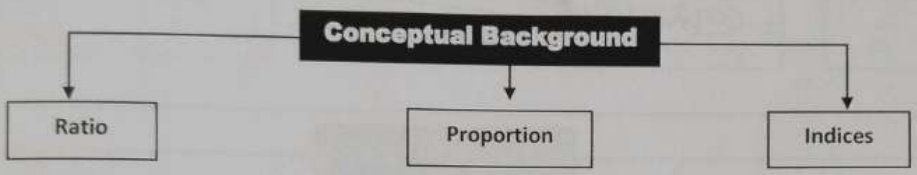
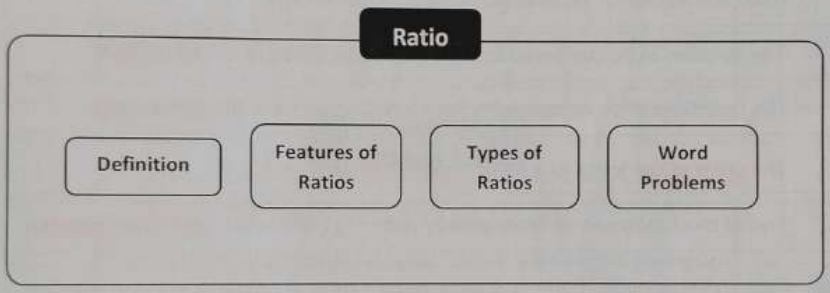
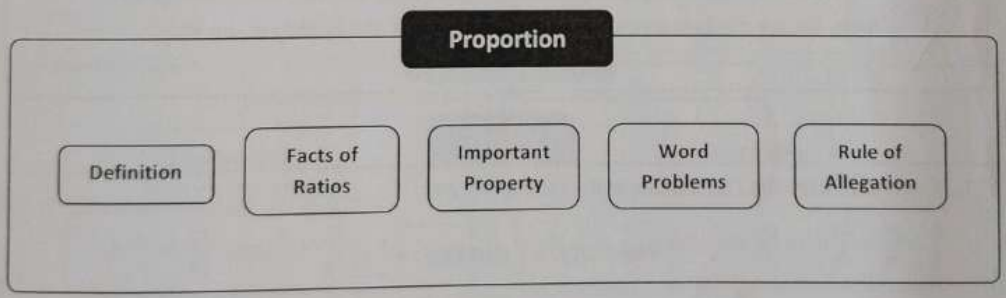
Ratio Proportion & Indices

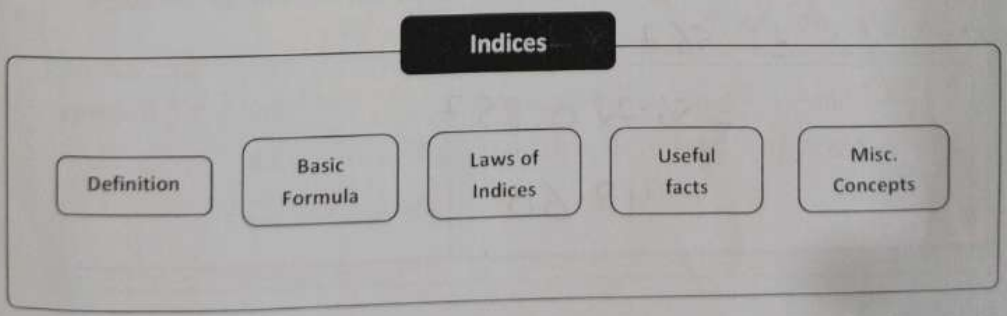
Chapter 1

Background of Chapter





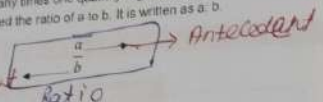




Ratios

Definition

Ratio is a comparison of two or more quantities of same kind, expressed in same unity, by division. A ratio expresses how many times one quantity is greater or less than the other. If 'a' and 'b' are two quantities then $\frac{a}{b}$ is called the ratio of a to b. It is written as a : b.



Features of Ratio

1. Ratio is a fraction or an abstract number. It has no unit.
2. The quantities to be compared to form a ratio should be of the same kind.
3. The quantities to be compared to form a ratio must be in the same units.
4. The order of the terms in a ratio is important, thus $\frac{x}{y} \neq \frac{y}{x}$.
5. Two or more ratio can be compared by reducing them to the same denominator.
6. Ratio in the Simplest Form or in the Lowest Terms: A ratio a : b is in the simplest form if a & b have no factor in common. Thus 200 : 280 is not in the simplest form. So, we divide both the terms by the H.C.F. of 200 & 280 i.e. by 40.

Note

1. If quantity increases / decreases in the ratio a: b then,

$$\text{New Qty.} = \left[\text{Old Qty.} \times \frac{b}{a}\right].$$

For e.g., 'R' who weighs 56.7 Kgs. reduces his weight in the ratio 7: 6, then his new weight

$= 56.7 \times \frac{6}{7}$

$= 56.7 \times 0.857$

$= 48.60$

Types of Ratios

Compound Ratio	Duplicate Ratio	Sub-Duplicate Ratio	Triplicate Ratio	Sub-Triplicate Ratio	Inverse Ratio
$\frac{a}{b}, \frac{c}{d}$ $\frac{a \times c}{b \times d}$	a/b $\frac{a^2}{b^2}$	a/b $\frac{\sqrt{a}}{\sqrt{b}}$	a/b $\frac{a^3}{b^3}$	a/b $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}$	$\frac{a}{b}$ $\frac{b}{a}$

Examples

$\frac{3}{4}, \frac{5}{7}$ $\frac{15}{28}$	$\frac{2}{3}$ $\frac{4}{9}$	$\frac{4}{9}$ $\frac{2}{3}$	$\frac{2}{3}$ $\frac{8}{27}$	$\frac{8}{27}$ $\frac{2}{3}$	$\frac{2}{3}$ $\frac{3}{2}$

Continued Ratio

Ratio Between 2 or more quantity is called continued ratio

(Solving from method is lengthy, so we will use the Shortcut Explained below)

Shortcut

3 Quantality

If $\frac{a}{b} = \frac{n_1}{d_1}$ and $\frac{b}{c} = \frac{n_2}{d_2}$

$a : b : c \Rightarrow n_1 \times n_2 : d_1 \times n_2 : d_2 d_1$

Example: If $\frac{a}{b} = \frac{3}{4}$ and $\frac{b}{c} = \frac{5}{6}$

$a : b : c \quad 3 \times 5 : 4 \times 5 : 4 \times 6$

$15 : 20 : 24$

4 Quantality

If $\frac{a}{b} = \frac{n_1}{d_1}$ and $\frac{b}{c} = \frac{n_2}{d_2}$ and $\frac{c}{d} = \frac{n_3}{d_3}$

$a : b : c : d \Rightarrow n_1 \times n_2 \times n_3 : d_1 \times n_2 \times n_3 : d_1 \times d_2 \times n_3 : d_1 \times d_2 \times d_3$

Example: If $\frac{a}{b} = \frac{3}{4}$ and $\frac{b}{c} = \frac{5}{6}$ and $\frac{c}{d} = \frac{7}{8}$

$a : b : c = 105 : 140 : 168$

Proportion

Definition

When two ratios are equal i.e., a : b = c : d, then the terms a, b, c, d are said to be in proportion.

- It is expressed as a : b :: c : d (read as a : b is proportional to c : d).
- a, b, c, d is called 1st, 2nd, 3rd, and 4th proportional respectively.

$\frac{a}{b} = \frac{c}{d}$ then a, b, c, d are terms of the Proportion

Facts Relating to Proportion

Cross Product Rule

If $\frac{a}{b} = \frac{c}{d}$ then

$a \times d = b \times c$

Product of Extreme = Product of mean

3 Quantity Proportion

a, b, c are in prop if

$\frac{a}{b} = \frac{b}{c}$

$b^2 = ac$

4 Quantity Proportion

a, b, c, d are in prop if

$\frac{a}{b} = \frac{c}{d}$

Thumb Rule

Continued Ratio

a : b, c : d, e

$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e}$

Cross = GP

Continue mean GP

Some Important Property

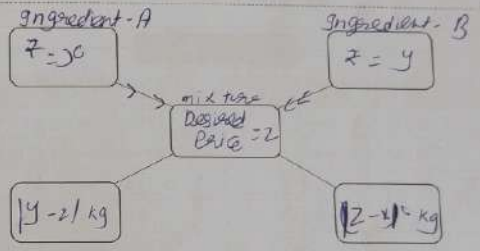
Property	If $\frac{a}{b} = \frac{c}{d}$, then
INVERTENDO	$\frac{b}{a} = \frac{d}{c}$
ALTERNENDO	$\frac{a}{c} = \frac{b}{d}$
COMPONENDO	$\frac{a+b}{b} = \frac{c+d}{d}$
DIVIDENDO	$\frac{a-b}{b} = \frac{c-d}{d}$
COMPONENDO & DIVIDENDO	$\frac{a+b}{a-b} = \frac{c+d}{c-d}$
ADDENDO	$\frac{a}{b} = \frac{c}{d}$ then $\frac{a+c}{b+d}$
SUBTRAHENDO	$\frac{a-c}{b-d}$

IIP $= \frac{a/b}{c/d} = \frac{a}{b} \times \frac{d}{c}$

Rule of Allegation

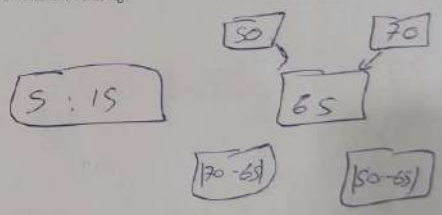
Allegation is a rule that enable us to find the ratio in which two or more ingredients must be mixed to produce a mixture of desired price. There are two types of Method used in Allegation.

