelligence is zero.

Hence, the correct option is (a) i.e., Zero.

Example 2. In case of "Insurance companies' profits" and "The number of claims they have to pay," there exists a:

(a) Positive correlation (b) Negative correlation

(c) No correlation (d) It cannot be predicted

Sol: (b) The number of claims that an insurance company has to pay increases, the profits of the company are likely to decrease. When the number of claims is high, the insurance company has to pay more money in claims, which reduces the money available for profits.

So as the number of claims increases, the profits of the company decrease, resulting in a negative correlation between the two variables.

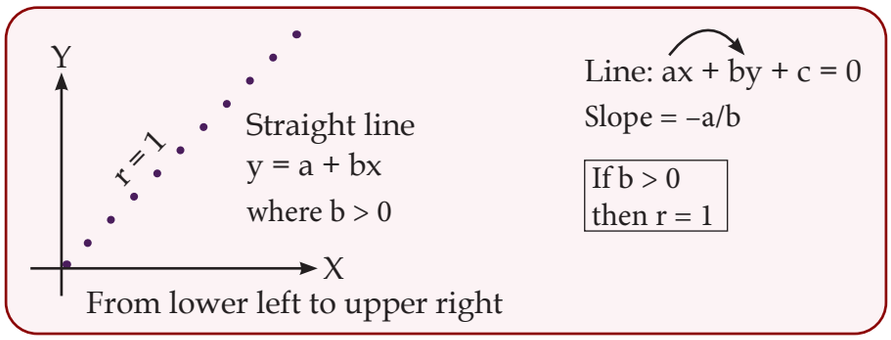
METHODS OF STUDYING CORRELATION

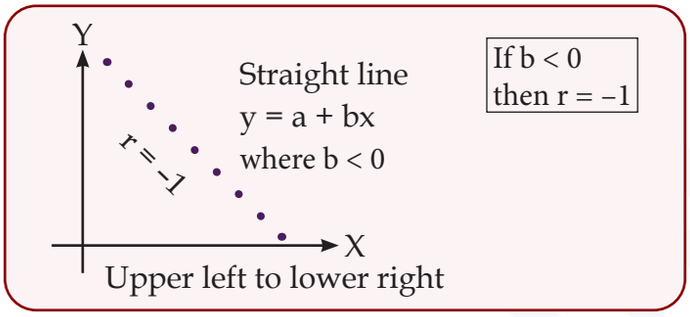
We shall discuss the following methods of measuring the linear relationship between two variables:

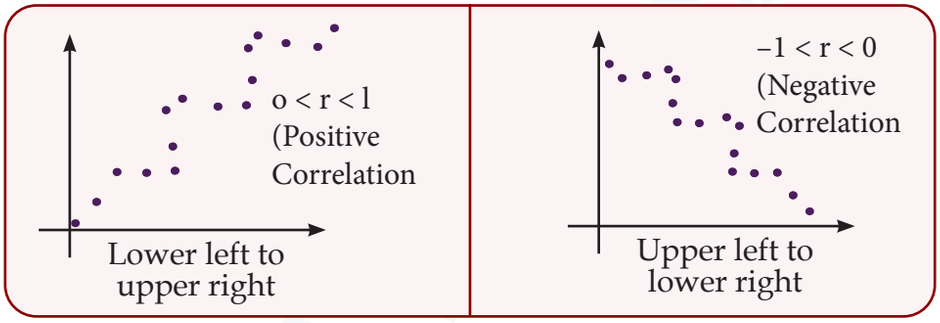
1. Scatter Diagram
2. Karl Pearson's Coefficient of Correlation
3. Spearman's Rank Correlation Coefficient
4. Coefficient of Concurrent deviations

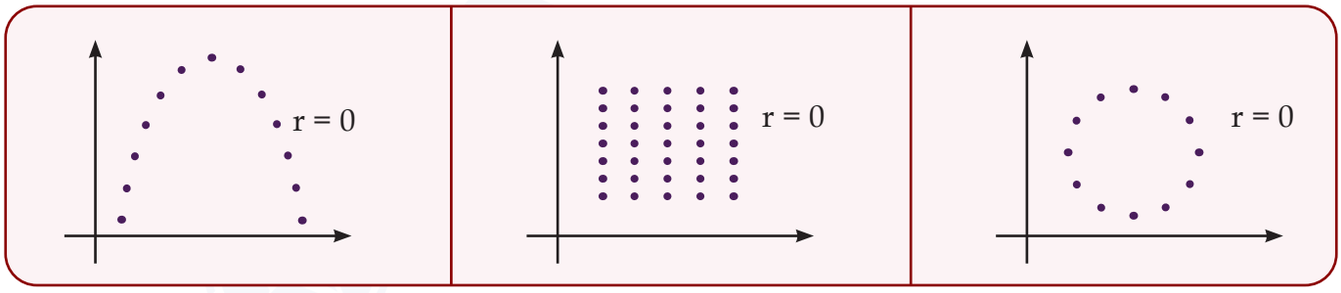
SCATTER DIAGRAM

A scatter diagram is a graphical presentation of bivariate data $\{(X_i, Y_i): i = 1, 2, \ldots n\}$ on two quantitative variables X and Y that allows us to show two variables together, one on each axis, each pair being represented by a point on the graph as in coordinate geometry.









1. If r = 1, then all the points of the scatter diagram lie on a straight line having positive slope and we say that a perfect positive linear relationship exists between the two variables. Similarly, if r = –1, then all the points of the scatter diagram lie on a straight line having a negative slope and we say that a perfect negative linear relationship exists between the two variables.
2. If r is close to +1, then all the points of the scatter diagram closely follow a straight line having positive slope and we say that a high positive correlation exists between the two variables. Similarly, if r is close to –1, then all the points of the scatter diagram closely follow a straight line having a negative slope and we say that a high negative correlation exists between the two variables.
3. If r is close to 0, the linear relationship between the two variables is weak or perhaps non-existent.

THE CORRELATION COEFFICIENT

Correlation analysis attempts to measure the strength or closeness of linear relationships between two variables by means of a single number called a correlation coefficient.

Definition. The quantitative measure of strength in the linear relationship between two variables is called the correlation coefficient. It is denoted by r.

- Thus, the correlation coefficient r measures the extent to which the points cluster about a straight line.
- The correlation coefficient ranges from + 1 to – 1 i.e. $-1 \le r \le 1$.
- If two variables have no linear relationship, the correlation between them is zero.
- Consequently, the more correlation differs from zero, the stronger the linear relationship between the two variables.

The following table shows degrees of correlation according to various values of r.

Degree of Correlation	Positive	Negative
Perfect correlation	+1	–1
Very high degree of correlation	+ 0.9 to + 1	- 0.9 to – 1
Fairly high degree of correlation	+ 0.75 to + 0.9	- 0.75 to - 0.9
Moderate degree of correlation	+ 0.50 to + 0.75	- 0.50 to - 0.75
Low degree of correlation	+ 0.25 to + 0.50	- 0.25 to - 0.5
Very low degree of correlation	0 to + 0.25	- 0.25 to 0
No correlation	0	0

Example 1. The covariance between two variables is **(ICAI)**

(a) Strictly positive (b) Strictly negative

(c) Always 0 (d) Either positive or negative or zero

Sol. (d) Correlation between two variables can be measured on a scale that varies from +1 through 0 to −1.

Covariance values are not standardized. Therefore, the covariance can range from negative infinity to positive infinity. Thus, the value for a perfect linear relationship depends on the data. Because the data are not standardized, it is difficult to determine the strength of the relationship between the variables.

When one variable increases as the other increases then the correlation is positive and when one decreases as the other increases then the correlation is negative.

Also, complete absence of correlation is represented as 0 (zero) correlation.

So, the correlation between two variable is either positive or negative or zero.

Hence, the correct option is (d).

COVARIANCE

- **Definition:** Consider a set of n pairs of observations (x_1, y_1), (x_2, y_2), ... (x_n, y_n) on two quantitative variables x and y, where x_1, x_2, ... denote observed values of the variable x, and y_1, y_2, ... those of y.

The covariance between x and y, denoted by Cov(x, y), is given by:

$$\text{Cov}(x, y) = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n} = \frac{\sum x_i y_i}{n} - \overline{xy}$$

Example 2. Find Cov(x, y) between x and y if

x	3	4	5	6	7
y	8	7	6	5	4

(a) 2 (b) 3 (c) –2 (d) –1

Sol. (c) According to the question, we have

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$
3	8	–2	2	–4
4	7	–1	1	–1
5	6	0	0	0
6	5	1	–1	–1
7	4	2	-2	–4
$\bar{x} = \frac{25}{5} = 5$	$\bar{y} = \frac{30}{5} = 6$			–10

Therefore, $\text{Cov}(x, y) = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n} = -10/5 = -2$

Hence, the correct option is (c).

Another Formula for Cov(x, y)

This formula is particularly useful when $\bar{x}$ or $\bar{y}$ is not an integer. The formula is:

$$\text{Cov}(x, y) = \frac{\sum xy}{n} - \left(\frac{\sum x}{n}\right)\left(\frac{\sum y}{n}\right)$$

Example 3. Find Cov(x, y) between x and y if

x	3	4	5	6	7
y	8	7	6	5	4

(a) 2 (b) 3 (c) –2 (d) –1

Sol. (c) $\text{COV}(x, y) = \frac{\sum xy}{n} - \left(\frac{\sum x}{n}\right)\left(\frac{\sum y}{n}\right)$

Therefore, the required table is,

x	y	xy
3	8	24
4	7	28
5	6	30

x	y	xy
6	5	30
7	4	28
$\sum x = 25$	$\sum y = 30$	$\sum xy = 140$

Here,

$\sum x = 25$

$\sum y = 30$

$\sum xy = 140$ and n = 5

Put these values in the above formula, then we get

$$\text{Cov}(x, y) = \frac{140}{5} - \left(\frac{25}{5}\right)\left(\frac{30}{5}\right)$$

Cov(x, y) = 28 – (5)(6)

Cov(x, y) = –2

Hence, the correct option is (c).

Example 4. Calculate the covariance between X and Y for the following data:

X	1	2	3	4	5	6	7	8	9	10
Y	6	9	6	7	8	5	12	3	17	1

(a) 2 (b) 0.2 (c) –0.4 (d) 0.4

Sol. (d) $\text{Cov}(x, y) = \frac{\sum xy}{n} - \left(\frac{\sum x}{n}\right)\left(\frac{\sum y}{n}\right)$

Therefore, the required table is,

x	y	xy
1	6	6
2	9	18
3	6	18
4	7	28
5	8	40
6	5	30
7	12	84
8	3	24
9	17	153
10	1	10
$\sum x = 55$	$\sum y = 74$	$\sum xy = 411$

Here,

$\sum x = 55$

$\sum y = 74$

$\sum xy = 411$ and n = 10

Put these values in the above formula. Then we get

$$\text{Cov}(x, y) = \frac{411}{10} - \left(\frac{55}{10}\right)\left(\frac{74}{10}\right) = \frac{411}{10} - \frac{407}{10} = \frac{4}{10} = 0.4$$

Hence, the correct option is (d).

Example 5. Find the covariance between X and Y, given that $\sum X = 60$ $\sum Y = 60$ $\sum XY = 574$ and n = 10.

(a) 2 (b) 3.4 (c) 3.2 (d) 3.6

Sol. (b) Given that, $\sum X = 60$ $\sum Y = 60$ $\sum XY = 574$ and n = 10

Put these values in $\text{Cov}(x, y) = \frac{\sum xy}{n} - \left(\frac{\sum x}{n}\right)\left(\frac{\sum y}{n}\right)$ then we get

$$\text{Cov}(x, y) = \frac{574}{10} - \left(\frac{60}{10}\right)\left(\frac{90}{10}\right)$$

$$\text{Cov}(x, y) = \frac{574}{10} - \frac{540}{10} = \frac{34}{10} = 3.4$$

Hence, the correct option is (b).

KARL PEARSON'S COEFFICIENT OF CORRELATION

The Karl Pearson's coefficient of correlation, also called the Pearson's product - moment correlation coefficient, is the most widely used method of measuring the linear correlation between two variables.

Definition: The Karl Pearson's coefficient of correlation between two variables x and y, denoted by r, is defined by

$$r = \frac{\text{Cov}(xy)}{\sigma_x * \sigma_y}$$

where var. x and var. y are the variances of the values of x and y respectively, while σ_x and σ_y are their standard deviations.

Example 6. If for two variable x and y, the covariance, variance of x and variance of y are 40, 16 and 256 respectively, what is the value of the correlation coefficient? **(ICAI)**

(a) 0.01 (b) 0.625 (c) 0.4 (d) 0.5

Sol. (b) We know that,

Correlation Coefficient, $r = \frac{\text{cov}(x, y)}{\sigma_x \times \sigma_y}$

where,

r = Correlation Coefficient

cov(x, y) = Covariance of x and y

σ_x = Standard deviation of x

σ_y = Standard deviation of y

Also, Standard Deviation $= \sqrt{\text{Variance}}$

$\Rightarrow$ Standard Deviation of x, $\sigma_x = \sqrt{16} = 4$

$\Rightarrow$ Standard Deviation of y, $\sigma_y = \sqrt{256} = 16$

Calculating Correlation Coefficient, we get

$$(r) = \frac{40}{4 \times 16} = \frac{40}{64} = 0.625$$

Hence, the correct answer is option (b) i.e. 0.625.

Example 7. The covariance between the length and weight of five items is 6 and their standard deviations are 2.45 and 2.61 respectively. Find the coefficient of correlation between length and weight.

(a) 0.9383 (b) 1.9385 (c) 0.2583 (d) 3.6353

Sol. (a) Given: Cov(x, y) = 6, σ_x = 2.45 and σ_y = 2.61

Using, $r = \frac{\text{Cov}(xy)}{\sigma_x * \sigma_y}$

Substituting the values in the above formula, we get

$$r = \frac{6}{(2.45)(2.61)} = 0.9383$$

Hence, the correct option is (a).

Example 8. The coefficient of correlation between x and y is 0.5, the covariance is 16. If the standard deviation of x is 4 then the standard deviation of y is **(Jan 2021)**

(a) 4 (b) 8 (c) 16 (d) 64

Sol. (b) Given: r = 0.5, cov(x, y) = 16, σ_x= 4 and σ_y= ?

We know that,

Coefficient of correlation is given by:

$$r = \frac{\text{cov}\ (x, y)}{\sigma_x\ \sigma_y}$$

$$\Rightarrow 0.5 = \frac{16}{4 \times \sigma_y}$$

$$\Rightarrow \sigma_y = \frac{16}{4 \times 0.5} = 8$$

Hence, the correct option is (b).

Example 9. If covariance of 10 pairs of items is 7, variance of X is 36 and $\sum(Y-\bar{Y})^2 = 90$. Find the value of r.

(a) 1.389 (b) 0.389 (c) 0.258 (d) 1.635

Sol. (b) Given: $\sum(Y-\bar{Y})^2 = 90$ n = 10, Cov(x, y) = 7 and Var x = 36

Then, $\sigma_x = \sqrt{\text{Var } x} = \sqrt{36} = 6$ and $\sigma_y = \sqrt{\frac{\sum(Y-\bar{Y})^2}{n}} = \sqrt{\frac{90}{10}} = 3$

Using, $r = \frac{\text{Cov}(xy)}{\sigma_x * \sigma_y}$

Substituting the values in the above formula, we get

$r = \frac{7}{(6)(3)} = 0.389$

Hence, the correct option is (b).

Example 10. The coefficient of correlation between two variables x and y is 0.4 and their covariance is 10. If the variance of x series is 9, find the standard deviation of y series.

(a) 4.389 (b) 69.445 (c) 8.33 (d) 7.635

Sol. (c) Coefficient of correlation, r = 0.4, Cov(x, y) = 10, var (x) = 9

$\Rightarrow \sigma_x = 3$

As we know that, $\rho(x,y) = \frac{\text{cov}(x,y)}{\sigma_x \times \sigma_y}$

$\Rightarrow 0.4 = \frac{10}{\sqrt{9} \times \sigma_y}$

$\Rightarrow \sigma_y = \frac{100}{4 \times 3} = 8.33$

Hence, the correct option is (c).

CHARACTERISTICS OF THE CORRELATION COEFFICIENT

- Correlation coefficient is independent of the units of measurement.
- The correlation coefficient r is independent of change of origin and scale under consideration depending on the sign of scale factors.

  That is, if the variables X and Y are replaced by the variables U and V such that U = aX + b and V = cY + d, where a and c are positive constants.

  However, if a and c are arbitrary constants, then.

  $r(aX+b, cY+d) = \frac{a \times c}{|a \times c|} r_{XY}$

  'or'

  $r_{UV} = \frac{|a \times c|}{a \times c} r_{XY}$

Thus, $r_{XY} = r_{UV}$ when a and c have the same signs and $r_{XY} = -r_{UV}$ when a and c have the opposite signs.

The correlation coefficient lies between −1 and 1, including both the limits.

That is, $-1 \le r \le 1$

Example 11. What are the limits of the correlation coefficient?

(a) No limit (b) -1 and 1, excluding the limits

(c) 0 and 1, excluding the limits (d) -1 and 1, including the limits

Sol. (d) We know that,

Correlation coefficient (r) lies between the range $-1 \le r \le 1$,

where −1 depicts the strong negative correlation and 1 depicts the strong positive correlation.

Thus, we can conclude that limits of the correlation coefficient are −1 and 1, including the limits.

Hence, the correct answer is option (d) i.e., −1 and 1, including the limits.

Example 12. Correlation coefficient is _____ of the units of measurement.

(a) Dependent (b) Independent (c) Both (d) None of these

Sol. (b) We know that,

The coefficient of Correlation is a unit-free measure.

Therefore, correlation coefficient is independent of the units of measurement.

Hence, the correct answer is option (b) i.e., Independent.

Example 13. If the correlation between X and Y is r, $U = \frac{X-5}{10}$ and $V = \frac{Y-7}{2}$ then r_{uv} is

(a) r (b) −r (c) $\frac{r-5}{2}$ (d) $\frac{r-7}{10}$

Sol. (a) Given: $U = \frac{X-5}{10}$ and $V = \frac{Y-7}{2}$

$U = \frac{X}{10} - \frac{5}{10}$ and $V = \frac{Y}{2} - \frac{7}{2}$

Thus, $a = \frac{1}{10}$ and $c = \frac{1}{2}$

Therefore, $r_{uv} = \frac{a \times c}{|a \times c|} \times r_{xy}$

$$\Rightarrow r_{uv} = \frac{\frac{1}{10} \times \frac{1}{2}}{\left|\frac{1}{10} \times \frac{1}{2}\right|} \times r_{xy}$$

$$\Rightarrow r_{uv} = r_{xy} = r$$

Hence, the correct option is (a) i.e. r.

Example 14. If r = 0.58, correlation coefficient of u = −5x + 3 and v = y + 2 is _____

(a) 0.58 (b) -0.58 (c) 0.62 (d) None

Sol. (b) We know that the value of correlation coefficient does not change with change in origin and scale.

Also, if the sign of x & y in given equations are opposite, then sign of r also changes.

Here, sign of x,y are opposite so $r_{uv} = -0.58$

Hence, option (b) is correct.

Example 15. The coefficient of correlation between X and Y is 0.6. U and V are two variables defined as $U = \frac{X-3}{2}, V = \frac{Y-2}{3}$, s then the coefficient of correlation between U and V is

(a) 0.6 (b) 0.4 (c) 0.8 (d) −0.6

Sol. (a) Given: Coefficient of correlation between X and Y = 0.6

Also, $U = \frac{X-3}{2}, V = \frac{Y-2}{3}$

We know that,

Correlation coefficient (Karl Pearson's) is independent of the change of scale and origin,

So, r(U, V) = r(X, Y) = 0.6

[Because X & Y have same sign]

Hence, the correct option is (a) i.e. 0.6.

PRACTICE QUESTIONS (PART A)

1. If x denotes height of a group of students expressed in cm and y denotes their weight expressed in kg, then the correlation coefficient between height and weight would be shown

(a) in kg (b) in cm (c) in kg and cm (d) free from any unit

2. The correlation coefficient between two variables X and Y is found to be 0.4. What is the correlation coefficient between 2X and (-Y)?

(a) 0.4 (b) –0.8 (c) 0.8 (d) –0.4

3. If r = 0.28, cov(x, y) = 7.6, V(x) = 9 then σ_y =

(a) 8.75 (b) 9.04 (c) 6.25 (d) None of these

4. If for two variables A and B, the covariance, variance of A, and variance of B are 60, 25, and 400 respectively, what is the value of the correlation coefficient?

(a) 0.2 (b) 0.5 (c) 0.4 (d) 0.6

5. Coefficient of correlation between X and Y is 0.6. If both X and Y are multiplied by −1, then resultant correlation coefficient is

(a) 0.6 (b) −0.6 (c) 1/0.6 (d) None of these

6. The covariance between two variable X and Y is 8.4 and their variances are 25 and 36 respectively. Calculate Karl Pearson's coefficient of correlation between them.

(a) 0.82 (b) 0.28 (c) 0.01 (d) 0.09

7. If correlation coefficient between x and y is 0.5, then the correlation coefficient between 2x − 3 and 3 − 5y is **(Dec 2019)**

(a) 0.5 (b) −0.5 (c) 2.5 (d) −2.5

8. The coefficient of correlation between x & y when Cov(x, y) = −16.5, Var(x) = 2.89, Var(y) = 100 is

(a) -0.97 (b) 0.97 (c) 0.89 (d) -0.89

9. The coefficient of correlation between X and Y series is −0.38. The linear relation between U and V are 3X + 5U = 3 and −8Y − 7V = 44, what is coefficient between U & V?

(a) 0.38 (b) -0.38 (c) 0.40 (d) None of these

Answer Key

1. (d) **2.** (d) **3.** (b) **4.** (d) **5.** (a) **6.** (b) **7.** (b) **8.** (a) **9.** (b)

COMPUTING THE CORRELATION COEFFICIENT

Various formulas for computing the correlation coefficient between the two variables X and Y:

- The correlation coefficient, r, between two variables X and Y is given by

$$r = \frac{\text{cov}(X, Y)}{\sqrt{\text{Var } X}.\sqrt{\text{var } Y}}$$

If the variable x takes on the values $x_1, x_2, \ldots, x_n$ and the variable y takes on the values $y_1, y_2, \ldots, y_n$ then we have

$$\text{Cov}(X, Y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} \text{ or } \frac{\Sigma xy}{n} - \bar{x}\bar{y}$$

$$\text{Var}(X) = \frac{\Sigma(x-\bar{x})^2}{n} \text{ or } \frac{\Sigma x^2}{n} - (\bar{x})^2$$

$$\text{Var}(Y) = \frac{\Sigma(y-\bar{y})^2}{n} \text{ or } \frac{\Sigma y^2}{n} - (\bar{y})^2$$

We obtain another formula for r:

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \cdot \sum(y-\bar{y})^2}}$$

Example 16. If the sum of the product of the deviation of X and Y from their means is zero, then the correlation coefficient between X and Y is **(July 2021)**

(a) Zero (b) Positive (c) Negative (d) 10

Sol. (a) According to the question, we have

$$\sum(X-\bar{X})(Y-\bar{Y}) = 0$$

Thus, correlation coefficient, $r = \frac{\sum(X-\bar{X})(y-\bar{Y})}{\sqrt{\sum(X-\bar{X})^2 \cdot \sum(Y-\bar{Y})^2}}$

$= \frac{0}{\sqrt{\sum(X-\bar{X})^2 \cdot \sum(Y-\bar{Y})^2}} = 0$

Hence, the correct option is (a).

Example 17. Calculate coefficient of correlation from the following results: n = 10,

$\sum X = 100, \sum Y = 150, \sum (X-10)^2 = 180$ $\sum (Y-15)^2 = 215$ and $\sum (X-10)(Y-15) = 60$

(a) 0.463 (b) 2.15 (c) 0.305 (d) –0.7618

Sol. (c) Given: $n = 10, \sum X = 100, \sum Y = 150, \sum (X-10)^2 = 180$

$\sum (Y-15)^2 = 215$ and $\sum (x-10)(y-15) = 60$

$r = \frac{\sum(X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum(X-\bar{X})^2 \sum(Y-\bar{Y})^2}} = \frac{60}{\sqrt{180 \times 215}} = \frac{60}{196.723} = 0.305$

Therefore, the coefficient of correlation is 0.305.

Hence, the correct option is (c).

Example 18. Find the covariance between X and Y for the following data:

X	66	67	68	69	70	71	72
Y	68	67	70	70	69	70	69

(a) 0.67 (b) 2.4 (c) 0.25 (d) 1.6

Sol. (a) $\bar{x} = \frac{\sum x}{n} = \frac{483}{7} = 69$

$\bar{y} = \frac{\sum y}{n} = 69$

x	66	67	68	69	70	71	72
y	68	67	70	70	69	70	69
$x-\bar{x}$	-3	-2	-1	0	1	2	3
$y-\bar{y}$	-1	-2	1	1	0	1	0
$(x-\bar{x})(y-\bar{y})$	3	4	1	0	0	2	0

$\sum (x-\bar{x})(y-\bar{y}) = 3+4+1+2 = 10$

$\sum (x-\bar{x})^2 = 9+4+1+0+1+4+9 = 28$

$$\sum(y-\bar{y})^2 = 1+4+1+1+0+1+0 = 8$$

Thus, $r = \dfrac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \cdot \sum(y-\bar{y})^2}} = \dfrac{10}{\sqrt{28.8}} = 0.67$

Hence, the correct option is (a).

Example 19. The coefficient of correlation between X and Y series from the following data is

	X series	Y series
Number of pairs of observations	15	15
Arithmetic mean	25	18
Standard Deviation	3.01	3.03
Sum of squares of dev. from mean	136	138

Sum of the product of the deviations of X and Y series from their respective means = 122

(a) 0.89 (b) 0.99 (c) 0.69 (d) 0.91

Sol. (a) Given that;

$\sigma_x = 3.01$

$\sigma_y = 3.03$

$\Sigma x^2 = 136$

$\Sigma y^2 = 138$

$\Sigma xy = 122$

where, $x = X - \bar{X}; y = Y - \bar{Y}$

Substituting the values in above formula, we get

$$r = \frac{\sum xy}{\sum x^2 \times \sum y^2} = \frac{122}{\sqrt{136 \times 138}} = 0.89$$

Hence, the correct option is (a).