Allegation Rule-1: It is modified form of finding weighted average. If two ingredients are mixed in a ratio and cost price of unit quantity of a **MIXTURE** the Mean Price it is given by



Example: In what ratio a shopkeeper mix two types of rice worth Rs.50 kg and Rs.70 kg, so that the average cost of the mixture is Rs.65 kg?

Solution



Indices

Definition

Indices are one of the forms of mathematical expression. Indices are used for expressing large numbers in terms of powers to a smaller base. $a = a \times a \times \ldots$ n times $= a^n$. Here 'a' is called the base and 'n' is called the index or exponent.

base $a^b = c$ value

Basic Formulas

Basic Formulae	Useful Results
$(a+b)^2 = a^2 + 2ab + b^2$	If $(a+b+c) = 0$, then $a^3 + b^3 + c^3 = 3abc$
$(a+b)^2 = a^2 + 2ab + b^2$	If $a^{1/3} + b^{1/3} + c^{1/3} = 0 \Rightarrow (a+b+c)^3 = 27abc$
$a^2 - b^2 = (a+b)(a-b)$	If $a^x = k$ then $a = k^{1/x}$
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	If $a^x = b^y$ then $a = b^{y/x}$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	If $ax = ay$ then $x = y$ $(a \neq 1)$
$(a+b)^3 = a^3 + 3ab(a+b) - b^3$	If $a^x = b^x$ then $a = b$ $(x \neq 0,\ a,\ b > 0)$
$(a-b)^3 = a^3 - 3ab(a-b) - b^3$	If $a^x b^y = a^m b^n \Rightarrow x = m$ & $y = n$ $(a \neq b)$
$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$	If $x = a^{1/3} - a^{-1/3}$, then $(x^3 + 3x) = (a - a^{-1})$
	If $x = a^{1/3} + a^{-1/3}$, then $(x^3 - 3x) = (a + a^{-1})$

Laws of Indices

Laws		
$a^m \times a^n$	=	a^{m+n}
$\frac{a^m}{a^n}$	=	a^{m-n}
$(a^m)^n$	=	$a^{m \times n}$
a^{-n}	=	$\frac{1}{a^n}$

Tip: $(a \times b)^n \Rightarrow a^m \times b^m$

$\left(\frac{a}{b}\right)^m \Rightarrow \frac{a^m}{b^m}$

Useful Facts

$\sqrt[n]{a}$	=	$(a)^{\frac{1}{n}}$
$\sqrt[n]{a^p}$	=	$(a^p)^{\frac{1}{n}} = a^{p \times \frac{1}{n}}$
a^0	=	1
a^1	=	a
$\left(\frac{a}{b}\right)^{m/n}$	=	$\left(\frac{b}{a}\right)^{-\frac{m}{n}}$
$\sqrt{a\sqrt{a\sqrt{a\ldots\ldots\infty}}}$	=	a
$\sqrt{a\sqrt{a\sqrt{a\ldots\ldots n\ times}}}$	=	$a^{\frac{2^n - 1}{2^n}}$

Clarification Note

Applying property between Multiple Base and trying to convert them into Single Base	Applying property between Split Single Base into Multiple Base
Property will apply only when there is [×, ÷] between multiple base. $2^3 \cdot 2^2 = 2^5$	Property will apply only when there is [+, −] between Powers of that Base. $2^{3+2} = 2^3 \times 2^2$
$2^3 + 2^2$ = No Property	$2^{3\times2}$ = No Property

$2^{-2} = \left(\frac{2}{3}\right)^2$ Such problem solved

Basic Tips

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$(a-b)^2 = a^2 + b^2 - 2ab$

$(a-b)(a+b) = a^2 - b^2$

$(-ve)^{odd} = -ve$

$(-ve)^{even} = +ve$

Misc. Notes

If $x = a^{\frac{1}{3}} - a^{-\frac{1}{3}}$ then $x^3 + 3x = a - \frac{1}{a}$

If $x = a^{\frac{1}{3}} + a^{-\frac{1}{3}}$ then $x^3 - 3x = a + \frac{1}{a}$

If $a^b = b^a$ only possible when a = b

$a \neq b$ $a = 2$ $b = 4$

'K' Concept

When Question says Something = Something = Something in that case we know they are equal but don't know exactly what they equal's to so we will assume it equals to "K"

Notice

So that we can have values of all the variable in the same form that is in the form of "K"

How to get 2 quantities when they are mixed in ratio?

Example: In 200kg of mixture if zinc and copper are in ratio of $\frac{3}{5}$ then calculate the amount of zinc and copper?

Sol:-

Example:

Other Useful Concepts

Note 1 : How to calculate any Power

$(a)^n = \sqrt[12]{a} - 1 \times \text{Power} + 1 \times$ (12 times) (12 times)

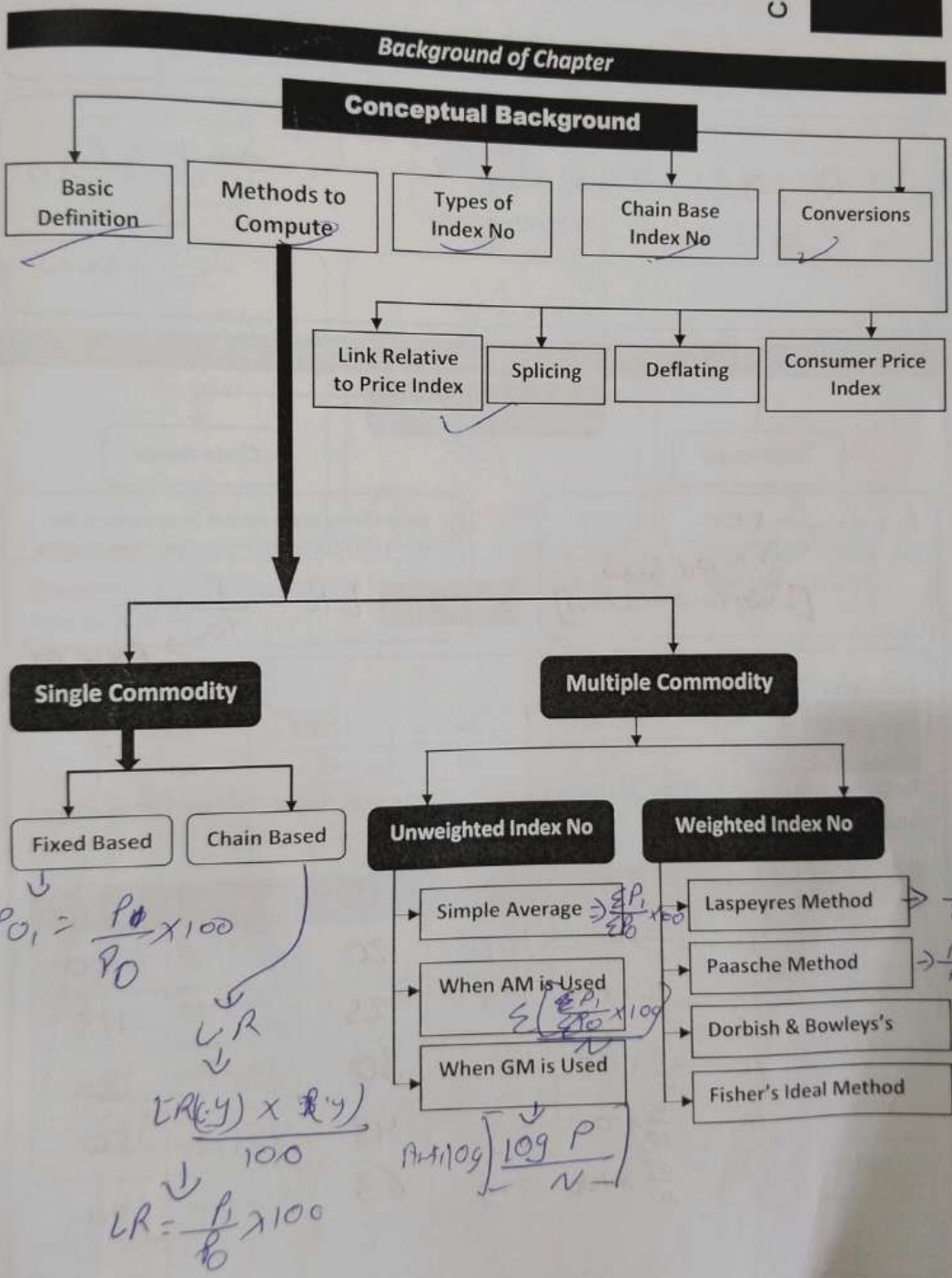
Note 2 : sum of all angle of $\Delta = 180$

Note 3 : Convert indices into log directly ; Reason in chapter log

$a^x = b$ → $x = \frac{\log a}{\log b}$

Index Number

Chapter 18



Definition

An index number is a ratio or an average of ratios expressed as a percentage. In an index number two or more time periods are involved, one of which is the base time period. The value at the base time period serves as the standard point of comparison.

Therefore, index number are supposed to be barometers which measures in the level of a phenomena.

Example → Petrol

20 P_0 → P_{2015} → Base year

100 P_1 → P_{2020} → Current year

~~100~~ $P_{01} \Rightarrow \frac{P_1}{P_0} \times 100$

$= \frac{100}{20} \times 100 = 500$.

Method to Compute Index number

Single Commodity

Fixed Based

$P_{01} = \frac{P_1}{P_0} \times 100$

P_0 → Fixed base [Given in question]

Chain Based

We will Study this in detail in Sometime but this formula will Require the Knowledge of **Link Relative**

Link Relative $L.R = \frac{P_1}{P_0} \times 100$

P_0 → Price of last year

Example: From the below table –

Year	2000	2001	2002	2003	2004
Price	20	25	30	45	63

Find the Index Numbers for all the years with 2000 as base, using the **Fixed base** and also find **Link Relative**

Year	**Fixed Base Method**			**Link Relative**		
2000	20	$\frac{20}{20} \times 100$	100	20	$\frac{20}{20} \times 100$	100
2001	25	$\frac{25}{20} \times 100$	125	25	$\frac{25}{20} \times 100$	125
2002	30	$\frac{30}{20} \times 100$	150	30	$\frac{30}{25} \times 100$	120
2003	45	$\frac{45}{20} \times 100$	225	45	$\frac{45}{30} \times 100$	150
2004	63	$\frac{63}{20} \times 100$	315	63	$\frac{63}{45} \times 100$	140

no gm Importance to qty

- **Multiple Commodity**
  - **Unweighted Index Number**
    - Simple Average
    - When AM is Used
    - When GM is Used
  - Weighted Index Number
    - We will Discuss it Next

use shortcut ↓ $\pi\left(\frac{P_1}{P_0}\right)^{\frac{1}{N}}$

Simple Average

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

When AM is Used

$$P_{01} \Rightarrow \frac{\Sigma\left[\frac{P_1}{P_0} \times 100\right]}{N}$$

$$\Downarrow \frac{\Sigma P}{N}$$

$$P = \frac{P_1}{P_0} \times 100$$

When GM is Used

$$P_{01} = \text{Antilog}\left[\frac{\Sigma \log\left(\frac{P_1}{P_0} \times 100\right)}{N}\right]$$

$$\Downarrow \text{Antilog}\left[\frac{\Sigma \log P}{N}\right]$$

Example: *From the following data construct Index Number for the year 2000 taking 2001 as Base* ***(a)*** *A.M* ***(b)*** *G.M*

Item	A	B	C	D	E
2001	6	2	4	10	8
2002	10	2	6	12	12

Solu:

Item	P_0 2001	P_1 2002	AM $\frac{P_1}{P_0} \times 100$ [P]	GM $\log P$
A	6	10	$\frac{10}{6} \times 100 \Rightarrow 166.66$	$\log 166.66 \Rightarrow 2.22$
B	2	2	$\frac{2}{2} \times 100 \Rightarrow 100$	$\log 100 \Rightarrow 2$
C	4	6	$\frac{6}{4} \times 100 = 150$	$\log 150 \Rightarrow 2.1769$
D	10	12	$\frac{12}{10} \times 100 = 120$	$\log 120 \Rightarrow 2.079$
E	8	12	$\frac{12}{8} \times 100 = 150$	$\log 150 \Rightarrow 2.1769$
	30	42	6866.66	10.6571

$$P_{01} = \frac{\Sigma P}{N} \Rightarrow \frac{6866.66}{5} \Rightarrow 137.33$$

$$P_{01} = \text{Antilog}\left[\frac{\Sigma \log P}{N}\right] \Rightarrow \text{Antilog}\left[\frac{10.6571}{5}\right]$$

$$P_{01} = \text{Antilog}[2.1312] \Rightarrow (10)^{2.1312}$$

$$\Rightarrow 135.058$$

$P_{01} \Rightarrow \frac{\Sigma P_1 w}{\Sigma P_0 w} \times 100$

Weighted Based

- Bridge index : Qty is weighted
- Qty index : Bridge is weighted

Laspeyres Method

Base year Q+y [q_0]

$P_{01} \Rightarrow \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$

Paasche Method

current year [q_1]

$P_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}$

Dorbish & Bowley's Method

A.M $\frac{L+P}{2}$

$\frac{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} + \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}}{2} \times 100$

$\frac{L+P}{2}$

Fisher's Ideal Method

G.M $\sqrt{L \times P}$

$\sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}} \times 100$

$\sqrt{L \times P}$

Marshall-Edgeworth Method

Avg of Q+y $\left(\frac{q_1 + q_0}{2}\right)$

$P_{01} = \frac{\Sigma P_1 (q_1 + q_0)}{\Sigma P_0 (q_1 + q_0)} \times 100$

Example: For the following data –

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	4	3	6	2
B	5	4	6	4
C	7	2	9	2
D	2	3	1	5

Compute the Laspeyere's Index, Fischer's Index, Dorbish Index Number

Laspeyres Method

$\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times 100$

$\Rightarrow \frac{63}{52} \times 100 \Rightarrow 121.15$

Paasche's Method

$\frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1} \times 100$

$= \frac{59}{52} \times 100 = 113.46$

Dorbish-Bowley Method

AM

$= \frac{L * P}{2} = \frac{121.15 + 113.46}{2} = 117.305$

Fisher Ideal Method

G.M

$\sqrt{L \times P}$

$\sqrt{121.15 * 113.46} \Rightarrow 117.24$

Marshall-Edgeworth Method

$P_{01} = \frac{\Sigma P_1 (Q_1 + Q_0)}{\Sigma P_0 (Q_1 + Q_0)} \times 100 = \frac{122}{104} \times 100$

117.30

Types of Index Number

- Price Index No → Already Discussed
- Quantity Index No → qty main muda hai Price will be taken as weight.
- Value Index No → value = P × Q

$V_{01} = P_{01} \times Q_{01}$

$q_{01} = \frac{q_1}{q_0} \times 100$

Quantity Index No

- **Laspeyres**: $q_{01} = \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times 100$
- **Paasch'e**: $= \frac{q_1 p_1}{q_0 p_1}$
- **Dorbish**: $\frac{L+P}{2}$
- **Fisher**: $\sqrt{L+P}$

Value Index No

$V_{01} = P_{01} \times q_{01} \Rightarrow V_{01} = \frac{V_1}{V_0} \times 100$

Example: The total value of retained imports into India in 1960 was Rs.71.5 million per month. The corresponding total for 1967 was Rs.87.6 million per month. The index of volume of retained imports in 1967 composed with 1960 (= 100) was 62.0. The price index for retained inputs for 1967 our 1960 as base?

Solution:

	value	Index
1960	71.5	100
1967	87.6	62

$V_{01} = \frac{V_1}{V_0} \times 100$

$= 122.51$

$V_{01} = P_{01} \times \Sigma q_1$

$122.51 = x \times 62$

$x = 1.97 \times 100$

$= 197.60$

Chain Index Number

Formula

$$C.B.I \Rightarrow \frac{L.R \text{ of } (C.y) \times CBI \text{ of } Py}{100}$$

Link Relative

$$LR = \frac{P_1}{P_0} \times 100$$

P_0 → Price of last year

Example:

Calculate Chain Index and Fixed Base Index with 2014 as base from following data:

Year	2014	2015	2016	2017	2018
Price of Rice	20	25	30	45	63

Solution:

Years	Price	$L.R = \frac{P_1}{P_0} \times 100$	$CBI = \frac{LR \text{ of } cy \times CBI \text{ of } pv}{100}$	$FBI = \frac{P_1}{P_0} \times 100$
2014	20	100	~~125~~ 100	$\frac{20}{20} \times 100 = 100$
2015	25	$\frac{25}{20} \times 100 = 125$	$\frac{125 \times 100}{100} = 125$	$\frac{25}{20} \times 100 = 125$
2016	30	$\frac{30}{25} \times 100 \Rightarrow 120$	$\frac{120 \times 125}{100} = 150$	$\frac{30}{20} \times 100 = 150$
2017	45	$\frac{45}{30} \times 100 \Rightarrow 150$	$\frac{150 \times 120}{100} = 225$	$\frac{45}{20} \times 100 = 225$
2018	63	$\frac{63}{45} \times 100 = 140$	$\frac{140 \times 150}{100} = 315$	$\frac{63}{20} \times 100 = 315$

Conversion

Fixed Based to Chain Based Index

$$CBI \Rightarrow \frac{FBI(C.y)}{FBI(Py)} \times 100$$

Chain Based to Fixed Based Index

$$FBI = \frac{C.B.I(Cy) \times FBI(Py)}{100}$$

Calculate the Chain Based Index

Years	2014	2015	2016	2017
Fixed Base Indices	80	100	120	180

Solution:

Years	Fixed Base Indices	$C.B.I = \frac{FBI(C.y)}{FBI(P.y)} \times 100$
2014	80	80
2015	100	125
2016	120	120
2017	180	150

Calculate the Fixed Base Index

Years	2014	2015	2016	2017
Chain Indices	80	125	120	150

Solution:

Years	Chain Indices	$FBI = \frac{CBI(C.y) \times FBI(Py)}{100}$
2014	80	100
2015	125	$\frac{125 \times 100}{100} = 125$
2016	120	$\frac{120 \times 125}{100} = 150$
2017	150	$\frac{150 \times 150}{100} = 225$

Link Relative to Price Relative

$$P.R \Rightarrow \frac{L.R \text{ of } (C.y) \times P.R \text{ of } (P.y)}{100}$$

Example: Convert the following into Price Relatives taking 2014 as base:

Years	2014	2015	2016	2017
Link Relative	80	100	120	150

Solution:

Years	Link Relative	$PR \Rightarrow \frac{L.R \text{ of } (C.y) \times P.R \text{ of } P.y}{100}$
2014	80	100
2015	125	$\frac{125 \times 100}{100} = 125$
2016	120	$\frac{120 \times 125}{100} = 150$
2017	150	$\frac{150 \times 150}{100} = 225$

Shifting of Bases

New g.N vs. g. New base

$$\frac{\text{old g.N using old base}}{\text{g.N of new base year}} \times 100$$

Example: Following are index number Of rice taking 2015 as base:

Years	2014	2015	2016	2017
Index Number	80	100	125	180

Shift the base 2016 and obtain new Index Number.