Example 20. Given the following information: r = 0.8, $\sum xy = 60$, $\sigma_y = 2.5$ and $\sum x^2 = 90$. where x and y are the deviations from the respective means, find the number of items (n).

(a) 8 (b) 11 (c) 14 (d) 10

Sol. (d) From the question, we know that

r = 0.8, $\Sigma xy = 60$, $\sigma_y = 2.5$ and $\Sigma x^2 = 90$

The formula for the standard deviation is given by:

$$\sigma_y = \sqrt{\frac{\sum y^2}{n}}$$

From this, we have

$$2.5 = \sqrt{\frac{\sum y^2}{n}}$$

Squaring both sides,

$6.25n = \Sigma x^2$

As we know that the formula for the coefficient of correlation r is: $r = \frac{\sum xy}{\sqrt{\left|\sum x^2 \sum y^2\right|}}$

Substituting the given values, we get

$$0.8 = \frac{60}{\sqrt{(90)(6.25n)}}$$

$$\Rightarrow \sqrt{n} = \frac{60}{0.8\sqrt{|(90)(6.25)|}}$$

$$\Rightarrow \sqrt{n} = \frac{600\sqrt{10}}{8 \times 3 \times 25}$$

$$\Rightarrow \sqrt{n} = \sqrt{10}$$

$$\Rightarrow n = 10$$

Hence, the correct option is (d).

ANOTHER FORMULA

Also,

$$r = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{\frac{\Sigma xy}{n} - \frac{\Sigma x}{n}\frac{\Sigma y}{n}}{\sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}\sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}}$$

$$r = \frac{\frac{n\Sigma xy - \Sigma x \Sigma y}{n^2}}{\sqrt{\frac{\Sigma x^2}{n} - \frac{(\Sigma x)^2}{n^2}}\sqrt{\frac{\Sigma y^2}{n} - \frac{(\Sigma y)^2}{n^2}}} = \frac{\frac{n\Sigma xy - \Sigma x \Sigma y}{n^2}}{\sqrt{\frac{n\Sigma x^2 - (\Sigma x)^2}{n^2}}\sqrt{\frac{n\Sigma y^2 - (\Sigma y)^2}{n^2}}} = \frac{\frac{n\Sigma xy - \Sigma x \Sigma y}{n^2}}{\frac{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}{\sqrt{n^2}}}$$

$$r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

FORMULAE OF CORRELATION COEFFICIENT

- $r = \frac{Cov(X,Y)}{\sqrt{VarX} \cdot \sqrt{VarY}}$
- $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$
- $r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$

Example 21. For the set of observations {(1, 2), (2, 5), (3, 7), (4, 8), (5, 10)}, the value of Karl-pearsons' coefficient of correlation is approximately given by **(Jan 2021)**

(a) 0.755 (b) 0.655 (c) 0.525 (d) 0.985

Sol. (d) According to the data of observations given, we have

X	Y	XY	X^2	Y^2
1	2	2	1	4
2	5	10	4	25
3	7	21	9	49
4	8	32	16	64
5	10	50	25	100
$\sum X = 15$	$\sum Y=32$	$\sum XY=115$	$\sum X^2=55$	$\sum Y^2=242$

Coefficient of correlation

$$= r = \frac{N\sum XY - \sum X \cdot \sum Y}{\sqrt{N\sum X^2 - (\sum X)} \cdot \sqrt{N\sum Y^2 - (\sum Y)^2}} = \frac{5\times 115 - 15\times 32}{\sqrt{5\times 55 - (15)^2} \cdot \sqrt{5\times 242 - (32)^2}}$$

$$= \frac{575 - 480}{\sqrt{275 - 225} \cdot \sqrt{1210 - 1024}} = \frac{+95}{\sqrt{50} \cdot \sqrt{186}} = +0.985$$

Therefore, the required value is 0.985.

Hence, the correct option is (d).

Example 22. From the following data, calculate Karl Pearson's coefficient of correlation:

Height of Fathers (in inches):	66	68	69	72	65	59	62	67	61	71
Height of Sons (in inches):	65	64	67	69	64	60	59	68	60	64

(a) 0.463 (b) 2.15 (c) 0.195 (d) 0.829

Sol. (d) Let Height of Fathers (in inches) be x

And Height of Sons (in inches) be y

Now, the required table is:

x	y	u = x – 66	v = y – 64	uv	u^2	v^2
66	65	0	1	0	0	1
68	64	2	0	0	4	0
69	67	3	3	9	9	9
72	69	6	5	30	36	25
65	64	-1	0	0	1	0
59	60	-7	-4	28	49	16
62	59	-4	-5	20	16	25
67	68	1	4	4	1	16
61	60	-5	-4	20	25	16
71	64	5	0	0	25	0

$\left(\sum_{i=1}^{N} x_i\right)$ = 660	$\left(\sum_{i=1}^{N} y_i\right)$ = 640	$\sum u = 0$	$\sum v = 0$	$\sum uv$ =111	$(\sum u^2)$ = 166	$(\sum v^2)$ = 108

Here, $\bar{x} = \frac{\sum_{i=1}^{N} x_i}{n} = \frac{660}{10} = 66$

$\bar{y} = \frac{\sum_{i=1}^{N} y_i}{n} = \frac{640}{10} = 64$

Now, Correlation coefficient, $r = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$

Substituting the values, we get

$r = \frac{111}{\sqrt{166 \times 108}} = 0.829$

Hence, the correct option is (d).

Example 23. Calculate Karl Pearson's coefficient of correlation between the variables X and Y using the following data:

X	5	4	3	2	1	5	1	7	9	8
Y	1	2	4	5	2	4	8	2	6	2

(a) 2.463 (b) 2.15 (c) –0.915 (d) –0.229

Sol. (d) According to the data given in the question, we have

x	y	xy	x^2	y^2
5	1	5	25	1
4	2	8	16	4
3	4	12	9	16
2	5	10	4	25
1	2	2	1	4
5	4	20	25	16
1	8	8	1	64
7	2	14	49	4
9	6	54	81	36
8	2	16	64	4
$\sum x = 45$	$\sum y = 36$	$\sum xy = 149$	$\sum x^2 = 275$	$\sum y^2 = 174$

Then, we have to use, $r = \dfrac{\sum xy - \dfrac{(\sum x)(\sum y)}{N}}{\sqrt{\sum x^2 - \dfrac{(\sum x)^2}{N}}\sqrt{\sum y^2 - \dfrac{(\sum y)^2}{N}}}$

$$r = \frac{149 - \dfrac{(45)(36)}{10}}{\sqrt{275 - \dfrac{(45)^2}{10}}\sqrt{174 - \dfrac{(36)^2}{10}}}$$

$$r = \frac{-13}{\sqrt{3219}} = -0.229$$

Therefore, the Karl Pearson's coefficient of correlation between the variables X and Y is –0.229.

Hence, the correct option is (d).

Example 24. The coefficient of correlation between x and y where

x	64	60	67	59	69
y	57	60	73	62	68

is **(ICAI)**

(a) 0.65 (b) 0.68 (c) 0.73 (d) 0.758

Sol. (a) Given, n = 5

To find correlation coefficient, we will prepare the table as follows:

x	y	$u = x - \bar{x}$ $= x - 64$	$v = y - \bar{y}$ $= y - 64$	uv	u^2	v^2
64	57	0	−7	0	0	49
60	60	−4	−4	16	16	16
67	73	3	9	27	9	81
59	62	−5	−2	10	25	4
69	68	5	4	20	25	16
$\left(\sum_{i=1}^{N} x_i\right)$ $=319$	$\left(\sum_{i=1}^{N} y_i\right)$ $=320$	$\sum u = -1$	$(\sum v) = 0$	$(\sum uv) =$ 73	$(\sum u^2) = 75$	$(\sum v^2) =$ 166

Here,

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{n} = \frac{319}{5} = 63.8 \text{ (Let's assume mean as 64)}$$

$$\bar{y}=\frac{\sum_{i=1}^{N} y_i}{n}=\frac{320}{5}=64$$

Correlation coefficient, $r=\frac{n\Sigma uv-\Sigma u\Sigma v}{\sqrt{n\Sigma u^2-(\Sigma u)^2}\sqrt{n\Sigma v^2-(\Sigma v)^2}}=573$

$$=\frac{5(73)-(-1)(0)}{\sqrt{5(75)-(-1)^2}\sqrt{5(166)-(0)^2}}$$

$$=\frac{365-0}{\sqrt{375-1}\sqrt{830-0}}$$

$= 0.655$

Hence, the correct answer is option (a) i.e., 0.655.

Example 25. What would be the correlation between u and v? **(ICAI)**

u	10	15	25	20	35
v	–24	–36	–42	–48	–60

(a) – 0.6 (b) –0.3224 (c) –0.93 (d) 0.93

Sol. (c)

u	$x = u - \bar{u} = u - 21$	v	$y = v - \bar{v} = v + 42$	x^2	y^2	xy
10	-11	-24	18	121	324	-198
15	-6	-36	6	36	36	-36
25	4	-42	0	16	0	0
20	-1	-48	-6	1	36	-6
35	14	-60	-18	196	324	-252
$\sum u = 105$	$\sum x = 0$	$\sum v = -210$	$\sum y = 0$	$\sum x^2 = 370$	$\sum y^2 = 720$	$\sum xy = -492$

Here, n = 5

Then $\bar{u}=\frac{\sum u}{n}=\frac{105}{5}=21$

And $\bar{v}=\frac{\sum v}{n}=\frac{-210}{5}=-42$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{\left(n\sum x^2 - (\sum x)^2\right)\left(n\sum y^2 - (\sum y)^2\right)}}$$

$$= \frac{5(-492) - (0)(0)}{\sqrt{\left(5(370) - (0)^2\right)\left(5(720) - (0)^2\right)}} = -0.93 \text{ (approx)}$$

Hence the correct option is (c).

Example 26. Calculate coefficient of correlation from the following data:

X	10,000	20,000	30,000	40,000	50,000	60,000	70,000
Y	0.3	0.5	0.6	0.8	1.0	1.1	1.3

(a) 9.7289×10^{-1} (b) 12.7289

(c) -12.7289×10^{-1} (d) –97.289

Sol. (a) Assume mean for x series is, $\frac{270000}{6} = 45000$.

And assume mean of y series is, $\frac{5.3}{6} = 0.8833 \approx 0.9$

The required table is.

x	y	u = x – 45000	v = y – 0.9	uv	u^2	v^2
10,000	0.3	-35,000	-0.6	21000	1225000000	0.36
20,000	0.5	-25,000	-0.4	10000	625000000	0.16
30,000	0.6	-15,000	-0.3	4500	225000000	0.09
40,000	0.8	-5,000	-0.1	500	25000000	0.01
50,000	1.0	5,000	0.1	500	25000000	0.01
60,000	1.1	15,000	0.2	3000	225000000	0.04
70,000	1.3	25,000	0.4	10000	625000000	0.16
		$\sum u = -35,000$	$\sum v = -0.7$	$\sum uv =$ 49500	$\sum u^2 =$ 2975000000	$\sum v^2 =$ 0.83

Now, Correlation coefficient, $r = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$

Then, we get

$$r = \frac{7 \times 49500 - (-35000) \times (-0.7)}{\sqrt{7 \times 2975000000 - (-35000)^2}\ \sqrt{7 \times (0.83) - (-0.7)^2}} = 9.7289 \times 10^{-1}.$$

Hence, the correct option is (a).

Example 27. Find Karl Pearson's coefficient of correlation between the age and the playing habits of the people from the following information

Age groups (in years)	No. of people	No. of players
15 and less than 20	200	150
20 and less than 25	270	162
25 and less than 30	340	170
30 and less than 35	360	180
35 and less than 40	400	180
40 and less than 45	300	120

(a) 0.463 (b) –0.9395 (c) 0.15 (d) 0.7618

Sol. (b)

Age groups (in years)	Mid Value (x)	No. of people	No. of players	Percentage of Players (%)	(y)	dx = x – 30	dy = x – 54	dxdy	dx^2	dy^2
15 - 19	17	200	150	$\frac{150}{200} \times 100 = 75\%$	75	-13	21	-273	169	441
20 - 24	22	270	162	$\frac{162}{270} \times 100 = 60\%$	60	-8	6	-48	64	36
25 - 29	27	340	170	$\frac{170}{340} \times 100 = 50\%$	50	-3	-4	12	9	16
30 - 34	32	360	180	$\frac{180}{360} \times 100 = 50\%$	50	2	-4	-8	4	16
35 - 39	37	400	180	$\frac{180}{400} \times 100 = 45\%$	45	7	-9	-63	49	81
40 - 44	42	300	120	$\frac{120}{300} \times 100 = 40\%$	40	12	-14	-168	144	196
	$\sum x$ = 177				$\sum y$ = 320	$\sum dx$ = –3	$\sum dy$ = –4	$\sum dxdy$ = –548	$\sum dx^2$ = 439	$\sum dy^2$ = 786

Now we have to find the Mean values of X and Y,

So, mean value of $\bar{x} = \frac{\sum x}{N} = \frac{177}{6} = 29.5 \approx 30$, then assumed mean is 30.

Now the Mean value of $\bar{y} = \frac{\Sigma y}{N} = \frac{320}{6} = 53.33 \approx 54,$, then assumed mean is 54.

As we know that,

$$r = \frac{\sum dxdy - \frac{\sum dx \sum dy}{N}}{\sqrt{\sum dx^2 - \frac{\sum dx^2}{N}} \sqrt{\sum dy^2 - \frac{(\sum dy)^2}{N}}}$$

$$r = \frac{-548 - \frac{(-3) \times (-4)}{6}}{\sqrt{439 - \frac{(-3)^2}{6}} \sqrt{786 - \frac{(-4)^2}{6}}} = \frac{550}{\sqrt{439 - \frac{3}{2}} \sqrt{786 - \frac{8}{3}}} = -\frac{22\sqrt{4935}}{1645} = -0.9395$$

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART B)

1. The coefficient of correlation between X and Y if cov(x, y) = 16.5, Var(X) = 8.25 and Var(Y) = 33.

(a) 0 (b) 1 (c) 2.5 (d) 4.3

2. The coefficient of correlation between X and Y for the following data:

$n = 25, \sum X = 55, \sum Y = 40, \sum X^2 = 385, \sum Y^2 = 192, \sum XY = 185$

(a) 0.068 (b) −0.068 (c) 0.186 (d) None of these

3. Find the covariance between X and Y, given that $\sum X = 60, \sum Y = 90, \sum XY = 574$ and n = 10.

(a) 1.5 (b) 2.7 (c) 3.4 (d) None of these

4. Take 200 and 150 respectively as the assumed mean for X and Y series of 11 values, then dx = X – 200, dy = Y − 150, ∑ dx = 13, ∑ dx^2 = 2667, ∑ dy = 42, ∑dy^2 = 6964; ∑dxdy = 3943

The value of r is

(a) 0.77 (b) 0.98 (c) 0.92 (d) 0.82

5. What is the coefficient of correlation between the ages of husbands and wives from the following data? **(ICAI)**

Age of Husband (Year)	46	45	42	40	38	35	32	30	27	25
Age of wife (Year)	37	35	31	28	30	25	23	19	19	18

(a) 0.58 (b) 0.98 (c) 0.89 (d) 0.92

6. Calculate the covariance between X and Y for the following data:

X	1	2	3	4	5	6	7	8	9	10
Y	6	9	6	7	8	5	12	3	17	1

(a) 0.5 (b) 0.75 (c) 0.4 (d) None of these

7. Find correlation between age and blindness using following data:

Age (years)	50 - 55	55-60	60-65	65-70	70-75	75-80
Number of Persons	25000	20000	15000	12000	10000	8000
Number of Blind	200	150	90	48	30	12

(a) 0.463 (b) 0.959 (c) –0.9894 (d) 0.7618

Answer Key

1. (b) **2.** (b) **3.** (c) **4.** (c) **5.** (b) **6.** (c) **7.** (b)

SPEARMAN'S COEFFICIENT OF RANK CORRELATION

- In situations where we have a series of items that cannot be measured numerically, such as qualities like beauty, intelligence, leadership ability, honesty, etc., we can still compare the rankings of individuals within a group.
- But the individuals in the group can be arranged in order thereby obtaining for each individual a number indicating its rank in the group.
- If we have a group of individuals ranked according to two different qualities, it is natural to ask the following question: "Is there an association between the rankings?"
- To answer this question, we need to use a formula known as Spearman's coefficient of rank correlation.
- The Spearman's correlation coefficient is nothing but Karl Pearson's correlation coefficient between the ranks and is interpreted in much the same way.

  The Spearman's correlation coefficient, denoted by "r" will range from - 1 to + 1. A value of + 1 indicates perfect association for identical rankings and a value of - 1 indicates perfect association for reverse rankings.

Rank R_1	Rank R_2
1	7
2	8
3	6
4	1
6	2
8	4
7	3
5	5

In this example, we have two sets of rankings, R_1 and R_2. The values in the table represent the ranks assigned to the individuals for each quality. By applying Spearman's coefficient of rank correlation formula, we can determine the extent of association between these rankings.

COMPUTING THE RANK CORRELATION COEFFICIENT

We shall consider the following three cases to compute the rank correlation coefficient.

Case I: When actual ranks are given

In this case the following steps are involved:

1. Compute D, the difference between the two ranks given to each individual.
2. Compute D^2 and obtain the $\sum D^2$.

Apply the formula: $r = 1 - \frac{6\sum D^2}{n\left(n^2 - 1\right)}$ where, n is the number of observations.

Example 28. Determine Spearman's rank correlation coefficient from the given data $\sum D^2 = 30$, N = 10. **(June 2019)**

(a) R = 0.82 (b) R = 0.32 (c) R = 0.40 (d) None of these

Sol. (a) Here, $\sum D^2 = 30$, N = 10

Spearman's rank correlation is;

$$r = 1 - \frac{6\Sigma D^2}{N\left(N^2 - 1\right)} = 1 - \frac{6 \times 30}{10\left(10^2 - 1\right)} = 1 - \frac{180}{990} = 1 - \frac{2}{11} = \frac{9}{11} = 0.82$$

Therefore, R = 0.82

Hence, the correct answer is option (a), i.e, R = 0.82

Example 29. If the sum of squares of the rank differences of 9 pairs of values is 80, find the correlation coefficient between them.

(a) 0.33 (b) 0.39 (c) –0.33 (d) –0.039

Sol. (a) Given,

Sum of squares of the rank differences = $\sum D^2 = 80$

Number of pairs n = 9

We know,

$$r = 1 - \frac{6\sum D^2}{n(n^2 - 1)}$$

$$r = 1 - \frac{6(80)}{9(9^2 - 1)} = 1 - \frac{480}{720} = 0.33 \text{ (approax)}$$

The correlation coefficient between them is 0.33.

Hence, the correct option is (a).

Example 30. Ten competitors in following in a beauty contest are ranked by two judges in the following order:

I Judge	1	6	5	10	3	2	4	9	7	8
II Judge	6	4	9	8	1	2	3	10	5	7

Calculate the Spearman's rank correlation coefficient.

(a) 0.6364 (b) 1.395 (c) –0.9894 (d) 0.7618

Sol. (a)

I Judge R_1	II Judge R_2	$D = R_1 - R_2$	D^2
1	6	-5	25
6	4	2	4
5	9	-4	16
10	8	2	4
3	1	2	4
2	2	0	0
4	3	1	1
9	10	-1	1
7	5	2	4
8	7	1	1
			$\sum D^2 = 60$

So, $\sum D^2 = 60$ and $n = 10$

Now we have to use, $r = 1 - \frac{6\sum D^2}{n(n^2-1)}$

$$r = 1 - \frac{6(60)}{10(10^2-1)} = \frac{7}{11} = 0.6364$$

The Spearman's rank correlation coefficient is 0.6364.

Hence, the correct option is (a).

Example 31. Rankings of 10 trainees at the beginning and at the end of a certain course are given as follows:

Trainees:	A	B	C	D	E	F	G	H	I	J
Rank at the beginning:	1	6	3	9	5	2	7	10	8	4
Rank at the end:	6	8	3	7	2	1	5	9	4	10

Calculate spearman's rank correlation coefficient

(a) 0.636 (b) 0.394 (c) –0.394 (d) 0.762

Sol. (b)

Trainees	Rank at the beginning R_1	Rank at the end R_2	$D = R_1 - R_2$	D^2
A	1	6	–5	25
B	6	8	–2	4
C	3	3	0	0

Trainees	Rank at the beginning R_1	Rank at the end R_2	$D = R_1 - R_2$	D^2
D	9	7	2	4
E	5	2	3	9
F	2	1	1	1
G	7	5	2	4
H	10	9	1	1
I	8	4	4	16
J	4	10	–6	36
				$\sum D^2 = 100$

So, $\sum D^2 = 100$ and $n = 10$

Now we have to use, $r = 1 - \frac{6\sum D^2}{n(n^2-1)}$

$$r = 1 - \frac{6(100)}{10(10^2-1)} = \frac{13}{33} = 0.3939$$

The Spearman's rank correlation coefficient is 0.394.

Hence, the correct option is (b).

Example 32. Ten competitors in a beauty contest are ranked by three judges in the following order:

I Judge:	1	4	8	9	6	10	7	3	2	5
II Judge:	4	8	7	5	9	6	10	2	3	1
III Judge:	6	7	1	8	10	5	9	2	3	4

Use the rank correlation method to determine which pair of judges has the nearest approach to common taste in beauty

(a) Judge I and Judge II (b) Judge I and Judge III

(c) Judge II and Judge III (d) None of these

Sol. (c)

I Judge R_1	II Judge R_1	III Judge R_1	$D_1 = R_1 - R_2$	$D_2 = R_1 - R_3$	$D_3 = R_2 - R_3$	D_1^2	D_2^2	D_3^2
1	4	6	–3	–5	–2	9	25	4
4	8	7	–4	–3	1	16	9	1
8	7	1	1	–7	6	1	49	36
9	5	8	4	1	–3	16	1	9
6	9	10	–3	–4	–1	9	16	1

I Judge R_1	II Judge R_1	III Judge R_1	$D_1 = R_1 - R_2$	$D_2 = R_1 - R_3$	$D_3 = R_2 - R_3$	D_1^2	D_2^2	D_3^2
10	6	5	4	5	1	16	25	1
7	10	9	–3	–2	1	9	4	1
3	2	2	1	1	0	1	1	0
2	3	3	–1	–1	0	1	1	0
5	1	4	4	1	–3	16	1	9
						$\sum D_1^2 = 94$	$\sum D_2^2 = 132$	$\sum D_3^2 = 62$

Now we have, $\sum D_1^2 = 94$ $\sum D_2^2 = 132$ $\sum D_3^2 = 62$ and n = 10

Then correlation coefficient between Judge I and Judge II is $r_1 = 1 - \frac{6\sum D_1^2}{n(n^2-1)}$

$$r_1 = 1 - \frac{6(94)}{10(10^2-1)} = \frac{71}{165} = 0.4303$$

Then correlation coefficient between Judge I and Judge III is $r_2 = 1 - \frac{6\sum D_2^2}{n(n^2-1)}$

$$r_2 = 1 - \frac{6(132)}{10(10^2-1)} = \frac{1}{5} = 0.2$$

Then correlation coefficient between Judge II and Judge III is $r_3 = 1 - \frac{6\sum D_3^2}{n(n^2-1)}$

$$r_3 = 1 - \frac{6(62)}{10(10^2-1)} = \frac{103}{165} = 0.6242$$

The Judge II and the Judge III have the nearest approach to common taste in beauty since the correlation coefficient is maximum in that case.

Hence, the correct option is (c).

Example 33. In a bivariate data of n pairs of observations, the sum of squares of differences between the ranks of observed values of two variables is 231 and the rank correlation coefficient is –0.4. Find the value of n.

(a) 10 (b) 13 (c) $-5 \pm \sqrt{74}i$ (d) 5

Sol. (a) Given: $\sum D^2 = 231$ and $r = -0.4$

Using formula, $r = 1 - \frac{6\sum D^2}{n\left(n^2-1\right)}$

$$\Rightarrow -0.4 = 1 - \frac{6(231)}{n(n^2-1)}$$

$$\Rightarrow -1.4 = -\frac{6(231)}{n(n^2-1)}$$

$\Rightarrow -1.4n\,(n^2 - 1) = -1386$

$\Rightarrow n\,(n^2-1) = 990$

$\Rightarrow n\,(n^2-1) = 10 \times 900 = 10\,(10^2 - 1)$

$\Rightarrow n = 10$

Therefore, the value of n is 10.

Hence, the correct option is (a).

Example 34. The coefficient of rank correlation of the marks obtained by 10 students in Statistics and Accountancy was found to be 0.2. It was later discovered that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 9 instead of 7. Find the correct value of coefficient of rank correlation.

(a) 0.3334 (b) 0.3939 (c) –0.3334 (d) –0.3939

Sol. (b) Given: r = 0.2, n=10

As we know, $r = 1 - \frac{6\sum D^2}{n\left(n^2-1\right)}$

$$\Rightarrow 0.2 = 1 - \frac{6\sum D^2}{10(10^2-1)}$$

$$\Rightarrow 0.8 = \frac{6\sum D^2}{990}$$

$$\Rightarrow \sum D^2 = \frac{0.8 \times 990}{6} = 132$$

Now Correct $\sum D^2 = 132 - 9^2 + 7^2 = 100$

Again using, $r = 1 - \frac{6\sum D^2}{n\left(n^2-1\right)}$

$$r = 1 - \frac{6(100)}{10(10^2-1)} = 1 - \frac{60}{99} = \frac{13}{33} = 0.3939$$

Therefore, the correct value of coefficient of rank correlation is 0.3939.

Hence, the correct option is (b).

Case II: When actual ranks are not given:

- Sometimes we are given the actual bivariate data on two variables and not the ranks. In such situations, it is necessary to assign the ranks.

- Ranks can be assigned by taking either the highest value as 1 or the lowest value as 1. The next highest or the next lowest value is given rank 2 and so on.
- But whether we start with the lowest value or the highest value, we must follow the same method in case of both the variables.