Years	Index Number (Base Year-2015)	new g.D $\Rightarrow \frac{\text{given g.N}}{\text{g.N of (New Base year)}}$
2014	80	$\frac{80}{125} \times 100 = 64$
2015	100	$\frac{100}{125} \times 100 = 80$
2016	125	$\frac{125}{125} \times 100 = 100$
2017	180	$\frac{180}{125} \times 100 = 144$

Splicing

By splicing of index number, we mean combining two or more series of overlapping index numbers to obtain a single index number on a common base.

Splicing **of numbers can be done only if the index numbers are constructed with the same items and have an overlapping year**

Index no of OLD Series Under Forward Splicing

given g.No of old series / overlapping year g.N ⇒ old series ka

Index no of NEW Series Under Backward Splicing

given g.No of ~~new series~~ / overlapping

given g.N of New Series × overlapping of g.N / 100 ⇒ old series

Example: Following are two index series A and B of price of rice taking 2012 as base and 2014 as base.

Years	2012	2013	2014	2015	2016	2017	2018
Index Series A	100	90	125				
Index Series B			100	120	150	180	200

Solution:

Years	Index Series A	Index Series B	Forward Splicing: old → New	Backward Splicing: New → old
2012	100		$\frac{100}{125} \times 100 = 80$	100
2013	90		$\frac{90}{125} \times 100 = 72$	90
2014	125	100	$\frac{125}{125} \times 100 = 100$	125
2015		120	$X = 120$	$\frac{120 \times 125}{100} = 150$
2016		150	$X = 150$	$\frac{150 \times 125}{100} = 1875$
2017		180	$X = 180$	$\frac{180 \times 125}{100} = 225$
2018		200	$X = 200$	$\frac{200 \times 125}{100}$ 250

Deflating

Deflated Value (Real Value)

$$\frac{\text{Current value}}{P.I} \times 100$$

Concept

B) 2015 — P: 100 — Q: 1000 ⇒ 100000

L) 2020 — P: 150 — Q: 1000 ⇒ 150000

Example: The following are the daily wages in rupees of a group of industrial workers and price indices: Calculate Real Wages.

Years	2013	2014	2015	2016	2017	2018
Daily Wages	80	108	125	147	216	230
Consumer Price Index	100	120	125	140	180	200

Solution

Years	Money Wages	Price Index	Real Wages = $\frac{C.V}{P.I} \times 100$
2013	80	100	$\frac{80}{100} \times 100 = 80$
2014	108	120	$\frac{108}{120} \times 100 = 90$
2015	125	125	$\frac{125}{125} \times 100 = 100$
2016	147	140	$\frac{147}{140} \times 100 = 105$
2017	216	180	$\frac{216}{180} \times 100 = 120$
2018	230	200	$\frac{230}{200} \times 100 \Rightarrow 115$

Consumer Price Index

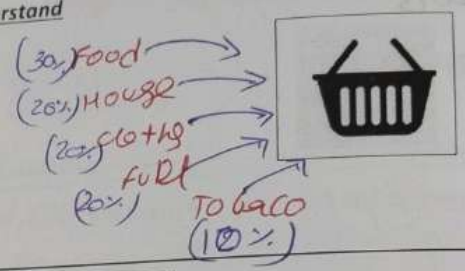
Consumer Price Index Measure change over time in general level of price of **goods and services** that household acquire for the purpose of consumption

E.g., Food, Housing, Clothing, Fuel, Tobacco

- C.P.I is always **weighted** index Number
- C.P.I is an **Inflation indicator** they are not same.
- Also Known **Cost of Living Index**
- To Calculate C.P.I **Price** are **Collected** from **Retail Traders.**

F + H + C + F + T = 100%

Let's Understand



Methods of Calculating C.P.I

Aggregate Expenditure Method

$$CPI = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$$

Family Budget Method

$$CPI = \frac{\Sigma PV}{\Sigma V}$$

$$P = \frac{P_1}{P_0} \times 100$$

V = weight

$V = P_0 q_0$ ×

Industrial Price Index

When **Weights** and **Index** of **Different industries** are given of Question is demanding **General Index** Number

$$C.P.I = \frac{\Sigma I w}{\Sigma w}$$

Clarification Note

ICAI Creates Variety of Questions in C.P.I which can create huge amount of confusion which method to apply in which situation so let's discuss this in detail

Real wages ↑ $\frac{C.V}{CPI} \times 100$ ⇒

Question Ask for Real Wages or Difference in real Wages

Just apply the formula of deflated value ⇒ and convert them into Real terms

Question Ask for Wages or Compensation or Additional DA to maintain Previous Standard of Living

Unitary method ↓

	C.P.I	wages
B	110	325
C	200	500

110 → 325

1 → $\frac{325}{110} \times 200$

= 590.90

Example: Consumer Price index number for the year 1957 was 313 with 1940 as the base year 96 the Average Monthly wages in 1957 of the workers into factory be Rs. 160/- their real wages is:
(a) Rs. 48.40 (b) Rs.51.12
(c) Rs. 40.30 (d) None of these
Solution:

1957 wages 160 C.P.I 313 Regal wages $\Rightarrow \frac{C.I}{C.P.I} \times 100 \Rightarrow \frac{160}{313} \times 100$ = 51.18

Example: During a certain period, the cost-of-living index number goes up from 110 to 200 and the salary of a worker is also raised from Rs. 325 to Rs. 500. The worker does not get really gain. Then the real wages decreased by:
Solution:

	CPI	wages	Real wages
Base	110	325	295.45
current	200	500	(250)
Diff in Real wages			(45.45)

Example: Consumer price index number goes up from 110 to 200 and the Salary of a worker is also raised from Rs. 325 to Rs.500. Therefore, in real terms, to maintain his previous standard of living he should get an additional amount of:
Solution:

	CPI	Salary
B	110	325
C	200	500

110 → 325
1 → $\frac{325}{110} \times 200$

Salary kitni honi chahiye ⇒ 590.90
Salary kitni hai ⇒ 500
Additional Salary 90.90

Example: When the cost of Tobacco was increased by 50%, a certain hardened smoker, who maintained his normal scale of consumption, said that the rise had increased his cost of living by 5%. Before the change in price, the percentage of his cost of living was due to buying Tobacco is
(a) 15% (b) 8% (c) 10% (d)None of these
Solution:

Tobacco Other

$x + y = 100$
$1.5x + y = 105$
$0.5x = 5$
$x = 10$

CPI

Example: The consumer price Index for April 1985 was 125. The food price index was 120 and other items index was 135. The percentage of food weight in the total weight of the index is

(a) 66.67 (b) 68.28 (c) 90.25 (d) None of these

Solution:

	x_i: q·N	r_i: weight	q w
food	120	x	$120x$
other	135	$(100-x)$	$13500-135x$
		100%	$13500-15x$

$$CPI = \frac{\Sigma I w}{\Sigma w}$$

$$125 = \frac{13500 - 15x}{100}$$

$$x = 66.66\%$$

Example: From the following data

Group	Weight	Index Number Base: 1952–53 = 100
Food	50	241
Clothing	2	221
Fuel and Light	3	204
Rent	16	256
Miscellaneous	29	179

The Cost-of-living index numbers is:

(a) 224.5 (b) 223.91

(c) 225.32 (d) None of these

Solution:

$$CPI = \frac{\Sigma I w}{\Sigma w} \times 100 \qquad \frac{22391}{100} = 223.91$$

Example: Given below are the data on prices of some consumer goods and the weights attached to the various items Compute price index number for the year 1985 (Base 1984 = 100)

Items	Unit	1984	1985	Weight
Wheat	Kg.	0.50	0.75	2
Milk	Litre	0.60	0.75	5
Egg	Dozen	2.00	2.40	4
Sugar	Kg.	1.80	2.10	8
Shoes	Pair	8.00	10.00	1

Then weighted average of price Relative Index is:

(a) 125.43 (b) 123.3 (c) 124.53 (d) 124.52

Solution:

$$P_{01} = \frac{\Sigma P w}{\Sigma w} \qquad \frac{24628}{200} = 123.14$$

Purchasing Power of Money

It is the **Reciprocal** of Price Index Number

Let's Understand Why?

	Price/kg of apple	You have Money	Kg you can buy	Purchasing Power
Base Year	100	Rs.500	5kg	100%
Current Year	125	Rs.500	4kg	80%

Example: If the consumer price index for 1994 is 800, then the purchasing power of a rupee is ___

(a) 0.125 paise (b) 12.5 paise (c) 8 paise (d) None of these

Types of Tests

S.no	Test	What it Means
1.	Unit Test	formula of index number should be free of unit
2.	Time Reversal Test	$P_{01} \times P_{10} = 1$
3.	Factor Reversal Test	$P_{01} \times Q_{01} = V_{01}$
4.	Circular Reversal Test	$P_{01} \times P_{12} \times P_{20} = 1$

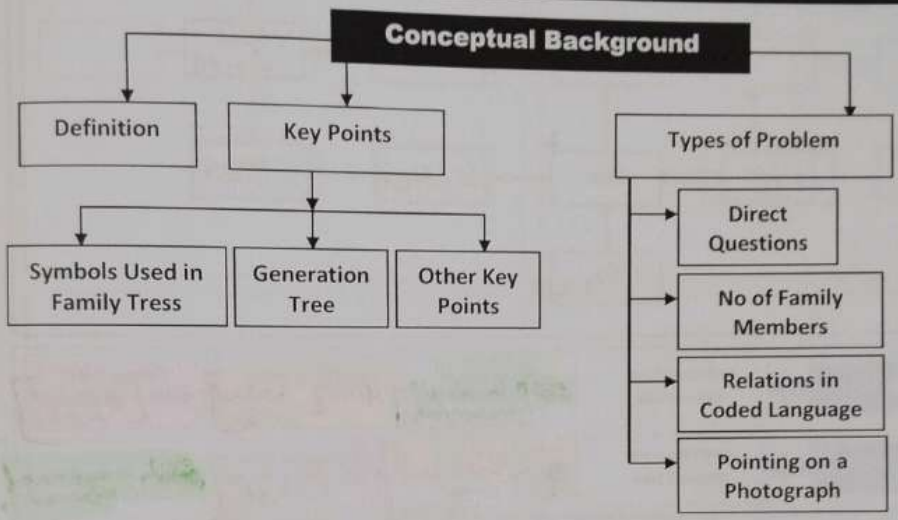
Extract of important Index Numbers which satisfy the test of Adequacy

Particulars	Unit Test	Time Reversal	Factor Reversal	Circular
Laspeyere's	✓	×	×	×
Paasche's	✓	×	×	×
Fisher's	✓	✓	✓	×
Marshall Edgeworth	✓	✓	×	×
Simple GM of Price Relatives	×	✓	×	✓

Blood Relations

Chapter 22

Background of Chapter



Meaning

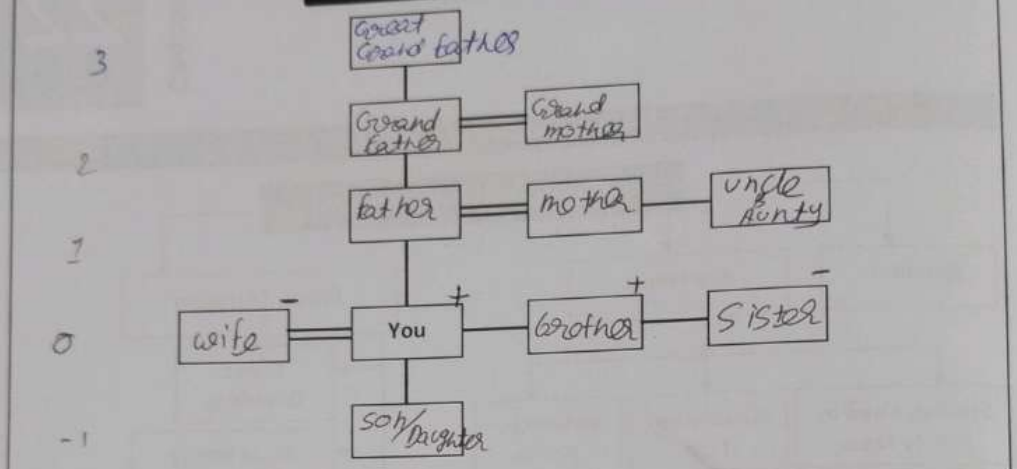
Blood Relation means relation among family members

Symbols to Make Family Tree

Member/Relation	Symbol
Male	+
Female	−
Couple	=
Other than Couple	———

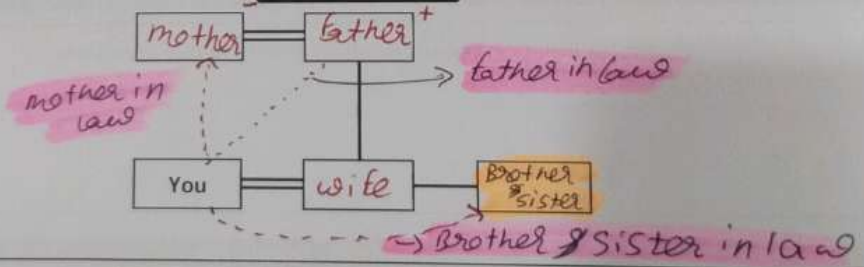
Generation Tree

Paternal/Maternal Side



Paternal Side	Relations From Fathers Side	Fathers Father (Paternal Grand father)	Fathers Brother (Paternal uncle)	Fathers Sister (Paternal Aunty)
Maternal Side	Relations From Mothers Side	Mothers Father (Maternal father)	Mothers Brother (Maternal uncle)	Mothers Sister (Maternal aunty)

In-Laws-Side



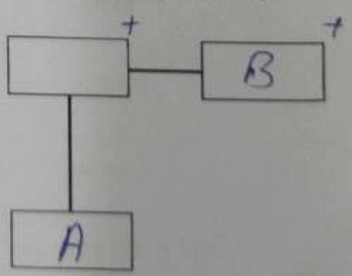
How to Check Relation in Tree Diagram?

Question always asks for 1st Person with Respect to 2nd Person

- How is A related to B?
- What is Relation of A to B?
- A is B's?

→ How is 'A' with Respect to 'B'

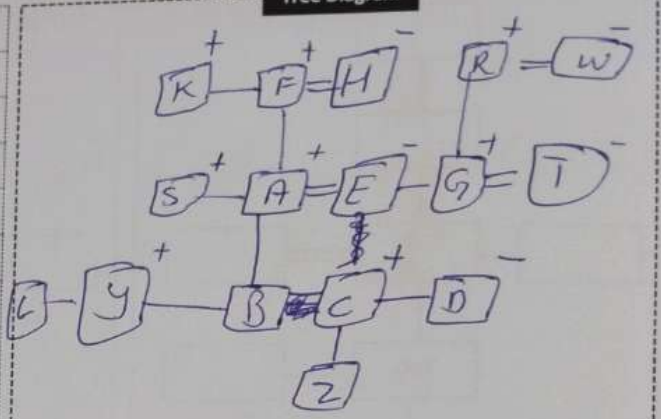
Draw the arrow from B (2nd Person) Towards A (1st Person)



Let's Understand Tree Diagram from Below Conditions

1. A is Father of B
2. C is the only Brother of B
3. D is the only Sister of C
4. C is Son of E
5. F is Father-in-Law of E
6. S is Son of F
7. H is Mother of S
8. G is Brother of E
9. K is Paternal Uncle of A
10. G is Son of R
11. W is Mother-in-Law of A
12. T is Daughter in Law of W
13. C is Father of Z
14. Y is Nephew of A (niece)
15. Y is Brother of L

Tree Diagram



How is **C** Related to **S**?

~~X~~ Uncle | Nephew ✓

How is **S** Related to **C**?

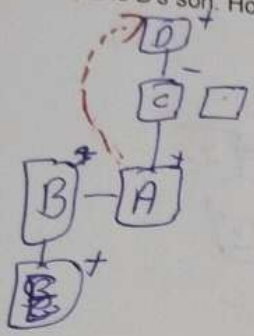
Uncle ✓ | Nephew

Answer the Following on the basis of above Family Tree Diagram

How is **E** Related to **D**	Mother
How is **H** Related to **S**	mother
How is **A** Related to **Z**	Grand father
How is **D** Related to **G**	~~Niece~~ niece
How is **D** Related to **R**	Grand daughter
How is **W** Related to **A**	~~Sister~~ mother in law
How is **K** Related to **S**	uncle
How is **H** Related to **B**	Grand mother
How is **G** Related to **B**	Maternal uncle
How is **Z** Related to **D**	Nephew/niece
How is **L** Related to **B**	Cousins
How is **L** Related to **S**	son / Daughter
How is **Y** Related to **F**	Grand son.
How is **R** Related to **S**	Brother's father in law

Examples on Tree Diagram Approach

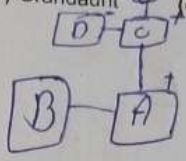
Example: A is B's brother. C is A's mother. D is C's father; E is B's son. How is D related to A?



Grand father

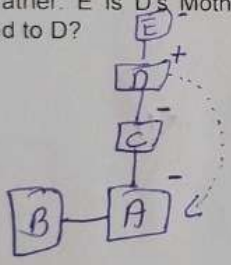
Example: As is B's brother. C is A's father. D is C's sister and E is D's mother. How is B related to E?

(a) Grand-daughter (b) Great grands daughter
(c) Grandaunt (d) Daughter



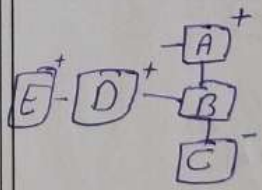
Grand Daughter

Example: A is B's Sister. C is B's Mother. D is C's Father. E is D's Mother. Then how is A related to D?



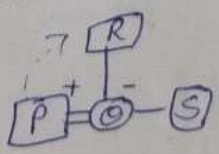
Grand Daughter

Example: A is the father of B. C is the daughter of B. D is the brother of B. E is the son of A. What is the relationship between C and E?



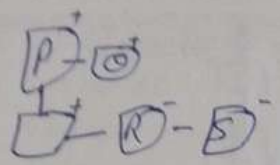
Uncle/ Niece

Example: If P is the husband of Q and R is the mother of S and Q. What is R to P?



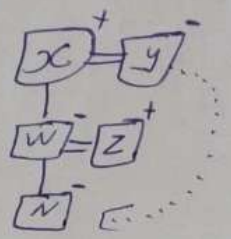
Mother in law

Example: P and Q are brothers. R and S are sister. P's son is S's brother. How is Q related to R?