Example 35. The marks obtained by 9 students in Mathematics and Accountancy are as follow:

Marks in Mathematics (X):	30	33	45	23	8	49	12	4	31
Marks in Accountancy (Y):	35	23	47	17	10	43	9	6	28

Calculate Spearman's rank correlation coefficient.

(a) 0.4 (b) 0.39 (c) –0.34 (d) 0.9

Sol. (d) Here we does not have ranks so we have to assign ranks first,

Marks in Mathematics (X)	Marks in Accountancy (Y)	Rank of (X) R_1	Rank of (Y) R_2	$D = R_1 - R_2$	D^2
30	35	5	3	2	4
33	23	3	5	–2	4
45	47	2	1	1	1
23	17	6	6	0	0
8	10	8	7	1	1
49	43	1	2	–1	1
12	9	7	8	–1	1
4	6	9	9	0	0
31	28	4	4	0	0
					$\sum D^2 = 12$

Now we have $\sum D^2 = 12$, and $n = 9$

Using $r = 1 - \dfrac{6\sum D^2}{n\left(n^2 - 1\right)}$

$$r = 1 - \frac{6(12)}{9(9^2 - 1)} = \frac{9}{10} = 0.9$$

Therefore, the spearman's rank correlation coefficient is 0.9.
Hence, the correct option is (d).

WHEN RANKS ARE EQUAL

- If there are two or more items with the same rank in either series, then it is customary to assign common rank to each repeated item.
- The common rank is the average of the ranks which these items would have got if they were different from each other and the next item will get the rank next to the rank used in computing the common rank.

- For example, suppose there are two items at rank 4. In this case, the common rank assigned to each item would be 4.5.
- The next item in line would then be assigned rank 5.
- Similarly, if there are three items at rank 7, the common rank assigned to each item will be 7.
- The next rank to be assigned will be 10.
- When equal ranks are assigned to certain items, an adjustment is made in the formula for calculating the Spearman's rank correlation coefficient. This adjustment involves adding a correction factor for each repeated item in both series.
- This correction factor is to be added for each repeated item in both the series. The formula can thus be written as follows:

$$r = 1 - \frac{6\sum D^2}{n\left(n^2 - 1\right)}$$

Example 36. Calculate Spearman's coefficient of rank correlation from the following data:

X	57	16	24	65	16	16	9	40	33	48
Y	19	6	9	20	4	15	6	24	13	13

(a) 0.7515 (b) 0.3009 (c) -0.3674 (d) 0.7899

Sol. (a)

(X)	(Y)	Rank of (X) R_1	Rank of (Y) R_2	$D = R_1 - R_2$	D^2
57	19	2	3	–1	1
16	6	8	8.5	–0.5	0.25
24	9	6	7	–1	1
65	20	1	2	–1	1
16	4	8	10	–2	4
16	15	8	4	4	16
9	6	10	8.5	1.5	2.25
40	24	4	1	3	9
33	13	5	5.5	–0.5	0.25
48	13	3	5.5	–2.5	6.25
					$\sum D^2 = 41$

Now we have, $\sum D^2 = 41$ and $n = 10$

Using, $r = 1 - \frac{6\sum D^2}{n\left(n^2 - 1\right)}$ we get

$$r = 1 - \frac{6(41)}{10(10^2 - 1)} = \frac{124}{165} = 0.7515$$

Therefore, the spearman's rank correlation coefficient is 0.7515.

Hence, the correct option is (a).

MERITS AND DEMERITS OF SPEARMAN'S RANK CORRELATION METHOD

Merits

The rank correlation method has the following merits

1. It is easy to understand and simple to apply.
2. The Spearman's rank correlation method is the only method that can be used to find correlation coefficients if we are dealing with data of qualitative characteristics like beauty, intelligence, honesty, etc.
3. This is the only method that can be used where we are given the ranks and not the actual bivariate data on two variables.

DEMERITS

The rank correlation method has the following limitations:

1. This method cannot be used for finding correlation in the case of bivariate frequency distribution.
2. This method is very difficult to apply when the number of items is more than 30.

PRACTICE QUESTIONS (PART C)

1. If the sum of square of differences of rank is 50 and the number of items is 8, then what is the value of the rank correlation coefficient?

   (a) 0.59 (b) 0.40 (c) 0.36 (d) 0.63

2. If the rank correlation coefficient between marks in Management and Mathematics for a group of students is 0.6 and the sum of the squares of the difference in rank is 66 . Then what is the number of students in the group?

   (a) 9 (b) 10 (c) 11 (d) 12

3. What is the value of the Rank correlation coefficient between the following marks in Physics and Chemistry:

Roll No:	1	2	3	4	5	6
Marks in Physics:	25	30	46	30	55	80
Marks in Chemistry:	30	25	50	40	50	78

   (a) 0.782 (b) 0.696 (c) 0.932 (d) 0.857

4. The ranks of five participants given by two judges are:

   Participants

	A	B	C	D	E
Judge I	1	2	3	4	5
Judge II	5	4	3	2	1

   (a) 1 (b) 0 (c) -1 (d) 12

5. The coefficient of rank correlation between debenture prices and share prices of a company is found to be 0.143. If the sum of the squares of the differences in ranks is 48, find the value of n.

(a) 5 (b) 6 (c) 7 (d) None of these

6. Calculate Spearman's rank correlation coefficient between advertisement cost and sales from the following data:

Advertisement cost (000 ₹)	39	65	62	90	82	75	25	98	35	78
Sales (lakhs ₹)	47	53	58	86	62	68	60	91	51	84

(a) 0.5 (b) 0.734 (c) 0.65 (d) None of these

Answer Key

1. (b) **2.** (b) **3.** (d) **4.** (c) **5.** (c) **6.** (b)

COEFFICIENT OF CONCURRENT DEVIATION

The coefficient of concurrent deviation is a statistical measure used to assess the degree of simultaneous variation or deviation in two variables. It helps in understanding how two sets of data move together or diverge from each other.

The formula for the coefficient of concurrent deviation is as follows

$$r_c = \pm\sqrt{\pm\frac{2c-m}{m}}$$

where, 'n' represents the number of concurrent deviation, 'm' represents the total number of deviations (which must be one less than the number of pairs of x and y values)

Also, if $(2c-m)>0$, then we take the positive sign both inside and outside the radical sign and if $(2c-m)<0$, we take the negative sign both inside and outside the radical sign.

Example 37. What is the coefficient of concurrent deviations for the following data **(ICAI)**

Supply	68	43	38	78	66	83	38	23	83	63	53
Demand	65	60	55	61	35	75	45	40	85	80	85

(a) 0.82 (b) 0.85 (c) 0.89 (d) -0.81

Sol. (c) Now prepare a table to find the coefficient of concurrence:

If the value increases then we will write +ve and −ve when decreases for both the variables.

Supply	Demand	Deviation sign for X	Deviation sign for Y	Deviation sign for XY
68	65			
43	60	−ve	−ve	+ve
38	55	−ve	−ve	+ve
78	61	+ve	+ve	+ve

Supply	Demand	Deviation sign for X	Deviation sign for Y	Deviation sign for XY
66	35	−ve	−ve	+ve
83	75	+ve	+ve	+ve
38	45	−ve	−ve	+ve
23	40	−ve	−ve	+ve
83	85	+ve	+ve	+ve
63	80	−ve	−ve	+ve
53	85	−ve	+ve	−ve

The number of positive signs is the value of c.

Here n = 11, m = n − 1 = 11 − 1 = 10, c = 9

We know that $r_c = \pm\sqrt{\pm\frac{(2c-m)}{m}}$

$\Rightarrow r_c = \pm\sqrt{\pm\frac{(2(9)-10)}{10}}$

$\Rightarrow r_c = \pm\sqrt{\pm\frac{(18-10)}{10}}$

$\Rightarrow r_c = \sqrt{\frac{8}{10}}$

$\Rightarrow r_c = 0.8944$

Hence, the correct answer is option (c) i.e., 0.89.

Example 38. For 10 pairs of observations, number of concurrent deviations was found to be 4. What is the value of the coefficient of concurrent deviation? **(ICAI)**

(a) $\sqrt{0.2}$ (b) $-\sqrt{0.2}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

Sol. (d) Given that,

No. of Concurrent deviations (c) = 4

Total number of observations (n) = 10

We know that the coefficient of concurrent deviation $\Rightarrow r_c = \pm\sqrt{\pm\frac{2c-m}{m}}$

Here, m = n − 1 = 10 − 1 = 9

Then,

$\Rightarrow r_c = \pm\sqrt{\pm\frac{2c-m}{m}}$

$\Rightarrow r_c = \pm\sqrt{\pm\frac{2\times4-9}{9}}$

$$\Rightarrow r_c = \pm\sqrt{\pm\frac{8-9}{9}}$$

$$\Rightarrow r_c = -\sqrt{\frac{1}{9}}$$

$$\Rightarrow r_c = \frac{-1}{3}$$

If $(2c - m) < 0$, then we take the negative sign both inside and outside the radical sign

Hence, the correct answer is option (d) i.e., −13.

Example 39. The coefficient of concurrent deviation for P pairs of observations was found to be 13. If the number of concurrent deviations was found to be 6, then the value of P is

(a) 10 (b) 9 (c) 8 (d) None of these **(ICAI)**

Sol. (a) Given:

Coefficient of concurrent deviation $(r_c) = \frac{1}{\sqrt{3}}$

No. of Concurrent deviations (c) = 6

Total number of observations (n) = ρ

We know that, coefficient of concurrent deviation

$$r_c = \pm\sqrt{\pm\frac{2c-m}{m}}$$

Here, m = P − 1

Then,

$$\Rightarrow r_c = \pm\sqrt{\pm\frac{2c-m}{m}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \pm\sqrt{\pm\frac{2\times 6-m}{m}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \pm\sqrt{\pm\frac{12-m}{m}}$$

Squaring on both sides, we get

$$\frac{1}{3} = \frac{12-m}{m}$$

$\Rightarrow m = 3\ (12 - m)$

$\Rightarrow m = 36 - 3m$

$\Rightarrow 4m = 36$

$$\Rightarrow m = \frac{36}{4} = 9$$

Here, m = P − 1

$\Rightarrow P = m + 1 = 9 + 1 = 10$

Hence, the correct answer is option (a) i.e., 10.

PREVIOUS YEAR QUESTION (PYQ'S)

1. Given that

X	–3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3
Y	9	$\frac{9}{4}$	0	$\frac{9}{4}$	9

Then Karl Pearson's coefficient of correlation is **(June 2019)**

(a) Positive (b) Zero (c) Negative (d) None

2. Given that

X	10	13	12	15	8	15
Y	12	16	18	16	7	18

The rank correlation coefficient R = **(June 2019)**

(a) $1-\frac{\sum d^2 \sum_{t=1}^{2} m_i(m_i^z-1)}{N(N^2-1)}$ (b) $1-\frac{6\left[\sum d^2+\frac{\sum_{i=1}^{2} m_i(m_i^2-1)}{12}\right]}{N(N^2-1)}$

(c) $1-6\sum d^2+\sum_{i=1}^{2}\frac{\frac{m_i(m_i^2-1)}{12}}{N(N^2-1)}$ (d) None

3. Determine Spearman's rank correlation coefficient from the given data $\Sigma D^2 = 30$, N = 10. **(June 2019)**

(a) R = 0.82 (b) R = 0.32 (c) R = 0.40 (d) None of these

4. Find correlation coefficient

X	5	4	3	2	1
Y	1	2	3	4	5

(Dec 2019)

(a) 1 (b) -1 (c) 0 (d) None of these

5. If scatter diagram from a line move from lower left to upper right corner then the correlation is **(Dec 2019)**

(a) Perfect positive (b) Perfect negative
(c) Simple positive (d) No correlation

6. If correlation coefficient between x and y is 0.5, then find the correlation coefficient between 2x – 3 and 3 – 5y is **(Dec 2019)**

(a) 0.5 (b) -0.5 (c) 2.5 (d) -2.5

7. Which of the following is a spurious correlation? **(Dec 2020)**

(a) Correlation between two variables having no casual relationship

(b) Negative correlation

(c) Bad relation between two variables

(d) Very low correlation between two variables

8. Scatter diagram does not help us to **(Dec 2020)**

(a) Find the type of correlation

(b) Identify whether variables correlated or not

(c) Determine the linear (or) non-linear correlation

(d) Find the numerical value of correlation coefficient

9. The covariance between two variables is **(Dec 2020)**

(a) Strictly Positive (b) Strictly Negative

(c) Always zero (d) Either positive or negative or zero

10. For the set of observations {(1, 2), (2, 5), (3, 7), (4, 8), (5, 10)}, the value of Karl Pearson's coefficient of correlation is approximately given by **(Jan 2021)**

(a) 0.755 (b) 0.655 (c) 0.525 (d) 0.985

11. The coefficient of correlation between x and y is 0.5, the covariance is 16 and the standard deviation of y is _____ if S.D of x is 4. **(Jan 2021)**

(a) 4 (b) 8 (c) 16 (d) 64

12. If the sum of the product of the deviation of X and Y from their means is zero, the correlation coefficient between X and Y is; **(Jan 2021)**

(a) Zero (b) Positive (c) Negative (d) 10

13. If the data points of (X, Y) series on a scatter diagram lie along a straight line that goes downwards as X-values move from left to right, then the data exhibit _____ correlation **(Dec 2021)**

(a) Direct (b) Imprefect indirect

(c) Indirect (d) Imperfect direct

14. If the plotted points in a scatter diagram lie from lower left to upper right, then the correlation is **(June 2022)**

(a) Negative (b) Perfect negative

(c) Zero (d) Positive

15. For finding correlation between two qualitative characteristics, we use **(June 2022)**

(a) Coefficient of rank correlation

(b) Scatter diagram

(c) Coefficient of concurrent deviation

(d) Product moment correlation coefficient

16. Karl Pearson's coefficient is defined from **(June 2022)**

(a) Grouped data (b) Ungrouped data

(c) Any data (d) Scattered data

17. For n pairs of observations, the coefficient of concurrent deviation is calculated as $\frac{1}{\sqrt{5}}$. If there are six concurrent deviations, then n=? **(June 2022)**

(a) 11 (b) 10 (c) 9 (d) 8

18. The coefficient of rank correlation between the ranking of following 6 students in two subjects, Mathematics and Statistics is:

Mathematics	3	5	8	4	7	10
Statistics	6	4	9	8	1	2

(Dec 2022)

(a) - 0.25 (b) - 0.35 (c) 0.38 (d) 0.20

19. Pearson's Correlation coefficient between x and y is **(Dec 2022)**

(a) $\frac{\text{cov}(x, y)}{s_x, s_y}$ (b) $\frac{\text{cov}^2(x, y)}{s_x, s_y}$ (c) $\frac{(s_x, s_y)}{\text{cov}(x, y)}$ (d) $\frac{s_x, s_y}{\text{cov}(x, y)}$

20. Given that r = 0.4 and n = 81, determine the limits from the population correlation coefficient. **(June 2023)**

(a) (0.333, 0.466) (b) (0.367, 0.433)

(c) (0.337, 0.463) (d) (0.373, 0.427)

21. Spearman's rank correlation coefficient r_R is given by: **(June 2023)**

(a) $1-\frac{6\sum d_i^2}{n(n^2+1)}$ (b) $1+\frac{6\sum d_i^2}{n(n^2-1)}$

(c) $1+\frac{6\sum d_i^2}{n(n^2+1)}$ (d) $1-\frac{6\sum d_i^2}{n(n^2-1)}$

22. If 4y – 6x = 8 is the regression line of y on x and the coefficient of correlation between x and y is 0.8. What is the value of x on y? **(Dec 2023)**

(a) 0.2448 (b) 0.4267 (c) 0.5733 (d) 0.7441

23. If the coefficient of correlation is 0.8, and regression coefficient b_{xy} = 0.32, then what is the value of regression coefficient? **(Dec 2023)**

(a) 2 (b) 1 (c) 0.52 (d) 0.48

24. If Cov (X, Y) = –2.15, σ_x= 1.30, σ_y = 2.50, then the correlation coefficient is **(June 2024)**

(a) –0.66 (b) 0.66 (c) 0.76 (d) 0.99

25. For a group of 10 students the sum of squares of difference in ranks for Physics and Chemistry marks was 60, what is the value of rank correlation coefficient. (Choose the nearest value) **(June 2024)**

(a) 0.636 (b) 0.725 (c) 0.698 (d) 0.842

26. Spearman's correlation coefficient is used to check **(June 2024)**

(a) The scattering of the data (b) The relationship in variables

(c) The median of a data (d) The range of a data

27. The variance of two variables 'x' and 'y' are 16 and 25 and covariance between 'x' and 'y' is 18.5. Another two variables 'u' and 'v' are defined as $u = \frac{(x-3)}{2}$ and $v = \frac{(y-2)}{3}$ then coefficient of correlation between 'u' and 'v' is: **(June 2024)**

(a) 0.85 (b) 0.875 (c) 0.90 (d) 0.925

28. In case of "Insurance companies profits" and "The number of claims they have to pay", there exists a **(Sept 2024)**

(a) Positive correlation (b) Negative correlation

(c) No correlation (d) It cannot be predicted

29. The variance of two variables 'x' and 'y' are 16 and 25 and covariance between 'x' and 'y' is 18.5. Another two variables 'u' and 'v' are defined as $u = \frac{(x-3)}{2}$ and $v = \frac{(y-2)}{3}$ then coefficient of correlation between 'u' and 'v' is: **(Sept 2024)**

(a) 0.85 (b) 0.875 (c) 0.90 (d) 0.925

30. You are given the following data relating to a frequency distribution of 10 observations: $\sum X = 50, \sum Y = 60, \sum X^2 = 300, \sum Y^2 = 350, \sum (X+Y)^2 = 1372$, then Cov (X, Y) is **(Jan 2025)**

(a) 4 (b) 2 (c) 6 (d) 8

31. If r = 0.8, then the coefficient of non-determination is **(Jan 2025)**

(a) 0.64 (b) 0.36 (c) 0.20 (d) –0.36

32. The correlation coefficient between X and Y is 0.8. If we add a number 10 in X-variable and subtract 20 from Y-variable, then the new correlation coefficient will be **(May 2025)**

(a) 0.4 (b) 0.6 (c) 0.9 (d) 0.8

33. If two variables move in the same direction i.e., an increase (or decrease) on the part of one variable introduces an increase (or decrease) on the part of the other variable, then the two variables are known to be **(Sept 2025)**

(a) Positive correlation (b) Negative correlation

(c) Zero correlation (d) Curvilinear correlation

34. The sum of squares of the differences between two ranks awarded by two judges on 10 candidates is _____ if the rank correlation coefficient is 0.8. **(Sept 2025)**

(a) 44 (b) 55 (c) 66 (d) 33

35. For a group of students, the sum of squares of differences in ranks for Maths and Physics marks are found to be 60, which is 120 times the value of rank correlation coefficient. How many students are there in the group? **(Sept 2025)**

(a) 8 (b) 10 (c) 9 (d) 12

Answer Key

1. (b)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (d)	**9.** (d)	**10.** (d)
11. (b)	**12.** (a)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (c)
21. (d)	**22.** (b)	**23.** (a)	**24.** (a)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (c)
31. (b)	**32.** (d)	**33.** (a)	**34.** (d)	**35.** (c)					

REGRESSION

Regression, in a literal sense, refers to a process of moving backward or returning to an average value. In the context of statistics and mathematics, it involves creating a mathematical equation to predict the value of one variable based on known values of one or more other variables. It is a method for understanding relationships and making forecasts.

E.g.: Consider predicting a student's final exam score based on the number of hours spent studying. Regression analysis could help develop an equation that models this relationship, allowing educators to make predictions about a student's potential performance based on their study hours. The regression equation would provide a tool for forecasting final exam scores using the known variable of study hours.

- **Dependent Variable:** The variable whose value is to be predicted is called the dependent variables or explained variable.
- **Independent Variable:** The variabless which are used to predict the values of a dependent variable are called independent variabless or explanatory variables.
- **Simple Regression Analysis and Simple Linear Analysis:** The regression analysis confined to the study of only two variables, a dependent varibale and an independent variable, is called simple regression analysis.

  When the relationship between the dependent variables and the independent variable is linear, the technique for prediction is called simple linear regression.

  If let say y depends on x, then equation will be:-

  $\Rightarrow y = a + bx$
- **METHOD OF LEAST SQUARE: REGRESSION**: Thus, if a line of best fit approximating the given data has the equation Y = a + bX then the method of least squares requires that we must determine constants a and b the so as to minimize

  $e_i = \sum(y_i - \widehat{y_1})^2 = \sum(y_i - (a + bx_i)^2$ where $\widehat{y_i} = a + bx_i$

  Where, e_i is the error of estimation.

- The equations known as the normal equation for estimating a and b, are given by

$$\Rightarrow b = \frac{n\sum xy - \sum x \sum y / n^2}{n\sum x^2 - \left(\sum x\right)^2 / n^2}$$

$$\Rightarrow b = \frac{\frac{\sum xy}{n^x} - \frac{\sum x \sum y}{n^2}}{\frac{\sum x^2}{n^2} - \frac{\left(\sum x\right)^2}{x^2}}$$

$$\Rightarrow b = \frac{\frac{\sum xy}{n} - \frac{\sum x}{n}\frac{\sum y}{n}}{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \frac{\text{cov}(x, y)}{\sqrt{x^2}}$$

Solving equation simultaneously for a and b, we obtain

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

Hence, the line of best fit approximating n pair of observations $(x_1, y_1,)$, $(x_{2,} y_2)$ ....., $(x_{n,} y_n)$ is y on x

y = a + bx, where a, b are the constants.

The line of best fit given is called the least squares line of regression of y on x. The constant b is called the regression coefficient of y on x is denoted by b_{yx}.

It measures the change in y corresponding to a unit change in x.

- Thus, b_{yx} represent the slope of the line of regression of Y on X and is given by

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

On the other hand, if we wish to estimate a value of X for a given value of Y, we have to obtain a regression line of X on Y:

X = bY + a.

The line of best fit given is called the least squares line of regression of X on Y. The constant b is called the regression coefficient of X on Y is denoted by b_{XY}.

It measures the change in X corresponding to a unit change in Y.

Thus, b_{yx} represent the slope of the line of regression of Y on X and is given by

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{N\sum y^2 - \left(\sum y\right)^2}$$

The equation of the line of regression of Y on X can also be written as $(y - \bar{y}) = b_{yx}(x - \bar{x})$

REGRESSION LINES

Since, $\sum y = an + b\sum x$

$$\left(\frac{\sum y}{n}\right) = \frac{na + b\sum x}{n}$$

$$\left(\frac{\sum y}{x}\right) = \frac{na}{n} + b\frac{\sum x}{n}$$

$$\Rightarrow \bar{y} = a + b\bar{x}$$

$$\Rightarrow a = \bar{y} - b\bar{x} \quad \text{...(i)}$$

$$\Rightarrow b_{yx} = b = \frac{x\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

(y on x)

$$b_{yx} = \frac{\text{cov}(x, y)}{(\sigma_x)^2}$$

$$b \text{ or } b_{yx} = r\left(\frac{\sigma_y}{\sigma_x}\right) \quad \text{...(ii)}$$

Regression line of y on x is given by

$y = a + bx$

From (i)

$$\Rightarrow y = (\bar{y} - b\bar{x}) + bx$$

$$\Rightarrow y - \bar{y} = b(x - \bar{x})$$

From (ii)

$$\Rightarrow y - \bar{y} = r\left(\frac{\sigma_y}{\sigma_x}\right)(x - \bar{x})$$

(Regression line in correlation coefficient)

$$\frac{r}{\sigma_x} = \frac{\text{cov}(x, y)}{\sigma_x\sigma_y(\sigma_x)}$$

$$\frac{r}{\sigma_x} = \frac{\text{cov}(x, y)}{(\sigma_x)^2}\sigma_y$$

$$\Rightarrow \frac{r}{\sigma_x} = \frac{b_{yx}}{\sigma_y}$$

$$\Rightarrow b_{yx} = \frac{\sigma_y r}{\sigma_x}$$

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Example 40. Calculate the regression coefficients from the following information:

$\sum X = 50, \sum Y = 30, \sum XY = 1000, \sum X^2 = 3000, \sum Y^2 = 1800$ and $n = 10$

(a) 0.497 and 0.309 (b) 0.307 and 0.3009

(c) 0.586 and –0.367 (d) None of these

Sol. (a) Given: $\sum X = 50, \sum Y = 30, \sum XY = 1000, \sum X^2 = 3000, \sum Y^2 = 1800$, and $n = 10$

Thus,

$$b_{xy} = \frac{n\sum XY - \sum X \sum Y}{n\sum Y^2 - \left(\sum Y\right)^2} = \frac{10(1000) - (50)(30)}{10(1800) - (30)^2} = \frac{10000 - 1500}{18000 - 900} = 0.497$$

Also, $b_{yx} = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - \left(\sum X\right)^2} = \frac{10(1000) - (50)(30)}{10(3000) - (50)^2} \frac{8500}{30000 - 2500} = \frac{17}{55} = 0.309$

Therefore, the regression coefficients are 0.497 and 0.309.

Hence, the correct option is (a).

Example 41. In the regression equation X on Y, $X = \frac{35}{8} - \frac{2Y}{5}$ b_{xy} is equal to

(a) $-\frac{2}{5}$ (b) $\frac{35}{8}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$

Sol. (a) We know that, equation of regression line X on Y is given by $X = a + bY$

where, $b = b_{xy}$ = slope of line and a= intercept of line

Comparing the given equation $X = \frac{35}{8} - \frac{2Y}{5}$ to the standard equation, we get

$$b_{xy} = -\frac{2}{5}$$

Hence, the correct answer is option (a) i.e., −2/5.

Example 42. In the estimation of egression equation of two variables X and Y, the following results were obtained:

$\sum X = 900, \sum Y = 700, \sum X^2 = 6360, \sum Y^2 = 2860, \sum XY = 3900, n = 10$

Obtain two regression equations.