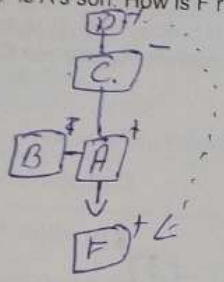
uncle

Example: X is the husband of Y. W is the daughter of X. Z is husband of W. N is the daughter of Z. What is the relationship of N to Y?



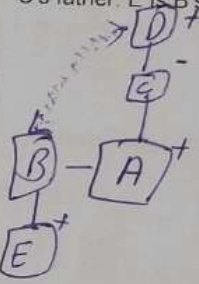
Granddaughter

Example: A is B's brother. C is A's mother. D is C's father. F is A's son. How is F related to D?



Great Grand Son

Example: A is B's brother. C is A's mother. D is C's father. E is B's son. How is B related to D?

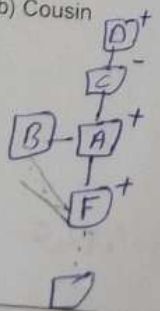


Grandson/Daughter

Example: A is B's brother. C is A's mother. D is C's father. F a is A's son. How is B related to F's child?

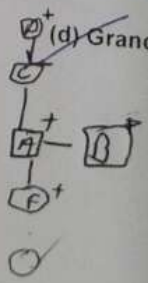
(a) Aunt

(b) Cousin

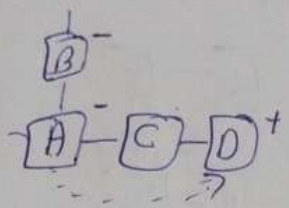


~~(c) Nephew~~

~~(d) Grandfather~~

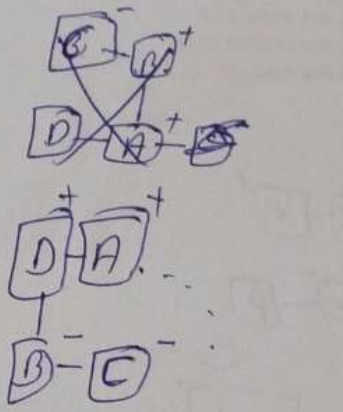


Example: A is B's daughter. B is C's mother. D is C's brother. How is D related to A?



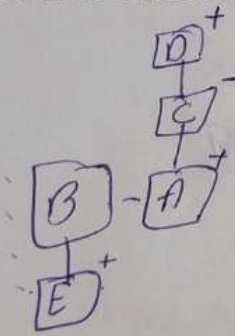
Brother

Example: A is D's brother. D is B's father. B and C are sisters. How is C related to A?



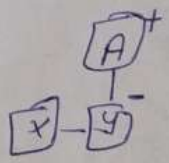
niece

Example: A is B's brother. C is A's mother; D is C's father. E is B's son. How is D related to E?



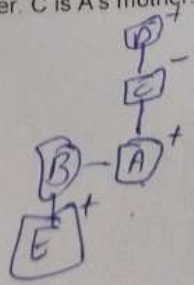
Great - Grandfather

Example: X and Y are the children of A. A is the father of X but Y is not his son. How is Y related to A?



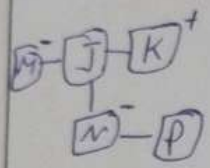
Daughter

Example: A is B's brother. C is A's mother. D is C's father. E is B's son. How is E related to A?



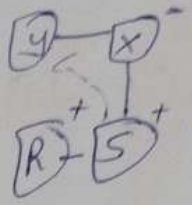
~~uncle~~ nephew

Example: Based on the statements given below, find out who is the uncle of P?
(i) K is the brother of J
(ii) M is the sister of K
(iii) P is the brother of N
(iv) N is the daughter of J

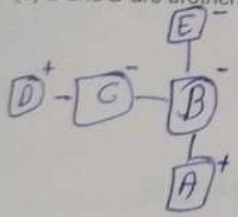


= K is uncle of P

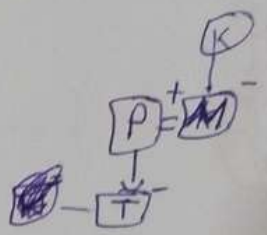
Example: R and S are brothers. X is the sister of Y and X is mother of R. What is Y to S?
(a) Uncle (b) Brother (c) Father (d) Mother



Example: A is the son of B while B and C are sisters to one another. E is the mother of C. If D is the son of E. which of the following statements is correct?
(a) D is the maternal uncle of A
(b) E is the brother of B
(c) D is the cousin of A
(d) B and D are brothers

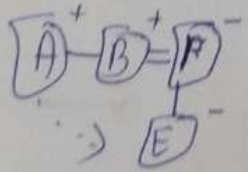


Example: P is the father of T. T is the daughter of M. M is the daughter of K. What is P to K?



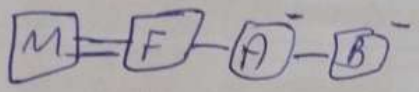
Son in law

Example: A and B are brothers. E is the daughter of F. F is the wife of B. What is the relation of E to A?



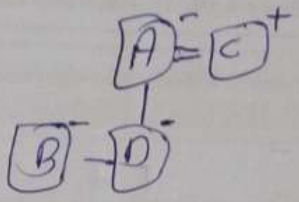
niece

Example: M and F are a married couple. A and B are sisters. A is the sister of F. Who is B to M?



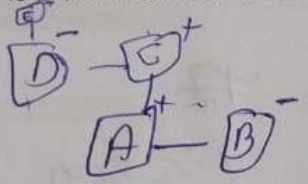
sister in law

Example: If A is the mother of D. B is not the son of C. C is the father of D, D is the sister of B, then how is A related to B?



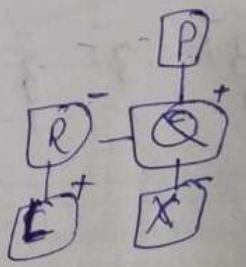
Mother

Example: A and B are brother and sister respectively. C is A's father. D is C's sister and E is D's mother. How is B related to E?



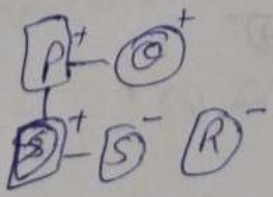
Grand ~~Son~~ daughter

Example: Q is the son of P. X is the daughter of Q. R is the aunty (Bua) of X and L is the son of R, then what is L to P?



Grand son

Example: P and Q are brothers. R and S are sisters. P's son is S's brother. How is Q related to R?



~~niece~~ uncle

Number of Family Members

In this Category of Question Sometimes Questions **Doesn't allocated Names** to family member's and there is no need of it Just **use Symbols'** and draw Diagram and count No of Members

Don't waste time **to Assume names** (It will not change the answer)

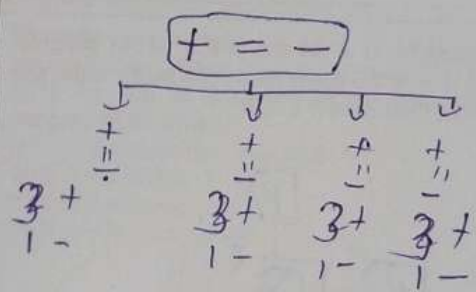
Example: In a group of friends, two men have wives, one is a bachelor, another wife is dead, two are divorced. They take four children with them to a picnic. How many have gone to picnic?

2 + = −
2 + = −
1 +
1 +
1 +

1 +

8 + 4 = 12

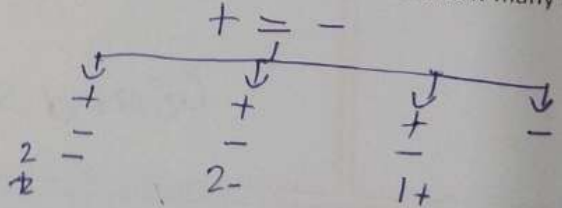
Example: A family has a man, his wife, there 4 sons and their wives. The family of every son also have 3 sons and one daughter each. Find out the total number of male members?



= ~~13~~

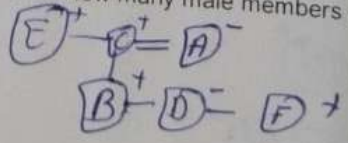
17 Person

Example: In a joint family there are father, mother, 3 married sons and one unmarried daughter, Of the sons, 2 have 2 daughter each, and one has a son. How many female members are there in a family?



⇒ 9 Persons

Example: Six members of a family namely A, B, C, D, E and F are travelling together. 'B' is the son of C but C is not the mother of B. A and C are married couple. E is the brother of C. D is the daughter of A. F is the brother of B. How many male members are there in the family?



= 4

Relations expressed in Coded Language

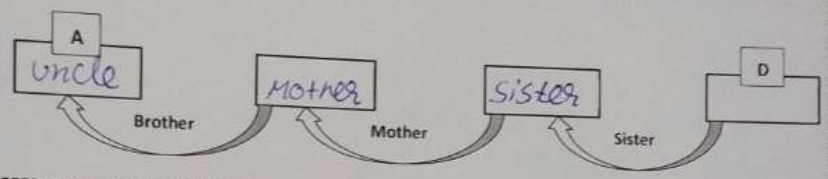
When Ever Relations are expressed on the based of **Coded Language** always refer to Start Solving from **Right** to **Left** (i.e., Decode it from end and move towards beginning)

1st Preference → NO DIAGRAM (Right to Left Approach)

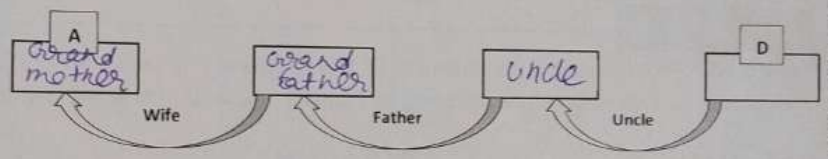
2nd Preference → TREE DIAGRAM APPROACH

Examples

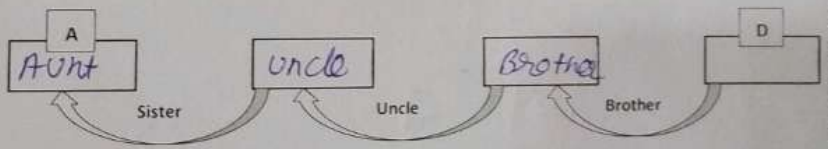
How A relates to D?