(a) $x - 1.81y + 6.67 = 0$ and $3.792x - y - 1.28 = 0$

(b) $4x - 1.31y + 6.67 = 0$ and $0.72x - 1.30y - 1.238 = 0$

(c) $1.3230x - 1.32y + 6.67 = 0$ and $0.792x - 1.033y - 1.228 = 0$

(d) $x - 1.28y + 0.4 = 0$ and $0.792x - y - 1.28 = 0$

Sol. (d) Given: $\sum X = 900, \sum Y = 700, \sum X^2 = 6360, \sum Y^2 = 2860, \sum XY = 390$

As we know,

$$\bar{X} = \frac{\sum X}{n} = \frac{900}{10} = 90 \text{ and } \bar{Y} = \frac{\sum Y}{n} = \frac{700}{10} = 70$$

y on x is $(y - \bar{y}) = b_{yx}(x - \bar{x})$ ...(i)

x on y is $(x - \bar{x}) = b_{xy}(y - \bar{y})$ ...(ii)

Since,

$$b_{yx} = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - \left(\sum X\right)^2} = \frac{10(3900) - (900)(700)}{10(6360) - (900)^2} \quad \frac{591000}{63600 - 810000}$$

$$= \frac{985}{1244} = 0.792$$

Therefore the equation of regression y on x is:

$(y - 70) = 0.792(x - 90)$

$y = 0.792x - 1.28$

$0.792x - y - 1.28 = 0$ ...(iii)

Also,

$$b_{xy} = \frac{n\sum XY - \sum X \sum Y}{n\sum Y^2 - \left(\sum Y\right)^2} = \frac{10(3900) - (900)(700)}{10(2860) - (700)^2} = \frac{3900 - 630000}{28600 - 490000} = 1.28$$

Therefore the equation of regression x on y is:

$(x - 90) = 1.381(y - 70)$

$x = 1.28y + 0.4$ ...(iv)

Therefore, the two equations are:

$x = 1.28y + 0.4$ and $0.792x - y - 1.28 = 0$

Hence, the correct option is (d).

Example 43. The regression equation of y on x for the following data:

x	41	82	62	37	58	96	127	74	123	100
y	28	56	35	17	42	85	105	61	98	73

is given by **(ICAI)**

(a) $y = 1.2x - 15$ (b) $y = 1.2x + 15$

(c) $y = 0.93x - 14.68$ (d) $y = 1.5x - 10.89$

Sol. (c) To find: regression equation y on x i.e. b_{yx}

The data according to the information is as follows:

x	y	$dx = x - \bar{X}$	$dy = y - \bar{Y}$	$dxdy = (x-\bar{X})(y-\bar{Y})$	$dx^2 = (x-\bar{X})^2$
41	28	−39	−32	1248	1521
82	56	2	−4	−8	4
62	35	−18	−25	450	324
37	17	−43	−43	1849	1849
58	42	−22	−18	396	484
96	85	16	25	400	256
127	105	47	45	2115	2209

x	y	$dx = x - \bar{X}$	$dy = y - \bar{Y}$	$dxdy = (x-\bar{X})(y-\bar{Y})$	$dx^2 = (x-\bar{X})^2$
74	61	−6	1	−6	36
123	98	43	38	1634	1849
100	73	20	13	260	400
$\sum x_i = 800$	$\sum y_i = 600$	$(\sum dx) = 0$	$(\sum dy) = 0$	$\sum dxdy =$ 8338	$\sum dx^2 =$ 8932

Here,

$$\bar{X} = \frac{\sum x_i}{N} = \frac{800}{10} = 80$$

$$\bar{Y} = \frac{\sum y_i}{N} = \frac{600}{10} = 60$$

We know that,

$$b_{yx} = \frac{N\sum dxdy - \sum dx \sum dy}{N\sum dx^2 - (\sum dx)^2} = \frac{10(8338) - 0}{10(8932) - (0)^2}$$

$= 83388/8932 = 0.9333$

$$\bar{Y} = a + b_{yx}\,\bar{X}$$

$60 = a + (0.9333)80$

$a = 60 - 74.64$

$a = -14.24$

So, regression equation y on x is given by $y = a + bx$

$y = 0.933x + 14.24$

Hence, the correct answer is option (c) i.e., $y = 0.93x - 14.68$.

Example 44. Following table gives the age of cars of a certain make and annual maintenance costs. Obtain the regression equation for costs related to age:

Age of cars in years:	2	4	6	8
Maintenance cost (in hundred)	10	20	25	30

Also estimate the annual maintenance cost for the ten-year-old car.

(a) $3.792x - y - 1.28 = 0$ and 30

(b) $0.72x - 1.30y - 1.238 = 0$ and 40

(c) $3.25x - y + 5.0 = 0$ and 37.5

(d) $0.792x - y - 1.28 = 0$ and - 37.5

Sol. (c) We have to obtain the regression equation for costs related to age. That means cost is dependent on age.

Let age of cars in years be x and Maintenance cost (in hundred) is y

Therefore, the regression equation for costs related to age will be given by $(y - \bar{y}) = b_{yx}(x - \bar{x})$

Age of cars in years (x)	Maintenance cost (in hundred) (y)	xy	x^2	y^2
2	10	20	4	100
4	20	80	16	400
6	25	150	36	625
8	30	240	64	900
$\sum x = 20$	$\sum y = 85$	$\sum xy = 490$	$\sum x^2 = 120$	$\sum y^2 = 2025$

Here, n = 4

Now, $\bar{X} = \frac{\sum X}{n} = \frac{20}{4} = 5$ and $\bar{Y} = \frac{\sum Y}{n} = \frac{85}{4} = 21.25$

Thus,

$$b_{yx} = \frac{n\sum XY - \sum X \sum Y}{N\sum X^2 - \left(\sum X\right)^2} = \frac{4(490) - (20)(85)}{4(120) - (20)^2} \frac{260}{360 - 400} = \frac{13}{4} = 3.25$$

Therefore, the equation of regression y on x is (y – 21.25) = 3.25(x – 5).

3.25x – y + 5.0 = 0

Now, the annual maintenance cost for the ten-year-old car is 3.25(10) – y + 5.0 = 0

$\Rightarrow$ y = 37.5

Hence, the correct option is (c).

Example 45. The following data relate to the heights of 10 pairs of fathers and sons:

(175, 173), (172, 172), (167, 171), (168, 171), (172, 173),

(171, 170), (174, 173), (176, 175), (169, 170), (170, 173)

The regression equation of height of son on that of father is given by **(ICAI)**

(a) y = 100 + 5x (b) y = 102.60 + 0.4055x

(c) y = 89.653 + 0.582x (d) y = 88.758 + 0.562x

Sol. (b) Let height of father be x and that of son be y.

To find the regression equation y on x, we will find b_{yx}. For that we prepare the table as follows:

Height of father (x)	Height of son (y)	u = x – 171	v = y – 172	uv	u^2
175	173	4	1	4	16
172	172	1	0	0	1
167	171	−4	−1	4	16
168	171	−3	−1	3	9
172	173	1	1	1	1
171	170	0	−2	0	0

Height of father (x)	Height of son (y)	u = x − 171	v = y − 172	uv	u^2
174	173	3	1	3	9
176	175	5	3	15	25
169	170	−2	−2	4	4
170	173	−1	1	−1	1
$(\sum x_i) = 1714$	$(\sum y_i) = 1721$	$(\sum u) = 4$	$(\sum v) = 1$	$(\sum uv) = 33$	$(\sum u^2) = 82$

Here, $\bar{x} = \frac{\sum x_i}{N} = \frac{1714}{10} = 171.4$

$\bar{y} = \frac{\sum y_i}{N} = \frac{1721}{10} = 172.1$

$b_{yx} = \frac{N\sum uv - \sum u \sum v}{N\sum u_i^2 - (\sum u)^2} = \frac{10(33) - 4(1)}{10(82) - (4)^2} = \frac{326}{804} = 0.4054$

$\bar{y} = a + b_{yx}\bar{x}$

$172.1 = a + 0.4054(171.4)$

$a = 172.1 - 69.486$

$a = 102.61$

Since, the regression equation is given by $y = a + bx$

Thus, the regression equation y on x is $y = 102.61 + 0.4054x$

Hence, the correct option is (b) i.e. $y = 102.60 + 0.405x$.

PRACTICE QUESTIONS (PART D)

1. If mean of X and Y variables is 20 and 40 respectively and the regression coefficient Y on X is 1.608, then the regression line of Y on X is:

(a) Y = 1.608 X + 7.84 (b) Y = 1.56 X + 4.84

(c) Y =1.608 X + 4.84 (d) Y =1.56 X + 7.84

2. If Y = 18X + 5 is the regression line of X on Y, the value of b_{xy} is

(a) $\frac{5}{18}$ (b) 18 (c) 5 (d) $\frac{1}{18}$

3. The regression equation of Y on X is, 2X + 3Y + 50 = 0. The value of b_{yx} is

(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{-3}{2}$ (d) None of these

4. The two regression coefficients for the following data are:

x	38	23	43	33	28
y	28	23	43	38	8

(a) 1.2 and 0.4 (b) 1.6 and 0.8 (c) 1.7 and 0.8 (d) 1.8 and 0.3

5. From the following data, obtain the two regression equations and estimate the value of X when Y is 130 and estimate the value of Y when X is 30.

X	39	33	30	31	32	36	41	49	46	43
Y	132	134	138	129	136	131	132	135	128	125

(a) X = 29.38 and Y = 143.66 (b) X = 35.15 and Y = 153.80
(c) X = 39.32 and Y = 133.84 (d) None of these

Answer Key
1. (a) **2.** (d) **3.** (b) **4.** (a) **5.** (c)

SOME MORE FORMULAS – REGRESSION

I. Formulas for Regression Coefficients in terms of Covariance and Variances:

By definition, the regression coefficient of Y on X is given by

$$b_{yx} = \frac{Cov(x, y)}{\sigma_x^2} \text{ or } b_{yx} = r.\frac{\sigma_y}{\sigma_x}$$

Similarly, the regression coefficient of X on Y is given by

$$b_{xy} = \frac{Cov(x, y)}{\sigma_y^2} \text{ or } b_{xy} = r.\frac{\sigma_x}{\sigma_Y}$$

The reader may also recall that the covariance between X and Y & the variances of X and Y values are respectively given by

$$Cov(X, Y) = \frac{\sum(x-\bar{x})(y-\bar{y})}{n} \text{ or } \frac{\sum xy}{n} - \bar{x}\cdot\bar{y}$$

$$\sigma_x^2 = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \text{ or } \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

$$\sigma_y^2 = \sqrt{\frac{\sum(y-\bar{y})^2}{n}} \text{ or } \sqrt{\frac{\Sigma y^2}{n} - (\bar{y})^2}$$

Formulas for regression coefficients in terms of deviation of X– and Y– values from their respective means:

By definition, the covariance between x and y is given by

$$Cov(x, y) = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{N} = \frac{\sum x_i y_i}{N} - \bar{x}\,\bar{y}$$

Further, the variances of x and y values are respectively given by

$$\sigma_X^2 = \frac{\text{Cov}(X, Y)}{b_{YX}} \text{ and } \sigma_y^2 = \frac{\text{Cov}(X, Y)}{b_{XY}}$$

Thus, using formulas, we obtain

$$\sigma_y^2 = \frac{\text{Cov}(X, Y)}{b_{XY}} \text{ and } b_{XY} = \frac{\text{Cov}(X, Y)}{\sigma_y^2}$$

RELATION OF REGRESSION COEFFICIENTS AND CORRELATION COEFFICIENT

$$r = \pm\sqrt{\pm b_{XY} \times b_{YX}}. \text{ and } r = \frac{\text{Cov}(x.y)}{\sigma_x \sigma_y}$$

Example 46. Given $\sigma_x = 20$, $\sigma_y = 20$ and cov(X, Y) = –100 find:

1. Correlation coefficient
2. Both the regression Coefficients.

(a) 0.25, 30, 78
(b) –0.25 –0.25, –0.25
(c) 25 0.55 0.10
(d) –0.45 0.25 0.72

Sol. (b) (1) Correlation coefficient:

$$r = \frac{\text{Cov}(x.y)}{\sigma_x \sigma_y} = \frac{-100}{(20)(20)} = -0.25$$

(2) Both the regression Coefficients.

$$\text{Now } b_{XY} = \frac{\text{Cov}(X, Y)}{\sigma_y^2} = \frac{-100}{20^2} = -0.25.$$

$$\text{And } b_{YX} = \frac{\text{Cov}(X, Y)}{\sigma_X^2} = \frac{-100}{20^2} = -0.25.$$

Hence the correct option is (b)

Example 47. Given $\bar{x} = 50, \bar{y} = 20$, $\sigma_x = 20$, and $\sigma_y = 20$, find both the regression Coefficients.

(a) 0.25x + y – 32.5 = 0, x + 0.25y – 55 = 0
(b) 0.25x + y – 32.5 = 0, x + 0.25y – 55 = 0
(c) x + 0.25y – 55 = 0, 0.792x – 1.033y – 1.228 = 0
(d) –0.45 x + 0.25y – 55 = 0, 0.792x – 1.0y – 1.28 = 0

Sol. (b) $b_{XY} = \frac{\text{Cov}(X, Y)}{\sigma_y^2} = \frac{-100}{20^2} = -0.25.$

$$\text{And } b_{YX} = \frac{\text{Cov}(X, Y)}{\sigma_X^2} = \frac{-100}{20^2} = -0.25.$$

Regression equation y on x is $(y - \bar{y}) = b_{yx}(x - \bar{x})$ ...(i)

Therefore, (y – 20) = –0.25(x – 50)

0.25x + y – 32.5 = 0

Regression equation x on y is $(x-\bar{x}) = b_{xy}(y-\bar{y})$ ...(ii)

Therefore, $(x - 50) = -0.25(y - 20)$

$x + 0.25y - 55 = 0$

Hence the correct option is (b).

Example 48. Find the regression coefficients b_{yx} and b_{xy} of Y on X and X on Y respectively, if standard deviations of X and Y are 4 and 3 respectively, and coefficient of correlation between X and Y is 0.8

(a) 2.4, 3.8 (b) 8.2, 4.8 (c) 1.2, 2.2 (d) 3.2, 2.4

Sol. (d) Given: $\sigma_x = 4$, $\sigma_y = 3$ and $r = 0.8$

As we know that $r = \dfrac{Cov(x.y)}{\sigma_x \sigma_y}$

Then $0.8 = \dfrac{Cov(x,y)}{(4)(3)}$

$Cov(x, y) = 12 \times 0.8 = 9.6.$

Therefore, $b_{XY} = \dfrac{Cov(X,Y)}{\sigma^2} = \dfrac{9.6}{3} = 3.2$

and $b_{YX} = \dfrac{Cov(X,Y)}{\sigma_X^2} = \dfrac{9.6}{4} = 2.4$

Hence the correct option is (d)

Example 49. The coefficient of correlation between the ages of husbands and wives in a community was found to be +0.8. The average of the husband's age of 25 years and that of the wives age is 22 years. Their standard deviations were 4 and 5 respectively. Find with the help of a regression equation, the expected age of husband when wife's age is 18 years.

(a) 12 (b) 13 (c) 10 (d) 11

Sol. (d) Given: $r = 0.8$, $\bar{x} = 25, \bar{y} = 22, \sigma_x = 4$ and $\sigma_y = 5$.

Let us consider the age of husband be x and the age of wives be y.

In the question the age of wives is independent variable and the age of husband is variable.

Therefore we have to find, y on x

As we know the standard equation of y on x is $(y-\bar{y}) = b_{yx}(x-\bar{x})$

Now, $b_{YX} = r \cdot \dfrac{\sigma_Y}{\sigma_X} = \dfrac{5^2}{4^2} = \dfrac{25}{18} = 1.5625.$

Then, $(y-\bar{y}) = b_{yx}(x-\bar{x})$

$(y - 22) = 1.5625(x - 25)$

$1.5625x - y = 17.0625$ ...(i)

The expected age of husband when wife's age is 18 years:

Put y = 18 in the 1st equation.

Then, $1.5625x - 18 = 17.0625$

$y = 11.0625 \approx 11$.

Hence the correct option is (d).

Example 50. From the following data obtain the two-regression equation and estimate the value of x when y is 130 and estimate the value of y when x is 30.

X	39	33	30	31	32	36	41	49	46	43
Y	132	134	138	129	136	131	132	135	128	125

(a) 39.328, 140.19 (b) 35.322, 120.139

(c) 89.38, 130.19 (d) 19.3228, 142.19

Sol. (a)

(X)	(Y)	$d_x = X - \bar{X} = X - 38$	$d_y = Y - \bar{Y} = Y - 132$	d_x^2	d_y^2	$d_x d_y$
39	132	1	0	1	0	0
33	134	-5	2	25	4	-10
30	138	-8	6	64	36	-48
31	129	-7	-3	49	9	21
32	136	-6	4	36	16	-24
36	131	-2	-1	4	1	2
41	132	3	0	9	0	0
49	135	11	3	121	9	33
46	128	8	-4	64	16	-32
43	125	5	-7	25	49	-35
$\sum X = 380$	$\sum Y = 1320$	$\sum d_x = 0$	$\sum d_y = 0$	$\sum d_x^2 = 398$	$\sum d_y^2 = 140$	$\sum d_x d_y = -93$

Here, $n = 10$

Then, $\bar{X} = \frac{\Sigma X}{n} = \frac{380}{10} = 38$

And $\bar{Y} = \frac{\sum Y}{n} = \frac{1320}{10} = 132$

Now, $b_{yx} = \frac{n\sum d_x d_y - \sum d_x \sum d_y}{n\sum d_x^2 - \left(\sum d_x\right)^2} = \frac{(10)(-93) - (0)(0)}{(10)(398) - (0)^2} = \frac{93}{398} = -0.234$

And $b_{xy} = \frac{n\sum d_x d_y - \sum d_x \sum d_y}{n\sum d_y^2 - \left(\sum d_y\right)^2} = \frac{(10)(-93) - (0)(0)}{(10)(140) - (0)^2} = \frac{93}{140} = -0.664$

Then the regression equation y on x is $(y - \bar{y}) = b_{yx}(x - \bar{x})$

$(y - 132) = -0.234(x - 38)$

$0.234x + y = 140.892$

$y = 140.892 - 0.234x$ ...(i)

And the regression equation is $(x - \bar{x}) = b_{xy}(y - \bar{y})$

$(x - 38) = -0.664(y - 132)$

$x + 0.664\ y = 125.648$

$x = 125.648 - 0.664y$ ...(ii)

Estimate the value of x when y is 130,

Put y = 130 in 2nd equation.

Then we get, $x = 125.648 - 0.664\ (130) = 39.328$.

Estimate the value of y when x is 30

Put x = 30 in 1st equation.

$y = 140.892 - 0.234(3) = 140.19$.

Hence the correct option is (a)

Example 51. The following data give the aptitude test score and productivity indicate 10 workers selected at random

Aptitude scores (X)	60	62	65	70	72	48	53	73	65	82
Productivity index (Y)	68	60	62	80	85	40	52	62	60	81

Calculate the two-regression equation and estimate the productivity index of a worker whose test score is 92.

(a) 100 (b) 95.321 (c) 85.321 (d) 105

Sol. (b)

(X)	(Y)	$d_x = X - \bar{X}$ = X - 65	$d_y = Y - \bar{Y}$ = Y – 65	d_x^2	d_y^2	$d_x d_y$
60	68	-5	3	25	9	-15
62	60	-3	-5	9	25	-15
65	62	0	-3	0	9	0
70	80	5	15	25	225	75
72	85	7	20	49	400	140
48	40	-17	-25	289	625	425
53	52	-12	-13	144	169	146
73	62	8	-3	64	9	-24
65	60	0	-5	0	25	0
82	81	17	16	289	256	272
$\sum X = 650$	$\sum Y = 650$	$\sum d_x = 0$	$\sum d_y = 0$	$\sum d_x^2 = 894$	$\sum d_y^2 = 1752$	$\sum d_x d_y = 1004$

Here n=10

Then, $\bar{X} = \frac{\Sigma X}{n} = \frac{650}{10} = 65$

And $\bar{Y} = \frac{\sum Y}{n} = \frac{650}{10} = 65$

Now $b_{yx} = \frac{n\sum d_x d_y - \sum d_x \sum d_y}{n\sum d_x^2 - \left(\sum d_x\right)^2} = \frac{(10)(1004) - (0)(0)}{(10)(894) - (0)^2} = \frac{1004}{984} = 1.123$

And $b_{xy} = \frac{n\sum d_x d_y - \sum d_x \sum d_y}{n\sum d_y^2 - \left(\sum d_y\right)^2} = \frac{(10)(1004) - (0)(0)}{(10)(1752) - (0)^2} = \frac{1004}{1752} = 0.573$

Then the regression equation y on x is $(y - \bar{y}) = b_{yx}(x - \bar{x})$

(y - 65) = 1.123(x - 65)

1.123x – y = 7.995

y = 1.123x - 7.995 ...(i)

And the regression equation is $(x - \bar{x}) = b_{xy}(y - \bar{y})$

1.0x - 0.573y = 27.755

x = 27.755 + 0.573y ...(ii)

Estimate the productivity index of a worker whose test score is 92.

Put x = 92 in 1st equation.

Then we get,

y = 1.123(92) – 7.995

y = 95.321

Hence the correct option is (b)

PROPERTIES OF REGRESSION COEFFICIENTS

SOME IMPORTANT PROPERTIES OF REGRESSION COEFFICIENTS

- **Property 1:** The coefficients of correlation and two regression coefficients have the same signs.
- **Property 2:** The coefficients of correlation are the geometric mean between the regression coefficients.
- **Property 3:** If one of the regression coefficients is greater than unity, the other must be less than the unity.
- **Property 4:** The two lines of regression intersect at the point $(\bar{x}, \bar{y})$ where x and y are the variables under consideration.
- **Property 5:** The regression coefficients are independent of change of origin but not for scale.

Determining the line of regression of y on x and that of x on y out of the given two regression lines:

Sometimes, it is required to find the line of regression of y on x or x on y out of the given two regression lines in such a case we below the following steps:

Step 1: Choose any one of the two regression lines as the line of regression of y on x and the other as the line of regression of x on y.

Step 2: Find two regression coefficients b_{xy} and b_{yx}

Step 3: Compute the product $b_{xy} \cdot b_{yx}$ if $b_{xy} \cdot b_{yx.} \leq 1$, then the assumption made in step 1 is correct, otherwise the assumption is wrong.

Example 52. Regression coefficient are _____.

(a) dependent of change of origin and of scale.

(b) independent of both change of origin and of scale.

(c) dependent of change of origin but not of scale.

(d) independent of change of origin but not of scale.

Sol. (d) We know, the regression coefficients are independent of the change of the origin. But, they are not independent of the change of the scale.

It means there will be no effect on the regression coefficients if any constant is subtracted from the value of x and y.

If x and y are multiplied by any constant, then the regression coefficient will change.

Hence, the correct option is (d) i.e. independent of change of origin but not of scale.

Example 53. If the regression line of y on x and of x on y are given by $2x + 3y = -1$ and $5x + 6y = -1$, then the arithmetic means of x and y are given by **(ICAI)**

(a) (1, –1) (b) (–1, 1) (c) (–1, –1) (d) (2, 3)

Sol. (a) Given equations,

$2x + 3y = -1$ ...(i)

$5x + 6y = -1$ ...(ii)

Multiply eq. (i) by (ii), we get

$4x + 6y = -2$ ...(iii)

Subtract equation (iii) from 2, we get

$x = 1$

Now, substitute $x = 1$ in eq (i),

$\Rightarrow 21 + 3y = -1$

$\Rightarrow 2 + 3y = -1$

$\Rightarrow 3y = -1-2$

$\Rightarrow 3y = -3$

$\Rightarrow y = -1$

We know, the two lines of regression intersects at (x, y), then the arithmetic means of x and y are given by 1, −1.

Hence, the correct answer is option (a) i.e 1, −1.

Example 54. Following are the two normal equations obtained for deriving the regression line of y and x:

$5a + 10b = 40$

$10a + 25b = 95$

The regression line of y on x is given by

(a) $2x + 3y = 5$ (b) $2y + 3x = 5$ (c) $y = 2x + 3$ (d) $y = 2x + 5$

Sol. (c) Given, $5a + 10b = 40$ ...(i)

$10a + 25b = 95$ ...(ii)

Multiplying eq (i) by 2, we get

$10a + 20b = 80$ ...(iii)

Subtracting eq (ii) and (iii), we get

$10a + 25b = 95$

$10a + 20b = 80$

$5b = 15$

$b = 3$

from (i)

$5a + 10b = 40$

$5a + 10(3) = 40$

$5a + 30 = 40(3)$

$5a = 40 - 30$

$5a = 10$

$a = 2$

As, "a" represents the y-intercept and "b" represents the slope

So the regression line of y on x is given by $y = 2x + 3$

Hence the correct option is (c).

Example 55. If $u = 2x + 5$ and $v = -3y - 6$ and regression coefficient of y on x is 2.4, what is the regression coefficient of v on u?

(a) 3.6 (b) –3.6 (c) 2.4 (d) –2.4

Sol. (b) Given, $u = 2x + 5$, $v = -3y - 6$

Regression coefficient of y on x = 2.4

We have to find the regression coefficient of v on u.

u and v can be written as

$$u = \frac{x + \frac{5}{2}}{\frac{1}{2}}$$

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$$v = \frac{y+2}{-\frac{1}{3}}$$

$$p = \frac{1}{2}, q = -\frac{1}{3}$$

$$b_{yx} = 2.4$$

$$b_{xy} = \frac{q}{p} b_{vu}$$

Substitute the values then we get

$$2.4 = -\frac{\frac{1}{3}}{\frac{1}{2}} b_{vu}$$

$$b_{vu} = -\frac{2.4 \times 3}{2} = 3.6$$

The regression coefficient of v on u = –3.6

Hence the correct option is (b)

Example 56. If 4y – 5x = 15 is the regression line of y on x and the coefficient of correlation between x and y is 0.75, what is the value of the regression coefficient of x and y?