How A relates to D?



How A relates to D?



Example: Read the following information carefully and answer the questions given below

P/Q means P is father of Q

P+Q means P is the mother of Q

P-Q means P is brother of Q

P*Q means P is the sister of Q

If **A/B*C-D**, then A is D's?

father
Brother
father sister

(a) Mother **(b)** Aunt **(c)** Uncle **(d)** Father ✓

Example: Read the following information carefully and answer the questions given below
P/Q means P is father of Q
P+Q means P is the mother of Q
P-Q means P is brother of Q
P*Q means P is the sister of Q
If **A+B/C-D**, then A is D's?

(a) Grand-Daughter (b) Grandson (c) Grand-Mother (d) Mother

November-2018

Example: If P + Q means P is the mother of Q; P ÷ Q means P is the father of Q; P - Q means P is the sister of Q; then which of the following relationship shows that M is the daughter of R?

(a) R - M + N
(b) R + N ÷ M
(c) R - M ÷ N
(d) None of the above

June-2022

Example: P + Q means P is brother of Q, P - Q means P is the mother of Q, P x Q means P is the sister of Q. Which of the following means M is the maternal uncle of R?

(a) M + K + R
(b) M - R + K
(c) M + K - R
(d) M + K × R

Pointing to a Photograph / When there is Introducer?

Whenever there is an **"Introducer in a Question"** or Question Says **"Pointing on a Photograph"**

In that case Prefer to **Decode Question from END** (**98%** Questions can be tackled from this approach)

in case its not possible to solve it with above approach Draw **Tree Diagram Approach**.

He/She/His/Her used for Person **who is Being Introduced**

Important Key Point

My word Represent the person who is giving **Introduction**

Suresh introduces a man as "**He** is the son of the woman who is the mother of the husband of **my** mother".

How is Suresh related to the man?

Suresh

If Relations are **Splitted** with word Multiple "**of**"

Go from last relation of question- decode it and move forward

If Relations are **NOT Splitted** with word multiple "**of**"

is ⇒ Break the question in Two Parts draw 2 diagrams and merge them.

January-2024

Example: When Mr. P saw Mr. Q, he recalled, "He is the son of the father of my daughter's mother."

Who is Mr. Q to Mr. P?

(a) Brother (b) Cousin (c) Nephew (d) Brother – in – law

January-2024

Example: Introducing a boy, Akshat said, "His mother is the only daughter of my mother – in – law."

How is Akshat related to the boy?

(a) Uncle (b) Father (c) Brother (d) Husband

Example: Introducing a boy, a girl said, "He is Son of the daughter of the father of my uncle." How is the girl related to a Boy?

(a) Son-in-law (b) Sister (c) Nephew (d) Uncle

Example: Suresh introduces a man as "He is the son of the woman who is the mother of the husband of my mother". How is Suresh related to the man?

(a) Uncle (b) Son/Nephew (c) Cousin (d) Grandson

Example: Pointing a women Ravi said "She is the only daughter of my wife's grandfather's only child." How is the women related to Ravi?

(a) Wife (b) Sister-in-law (c) Sister (d) Data is Inadequate

Example: Rahul's Mother is the only daughter of Mounica's father. How is Mounica husband related to Rahul?

(a) Uncle (b) Father (c) Grandfather (d) Son

Example: Pointing a women man said "She is the sister-in-law of the only daughter of my mother-in-law husband". How is the women related to Man?

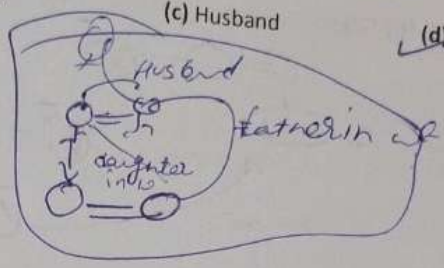
(a) Wife (b) Sister-in-law (c) Sister (d) Data is Inadequate

Ch. 22: Blood Relations

Aman Khedia

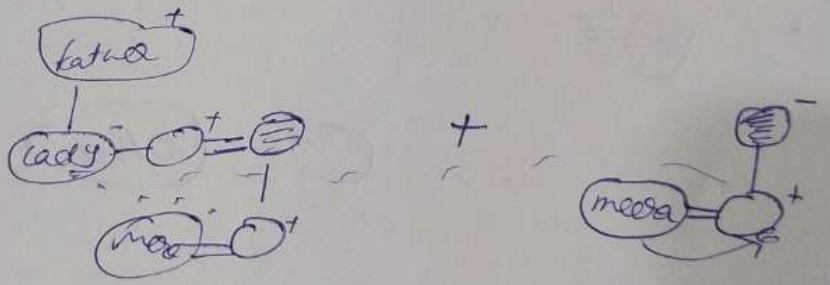
Example: Pointing to a man in a photograph a woman says "He is the father of my only daughter-in-law's father-in-law." How is man related to woman?

(a) Father **(b)** Brother **(c)** Husband **(d)** Father-in-law



Example: Pointing to a lady in a photograph Meera said. "Her father's only son's wife is my mother-in-law "How is Meera's husband related to that lady in the photo?

(a) Nephew **(b)** Uncle **(c)** Son **(d)** Father

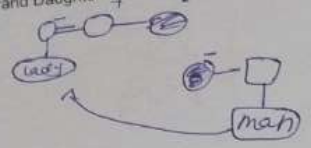


Example: Pointing to a lady in a photograph. Meera said. "Her father's only son's wife is my mother-in-law "How is Meera's husband related to that lady in the photo?

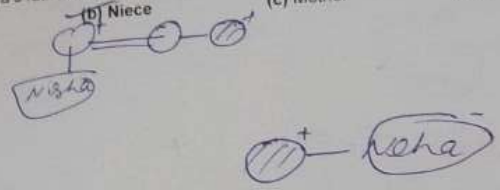
(a) Nephew **(b)** Uncle **(c)** Son **(d)** Father

Miscellaneous Questions

Example: A man said to a lady your mothers husband sister is my Aunt. How is lady related to the man?
(a) Daughter **(b)** Grand Daughter **(c)** Mother **(d) Sister**



Example: Nisha's father's brother-in-law is the brother of Neha. How is Nisha related to Neha?
(a) Sister **(b) Niece** **(c)** Mother **(d)** Mother-in-law



January-2024

Example: Showing the man playing the cricket, Ms. P said, "He is the brother of my uncle's daughter." Who is the man to Ms. P?
(a) Son **(b) Cousin** (c) Uncle (d) Brother – in -law

January-2024

Example: In a family there are six members A, B, C, D, E and F. A & B are married couple. A being the male member. D is only son of C who is brother of A. E is sister of D. B is daughter-in-law of F, whose husband is died. How C is related to B?
(a) Brother (b) Nephew
(c) Brother-in-law (d) Sister-in-law

January-2024

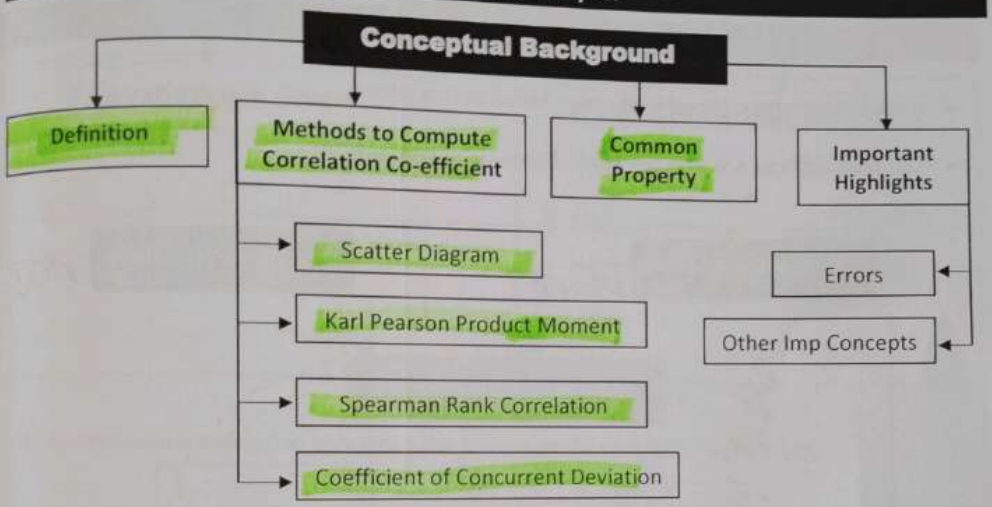
Example: Pointing to a lady, Suresh said "She is the mother of my son's wife's daughter." How is Suresh related to lady?
(a) Uncle (b) Cousin
(c) Daughter-in-law **(d) Father-in-law**

Example: P, Q, R, S, T, U are 6 members of a family in which there are two married couples. T, a teacher is married to a doctor who is mother of R and U. Q the lawyer is married to P. P has one son and one grandson. Of the two married ladies one is a housewife. There is also one student and one male engineer in the family. Which of the following is true about the grand-daughter of the family?
(a) She is a lawyer (b) She is an engineer
(c) She is a student (d) She is a doctor