(a) 0.45 (b) 0.9375 (c) 0.6 (d) None of these

Sol. (a) Given $r = \sqrt{b_{XY} \times b_{YX}}$ ...(i)

Now regression line of y on x is given by:

4y = 15 + 5x

$$\Rightarrow y = \frac{15}{4} + \frac{5}{4}x$$

$$\Rightarrow b_{yx} = \frac{5}{4}$$

Also, coefficient of correlation r = 0.75

Now substituting in eq (i), we get

$$0.75 = +- \sqrt{b_{xy} \times \frac{5}{4}x}$$

Squaring both sides we get

$$(0.75)^2 = \frac{5}{4} b_{XY}$$

$$\Rightarrow 0.5625 = \frac{5}{4} b_{XY}$$

$$\Rightarrow \text{or } b_{XY} = 0.5625 \times \frac{4}{5}$$

$$\Rightarrow \text{or } b_{XY} = 0.45$$

Hence, the correct option is (a).

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Example 57. If the regression line of y on x and of x on y are given be $2x + 3y = -1$ and $5x + 6y = -1$, then the arithmetic means of x and y are given by

(a) (1, -1) (b) (-1, 1) (c) (-1, -1) (d) (2, 3)

Sol. (a) Given:

$2x+3y= -1$ ...(i)

$5x + 6y = -1$ ...(ii)

Multiplying eq (i) by 5 and eq (ii) by 2, we get:

$10x + 15y = -5$

$10x + 12y = -2$

Subtracting the above equations, we get:

$3y = -3$

$y = -1$

Substituting $y = -1$ in the eq (i), we get

$2x + 3(-1) = -1$

$2x = 2$

$x = 1$

Therefore, $x = 1$ and $y = -1$.

Hence the correct option is (a).

PRACTICE QUESTIONS (PART E)

1. The regression equation are $2x + 3y + 1 = 0$ and $5x + 6y + 1 = 0$, then Mean of x and y respectively are **(Dec 2022)**

(a) –1,–1 (b) –1,1 (c) 1, –1 (d) 2,3

2. The correlation coefficient between x and y is $\frac{-1}{2}$. The value of $\frac{-1}{8}$ Find b_{yx}.

(a) – 2 (b) – 4 (c) 0 (d) 2

3. If $y = 3x + 4$ is the regression line of y on x and the arithmetic mean of x is −1, what is the arithmetic mean of y?

(a) 1 (b) – 1 (c) 7 (d) None of these

4. The equations of the two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the correlation coefficient between x and y. **(Dec 2022)**

(a) –0.75 (b) 0.25 (c) –0.92 (d) 1.25

Answer Key

1. (c) **2.** (a) **3.** (a) **4.** (a)

COEFFICIENT OF DETERMINATION AND NON-DETERMINATION

As we know, Correlation coefficient measures a linear relationship between the two variables which tells the amount of variation in one variable with respect to another variable. Thus, a better measure of correlation is indicated by square of the correlation coefficient, which is known as 'Coefficient of Determination'' given by

Coefficient of determination = r^2

For coefficient of correlation, r = 0.4, then $r^2 = (0.4)^2 = 0.16$ indicated 16% of variation which is accounted by ther factor under consideration and remaining 84% is due to other factors which we call it as ''Coefficient of Non-determination'' given by $1 - r^2$.

i.e., Coefficient of Non-determination = $1 - r^2$

Example 58. What is a spurious correlation? **(ICAI)**

(a) It is a bad relation between two variables.

(b) It has very low correlation between the two variables.

(c) It is the correlation between two variables having no causal relation.

(d) It is a negative correlation.

Sol. (c) Spurious correlation refers to the correlation between two variables that do not have any causal relationship between them. It is a statistical term used to describe a situation where two variables appear to be related to each other, but actually, they are not.

Hence the correct option is (c).

Example 59. Scatter diagram helps us to **(ICAI)**

(a) Find the nature correlation between two variables

(b) Compute the extent of correlation between two variables

(c) Obtain the mathematical relationship between two variables

(d) Both (a) and (c)

Sol. (d) Scatter plot (scatter graph, scatter chart) uses dots to represent values for two different numeric variables.

Scatter diagram is a tool for analyzing the relationship between two variables, i.e., whether variables are correlated or not.

Also, it will help to identify the types of correlation

And from a diagram we can observe that the relation is linear or non linear.

Hence the correct option is (d).

Example 60. When r = 1, all the points in a scatter diagram would lie **(ICAI)**

(a) On a straight line directed from lower left to upper right

(b) On a straight line directed from upper left to lower right

(c) On a straight line

(d) Both (a) and (b)

Sol. (d) Perfectly positive correlation if all the points on the scatter diagram are on a straight line with positive slope then the correlation is said to be perfectly positive, that is r = +1.

Perfectly negative correlation if all the points of the scatter diagram fall on a straight line with negative slope in the correlation is said to be perfectly negative or r = –1.

Hence the correct option is (d).

Example 61. What are the limits of the correlation coefficient? **(ICAI)**

(a) No limit (b) –1 and 1, excluding the limits

(c) 0 and 1, including the limits (d) –1 and 1, including the limits

Sol. (d) Either two variables can be completely related i.e. 1 or we can say positive correlation or the two variables are completely opposite i.e. negatively correlated -1 situation case.

Hence the correct option is (d).

Example 62. If the coefficient of correlation between two variables is –0.9, then the coefficient of determination is **(ICAI)**

(a) 0.9 (b) 0.81 (c) 0.1 (d) 0.19

Sol. (b) Coefficient of Correlation and Coefficient of Determination

The coefficient of correlation is a statistical measure that indicates the degree of association between two variables. It ranges from –1 to +1.

Coefficient of Correlation = –0.9

The formula for the coefficient of determination, denoted by r^2 where r is Coefficient of Correlation.

The coefficient of determination is $(-0.9)^2 = 0.81$

Hence the correct option is (b).

Example 63. If the coefficient of correlation between two variables is 0.7, then the percentage of variation unaccounted for is **(ICAI)**

(a) 70% (b) 30% (c) 51% (d) 49%

Sol. (d) The coefficient of correlation between two variables is 0.7,

Then the percentage of variation unaccounted = $r^2 \times 100 = 0.49(100) = 49\%$.

Hence the correct option is (d).

Example 64. What are the limits of the two regression coefficients? **(ICAI)**

(a) No limit

(b) Must be positive

(c) One positive and the other negative

(d) Product of the regression coefficient must be numerically less than unity

Sol. (d) The product of the regression coefficients must be numerically less than unity to avoid problems with multicollinearity. Multicollinearity occurs when there is a high correlation between two or more independent variables in a multiple regression model. This can lead to unstable and unreliable estimates of the regression coefficients. To avoid this problem, the product of the regression coefficients must be less than one.

Hence the correct option is (d).

FULL CHAPTER PRACTICE QUESTIONS

1. If the regression line of Y on X is given by Y = X + 2 and Karl Pearson's coefficient is 0.5 then $\frac{\sigma_y^2}{\sigma_x^2} = ?$

(a) 3 (b) 2 (c) 4 (d) None

2. Given the following series:

x	10	13	12	15	8	15
y	12	16	18	16	7	18

The rank correlation coefficient r = ?

(a) $1-\frac{6\sum d^2+\sum_{i=1}^{2}\frac{m_1\left(m_1^2-1\right)}{12}}{n\left(n^2-1\right)}$ (b) $1-\frac{6\left[\sum_{1=i}^{2}\frac{m_1\left(m_1^2-1\right)}{12}\right]}{n\left(n^2-1\right)}$

(c) $1-6\sum d^2+\sum_{i=1}^{2}\frac{\frac{m_t^2\left(m_1^2-1\right)}{12}}{n\left(n^2-1\right)}$ (d) $1-6\sum d^2+\sum_{i=1}^{2}\frac{\frac{m_1\left(m_1^2-1\right)}{12}}{n\left(n^2-1\right)}$

3. Determine spearman's rank correlation coefficient from the given data $\sum d^2 = 30, n = 10$

(a) r = 0.82 (b) r = 0.32 (c) r = 0.40 (d) None of these

4. Which of the following statements is not true about scatter diagrams?

(a) It finds the type of correlation
(b) It helps to identify whether variables are correlated or not
(c) It determines the linear or non-linear correlation
(d) It finds the numerical value of correlation coefficient

5. Given that

X	-3	-3/2	0	3/2	3
Y	9	9/4	0	9/4	9

Then Karl pearson's coefficient of correlation is

(a) Positive (b) Zero (c) Negative (d) None of these

6. A.M of regression coefficient is

(a) Equal to r (b) Greater than or equal to r
(c) Half of r (d) None

7. The regression coefficient is independent of the change of

(a) Scale (b) Origin
(c) Scale and origin both (d) None of these

8. If the correlation coefficient between the variables X and Y is 0.5, then the correlation coefficient between the variables 2x – 4 and 3- 2y is

(a) 1 (b) 0.5 (c) –0.5 (d) 0

9. If the two regression lines are 3x = y and 8y = 6x, then the value of correlation coefficient is

(a) 0.5 (b) -0.5 (c) 0.75 (d) -0.80

10. If r = 0.6 then the coefficient of non – determination will be:

(a) 0.40 (b) -0.60 (c) 0.36 (d) 0.64

11. The correlation coefficient (r) is the _____ of the two regression coefficient (b_{yx} and b_{xy})

(a) AM (b) GM (c) HM (d) Median

12. If there is a constant increase in a series, then the corresponding graph will be

(a) Convex curve (b) Concave curve

(c) Parabola (d) Straight line from the left to the right

13. If the plotted points are scatter diagram are evenly distribution, then the correlation is

(a) Zero (b) Negative (c) Positive (d) (a) or (b)

14. The coefficient of determination is defined by the formula

(a) $r^2 = \frac{1 - \text{unexplained variance}}{\text{Total variance}}$ (b) $r^2 = \frac{\text{explained variance}}{\text{Total variance}}$

(c) Both (a) and (b) (d) None of these

15. In the method of concurrent Deviations, only the directions of change (positive direction/ negative direction) in the variables are taken into account for calculation of

(a) Coefficient of SD (b) Coefficient of regression

(c) Coefficient of correlation (d) None of these

16. Correlation coefficient is _____ of units of measurement.

(a) Dependent (b) Independent (c) Both (d) None of these

17. In case speed of an automobile and the distance required to stop the car after applying brakes correlation is

(a) Positive (b) Negative (c) Zero (d) None of these

18. A relationship $r^2 = 1 - \frac{500}{300}$ is not possible

(a) True (b) False (c) Both (d) None of these

19. Rank correlation coefficient lies between

(a) 0 to 1 (b) –1 to +1 inclusive of these value

(c) –1 to 0 (d) Both

20. The two line of regression intersect at the point

(a) Mean (b) Mode (c) Median (d) None of these

21. If the two line of regression are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$, then the regression line of y on x is:

(a) $x + 2y - 5 = 0$ (b) $2x + 3y - 8 = 0$

(c) $x + 2y = 0$ (d) $2x + 3y = 0$

22. If the sum of squares of the differences of ranks, given by two judges X and Y of 10 students is 28, what is the value of the rank correlation coefficient?

(a) 0.725 (b) 0.650 (c) 0.750 (d) 0.873

23. The ranks of five cars given by two critics are:

Car	Critic 1	Critic 2
A	2	1
B	1	3
C	4	2
D	5	4
E	3	5

What is the rank correlation coefficient between the ranks assigned by the two critics?

(a) 1 (b) 0.3 (c) -1 (d) 0.5

24. For 12 pairs of observations, the number of concurrent deviations was found to be 3. What is the value of the coefficient of concurrent deviation?

(a) $\sqrt{\frac{5}{11}}$ (b) $\frac{5}{11}$ (c) $-\sqrt{\frac{5}{11}}$ (d) None of these

25. The sum of squares of rank differences in the ages of 12 individuals is 64. Calculate the coefficient of rank correlation.

(a) 0.78 (b) 0.73 (c) 0.87 (d) 0.80

26. If the two line of regression are $2Y - X = 35$ and $10X - Y = 70$. The regression line of Y on X is

(a) $2Y - X = 35$ (b) $10X - Y = 70$

(c) Any of the two line (d) None of the two line

27. Consider the regression line of y on x as $y = 20 + 0.4(x - 15)$ and the regression line of x on y as $x = 30 + 0.9y - 40$, what is the correlation coefficient between x and y?

(a) 0.36 (b) 0.4

(c) 0.6 (d) None of these

28. If the two regression lines are $3X = Y$ and $8Y = 6X$, then the value of correlation coefficient is

(a) -0.5 (b) 0.5 (c) 0.75 (d) -0.80

Answer Key

1. (c)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (d)
11. (b)	**12.** (d)	**13.** (a)	**14.** (c)	**5.** (c)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (a)
21. (a)	**22.** (d)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (c)	**28.** (b)		

PREVIOUS YEAR QUESTION (PYQ'S)

1. A.M. of regression coefficient is **(June 2019)**

(a) Equal to r (b) Greater than or Equal to r

(c) Half of r (d) None

2. If the regression line of Y on X is given by Y = X + 2 and Karl Pearson's coefficient of correlation is 0.5, then $\frac{\sigma_y^2}{\sigma_x^2}$ = _____ . **(June 2019)**

(a) 3 (b) 2 (c) 4 (d) None

3. Consider the regression line 3x + 2y = 26, 6x + y = 31. Find the correlation coefficient between x and y. **(Dec 2019)**

(a) 0.5 (b) –0.5 (c) 0.25 (d) –0.25

4. The interesecting point of the two regression lines: y on x and x on y is **(Jan 2021)**

(a) (0, 0) (b) $(\bar{x}, \bar{y})$ (c) (b_{yx}, b_{xy}) (d) (1, 1)

5. Given that the variance of x is equal to the square of standard deviation x and the regression line of y on x is y = 40 + 0.5 (x – 30), then regression line of x on y is **(Jan 2021)**

(a) y = 40 + 4(x – 30) (b) y = 40 + (x – 30)

(c) y = 40 + 2(x – 30) (d) x = 30 + 2(y – 40)

6. The regression coefficients remain unchanged due to **(Jan 2021)**

(a) A shift of scale (b) A shift of origin

(c) Replacing x-values by $\frac{1}{x}$ (d) Replacing y-values by $\frac{1}{y}$

7. If the slope of the regression line is calculated to be 5.5 and the intercept 15 then the value of Y when X is 6 is **(July 2021)**

(a) 88 (b) 48 (c) 18 (d) 78

8. If Y = 6X and X = 0.01Y, then r is equal to **(July 2021)**

(a) –0.1 (b) 0.1 (c) 0.3 (d) –0.3

9. The straight line graph of the linear equation Y = a + bX, slope is horizontal if **(July 2021)**

(a) b = 1 (b) b ≠ 0 (c) b = 0 (d) a = b ≠ 0

10. If $b_{yx} = -1.6$ and $b_{xy} = -0.4$, then r_{xy} will be **(July 2021)**

(a) 0.4 (b) –0.8 (c) 0.64 (d) 0.8

11. For any two variables x and y, the regression, equations are given as $2x + 5y - 9 = 0$ and $3x - y - 5 = 0$. What are the A.M. of x and y? **(Dec 2021)**

(a) 2, 1 (b) 1, 2 (c) 4, 2 (d) 2, 4

12. The intersecting point of two regression lines falls on x-axis. If the mean of X-values is 16, the standard deviation of X and Y are respectively 3 and 4, then the mean of Y-values is **(Dec 2021)**

(a) $\frac{16}{3}$ (b) 4 (c) 0 (d) 1

13. The regression coefficients remain unchanged due to **(Dec 2021)**

(a) Shift of origin (b) Shift of scale

(c) Always (d) Never

14. For positive and perfectly correlated random variables, one of the regression coefficient is 1.3 and the standard deviation of X is 2, the variance of Y is **(June 2021)**

(a) 2.66 (b) 6.76 (c) 6.56 (d) 3.16

15. The equations of the two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the correlation coefficient between x and y. **(Dec 2022)**

(a) –0.75 (b) 0.25 (c) –0.92 (d) 1.25

16. If the regression equations are $2x + 3y + 1 = 0$ and $5x + 6y + 1 = 0$, then mean of x and y respectively are **(Dec 2022)**

(a) –1, –1 (b) –1, 1 (c) 1, –1 (d) 2, 3

17. If $b_{yx} = 0.5$ and $b_{xy} = 0.46$, then the value of correlation coefficient r is **(Dec 2022)**

(a) 0.23 (b) 0.25 (c) 0.39 (d) 0.48

18. For variables X and Y, we collect the four observations with $\Sigma X = 10$; $\Sigma Y = 14$, $\Sigma X^2 = 65$; $\Sigma Y^2 = 5$ and $\Sigma XY = 3$. What is the regression line of Y on X?

(a) $Y = -0.8X - 5.5$ (b) $Y = 0.8X - 5.5$

(c) $Y = -0.8X + 5.5$ (d) $Y = 0.8X + 5.5$

19. The regression lines will be perpendicular to each other when the value of r is

(a) 1 (b) –1 (c) $\frac{1}{2}$ (d) 0

20. If the regression equations are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$, then the mean of x and the mean of y are _____ respectively.

(a) –3 and 4 (b) 2 and 4 (c) 1 and 2 (d) 2 and 1

21. If the equation of the two regression lines are $2x - 3y = 0$ and $4y - 5x = 8$, then the correlation coefficient between x and y is equal to **(Dec 2019)**

(a) $\sqrt{\frac{15}{8}}$ (b) $\sqrt{\frac{8}{15}}$ (c) $\sqrt{\frac{6}{15}}$ (d) $\sqrt{\frac{1}{15}}$

22. If the regression line of y on x and of x on y are given by $10x - 290 = -20y$ and $7y - 104 = -4x$, then the arithmetic means of x and y are given by; **(Dec 2023)**

(a) 5, 12 (b) 7, 12 (c) 12, 5 (d) 5, 7

23. If the regression coefficient (r_{yx}) of y on x is greater than unity, then other regression coefficient (r_{xy}) of x on y is: **(Dec 2023)**

(a) Less than x (b) Greater than x

(c) Equality to one (d) Equal to zero

24. If the regression lines are $3x - 4y + 8 = 0$ and $4x - 3y = 1$, then the correlation coefficient between x and y is **(June 2024)**

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{1}{8}$

25. Which one of the following statement is correct regarding limit of the two regression coefficients? **(June 2024, Sept 2024)**

(a) No limit.

(b) Must be positive.

(c) One positive and the other negative.

(d) Product of the regression coefficients must be numerically less than unity.

26. Which of the following statements is correct? **(Sept 2024)**

(a) Both regression coefficients must be less than unity

(b) Regression coefficients are independent of origin but not of scale.

(c) The regression lines of two independent variables are parallel to each other.

(d) If two regression lines coincide with each other, there is no correlation between the variates.

27. The correlation coefficient between X and Y is 0.2 and Var (X) = 5 Var (Y), then regression coefficient of X on Y is **(Jan 2025)**

(a) $\sqrt{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{\sqrt{5}}$ (d) 5

28. If $r = 0.8$, $b_{yx} = 0.6$, $b_{xy} = 0.5$, $\bar{x} = 5$ & $\bar{y} = 3$, then the regression equation y on x is **(Jan 2025)**

(a) $y = 0.96x - 3.7$ (b) $y = 0.6x - 6$

(c) $y = 0.8x$ (d) $y = 0.6x$

29. When both the regression coefficient are $b_{xy} = 0.7$ and $b_{yx} = 0.8$ respectively, then the correlation coefficient between x and y is **(May 2025)**

(a) 0.75 (b) 0.56 (c) 0.28 (d) 0.87

30. Given $x = 2y + 4$ and $y = kx + 6$ are the two lines of regression x on y and y on x respectively. If the value of correlation coefficient (r) is 0.5, then the value of k is **(May 2025)**

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

31. For 9 college students group, the sum of squares of differences in ranks for History and Hindi marks was found to be 62, then what is the value of rank correlation coefficient? **(May 2025)**

(a) 1 (b) 0.48 (c) 0.52 (d) 0.87

32. If m + 3x = 10 and 2y +5n = 25 and regression coefficient of y on x is 0.80, what is the regression coefficient of n on m? **(Sept 2025)**

(a) –0.106 (b) 9.375 (c) 0.106 (d) 0.0106

33. If the regression coefficient b_{yz} is greater than one, then the regression coefficient b_{xy} is **(Sept 2025)**

(a) cannot be less than one (b) cannot be greater than one

(c) can be equal to one (d) can be equal to zero

Answer Key

1. (b)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (c)	**10.** (b)
11. (a)	**12.** (c)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (d)	**20.** (c)
21. (b)	**22.** (a)	**23.** (a)	**24.** (a)	**25.** (d)	**26.** (b)	**27.** (c)	**28.** (d)	**29.** (a)	**30.** (a)
31. (b)	**32.** (c)	**33.** (b)							

SUMMARY

There are four ways to find Correlation:

- **Scatter Diagram:** A scatter diagram is a graphical presentation of bivariate data {(Xi, Yi): i = 1, 2, … n} on two quantitative variables X and Y that allows us to show two variables together.
- **Perfect Positive Correlation:** If the points of the scatter diagram fall on a straight line and have a positive(upward) slope, then the correlation is said to be perfectly positive;
- **Positive Correlation:** When the points of the scatter diagram cluster around a straight line (upward slope from left to right), then the correlation is said to be positive.
- **Perfect Negative Correlation:** If the points of the scatter diagram fall on a straight line and have a negative(downward) slope, then the correlation is said to be perfectly negative
- **Negative Correlation:** When the points of the scatter diagram cluster around a straight line (downward/negative slope), then the correlation is said to be negative.
- **No Correlation:** When the points of the scatter diagram are scattered in a haphazard manner, then there is zero or no correlation.
- **Correlation coefficient (r):**

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{\left(n\sum x^2 - (\sum x)^2\right)\left(n\sum y^2 - (\sum y)^2\right)}}$$

Degree of Correlation	Positive	Negative
Perfect correlation	+1	-1
Very high degree of correlation	+ 0.9 to + 1	- 0.9 to – 1
Fairly high degree of correlation	+ 0.75 to + 0.9	- 0.75 to - 0.9
Moderate degree of correlation	+ 0.50 to + 0.75	- 0.50 to - 0.75
Low degree of correlation	+ 0.25 to + 0.50	- 0.25 to - 0.5
Very low degree of correlation	0 to + 0.25	- 0.25 to 0
No correlation	0	0

REGRESSION

- Linear regression establishes the linear relationship between two variables based on a line of best fit. Linear regression is thus graphically depicted using a straight line with the slope defining how the change in one variable impacts a change in the other.
- The y-intercept of a linear regression relationship represents the value of one variable when the value of the other is zero
- Formulated by : $y = mx + b$

  where m is the slope and b is the intercept

  $r = \pm\sqrt{b_{yx} \times b_{xy}}$

  $b_{yx} = \frac{q}{p} \times b_{vu}$ where $u = \frac{x-a}{p}$ and $v = \frac{y-c}{q}$

  Coefficient of determination: r^2

❑❑❑

CHAPTER

18

Index Number

INDEX NUMBER

An Index Number is a statistical measure that is used to track and represent changes in a variable or a collection of related variables over time, space, or other factors. It provides a way to summarize and compare data to understand the overall trend or performance.

In the context of financial markets, Indices play a crucial role in measuring the performance of stock markets. For example, the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE) in India provide indices such as the Sensex and Nifty, respectively. These indices represent the collective performance of a specific set of stocks listed on the respective exchanges.

Market Summary > NIFTY 50 **Market Summary > BSE SENSEX**

Definition

An Index number is a ratio of two or more time periods involved, one of which is the base time period. The value at the base time period serves as the standard point of comparison.

Types of Index Numbers

Types	Significance
Price Index Numbers	Shows movements in the price levels between the base year and other periods. Increase in price Level is called Inflation whereas decrease refers to Deflation. The Consumer Price Index (CPI) & Wholesale Price Index (WPI) are common examples. It in denoted by P_{01}

Quantity Index Numbers	Shows the movement in Quantity levels between two periods. Used in calculating Industrial Production Indices etc. It is denoted by P_{01}
Value Index Numbers	Shows the movement in value levels between two periods. It is denoted by V_{01} Value = Price x Quantity It is used for computing Growth rate of an Economy etc.

Index Time Series: An Index Time Series is a list of index numbers for two or more periods of time, where each Index number employs the same base year.

- **Issues involved in Construction of Index Numbers:**

I. **Selection of Data:** Choosing the right data is crucial. For instance, if you're making an index to measure the cost of living, you should focus on prices that directly affect living expenses while excluding prices for things like machinery.

II. **Base Period:** The base period serves as your reference point. It should be a stable time, not influenced by unusual events like wars. A relatively recent base period is often better, and there are different methods for choosing it.

III. **Selection of Weights:** Every variable in your index needs a weight based on its importance for your specific purpose. For example, if you're calculating a cost of living index, essentials like cereals might weigh more than non-essentials like sugar.

IV. **Use of Averages:** Averages are vital. The type of average you pick, be it geometric or arithmetic, depends on the nature of your index.

V. **Choice of Variables:** Deciding what variables to use, like price or quantity, is essential. For a price index, you need to choose between wholesale or retail prices and decide on the relevant time frame for prices.

VI. **Selection of Formula:** The formula you use is critical. Different formulas applied to the same data can yield different results. So, it's important to choose the right one for your index.

- **CONSTRUCTION OF INDEX NUMBERS:**

Notation: If we have to take the prices of 3 different commodity for n^{th} period then it can be written as $P_n(1)$, $P_n(2)$, $P_n(3)$

While if we take price corresponding to base period for 3 different commodity, it will be written as $P_0(1)$, $P_0(2)$, $P_0(3)$

Now let's say we have commodity 'j' where j is variable and varying from 1 to k then:

Summation for all prices of nth period can be written as $\sum_{j=1}^{k} P_n(j)$ or $\sum P_n(j)$.

RELATIVES

1. **Price relative:** As we are discussing the prices, let's talk about relative prices which is called as price relative. Price relative can be defined as the ratio of Price of a single commodity in one time period to the price of base period or reference period.

   It is written as: Price relative $= \frac{P_n}{P_0}$

   If we have to express it in form of percentage, it can be multiplied by 100:

   Price relative $= \frac{P_n}{P_0} \times 100$

   E.g.: Let's consider the price of a particular product, a smartphone, in two time periods: the base period (2010) and the current period (2021). In 2010. the smartphone was priced at ₹20,000. while in 2021, it was priced at ₹48,000. To calculate the price relative, we can use the formula:

   $\Rightarrow$ Price Relative $= \frac{\text{Price in 2021}}{\text{Price in 2010}} \times 100 = \frac{48,000}{20,000} \times 100 = 2.4 \times 100 = 240\%$

   This indicates that the price of the smartphone has increased by 240% from the base period to the current period in terms of Indian Rupees.

   As we discussed for Price relative, the same discussion can happen in terms of quantity, volume of consumption etc.

   Then, in that case relatives will be:

2. **Quantity Relative:** The quantity relative compares the quantity of a particular commodity in one time period to the quantity in a base or reference period.

   Quantity Relative $= \frac{Q_n}{Q_0}$

3. **Value relative:** The value relative considers the value of a commodity by multiplying the price and quantity relatives together.

   Value Relative $= \frac{V_n}{V_0} = \frac{P_n}{P_0} \times \frac{Q_n}{Q_0} = \frac{P_n Q_n}{P_0 Q_0}$

   Link Relative: When we take ratio of successive prices or quantities, then it is called the link relatives i.e., $\frac{P_1}{P_0}, \frac{P_2}{P_1}, \frac{P_3}{P_2}, \frac{P_n}{P_{n-1}}$

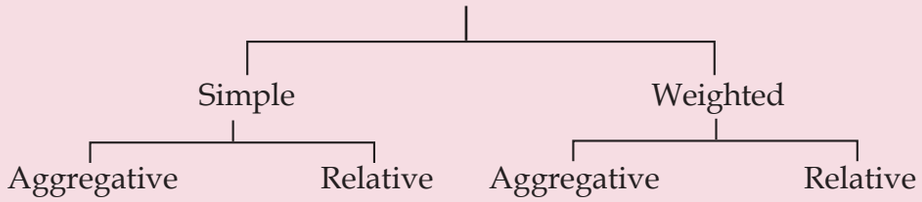
Commodity	Milk (per litre)	Link Relative
2005	50	$\frac{50}{50} \times 100 = 100$
2010	55	$\frac{55}{50} \times 100 = 110$

2015	60	$\frac{60}{55}\times 100 = 109.09$
2020	70	$\frac{70}{60}\times 100 = 116.67$

Chain Relative: When the ratio is taken in respect to base price then it is called the chain relatives i.e., $\frac{P_1}{P_0}, \frac{P_2}{P_0}, \frac{P_3}{P_0}, \frac{P_n}{P_0}$

Commodity	Milk (per litre)	Link Relative
2005	50	$\frac{50}{50}\times 100 = 100$
2010	55	$\frac{55}{50}\times 100 = 110$
2015	60	$\frac{60}{50}\times 100 = 120$
2020	70	$\frac{70}{50}\times 100 = 140$

Methods for Constructing Index Numbers



Let's start our discussion by understanding methods:

SIMPLE AGGREGATIVE

In this method of computing a price index, we express the total of commodity prices in a given year as a percentage of total commodity price in the base year.

Simple Aggregative price index $= \frac{\sum P_n}{\sum P_0}\times 100$

where, $\sum P_n$ is the sum of all commodity prices in the current year and $\sum P_0$ is the sum of all commodity prices in the base year.

Example of Simple Aggregate

Commodity	2010	2015	2020
Milk (per litre)	50	60	70
Atta (per kg)	10	12	15
Banana (dozen)	30	45	50
Aggregate	90	117	135
Index	100	130	150

Simple Aggregative Index for 2015 over $2010 = \frac{117}{90} \times 100 = 130$ and for 2020 over $2010 = \frac{135}{90} \times 100 = 150$

DEMERITS OF ABOVE METHOD

- It shows that the first commodity exerts greater influence than the other two because the price of the first commodity is higher than that of the other two.
- Further, if units are changed then the Index numbers will also change.

SIMPLE RELATIVE OR SIMPLE AVERAGE

If we change the actual price for each variable into percentage of the base period. These percentages are called relatives because they are relative to the value for the base period and the index number formed is simple relative.

E.g.:

Simple Relative Method

↓

Find Price Relative of each product

↓

$$\text{Price Relative} = \frac{P_1}{P_0} \times 100$$

$$P_{01}(AM) = \frac{\sum\left(\frac{P_1}{P_0} \times 100\right)}{N} \qquad P_{01}(GM) = AL\left[\frac{\sum \log\left(\frac{P_1}{P_0} \times 100\right)}{N}\right]$$

Product	2014 (P_0)	2024 (P_1)	$\frac{P_1}{P_0}\times 100$	$\text{Log}\left(\frac{P_1}{P_0}\times 100\right)$
Potato	12	24	$\frac{24}{12}\times 100 = 200$	2.3010
Tomato	15	30	$\frac{30}{15}\times 100 = 200$	2.3010
Onion	10	40	$\frac{40}{10}\times 100 = 400$	2.6020
			800	7.204

$$P_{01}(AM) = \frac{\sum\left(\frac{P_1}{P_0}\times 100\right)}{N} = \frac{800}{3} = 266.66\%$$

$$P_{01}(GM) = AL\left[\frac{\sum \log\left(\frac{P_1}{P_0}\times 100\right)}{N}\right] = AL\left(\frac{7.204}{3}\right) = 251.95$$

ADVANTAGE OF SIMPLE RELATIVE METHOD

- Index number computed from relatives will remain the same regardless of the units by which the prices are quoted.

DISADVANTAGE OF SIMPLE RELATIVE METHOD

- This amounts to giving undue weight to a commodity which is used in a small quantity because the relatives which have no regard to the absolute quantity will give weight more than what is due from the quantity used.

WEIGHTED AGGREGATIVE INDEX

While calculating the index number we will take care of the number of quantities or portions of the commodity. $P_{01} = \frac{\sum P_1 W}{\sum P_0 W}\times 100$

E.g.:

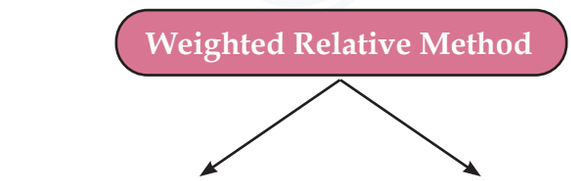
Commodity	Quantity for 2010	2010 (Price)	Quantity for 2020	2020 (Price)
Milk (per litre)	2 litres	50	2.5 litres	70
Atta (per kg)	1.5 kg	10	1 kg	15
Banana (per dozen)	3 dozen	30	4 dozen	50

SOME OF THE IMPORTANT FORMULA TO CALCULATE INDEX ARE

- Laspeyre's Index number $= \frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$
- Paasche's Index number $= \frac{\sum P_n Q_n}{\sum P_0 Q_n} \times 100$
- Marshall-Edgeworth Index number $= \frac{\sum P_n (Q_0 + Q_n)}{\sum P_0 (Q_0 + Q_n)} \times 100$
- Fisher's Index number $= \sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{\sum P_n Q_n}{\sum P_0 Q_n}} \times 100$
- Dorbish and Bowley's Price Index Number $P_{01} = \frac{L+P}{2} = \frac{\frac{\sum P_n Q_o}{\sum P_o Q_o} + \frac{\sum P_n Q_n'}{\sum P_o Q_n'}}{2} \times 100$
- Walsch's Index $P_{01} = \frac{\sum P_1 \sqrt{Q_0 Q_1}}{\sum P_0 \sqrt{Q_0 Q_1}} \times 100$
- Kelly's Index $P_{01} = \frac{\sum P_1 Q}{\sum P_0 Q} \times 100$

WEIGHTED AVERAGE OF RELATIVE METHOD

To overcome the disadvantage of a simple average of relative method, we can use weighted average of relative method.



P_{01} (AM) $= \frac{\sum W \left(\frac{P_1}{P_0} \times 100 \right)}{\sum W}$

P_{01}(GM) $= \text{AL} \frac{\sum W_i \log \left(\frac{P_1}{P_0} \times 100 \right)}{N}$

Example:

	2014 (P_0)	2024 (P_1)	Weight (W)	$\frac{P_1}{P_0}\times 100$	$W\left(\frac{P_1}{P_0}\times 100\right)$
A. Food	100	160	50	160	8000
B. Rent	150	450	40	300	12000
C. Medical	20	40	10	200	2000
			100		22000

$$P_{01}(AM) = \frac{\sum Wi\left(\frac{P_1}{P_0}\times 100\right)}{\sum Wi} = \frac{22000}{100} = 220\%$$

Relationship between Fisher's Index, Paasche's Index number and Laspeyre's Index:

$$\text{Fisher's Ideal Index Number} = \sqrt{\text{Paasche index number} \times \text{Laspeyre's index number}}$$

Relationship between Dorbish and Bowley's Price Index, Paasche's Index number and Laspeyre's Index:

$$\text{Dorbish and Bowley's Price Index Number} = \frac{\text{Laspeyre's Price index} + \text{Paasche's Price Index}}{2}$$

PRICE AND QUANTITY INDEX

	Method	Price Index	Quantity Index
1.	Simple Aggregate	$\frac{\sum P_n}{\sum P_0}$	$\frac{\sum Q_n}{\sum Q_0}$
2.	Simple Average of Relative	$\frac{\frac{\sum P_n}{\sum P_0}}{n}$	$\frac{\frac{\sum Q_n}{\sum Q_0}}{n}$
3.	Weighted Aggregate		
	(a) With base year weight (Laspeyre's index)	$\frac{\sum P_nQ_0}{\sum P_0Q_0}$	$\frac{\sum Q_nP_0}{\sum Q_0P_0}$
	(b) With current year weight (Paasche's index)	$\frac{\sum P_nQ_n}{\sum P_0Q_n}$	$\frac{\sum Q_nP_n}{\sum Q_0P_n}$
	(c) Fisher's Ideal [Geometric mean of Laspeyre's and Paasche's]	$\sqrt{\frac{\sum P_nQ_0}{\sum P_0Q_0}\times\frac{\sum P_nQ_n}{\sum P_0Q_n}}$	$\sqrt{\frac{\sum Q_nP_0}{\sum Q_0P}\times\frac{\sum Q_nP_n}{\sum Q_0P_n}}$

	Method	Price Index	Quantity Index
4.	Weighted Average of Relative W = Weights = Base Year or Current Year Price Weight	$\frac{\sum\left(\frac{P_n}{P_0}W\right)}{\sum W}$	$\frac{\sum\left(\frac{Q_n}{Q_0}\right)}{\sum W}$

CHAIN INDEX NUMBERS

- So far we concentrated on a fixed base but it does not suit when conditions change quite fast.
- Under this method the relatives of each year are first related to the preceding year called the link relatives and then they are chained together by successive multiplication to form a chain index.

$$\text{Chain Index} = \frac{\text{Link Relative of current year} \times \text{Chain index of the previous year}}{100}$$

Let us understand from the below example:

Year	Price of commodity	Link relative	Chain index
2010	10	$\frac{10}{10}\times 100 = 100$	100
2012	12	$\frac{12}{10}\times 100 = 120$	$\frac{120\times 100}{100} = 120$
2014	14	$\frac{14}{12}\times 100 = 116.67$	$\frac{116.67\times 120}{100} = 140$
2016	15	$\frac{15}{14}\times 100 = 107.14$	$\frac{107.14\times 140}{100} = 150$
2018	17	$\frac{17}{15}\times 100 = 113.33$	$\frac{113.33\times 150}{100} = 170$
2020	20	$\frac{20}{17}\times 100 = 117.65$	$\frac{117.65\times 170}{100} = 200$
2022	22	$\frac{22}{20}\times 100 = 110$	$\frac{200\times 110}{100} = 220$

VALUE INDEX

We know, Value = Price × Quantity i.e., $\frac{\sum V_n}{\sum V_0} = \frac{\sum P_n \times Q_n}{\sum P_0 \times Q_0}\times 100$

E.g.: The Value Index for the following data:

Commodity	Quantity (units)		Prince in (₹)	
	1995 Q_0	1999 Q_n	1995 P_0	1999 P_n
A	100	150	500	900
B	80	100	320	500
C	60	72	120	360
D	30	33	360	297

We have,

Commodity	Quantity (units)		Prince in (₹)			
	1995 Q_0	1999 Q_n	1995 P_0	1999 P_n	P_0Q_0	P_nQ_n
A	100	150	500	900	50000	135000
B	80	100	320	500	25600	50000
C	60	72	120	360	7200	25920
D	30	33	360	297	10800	9801
					93600	220721

We know, Value is Price multiplied by Quantity

Thus, $\frac{\Sigma V_n}{\Sigma V_0} = \frac{\Sigma P_n \times Q_n}{\Sigma P_0 \times Q_0} \times 100$

$= \frac{220721}{93600} \times 100 = 235.813$

Deflating time series using Index: Deflated Value $= \frac{\text{Current value}}{\text{Price index of current year}}$

E.g.: From the table, compute the real GNP.

Year	Wholesale Price Index	GNP at current Prices
1970	113.1	7499
1971	116.3	7935
1972	121.2	8657
1973	127.7	9323

Thus, we have

Year	Wholesale Price Index	GNP at current Prices	GNP (Real)
1970	113.1	7499	$\frac{7499}{113.1} \times 100 = 6630$
1971	116.3	7935	$\frac{7935}{116.3} \times 100 = 6823$

Year	Wholesale Price Index	GNP at current Prices	GNP (Real)
1972	121.2	8657	$\frac{8657}{121.5}\times 100 = 7143$
1973	127.7	9323	$\frac{9323}{127.7}\times 100 = 7300$

SHIFTING AND SPLICING OF INDEX NUMBERS

Shifting of Index number means that base period of the index has to be shifted:

$$\text{Shifted Price Index} = \frac{\text{Original Price index}}{\text{Price Index of the year on which it has to be shifted}}\times 100$$

From the following Index numbers with 1980 = 100 Shift the Index numbers to 1990 as the base:

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991
Original Price Index	100	104	106	108	110	112	115	117	125	131	140	147

Thus, on shifting the index number to 1990 as the base, we get

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991
Original Price Index	100	104	106	108	110	112	115	117	125	131	140	147
Shifting Base index (1990)	$\frac{100}{140}\times 100 = 71.4$	$\frac{104}{140}\times 100 = 74.28$	75.7	76.4	78.57	80	82.1	83.6	89.28	93.5	100	105

SPLICING OF INDEX NUMBERS

The following represent two series of index numbers:

1. One series with 1990 as base which is discontinued with the year 1995.
2. The second series with 1995 as the base is started from the year 1995, the year in which the old index is discontinued.

Year	Old Price Index [1990 = 100]	Revised Price Index [1995 = 100]
1990	100.00	—
1991	102.30	—

Year	Old Price Index [1990 = 100]	Revised Price Index [1995 = 100]
1992	105.30	–
1993	107.60	–
1994	111.90	–
1995	114.20	100.00
1996	–	102.50
1997	–	106.40
1998	–	108.30
1999	–	111.70
2000	–	117.80

LIMITATION OF INDEX NUMBER

1. As the index are constructed mostly from deliberate samples, chances of errors creeping in cannot be always avoided.
2. Since index numbers are based on some selected items, they simply depict the broad trend and not the real picture.
3. Since many methods are employed for constructing index numbers, the result gives different values and this at time create confusion.

USEFULNESS OF INDEX NUMBERS

1. Framing suitable policies in economics and business. They provide guidelines to make decisions in measuring intelligence quotients, research etc.
2. They reveal trends and tendencies in making important conclusions in cyclical forces, irregular forces etc.
3. They are important in forecasting future economic activity. They are used in times series analysis to long term trends, seasonal variations and cyclical developments.
4. Index numbers are very useful in deflating i.e., they are used to adjust the original data for price changes and thus transform nominal wages into real wages.

TEST OF ADEQUACY

1. **Unit Test:** This test requires that the formula should be independent of the unit in which or for which prices and quantities are quoted. Except for the simple (weighted) aggregative index all other formulae satisfy this test.
2. **Time Reversal Test:** It means if periods are reversed and indices are multiplied it should result in unity. $P_{01} \times P_{10} = 1$ where P_{01} is the index for time 1 on 0 and P_{10} is the index for time 0 on 1. Laspeyre's method and Paasche's method do not satisfy this test, but Fisher's Ideal formula does.

Proof:

I. Laspeyre's Index:

We know that, $P_{01} = \frac{\sum P_1Q_0}{\sum P_0Q_0}$ & $P_{10} = \frac{\sum P_0Q_1}{\sum P_1Q_1}$

Thus, $P_{01} \times P_{10} = \frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_0Q_1}{\sum P_1Q_1} \neq 1$

II. Paasche's Index:

We know that, $P_{01} = \frac{\sum P_1Q_1}{\sum P_0Q_1}, P_{10} = \frac{\sum P_0Q_0}{\sum P_1Q_0}$

$\therefore P_{01} \times P_{10} = \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0} \neq 1$

III. Fisher's Index:

We know, $P_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}$, $P_{10} = \sqrt{\frac{\sum P_0Q_1}{\sum P_1Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0}}$

$\therefore P_{01} \times P_{10} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum P_0Q_1}{\sum P_1Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0}} = 1$

3. **Factor Reversal Test:** This holds when the product of the price Index and the quantity index should be equal to the corresponding value index, i.e. $P_{01} \times Q_{01} = V_{01}$

Proof:

(I) Paasche's Index:

To check: $P_{01} \times Q_{01} = V_{01}$

We know,

$$P_{01} = \frac{\sum P_1Q_1}{\sum P_0Q_1}$$

$$Q_{01} = \frac{\sum Q_1P_1}{\sum Q_0P_1}$$

$$P_{0,} \times Q_{01} = \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum Q_1P_1}{\sum Q_0P_1} \neq V_{01}$$

Similarly, we can prove that Laspeyre's Index number do not satisfy the Factor Reversal test.

(II) Fisher's Index:

We know, $P_{0_1} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}$

$$Q_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}$$

$$\Rightarrow \sqrt{\frac{\sum (P_1Q_1)^2}{\sum (P_0Q_0)^2}} = \frac{\sum P_1Q_1}{\sum P_0Q_0}$$

Fisher's Index satisfies Factor Reversal test.

Since, Fisher's Index number satisfies both the tests in (2) and (3), thus it is called an Ideal Index number.

Fisher's index is the Ideal Index Number.

4. **Circular Test:** It is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base. i.e., $P_{01} \times P_{12} \times P_{20} = 1$

   It is satisfied by:

   (I) Weighted aggregative with fixed weighted average.

   (II) Simple geometric mean of price relatives.

Example 1. A series of numerical figures which show the relative position is called **(ICAI)**

(a) Index number (b) Relative number

(c) Absolute number (d) None of these

Sol. (a) An index number is a ratio of two or more time periods involved, one of which is the base time period. The value at the base time period serves as the standard point of comparison.

Hence, the correct option is (a).

Example 2. Index number for the base period is always taken as **(ICAI)**

(a) 200 (b) 50 (c) 1 (d) 100

Sol. (d) An Index Number is a statistical tool for determining the degree of changes in a set of connected variables.

Let $\sum P_1$ be the value of commodity in current period and $\sum P_0$ be the value of commodity in base period

Then. $P = \frac{\sum P_1}{\sum P_0} \times 100$

Base period = current period for determining the Index number in the base period.

$\sum P_1 = \sum P_0$

$\Rightarrow P = 1 \times 100 = 100$

Hence. Index Number for the base period is always taken as hundred.

Hence, the correct option is (d).

Example 3. Price relative is equal to **(ICAI)**

(a) $\frac{\text{Price in the given year} \times 100}{\text{Price in the base year}}$ (b) $\frac{\text{Price in the base year} \times 100}{\text{Price in the base year}}$

(c) Price in the given year × 100 (d) Price in the base year × 100

Sol. (a) The percentage difference between the current year's price and the base year's price is known as a price relative.

The approach uses the current-year price of each commodity as a percentage of the base-year price.

Price relative $= \frac{P_1}{P_0} \times 100$

Hence, the correct option is (a).

Example 4. _____is an extension of the time reversal test. **(ICAI)**

(a) Factor Reversal test (b) Circular test

(c) Both (d) None of these

Sol. (b) The circular test is a statistical technique in which the index of the later period on the earlier period is exactly the opposite of the index of the earlier period on the later period, however this test is applicable when there are more than two periods involved.

Hence, the correct option is (b).

Example 5. Index numbers show _____ changes rather than absolute amounts of change.

(a) Relative (b) Percentage (c) Both (d) None of these **(ICAI)**

Sol. (a) Instead of displaying exact quantities of change, index numbers are utilized to display relative changes. This implies that index numbers calculate the difference between a variable's current value and its base value. The Consumer Price Index (CPI), for instance, is used to track variations in the cost of products and services over time.

Hence, the correct option is (a).

Example 6. The _____ makes index numbers time-reversible. **(ICAI)**

(a) A.M. (b) G.M. (c) H.M. (d) None of these

Sol. (b) This test determines if the procedure is time-reversible in both directions. Prof. Fisher asserts that regardless of which of the two points of time is used as the base, the method for computing an index number should provide the same ratio between one point of time and the other.

In other words, if the base is inverted and the data for any two years are analysed using the same methodology 7, the two index numbers should be the reciprocals of one another.

$P_{01} \times P_{10} = 1$

Hence, the correct option is (b).

Example 7. _____ play a very important part in the construction of index numbers.

(a) Weights (b) Classes (c) Estimations (d) None of these **(ICAI)**

Sol. (a) In conclusion, weights are very important when creating index numbers.

Each item's weight is allocated to indicate its relative importance, maintain correctness, allow for comparison, and aid in classifying the objects.

It is crucial for each of the factors in the composite index to have a unique impact on the index. As a result, the importance should extend to those factors that are crucial to an index's goal.

Hence, the correct option is (a).

Example 8. Index number is equal to **(ICAI)**

(a) Sum of price relatives (b) Average of the price relative

(c) Product of Price Relative (d) None of these

Sol. (b) The Index number is determined by dividing the total of the actual prices for the base year by the total of the prices for the year for which the index number is to be obtained.

In terms of percentages, Index numbers are expressed.

The base period is the time frame between the two that will be used for comparison.

The base period's index number is always set at 100.

$$\text{Price relative} = \frac{\text{Price in the given year}}{\text{Price in the base year}} \times 100$$

Hence, the correct option is (b).

Example 9. The _____ of group indices gives the General Index. **(ICAI)**

(a) H.M. (b) G.M. (c) A.M. (d) None of these

Sol. (c) A weighted average of the financial accounts of the group firms is used to construct group indices. Each firm is assigned a weight based on how big it is in comparison to the other companies in the group.

For instance, if Company B has $20 million in sales and Company A has $10 million, Company B would be given more weight for determining the group revenue index.

As a result, it is an average measure. In essence, it depicts change over time more precisely than a basic index number, making it more realistic.

Hence, the correct option is (c).

Example 10. In the price index, when a new commodity is required to be added, which of the following index is used?

(a) Shifted price index (b) Splicing price index

(c) Deflating price index (d) Value price index

Sol. (b) We know that,

When a new commodity is required to be added in the price index, the index used is called the Splicing price index. This involves combining the old and new data into a single index to maintain the continuity of the index while accounting for changes in the basket of goods and services used to calculate the index.

Hence, the correct option is (b).

Example 11. Circular Test is one of the tests of **(ICAI)**

(a) Index numbers (b) Hypothesis (c) Both (d) None of these

Sol. (a) The 'Circular Test' is one of the tests of Index Number.

Symbolically, the test is represented as $P_{01} \times P_{12} \times P_{20} = 1$

Hence, the correct option is (a).

Example 12. The cost of living index is always **(July 2021)**

(a) Price index number (b) Quantity index number

(c) Weighted Index number (d) value index number

Sol. (c) Cost of living Index Number is nothing but weighted Index Number.

Hence the correct answer is option(c) i.e., Weighted index number.

Example 13. _____ is particularly suitable for the construction of index numbers.

(a) H.M. (b) A.M. (c) G.M. (d) None of these **(ICAI)**

Sol. (c) The Geometric Mean (G.M.) is particularly suitable for constructing index numbers. This choice is justified because:

1. It reflects proportionate changes: G.M. accurately represents changes in a series, regardless of the size of those changes.
2. Less sensitivity to extreme values: G.M. is less influenced by extreme values, as it multiplies rather than adds data points.
3. Suitable for logarithmic series: It works well with logarithmic data, simplifying the calculation of the average.
4. Supports consumer behavior theory: G.M. helps analyze average price changes, a key element in predicting consumer behavior.

Overall, the G.M. is a strong and trustworthy central tendency measure that is especially well suited for the creation of index numbers. It is the perfect tool for analysing economic and financial data since it can handle logarithmic series, represent proportional changes, and lessen the impact of extreme values. Hence, the correct option is (c).

Example 14. In the data group Bowley's and Laspeyre's index number is as follows:

Bowley's index number = 150, Laspeyre's index number = 180, then Paasche's index number

(a) 120 (b) 30 (c) 165 (d) None of these

Sol. (a) Given: Bowley's index number = 150

Laspeyre's index number = 180

As we know, Bowley's index number is expressed in the form of sum of Laspeyre's and Paasche's divided by 2.

$$\text{So, Bowley s index number} = \frac{\text{Laspeyres} + \text{Paasches}}{2}$$

Let Paasche's index number be x, then

$$\Rightarrow 150 = \frac{180 + x}{2} \Rightarrow 300 = 180 + x \Rightarrow x = 120$$

Hence, the correct answer is option (a) i.e., 120.

Example 15. Weighted G.M. of relative formula satisfy _____ test. **(ICAI)**

(a) Time Reversal Test (b) Circular test

(c) Factor Reversal Test (d) None of these

Sol. (a) We know that,

Weighted G.M. of relative formula satisfy Time Reversal test.

Hence, the correct option is (a).

Example 16. Factor Reversal test is satisfied by **(ICAI)**

(a) Fisher's Ideal Index (b) Laspeyre's Index

(c) Paasche's Index (d) None of these

Sol. (a) Factor Reversal test holds when the product of the price Index and the quantity index should be equal to the corresponding value index. i.e. $P_{01} \times Q_{01} = V_{01}$

This test is satisfied by Fisher's method .

Hence, the correct option is (a).

Example 17. G.M of Laspeyre's and Paasche's Price Index number is _____ price index number.

(a) Kelly's (b) Fisher's (c) Bowley's (d) None of these

Sol. (b) We know that,

Fisher's Ideal Index is the geometric mean of Laspeyre's and Paasche's Price Index number.

Hence, the correct option is (b).

Example 18. Laspeyre's formula does not satisfy **(ICAI)**

(a) Factor Reversal Test (b) Time Reversal Test

(c) Circular Test (d) All of the above

Sol. (d) We know that,

Laspeyre's formula do not satisfy Factor Reversal Test, Time Reversal Test and Circular Test.

Hence, the correct option is (d).

Example 19. A ratio or an average of ratios expressed as a percentage is called **(ICAI)**

(a) A relative number (b) An absolute number

(c) An index number (d) None of these

Sol. (c) A ratio or an average of ratios stated as a percentage is called an index number. It involves two or more time periods, one of which is the base period.

Hence the correct option is (c).

Example 20. The consumer price index goes up from 120 to 180 when salary goes up from 240 to 540 , what is the increase in real terms? **(July 2021)**

(a) 80 (b) 150 (c) 120 (d) 240

Sol. (c) Given;

The consumer price index goes up from 120 to 180.

Salary goes up from 240 to 540

$$\text{Actual salary} = \frac{180}{120} \times 240 = 360$$

Salary increase in real terms = 360 - 240 = 120

Hence, the correct option is (c) i.e. 120.

Example 21. An index time series is a list of numbers for two or more periods of time.

(a) Index (b) Absolute (c) Relative (d) None of these **(ICAI)**

Sol. (a) A time series is a collection of data points that appear in a particular order over a certain amount of time. Cross-sectional data, which records a moment in time, can be compared to this.

A set of index numbers for two or more time periods that all use the same base years is known as an index time series.

Hence the correct option is (a).

Example 22. Index numbers are often constructed from the **(ICAI)**

(a) Frequency (b) Class (c) Sample (d) None of these

Sol. (c) (i) **Simple or Unweighted Index Numbers:** Unweighted index numbers are index numbers in which each item must have some weight even when no specific weight is given to any item. It may be created using two methods, namely the simple aggregate method and the simple average of price relatives method, and is also referred to as a simple index number.

(ii) **Simple Aggregative Method:** This approach expresses the aggregate price of all the selected commodities in the current year as the aggregate price of all the commodities in the base year. The following formula is used to create an index number:

$$\text{Index} = \frac{\sum P_1}{\sum P_0} \times 100$$

Hence, the correct option is (c).

Example 23. Fisher's index number is called as ideal index number because it satisfies

(a) Factor reversal test **(Dec 2022)**

(b) Time reversal test

(c) Both factor and time reversal test

(d) Circular test

Sol. (c) We know,

Fisher's index number is called the ideal index number because it satisfies both factor and time reversal test. Hence, the correct option is (c).

Example 24. Among the following index numbers, which one satisfies the circular test?

(a) Laspeyre's Index (b) Paasche's Index

(c) Fisher's Index (d) None of these

Sol. (d) We know that,

The circular test is satisfied if $P_{01} \times P_{12} \times P_{20} = 1$

Circular test is only satisfied by:

- Simple Aggressive method
- Simple Geometric mean of price relatives

Thus, Fisher's ideal index, Laspeyre's Index, Paasche's Index does not satisfy the Circular test. Hence, the correct answer is option (d) i.e., None of the above.

Example 25. If $\sum P_0Q_0 = 406, \sum P_0Q_1 = 451, \sum P_1Q_0 = 456, \sum P_1Q_1 = 506$ then Fisher's ideal index number is

(a) 184.50 (b) 112.26 (c) 118.66 (d) 120.50

Sol. (b) Given: $\sum P_0Q_0 = 406, \sum P_0Q_1 = 451, \sum P_1Q_0 = 456, \sum P_1Q_1 = 506$

As we know, Fisher's Ideal index is given by the formula,

$$\text{Fisher's Index Number} = \sqrt{\frac{\sum p_1q_0 \times \sum p_1q_1}{\sum p_0q_0 \times \sum p_0q_1}} \times 100$$

$$= \sqrt{\frac{456}{406} \times \frac{506}{451}} \times 100 = \sqrt{\frac{11349}{16240}} \times 100$$

$$= \sqrt{1.260122552} \times 100 = 112.26$$

Hence, the correct answer is option (b) i.e., 112. 26.

Example 26. _____ is a point of reference in comparing various data describing individual behavior.

(a) Sample (b) Base period (c) Estimation (d) None of these **(ICAI)**

Sol. (b) We know that,

Base period is a point of reference in comparing various data describing individual behavior.

Hence, the correct option is (b).

Example 27. The ratio of price of single commodity in a given period to its price in the preceding year price is called the **(ICAI)**

(a) Base period (b) Price ratio (c) Relative price (d) None of these

Sol. (c) The relative price is the ratio in the cost of a particular good in two different time periods. This idea is frequently applied in economics to compare the cost of an item or service over different time periods. Amount of Relative Price Matters

In order to compare the cost of products and services across time, relative pricing is crucial. Economists may establish how the cost of an item or service has changed over time by comparing the price of that good or service in one era to the price of that same good or service in another period.

E.g.: The relative price of milk in 2020, for instance, would be $I.33 ($4 divided by $3), if the price of a gallon of milk was $3 in 2010 and $4 in 2020. Accordingly, milk will cost 33% higher in 2020 than it did in 2010.

Hence, the correct option is (c).

Example 28. $\frac{\text{Sum of all com mod ity price in the current year} \times 100}{\text{Sum of all com mod ity price in the base year}}$

(a) Relative Price Index

(b) Simple Aggregative Price Index

(c) Both

(d) None of these

Sol. (b) In this method aggregate price of commodities in current year $\sum P_1$ are divided by the aggregate price of these commodities in the base year $\sum P_0$ and expressed in percentage

$$\sum P_0 = \frac{\sum P_1}{\sum P_0} \times 100$$

where $\sum P_1$– Sum of prices of commodities of current year

$\sum P_0$– Sum of prices of commodities of base year

Hence, the correct option is (b).

Example 29. Chain index is equal to **(ICAI)**

(a) $\frac{\text{Link relative of current year} \times \text{chain index of the current year}}{100}$

(b) $\frac{\text{Link relative of previous year} \times \text{chain index of the current year}}{100}$

(c) $\frac{\text{Link relative of Current year} \times \text{chain index of the Pr evious year}}{100}$

(d) $\frac{\text{Link relative of Pr evious year} \times \text{chain index of the Pr evious year}}{100}$

Sol. (b) The base is constant and unchanging throughout the series according to fixed base approaches. However, as time goes on, certain items in the series could be added while others might be taken out.

As a result, it is difficult to compare the outcome of the current circumstances with that of the previous one. With this approach, we first represent each year's numbers as a percentage of the year before. Link Relatives are what these are called. The next step is to multiply each one successively to link them together to create a chain index.

$$\text{Chain Index} = \frac{\text{Current Year Relative} \times \text{Previous Year Link Relative}}{100}$$

Hence, the correct option is (b).

Example 30. If Laspeyre's Index is 119 and Paasche's Index is 112. Then Fisher's index number will be **(Dec 2022)**

(a) 113.99 (b) 115.45 (c) 115.89 (d) 151.98

Sol. (b) Given: Laspeyre's Index number = 119

Paasche's Index Number = 112

As we know, Paasche's and Laspeyre's index numbers are combined to create the Fisher index, which is the square root of that product.

Fisher's ideal index number $= \sqrt{\text{Paasche's index number} \times \text{Laspeyre's index numbers}}$

$= \sqrt{112 \times 119} = 115.45$

Hence, the correct answer is option (b).

Example 31. P_{01} is the index for time **(ICAI)**

(a) 1 on 0 (b) 0 on 1 (c) 1 on 1 (d) 0 on 0

Sol. (a) A ratio of two or more time periods, one of which being the base time period, is called an index number. The standard point of comparison is the value from the base time period.

Symbolically, $P_{01} \times P_{10} = 1$

where P_{01} is the index for time 1 on 0 and P_{10} is the index for time 0 on 1.

Hence, the correct option is (a).

Example 32. P_{10} is the index for time **(ICAI)**

(a) 1 on 0 (b) 0 on 1 (c) 1 on 1 (d) 0 on 0

Sol. (b) We know that, $P_{01} \times P_{10} = 1$

where P_{01} is the index for time 1 on 0 and P_{10} is the index for time 0 on 1.

Hence, the correct option is (b).

Example 33. If $\sum P_0Q_0 = 240, \sum P_1Q_1 = 480, \sum P_1Q_0 = 600, \sum P_0Q_1 = 192$ then the Laspeyre's Index number is **(Nov 2018)**

(a) 250 (b) 300 (c) 350 (d) 200

Sol. (a) As we know, Laspeyre's Index number is formulated as,

Laspeyre's Index number

$$= \frac{\sum P_1Q_1}{\sum P_0Q_0} \times 100 = \frac{480}{192} \times 100 = 2.5 \times 100 = 250$$

Hence, the correct answer is option (a) i.e., 250.

Example 34. In the year 2010, the price index for a particular item is 150 with the base year 2005. What does this index value indicate?

(a) The prices of the item have decreased by 50% since 2005.

(b) The prices of the item have increased by 50% since 2005.

(c) The prices of the item have increased by 150% since 2005.

(d) The prices of the item have increased by 50 units since 2005.

Sol. (b) Let the price of the base year (2005) be 100.

Given as per the question,

Price of the current year (2010) = 150

Then, the percentage increase = 150 - 100 = 50%

Hence, the correct answer is option (b) i.e., the prices of the item have increased by 50% since 2005.

Example 35. When the product of price index and the quantity index is equal to the corresponding value index then the test that holds is **(ICAI)**

(a) Unit Test (b) Time Reversal Test

(c) Factor Reversal Test (d) None holds

Sol. (c) It states that the value index should equal the product of a price index and a quantity index. According to Fisher, each formula must allow for the interchange of two times without producing inconsistent results, and it must also allow for the interchange of prices and quantities without producing inconsistent results. This means that multiplying two results together should produce the true value ratio.

Hence, the correct option is (c) i.e., Factor Reversal Test.

Example 36. The Index number in wholesale prices is 152 for August 1999 compared to August 1998. During the year there is net increase in prices of wholesale commodities to the extent of

(a) 45% (b) 35% (c) 52% (d) 48%

Sol. (c) Let the base price be 100

Given: Current Price =152

Percentage Increase = (152 - 100)% = 52%

Hence, the correct answer is option (c) i.e., 52%.

Example 37. Laspeyre's method and Paasche's method do not satisfy **(ICAI)**

(a) Unit Test (b) Time Reversal Test

(c) Circular test (d) both (b) and (c)

Sol. (d) We know that,

Laspeyre's method and Paasche's method do not satisfy both Time Reversal Test and Circular test. Hence, the correct option is (d).

Example 38. An index number that can serve many purposes is known as a

(a) General purpose index (b) Special purpose index

(c) Both (a) and (b) are incorrect (d) Both (a) and (b) are correct

Sol. (a) To get the best results, it is important to make a clear decision on the index number's construction, its scope, and the variable it will be used to measure. choosing the base year: The reference year is another name for the base year. It serves as the baseline for comparisons.

Therefore, an index number that can serve many purposes is known as a General purpose index.

Hence the correct option is (a).

Example 39. Which of the following statements is true? **(Nov 2018)**

(a) Paasche's Index number is based on the base year quantity

(b) Fisher's Index number is the arithmetic mean of Laspeyre's Index number and Paasche's Index Numbers

(c) Arithmetic Mean is the most appropriate average for constructing the index number

(d) Fisher's Index number is an Ideal Index Number

Sol. (d) As we know, Paasche's and Laspeyre's index numbers are combined to create the Fisher index, which is the square root of that product.

$$\text{Fisher's ideal index number} = \sqrt{\text{Paasche index number} \times \text{Laspeyre's index numbers}}$$

Thus, The statement which is true is Fisher's Index number is an Ideal Index Number.

Hence, the correct answer is option (d) i.e., Fisher's Index number is an Ideal Index Number.

Example 40. The index number is a special type of average. **(ICAI)**

(a) False (b) True (c) Both (d) None of these

Sol. (b) An exclusive class of averages are index numbers. The technique of index numbers is used to measure the relative changes in the level of a phenomenon when the measurement of absolute change is not possible and the series are expressed in different types of items, whereas mean, median, and mode measure the absolute changes and are used to compare only those series which are expressed in the same units.

The use of index numbers allows for the measurement of changes in a single variable or set of linked variables. For instance, a set of variables may include the price of sugar, milk, and rice, whereas the price of wheat could be one of the variables.

Hence, the correct option is (b) i.e., True.

Example 41. The cost of living index numbers in years 2015 and 2018 were 97.5 and 115 respectively. The salary of a worker in 2015 was ₹19,500. How much additional salary was required for him in 2018 to maintain the same standard of living as in 2015 ?

(a) 3000 (b) 4000 (c) 3500 (d) 4500

Sol. (c) Let the salary in 2018 be x,

Make a data table according to the question,

Year	Cost of Living Index	Salary
2015	97.5	19500
2018	115	x

Now, Salary in 2018 will be given as,

$$= \frac{\text{Cost of Living Index in 2018}}{\text{Cost of Living Index in 2015}} \times \text{Salary in 2015} = \frac{115}{97.5} \times 19500 = 23{,}000$$

So, salary of the worker in 2015 was 19500 and the additional salary required will be given by,

= 23000-19500 = 3500

Hence, the correct option is (c).

Example 42. Fisher's Ideal Formula does not satisfy _____ **(July 2021)**

(a) Unit Test (b) Circular Test

(c) Time Reversal Test (d) Factor Reversal Test

Sol. (b) As we know, the circular test is not satisfied by Fisher's index number.

Each method has a unit test, with the exception of the simple aggregative method.

Thus, Fisher's Ideal Formula does not satisfy the Circular test.

Hence, the correct answer is option (b).

Example 43. _____ satisfies circular test. **(ICAI)**

(a) G.M. of price relatives or the weighted aggregate with fixed weights

(b) A.M. of price relatives or the weighted aggregate with fixed weights

(c) H.M. of price relatives or the weighted aggregate with fixed weights

(d) None of these

Sol. (a) It is well known, the circular test is satisfied by the weighted aggregate with fixed weights or the geometric mean of the price relatives but not by perfect indices.

As a result, the circular test is satisfied by G.M. of price relatives or the weighted aggregate with fixed weights.

Hence, the correct answer is option (a).

Example 44. Laspeyre's and Paasche's method _____ time reversal test. **(ICAI)**

(a) Satisfy (b) Do not satisfy

(c) Are (d) Are not

Sol. (b) The index for the later era based on the earlier period should equal the earlier period based on the later period's index, according to the time-reversal test.

i.e., $P_{01} \times P_{10} = 1$

As a result, the time-reversal test using Laspeyres and Paasche's technique is not satisfied.

Hence, the correct answer is option (b) i.e., do not satisfy.

Example 45. The number of test of Adequacy is **(ICAI)**

(a) 2 (b) 5 (c) 3 (d) 4

Sol. (d) We know, Test of Adequacy are as follow:

I. Unit Test

II. Time Reversal Test

III. Factor Reversal Test

IV. Circular Test

Thus, there are 4 test of Adequacy.

Hence, the correct answer is option (d).

Example 46. Theoretically, G.M. is the best average in the construction of index numbers but in practice, mostly the A.M. is used.

(a) False (b) True (c) Both (d) None of these

Sol. (b) Although the geometric mean is known to be more difficult to compute than the arithmetic mean, the outcomes it generates are more accurate and powerful.

Therefore, while it is true that, in theory, the G.M. is the optimum average for constructing index numbers, in practice, the A.M. is typically utilized.

Hence, the correct answer is option (b) i.e., True.

Example 47. Laspeyre's or Paasche's or the Fisher's ideal index do not satisfy **(ICAI)**

(a) Time Reversal Test (b) Unit Test

(c) Circular Test (d) None of these

Sol. (c) As we know,

The Laspeyre's, Paasche's, or Fisher's ideal index does not satisfy the Circular Test.

Hence, the correct answer is option (c).

Example 48._____ is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base. **(ICAI)**

(a) Unit Test (b) Circular Test

(c) Time Reversal Test (d) None of these

Sol. (b) The Circular test can be used to gauge price changes over a period of years when the base has to be adjusted.

The measuring of price changes over a number of years when it is appropriate to relocate the base is thus the focus of the circular test.

Hence, the correct answer is option (b).

Example 49. The test of shifting the base is called **(ICAI)**

(a) Unit Test (b) Time Reversal Test

(c) Circular Test (d) None of these

Sol. (c) The Circular test can be used to gauge price changes over a period of years when the base has to be adjusted.

When index numbers are utilized, it is also possible to measure price changes over a longer period of time rather than only comparing prices over two years. In many cases, changing the foundation is desirable. Thus, the test of shifting the base is called the Circular test.

Hence, the correct answer is option (c).

Example 50. The formula for conversion to current value

(a) Deflated value $= \frac{\text{Price Index of the current year}}{\text{Previous value}}$

(b) Deflated value $= \frac{\text{Current value}}{\text{Price Index of the current year}}$

(c) Deflated value $= \frac{\text{Price Index of the Previous year}}{\text{previous value}}$

(d) Deflated value $= \frac{\text{Price Index of the Previous year}}{\text{previous value}}$

Sol. (b) As we know, the current value can be expressed mathematically as the result of the deflated value and the current year's price index.

It is formulated as, Deflated Value $= \frac{\text{Current value}}{\text{Price Index of the current year}}$

Hence, the correct answer is option (b).

Example 51. Shifted price Index $= \frac{\text{Original Price} \times 100}{\text{Price Index of the year on which it has to be shifted}}$

(a) True (b) False (c) Both (d) None of these

Sol. (a) The ratio of the original price and the price index of the year on which it must be shifted full, multiplied by 100, is the formula for the "Shifted Price Index."

It is formulated as, Shifted Price Index

$= \frac{\text{Original Price} \times 100}{\text{Price Index of the year on which it has to be shifted}}$

Hence, the correct answer is option (a) i.e., True.

Example 52. The weighted aggregative price index numbers for 2001 with 2000 as the base year using Paasche's index number is **(July 2021)**

Commodity	Price in ₹		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(a) 112.32 (b) 112.38 (c) 112.26 (d) 112.20

Sol. (d) According to the data given,

Commodity	Price in (₹)		Quantities			
	2000	2001	2000	2001		
	P_0	P_1	Q_0	Q_1	P_1Q_1	P_0Q_1
A	10	12	20	22	264	220
B	8	8	16	18	144	144
C	5	6	10	11	66	55

Commodity	Price in (₹)		Quantities			
D	4	4	7	8	32	32
					ΣP_1Q_1=506	ΣP_0Q_1= 451

We know that, Paasche's index number

$$P_{01} = \frac{\sum P_1Q_1}{\sum P_0Q_1} \times 100 = \frac{506}{451} \times 100 = 112.195 = 112.20$$

Hence, the correct option is (d).

Example 53. The weighted aggregative price index numbers for 2001 with 2000 as the base year using Marshal -Edgeworth Index number is **(July 2021)**

Commodity	Price in (₹)		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(a) 112.26 (b) 112.20 (c) 112.32 (d) 112.38

Sol. (a)

Commodity	Price in (₹)		Quantities			
	2000	2001	2000	2001		
	P_0	P_1	Q_0	Q_1	$P_1(Q_0 + Q_1)$	$P_0(Q_0 + Q_1)$
A	10	12	20	22	504	420
B	8	8	16	18	272	272
C	5	6	10	11	126	105
D	4	4	7	8	60	60
Total					962	557

Marshal -Edgeworth index number

$$= \frac{\sum P_1(Q_0 + Q_1)}{\sum P_0(Q_0 + Q_1)} \times 100 = \frac{962}{857} \times 100 = 112.25 = 112.26$$

Hence, the correct option is (a).

Example 54. From the following data

	Commodity	A	B	C	D
1992 Base year	Price	3	5	4	1
	Quantity	18	6	20	14
1993 Current year	Price	4	5	6	3
	Quantity	15	9	26	15

The Paasche's price index number is

(a) 146.41 (b) 120.50 (c) 164.82 (d) None of these

Sol. (a) According to the data given,

Commodity	P_0	Q_o	P_1	Q_1	P_0Q_0	P_1Q_0	P_0Q_1	P_1Q_1
A	3	18	4	15	54	72	45	60
B	5	6	5	9	30	30	45	45
C	4	20	6	26	80	120	104	156
D	1	14	3	15	14	42	15	45
					ΣP_0Q_0 = 178	ΣP_1Q_0 = 264	ΣP_0Q_1 = 209	ΣP_1Q_1 = 306

We know that,

Paasche's Price index $P_{02} = \frac{\sum P_1Q_1}{\sum P_0Q_1} \times 100 = \frac{306}{209} \times 100$

$\Rightarrow 1.46411 \times 100$

$\Rightarrow P_{02} = 146.41$

Therefore, Paasche's Price index = 146.41

Hence, the correct answer is option (a) i.e., 146.41.

Example 55. The prices and quantities of 3 commodities in base and current years are as follows: **(June 2019)**

p_0	p_1	q_0	q_1
12	14	10	20
10	8	20	30
8	10	30	10

The Laspeyre's Index number is:

(a) 118.13 (b) 107.14 (c) 120.10 (d) None of these

Sol. (b) Make a data table according to question,

p_o	p_1	q_0	q_1	p_0q_0	p_1q_0
12	14	10	20	120	140
10	8	20	30	200	160
8	10	30	10	240	300
				Σp_0q_0 = 560	Σp_1q_0= 600

As we know, Laspeyre's Index number is formulated as,

$$\text{Laspeyre's Index number} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Put the values in the formula and compute,

$$= \frac{600}{560} \times 100 = 107.14$$

Hence, the correct answer is option (b).

Example 56. With the base year 1960 the C.L.I in 1972 stood at 250. x was getting a monthly salary of ₹ 500 in 1960 and ₹750 in 1972. In 1972 to maintain his standard of living in 1960, x has to receive as extra allowances of

(a) ₹600 (b) ₹500 (c) ₹ 300 (d) None of these

Sol. (b) Given: Base Price = ₹500; Index Number = 250

Index number is given by the formula,

$$\text{Index number} = \frac{\text{Current Year Price}}{\text{Base Year Price}} \times 100$$

Put the values and compute for Current year price,

$$\text{Current Year Price} = \frac{250(500)}{100} = 1250$$

Now, D.A (x) will be given as,

D.A (x) = 1250-750 = 500

Hence, the correct answer is option (b) i.e., ₹500.

Example 57. If the ratio between Laspeyres index number and Paasche's index number is 28 : 27. Then the missing figure in the following table P is **(ICAI)**

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
X	L	10	2	5
Y	L	5	P	2

(a) 1 (b) 4 (c) 3 (d) 0

Sol. (b)

Commodity	P_0	Q_0	P_1	Q_1	P_0Q_0	P_1Q_0	P_0Q_1	P_1Q_1
X	L	10	2	5	10L	20	5L	10
Y	L	5	P	2	5L	5P	2L	2P
					ΣP_0Q_0 = 15 L	ΣP_1Q_0 = 20 + 5P	ΣP_0Q_1 = 7 L	ΣP_1Q_1 = 10 + 2P

As we know, Laspeyre's index number is formulated as,

$$= \frac{\sum P_1Q_0}{\sum P_0Q_0} \times 100 = \frac{20+5P}{15L} \times 100$$

Paasche's index number is given by the formula,

$$= \frac{\sum P_1Q_1}{\sum P_0Q_1} \times 100 = \frac{10+2P}{7L} \times 100$$

Now, as per the question,

$$\Rightarrow \frac{28}{27} = \frac{\frac{20+5P}{15L} \times 100}{\frac{10+2P}{7L} \times 100}$$

$\Rightarrow 9(20 + 5P) = 20(10 + 2P) \Rightarrow 180 + 45P = 200 + 40P \Rightarrow 5P = 20 \Rightarrow P = 4$

Hence, the correct option is (b).

FULL CHAPTER PRACTICE QUESTIONS

1. The most commonly used mathematical method for finding secular trends is

(a) Moving (b) Simple average

(c) Exponential (d) None of these

2. For year 2015, price index was 267% with base year 2005. The percentage increase in price index over base year 2005 is

(a) 26% (b) 67% (c) 167% (d) None of these

3. The cost of living index numbers in 2015 and 2018 were 97.5 and 115 respectively. The salary of a worker in 2015 was ₹19500. How much additional salary was required for him in 2018 to maintain the standard of living as in 2015?

(a) 3000 (b) 4000 (c) 3500 (d) 4500

4. The prices and quantities of 3 commodities in base and current years are as follows:

P_0	P_1	Q_0	Q_1
12	14	10	20
10	8	20	30
8	10	30	10

The Laspeyre price index is

(a) 118.13 (b) 107.14 (c) 120.10 (d) None of these

5. Which is not satisfied by fisher's ideal index number?

(a) Factor Reversal Test (b) Time Reversal Test

(c) Circular Test (d) None of these

6. Which is called an ideal index number?

(a) Laspeyre's index numbers

(b) Pasche's index number

(c) Fisher's index number

(d) Marshall Edgeworth index number

7. If $\sum p_0q_0 = 240, \sum p_1q_1 = 480, \sum p_1q_0 = 600$ and $\sum p_0q_1 = 192$ then laspeyre's Index number is

(a) 250 (b) 300 (c) 350 (d) 200

8. If Laspeyre's index number is 250 and paasche's index number is 160, then fisher's index number is

(a) 40000 (b) $\frac{25}{16}$ (c) 200 (d) $\frac{16}{25}$

9. Which of the following statements is true? **(Nov 2018)**

(a) Paasche's index number is based on the base year quantity

(b) Fisher's index number is the Arithmetic mean of Laspeyre's index number and Paasche's index number

(c) Arithmetic mean is the most appropriate average for constructing the index number

(d) Fisher's index number is an ideal index number

10. The number of test of Adequacy is **(May 2018)**

(a) 2 (b) 5 (c) 4 (d) 3

11. If the 1970 index with base with 1965 is 200 and 1965 index with base 1900. Is 150, the index 1970 on base 1960 will be

(a) 700 (b) 300 (c) 500 (d) 600

12. Circular test is satisfied by

(a) Laspeyre's index number

(b) Paasche's index number

(c) The simple geometric mean of price relatives and the weighted aggregative with fixed weights.

(d) None of these

13. Price relative is expressed in term of

(a) $p = \frac{P_n}{P_0}$ (b) $p = \frac{P_0}{P_n}$ (c) $p = \frac{P_n}{P_0} \times 100$ (d) $p = \frac{P_0}{P_n} \times 100$

14. The circular test is an extension of **(May 2018)**

(a) The time reversal test (b) The factor reversal test

(c) The unit test (d) None of these

15. If $\sum P_0Q_0 = 1360, \sum P_nQ_0 = 1900, \sum P_0Q_n = 1344, \sum P_nQ_n = 1880$ then Laspeyre's index number is **(May 2018)**

(a) 0.71 (b) 1.39 (c) 1.75 (d) None of these

16. The cost of living index is always

(a) Price index number (b) Quantity index number

(c) Weighted Index number (d) Value index number

17. P_{01} is the index for time **(May 2018)**

(a) 1 on 0 (b) 0 on 1 (c) 1 on 1 (d) 0 on 0

18. If Laspeyre's index number is 110 and Fisher's ideal number is 109, then Paasche's index number is

(a) 108 (b) 110 (c) 109 (d) 18

19. When the prices for quantities consumed of all commodities are changing in the same ratio, then the index numbers due to Laspeyre's and Paasche's will be

(a) Equal

(b) Unequal

(c) Reciprocal of Marshall Edge worth index number

(d) Reciprocal of fisher index number

20. If $\sum P_0Q_0 = 116, \sum P_0Q_1 = 140, \sum P_1Q_0 = 97, \sum P_1Q_1 = 117$ then Fisher's ideal index number is

(a) 184 (b) 83.59 (c) 119.66 (d) 120

21. If the 1970 index with base 1965 in 200 and 1965 index with base 1960 is 150, the index 1970 on the base 1960 will be

(a) 700 (b) 300 (c) 500 (d) 600

22. The index number for the year 2012 taking 2011 as base using simple average of price relatives method from data given below is

Commodity	A	B	C	D	E	
Price in 2011 (P_0)	115	108	95	80	90	$\sum P_0$ =488
Price in 2012 (P_1)	125	117	108	95	95	$\sum P_1$ =540

(a) 112 (b) 117 (c) 120 (d) 111

23. Bowley's index number is expressed in terms of

(a) $\frac{\text{Laspeyre's} + \text{Paasche's}}{2}$ (b) $\frac{\text{Laspeyre's} \times \text{Paasche's}}{2}$

(c) $\frac{\text{Laspeyre's} - \text{Paasche's}}{2}$ (d) None of these

24. Fisher's ideal formula for calculating index number satisfies the

(a) Unit Test (b) Factor Reversal Test

(c) Both (a) and (b) (d) None of these

25. Circular Test is satisfied by:

(a) Paasche's Index Number

(b) The simple geometric mean of price relatives and the weighted aggregative fixed weights

(c) Laspeyre's Index Number

(d) None of these

26. Cost of living index numbers are also used to find real wages by the process of

(a) Base shifting (b) Splicing of index numbers

(c) Deflating of index numbers (d) None of these

27. The prices of a commodity in the years 1975 and 1980 were 25 and 30 respectively then Price relative of 1975 on 1980 is:

(a) 113.25 (b) 83.33 (c) 109.78 (d) None of these

28. Laspeyre's and Paasche's Method _____ Time Reversal Test.

(a) Do not satisfy (b) Satisfy

(c) Depends on the case (d) Can't say Time Reversal Test

29. Consumer Price Index number goes up from 100 to 200 and salary of a worker is also raised from 300 to 500. The Real wage is

(a) 300 (b) 250 (c) 600 (d) 350

30. Paasche's price index is a weighted aggregate price index in which _____quantities are taken as weights.

(a) Base year

(b) Current year

(c) Average of base year and current year

(d) None of these

31. From the following data, calculate price index number for 2010 with 2000 as base by Fisher's Index Number.

Commodity	2000		2010	
	Price	Quantity	Price	Quantity
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

(a) 150.50 (b) 123.24 (c) 130.65 (d) None of these

32. From the following data, calculate price index number for 2007 with 2006 as base by Marshall-Edgeworth method.

Commodity	2000		2010	
	Price	Quantity	Price	Quantity
A	10	100	12	150
B	8	80	10	100
C	5	60	10	72
D	24	30	18	33

(a) 118.64 (b) 132.40 (c) 140.75 (d) None of these

Answer Key

1. (b)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (d)	**10.** (c)
11. (b)	**12.** (c)	**13.** (c)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (b)
21. (b)	**22.** (d)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (b)
31. (b)	**32.** (a)								

PREVIOUS YEAR QUESTION (PYQ'S)

1. Which one of the following is an Ideal index number? **(June 2019)**

(a) Laspeyre's index number
(b) Paasche's index number
(c) Fisher's index number
(d) Marshall Edgeworth index number

2. Which is not satisfied by Fisher's Index number? **(June 2019)**

(a) Factor Reversal Test (b) Time Reversal Test
(c) Circular Test (d) None of the above

3. The prices and quantities of 3 commodities in base and current years are as follows: **(Dec 2019)**

p_0	p_1	q_0	q_1
12	14	10	20
10	8	20	30
8	10	30	10

(a) 118.13 (b) 107.14 (c) 120.13 (d) None

4. The cost of living index number in years 2015 and 2018 were 97.5 and 115 respectively. The salary of a worker in 2015 was Rs 19,500. How much additional slary was required for him in 2018 to maintain the ame standard of living as in 2015? **(June 2019)**

(a) 3000 (b) 4000 (c) 3500 (d) 4500

5. For year 2015, price index was 267% with base year 2005. The percentage increase in price index over base year 2005 is **(Dec 2019)**

(a) 267% (b) 67% (c) 167% (d) None of these

6. The value of the base time period serves as a standard point of comparison. **(Dec 2019)**

(a) True (b) False (c) Both (d) None of these

7. Fisher's ideal formula does not satisfy _____ test. **(Dec 2019)**

(a) Circular (b) Time Reversal

(c) Factor Reversal (d) Unit

8. Index Number are expressed as _____ . **(Dec 2020)**

(a) Squares (b) Ratios (c) Percentages (d) Combinations

9. If Laspeyre's index number is 110 and Fisher's ideal index number is 109. Then Paasche's index number is **(Dec 2020)**

(a) 108 (b) 110 (c) 109 (d) 118

10. The cost of living index is always **(Jan 2021)**

(a) Price index number (b) Quantity index number

(c) Weighted index number (d) Value index number

11. Fisher's index number does not satisfy **(Jan 2021)**

(a) Unit test (b) Circular test

(c) Time reversal test (d) Factor reversal test

12. When the prices for quantities consumed of all commodities are changing in the same ratio, then the index numbers due to Laspeyre's and Paasche's will be **(Jan 2021)**

(a) Equal

(b) Unequal

(c) Reciprocal of Marshall Edgeworth Index number

(d) Reciprocal of Fisher Index number

13. The consumer price index goes up from 120 to 180 when salary goes from 240 to 540, what is the increase in real terms? **(July 2021)**

(a) 80 (b) 150 (c) 120 (d) 240

14. The weighted aggregative price index numbers fro 2001 with 2000 as the base year using Paasche's index number is **(July 2021)**

Commodity	Price (in Rs)		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(a) 112.32 (b) 112.38 (c) 112.26 (d) 112.20

15. The weighted aggregative price index numbers for 2001 with 2000 as the base year using Marshal-Edgeworth Index numbers is **(July 2021)**

Commodity	Price (in Rs)		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(a) 112.26 (b) 112.20 (c) 112.32 (d) 112.38

16. The weighted aggregative price index numbers for 2001 with 2000 as the base year using Fisher's Index number is **(July 2021)**

Commodity	Price (in Rs)		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(a) 112.32 (b) 112.20 (c) 112.38 (d) 112.26

17. The test of shifting the base is called **(June 2022)**

(a) Unit Test (b) Time Reversal Test

(c) Factor Reversal Test (d) Circular Test

18. Let p_0 and p_1 be the prices of a commodity in the base and current year respectively. The price relative with respect to base year is **(June 2022)**

(a) $\frac{p_1}{p_0}$ (b) $\frac{p_0}{p_1}$ (c) $\frac{p_1 - p_0}{p_0}$ (d) $\frac{p_1 - p_0}{p_1}$

19. The Laspeyre's index number is a weighted aggregate method by taking _____ as weights. **(June 2022)**

(a) Quantity consumed in the base year

(b) Quantity consumed in the current year

(c) Value of items consumed in the base year

(d) Value of items consumed in the current year

20. Which one of the following methods is based on geometric mean for calculating as index number? **(June 2022)**

(a) Fisher's method (b) Kelley's method

(c) Paasche's method (d) Laspeyre's method

21. Which one of the following test is not applied for selecting an index number?

(June 2022)

(a) Time Reversal (b) Price Relative

(c) Factor Reversal (d) Circular

22. From the following data construct the index number of Laspeyre's method

$\Sigma P_1Q_1 = 99$, $\Sigma P_0Q_1 = 76$, $\Sigma P_0Q_0 = 73$, $\Sigma P_1Q_0 = 96$ **(Dec 2022)**

(a) 136.25 (b) 131.51

(c) 130.88 (d) 76.04

23. Which of the following index measures the change from month to month in the cost of a representative "basket" of goods and services of the type which are brought by a typical household? **(Dec 2022)**

(a) Retail Price Index (b) Laspeyre's Index

(c) Fisher's Index (d) Paasche's Index

24. Fisher's index number is called as ideal index number because it is satisfying

(Dec 2022)

(a) Factor Reversal Test

(b) Time Reversal Test

(c) Both Factor and Time Reversal Test

(d) Circular Test

25. If Laspeyre's Index is 119 and Paasche's Index is 112, then Fisher's Index is

(Dec 2022)

(a) 113.99 (b) 115.45

(c) 115.89 (d) 151.98

26. In price index, when a new commodity is required to be added, which of the following index is used? **(Dec 2022)**

(a) Shifted price index (b) Splicing price index

(c) Deflating price index (d) Value price index

27. Consider the data **(June 2023)**

Year	Base Year		Current Year	
Commodities	Price	Quantity	Price	Quantity
A	10	5	20	2
B	15	4	25	8
C	40	2	60	6
D	25	3	40	4

Laspeyre's Index is

(a) 166.04 (b) 156.04 (c) 164.06 (d) 154.06

28. Which of the following index is computed by taking average of base year and current year? **(June 2023)**

(a) Marshall Edgeworth's Index (b) Paasche's Index
(c) Laspeyre's Index (d) Fisher's Index

29. The index number of prices for a country at a given data is 250. In comparison to the base period price the price of all commodities in the country has increased by _____ times **(June 2023)**

(a) 1.25 (b) 1.5 (c) 2 (d) 2.5

30. If Fisher's index number is 160 and Paasche's index number is 140 then the Laspeyre's index number is **(June 2023)**

(a) 187.77 (b) 182.86 (c) 183.25 (d) 186.25

31. Weighted geometric mean of relative formula satisfies _____ test while Factor Reversal Test is satisfied by _____ . **(June 2023)**

(a) Time Reversal, Fisher's Ideal Index
(b) Time Reversal, Laspeyre's Index
(c) Factor Reversal, Paasche's Index
(d) Factor Reversal, Fisher's Ideal Index

32. Fisher's Index number does not satisfy following test": **(Dec 2023)**

(a) Unit Test (b) Time Reversal Test
(c) Circular Test (d) Factor Reversal Test

33. An index number constructed to measure the relative change in the price of an item or group of items is called **(Dec 2023)**

(a) Quantity index number (b) Price index number
(c) Volume index number (d) Composite index number

34. The gross monthly pay of an employee was Rs 15,000 in the year 2020. The consumer price index number in 2023 is 155 with 2020 as base year. If an employee is to be rightly compensated what dearness allowance is required to be paid? **(Dec 2023)**

(a) Rs 8000 (b) Rs 8250 (c) Rs 8500 (d) Rs 8750

35. If the Laspeyre's index is 110 and Paasche's index number is 108, then what is the value of Fisher's index? **(Dec 2023)**

(a) 106.50 (b) 107.60 (c) 108.99 (d) 109.88

36. From the year 2013 to 2023, consumer price index number is increased from 135 to 180. During the period, the salary of the employee as per pay commission recommendations was revised from Rs 23,000 to Rs 29,500. In real terms, an employee should get following additional amount (up to nearest whole number) to maintain his previous standard of living. **(Dec 2023)**

(a) Rs 1168 (b) Rs 666 (c) Rs 909 (d) Rs 6500

37. Which of the following is not a test of adequacy in the context of index numbers?

(June 2024)

(a) Unit test (b) Square test (c) Circular test (d) Factor reversal test

38. If the prices of all commodities in the base year are twice the values of the respective commodities in the current year, then the Fisher's ideal index number is equal to

(June 2024)

(a) 200 (b) 50 (c) 25 (d) 400

39. The average of base year and current year is used in _____ index number **(June 2024)**

(a) Laspeyre's (b) Paasche's (c) Fisher's ideal (d) Marshall-Edgeworth

40. Which index number satisfies both the time reversal and factor reversal tests? **(June 2024)**

(a) Fisher's Ideal index (b) Laspeyre's index

(c) Paasche's index (d) Marshall-Edgeworth's index

41. Which of the following does not satisfy the time reversal test? **(June 2024)**

(a) Fisher's Ideal index and Laspeyre's index

(b) Laspeyre's index and Paasche's index

(c) Paasche's index and Fisher's Ideal index

(d) Laspeyre's index, Paasche's index & Fisher's Ideal index

42. The value index is equal to: **(Sept 2024)**

(a) The total sum of the values of a given year multiplied by the sum of the values of the base year.

(b) The total sum of the values of a given year plus the sum of the values of the base year.

(c) The total sum of the values of a given year divided by the sum of the values of the base year.

(d) The total sum of the values of a given year minus the sum of the values of the base year.

43. Which one of the following tests of adequacy is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base? **(Sept 2024)**

(a) Unit test (b) Time Reversal test

(c) Circular test (d) Factor Reversal test

44. The consumer price index for the year 2023 is 273 with 2010 as base year. The average monthly wages of industrial worker in year 2023 is ₹ 8,190. What is the real wage? **(Sept 2024)**

(a) ₹ 2,800 (b) ₹ 3,000 (c) ₹ 3,200 (d) ₹ 3,400

45. During a certain period, the cost of living index goes up from 110 to 200 and the salary of a worker is also raised from ₹ 330 to ₹ 500, then in the real terms, the raise in the salary is effectively- **(Sept 2024)**

(a) Gain by ₹ 50 (b) Gain by ₹ 75 (c) Loss by ₹ 90 (d) Loss by ₹ 50

46. Time Reversal test is satisfied by: **(Sept 2024)**

(a) Laspeyre's method but not Fisher's method

(b) Paasche's method but not Laspeyre's method

(c) Fisher's method

(d) Laspeyre's method and Fisher's method

47. When the cost of Beverages increased by 40%, the person said that the rise had increased his cost of living by 8%. Before the change in price, the percentage of his cost of living was due to buying Beverages is: **(Jan 2025)**

(a) 20% (b) 15% (c) 5% (d) 2%

48. If the prices of all the goods change in the same ratio, then **(Jan 2025)**

(a) Laspeyre's index and Paasche's index numbers are not equal.

(b) Laspeyre's index and Paasche's index numbers are equal.

(c) Laspeyre's index is greater than Paasche's index number.

(d) Laspeyre's index is less than Paasche's index number.

49. Given that $\Sigma p_n q_n = 300$, $\Sigma p_o q_o = 125$ and Paasche's index number is 200 then the value of $\Sigma \rho_o q_n$ is **(Jan 2025)**

(a) 150 (b) 125 (c) 250 (d) 100

50. If Laspeyre's index number is 125 and Paasche's index number is 500 then Fisher's index number is **(Jan 2025)**

(a) 250.0 (b) 312.5 (c) 62.5 (d) 147.5

51. From the following data, find out an index number for 2022 taking 2021 as base (using simple aggregative method): **(May 2025)**

Commodities	Price in 2021	Price in 2022
A	80	120
B	220	200
C	300	400

(a) 100 (b) 120 (c) 108 (d) 190

52. From the following chain base index numbers based on 2015, find out new chain base index number for the year 2022 by shifting the base year 2019. **(May 2025)**

Years	2015	2016	2017	2018	2019	2020	2021	2022
Index No. (Base 2015)	100	105	95	85	120	110	130	150

(a) 125 (b) 180 (c) 100 (d) 150

53. If $\Sigma p_n q_n = 249$, $\Sigma p_o q_o = 150$, $\Sigma p_n q_o = 145$ and Paasche's Index Number = 150, then Fisher's Ideal Price Index number is **(May 2025)**

(a) 75 (b) 126.9 (c) 120.62 (d) 171

54. The prices of a commodity in the years 2015 and 2020 were 50 and 60 respectively. Price relative of 2015 on 2020 is

(a) 100 (b) 110 (c) 83.33 (d) 120

55. Chain Index is equal to **(May 2025)**

(a) $\frac{\text{Link relative of current year} \times \text{Chain Index of current year}}{100}$

(b) $\frac{\text{Link relative of previous year} \times \text{Chain Index of current year}}{100}$

(c) $\frac{\text{Link relative of current year} \times \text{Chain Index of previous year}}{100}$

(d) $\frac{\text{Link relative of previous year} \times \text{Chain Index of previous year}}{100}$

56. If $\Sigma p_0 q_0 = 160.7$ and $\Sigma p_1 q_0 = 178.6$, then the cost of living index is **(Sept 2025)**

(a) 111.14 (b) 93.10 (c) 104.71 (d) 105.7

57. Two indices this is current on base and base on current should be reciprocals of each other in **(Sept 2025)**

(a) Unit test (b) Time reversal test

(c) Circular test (d) Average weighted test

58. If the consumer price index number is 750, then the purchasing power of one rupee is _____ **(Sept 2025)**

(a) 12.5 paise (b) 15 paise (c) 13.3 paise (d) 16.5 paise

59. Which sampling technique can be used for the construction of Index numbers? **(Sept 2025)**

(a) Systematic sampling (b) Quota sampling

(c) Cluster sampling (d) Random sampling

60. From the year 2015 to 2025, Consumer price index increased from 125 to 196. During this period, salary of the employees as per 7th pay commission recommendations was revised from ₹25,000 to ₹37,250. In real terms, an employee should get following amount as an additional amount to maintain his previous standard of living: **(Sept 2025)**

(a) ₹1,965 (b) ₹1,950 (c) ₹1,945 (d) ₹14,200

Answer Key

1. (c)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (c)
11. (b)	**12.** (a)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (a)
21. (b)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (a)	**29.** (b)	**30.** (b)
31. (a)	**32.** (c)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (a)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (a)
41. (b)	**42.** (c)	**43.** (c)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (a)	**48.** (b)	**49.** (a)	**50.** (a)
51. (b)	**52.** (a)	**53.** (c)	**54.** (c)	**55.** (c)	**56.** (a)	**57.** (b)	**58.** (c)	**59.** (d)	**60.** (b)

SUMMARY

Index numbers are the most important need in the study of statistics. If you were to modify the variable in the estimation of any particular statistics, just imagine how it would be without these data! The technique will be wholly ineffectual in and of itself. The measurement of any change in a variable or variables over a predetermined period is therefore done via index numbers. These figures represent a broad relative shift rather than a specific quantifiable value. A percentage is used to express an index number.

Learn more about the Index numbers so that we may explore their significance, traits, categories, and restrictions. Continue reading the material to learn more about the supplementary part that we have included.

IMPORTANCE OF INDEX NUMBER

In studies of a region's economic situation, index numbers are most frequently utilised. The level of a variable in relation to its level across a specific time period is defined by the index number, as was previously indicated. These index values are used to analyse how the impact of all the variables that cannot be directly measured or approximated vary over time.

Due to their effectiveness in determining the magnitude of economic changes over a given time period, index numbers therefore have a significant position. The impact of such changes owing to variables that cannot be assessed directly are studied.

THE FOLLOWING ARE SOME OF THE KEY DISTINGUISHING CHARACTERISTICS OF INDEX NUMBERS

- When absolute measurement cannot be done, this specific category of average is used to assess relative changes.
- Only speculative changes in variables that may not be immediately measurable are shown by index numbers. It provides a broad overview of the relative changes.
- The index number measurement technique varies from one connected variable to another.
- It facilitates comparing the levels of a phenomena on a given day to those from earlier dates.

- It serves as an example of an uncommon average, particularly for a weighted average.
- Index numbers are useful everywhere. You may also utilise the index that is used to determine price fluctuations.

SIMPLE AGGREGATE METHOD

This approach is predicated on the idea that different things and their pricing are given in the same units. Each thing is given the same importance. The following is the formula for a straightforward aggregative pricing index: $P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$

where,

$\sum P_1$ is the total of current year's prices for the various items.

$\sum P_2$ is the total of base year's prices for the various items.

VARIOUS INDEX NUMBER TYPES

Different sorts of index numbers are used in certain ways. To understand the same, we shall study the various index numbers. The students will get an understanding of the significance of each form of Index number in relation to the assignment they are practicing for from this lesson on the many sorts of Index numbers.

- **Price Index:** The ratio of the aggregate value for a given period to the aggregate value found in the base period yields a value index number. The value index is used, among other things, for inventory, sales, and international commerce.

  Price relative $= \frac{P_n}{P_0} \times 100$

- **Quantity Index:** When measuring changes in the volume or amount of items produced, consumed, and sold within a given time frame, a quantity index number is utilised. It displays the relative change over a time period for certain product quantities. An illustration of a quantity index is the Index of Industrial Production (IIP)

  Quantity Relative $= \frac{Q_n}{Q_0} \times 100$

- **Value Relative:** The value relative considers the value of a commodity by multiplying the price and quantity relatives together.

  Value Relative $= \frac{V_n}{V_0} = \frac{P_n}{P_0} \times \frac{Q_n}{Q_0} = \frac{P_n Q_n}{P_0 Q_0}$

USES OF INDEX NUMBER IN STATISTICS

In several straightforward to complex research, index numbers are helpful. Like it is used in the basic study of human population in a country and also it is used to determine the extinction rate of the rare animals in a particular region. There are many more applications for index numbers; here are some examples:

- It aids in gauging changes in both pricing levels and living standards.
- Regulation of wage rates is compatible with shifts in the level of prices. Wage rates may change when pricing levels are established.
- The index number of prices is used to frame government policy. Index numbers serve as the foundation for the built-in price stability of fiscal and economic policy.
- It provides a starting point for comparing many economic factors internationally, such as the living standards of two nations.

ADVANTAGES OF INDEX NUMBER

- Index numbers' benefits are closely related to how they are used. In conclusion, the benefits are as follows: It makes cost-effective adjustments to primary data, which is helpful for deflation. It makes the switch from nominal wage to real wage easier.
- In economics, index numbers are frequently used to aid in the formulation of effective policies. These results also aid in the development of research.
- It is useful for patterns like making conclusions for cyclical and irregular forces.
- In the event that economic activity develops in the future, index numbers can be useful. To identify patterns and cyclical processes, this time series analysis is used.
- The figure is helpful in gauging changes in living standards across various nations over a predetermined time frame.

ISSUES IN THE CONSTRUCTION OF INDEX NUMBERS

The following are some considerations for the development of an index number that should be made:

- Purpose of Index number
- Selection of Items
- Choice of Average
- Assignment of weights
- Choice of Base year

LASPEYRE'S METHOD: $= \left(\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \right) \times 100$

Here, $\sum P_1 Q_0$ represents the sum of prices for the current year times the base year's quantity weights, and $\sum P_0 Q_0$ represents the sum of prices for the base year times the base year's quantity weights.

PAASCHE'S METHOD: $= \left(\frac{\sum P_1 Q_1}{\sum P_0 Q_1} \right) \times 100$

Here, $\sum P_1 Q_1$ is the sum of current year prices multiplied by current year quantities taken as weights, and $\sum P_0 Q_1$ is the sum of base year prices multiplied by current year quantities taken as weights.

MARSHALL-EDGEWORTH METHOD: $= \frac{\sum P_n (Q_0 + Q_n)}{\sum P_0 (Q_0 + Q_n)} \times 100$

FISHER'S METHOD: $= \sqrt{\left[\left(\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \right) \times \left(\frac{\sum P_1 Q_1}{\sum P_0 Q_1} \right) \right]} \times 100$

Fisher's ideal index number is an ideal index number.

DORBISH AND BOWLEY'S PRICE INDEX

Dorbish and Bowley's Price Index $= \frac{\frac{\sum P_n Q_0}{\sum P_0 Q_0} + \frac{\sum P_n Q_n}{\sum P_0 Q_n}}{2} \times 100$

LIMITATIONS OF INDEX NUMBER

We are aware that everything in the world has both benefits and drawbacks. Although index numbers offer many benefits, this is also where some of its drawbacks appear. The following are some index numbers' restrictions:

- Given that index numbers are generated from samples, there is a risk for inaccuracy. These samples are assembled after careful consideration, which increases the possibility of mistakes. It can also be found in base periods, weights, etc.
- It is always determined using the things. Items that are so carefully chosen may not accurately reflect current trends, which leads to erroneous analysis.
- The index numbers provide a rough idea of the relative changes that take place.
- The choice of representative goods could be biased.

MAIN CHARACTERISTICS OF INDEX NUMBERS

Index numbers are a particular kind of average that

1. give a measurement of relative changes in the frequency of a certain phenomena;
2. are stated in terms of percentages to demonstrate the magnitude of relative change
3. quantify relative changes.
4. They are also capable of measuring changes that are not readily quantifiable.

Economic barometers are index numbers. They aid in the creation of economic policy, planning, and other things. They are employed in the analysis of trends and patterns. Indicators like index numbers can be used to predict future economic activity. They gauge the value of money in terms of purchases.

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