Correlation Analysis

Chapter **13**

Background of Chapter



Definition

- It is the extent of **Relationship** Between two Variables

Eg. Relationship Between Demand & Supply	Eg. Relationship Between Demand, Supply & Gov Policies
Correlation ✓	Correlation = X

- It is Represented "γ"

Important Point to Remember

1. $-1 \leq \gamma \leq 1$
2. "r" is a Unit Free Measure

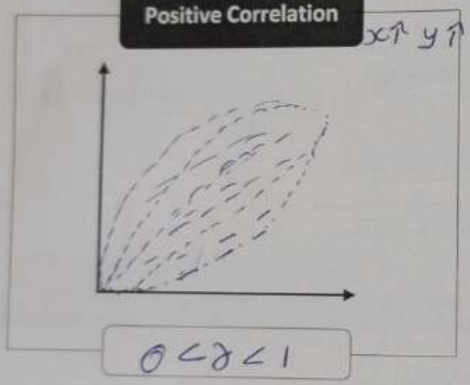
Methods to Compute Correlation Coefficient

- **A** — Scatter Diagram
- **B** — Karl Pearson
- **C** — Spearman Rank
- **D** — Coefficient of Concurrent Deviation

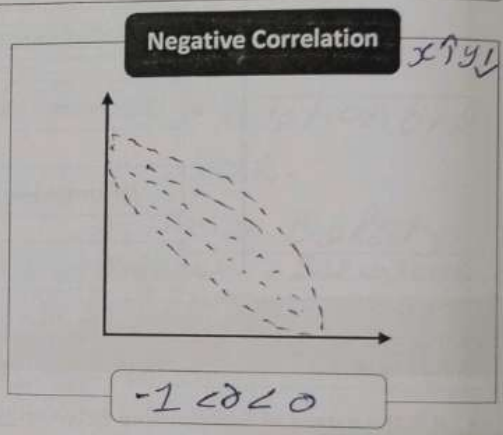
Scatter Diagram

- It is a Diagrammatic **{Graphical}** presentation of two data.
- This method Does not Give any **kind of formula** to Compute "r"

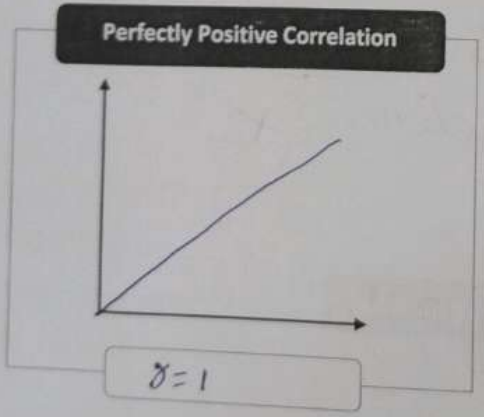
Positive Correlation



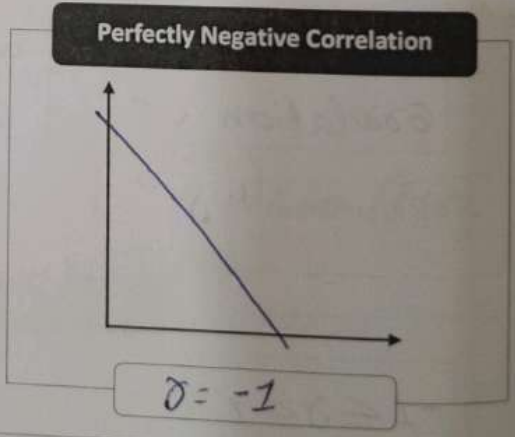
Negative Correlation



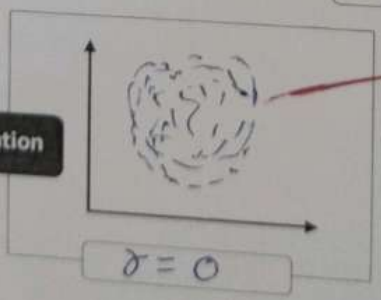
Perfectly Positive Correlation



Perfectly Negative Correlation



No Correlation



→ Curvilinear graph

~~r = 0~~

- non sense correlation
- suspicious correlation

Karl Pearson Product Moment Correlation Coefficient

- It is the **Best Method** to Compute Coefficient of Correlation.
- It Consider **All the Observation** (Based on Actual Observation)

Formula

If Covariance is Given

$$r = \frac{Cov(x,y)}{\sigma_x \times \sigma_y}$$

If all Observations are Given

$$r = \frac{N\Sigma xy - \Sigma x \Sigma y}{\sqrt{N\Sigma x^2 - (\Sigma x)^2}\sqrt{N\Sigma y^2 - (\Sigma y)^2}}$$

Important Highlights

1. Karl Pearson is applicable only when the relation between two variable is **linear**

Eg. $x = 0.2y$ ✓✓ Eg. $x^2 = y$ ✗✗

Covariance

LDR — mostly ↑ — less change ↑

$$Cov(x,y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{N}$$

$$Cov(x,y) = \frac{N\Sigma xy - \Sigma x \Sigma y}{N^2}$$

Example: Compute the co-efficient between x & y from the following data n = 10,

$$\sum xy = 220, \sum x^2 = 200, \sum y^2 = 262, \sum x = 40, \sum y = 50$$

$$\frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{N\Sigma x^2 - (\Sigma x)^2}\sqrt{N\Sigma y^2 - (\Sigma y)^2}} \Rightarrow \frac{10 \times 220 - 40 \times 50}{\sqrt{10 \times 200 - 1600}\sqrt{10 \times 262 - 2500}}$$

$$\Rightarrow \frac{200}{20 \times 10.95} = 0.91$$

Examples

Example: Find PRODUCT MOMENT CORRELATION COEFFICENT from the following information:

X:	2	3	5	5	6	8
Y:	9	8	8	6	5	3

x	y	xy	x^2	y^2
2	9	18	4	81
3	8	24	9	64
5	8	40	25	64
5	6	30	25	36
6	5	30	36	25
8	3	24	64	9
29	39	166	163	279

$$= \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{N\Sigma(x)^2 - (\Sigma x)^2} \times \sqrt{N\Sigma(y)^2 - (\Sigma y)^2}}$$

$$\frac{6 \times 166 - 29 \times 39}{\sqrt{6 \times 163 - 841} \times \sqrt{6 \times 279 - 1521}}$$

$$\frac{-135}{11.70 \times 12.36}$$

$$\frac{-135}{144.79} = \boxed{-0.932}$$

Example: A Computer operator while calculating the coefficient between two variables x and y for 25 pairs of observations obtained the following constants:

$$n = 25, \Sigma x = 125, \Sigma x^2 = 650, \Sigma y = 100, \Sigma y^2 = 460, \Sigma xy = 508,$$

It was later discovered at the time of checking that he had copied down two pairs as (6,14) and (8,6) while the correct pairs were (8,12) and (6,8). Obtain the correct value of the correlation coefficient.

Incorrect $\Sigma x = 125 - 6 - 8 + 8 + 6 = 125$

Incorrect $\Sigma y = 100 - 14 - 6 + 12 + 8 = 100$

Incorrect $\Sigma xy = 508 - 84 - 48 + 96 + 48 = 520$

Incorrect $\Sigma x^2 = 650 - 36 - 64 + 64 + 36 = 650$

Incorrect $\Sigma y^2 = 460 - 196 - 36 + 144 + 64 = 436$

$$\frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{N\Sigma(x^2) - (\Sigma x)^2} \cdot \sqrt{N\Sigma(y)^2 - (\Sigma y)^2}}$$

$$= r = 0.66$$

om the following

6	8
5	3

$\Sigma x \cdot \Sigma y$

$\sqrt{N\Sigma(y)^2 - (\Sigma y)^2}$

29×39

$\sqrt{6 \times 279 - 152}$

$\times 12 \cdot 36$

$77 = -0.932$

wo variables x and y

08,

wn two pairs as

the correct value of

436

Spearmen Rank Correlation

- This Method is used when we want to compare two **Qualitative Data**, not the **Quantitively Data.**
- It is **not** Based on **Actual Observation**; it is Based on **Ranks.**

Formula

$$\gamma_R = 1 - \frac{6\Sigma d_i^2}{n[n^2-1]}$$

here : $d_i = x_i - y_i$

n = no of observ.

Steps to Solve Questions

1. If the Question have **given** you Rank's, then No issues, but if Ranks are not given then **compute Ranks 1st**
2. Now find $\Sigma d_i^2 = (x_i - y_i)^2$
3. Now Apply the **Formula**

If the Ranks are Repeated

Example: Compute the coefficient of rank correlation between sales and advertisement (in thousands of rupees) from the following data:

Sales:	90	85	68	75	82	80	95	70
Advertisement:	7	6	2	3	4	5	8	1

Solution:

Sales	A·D	R_{sale} (x_i)	$R_{A·d}$ (y_i)	$(x_i - y_i)^2$ d_i^2
90	7	2	2	0
85	6	3	3	0
68	2	8	7	1
75	3	6	6	0
82	4	4	5	1
80	5	5	4	1
95	8	1	1	0
70	1	7	8	1
				4

$$\gamma_R = 1 - \frac{6 \Sigma d_i^2}{n[n^2-1]} \qquad \frac{1 - 6 \times 4}{8[64-1]} = 0.95$$

BASED ON SUMMARY DATA

Example: For a group of 8 students, the sum of squares of differences in ranks for Math's and Stats marks was found to be 50 what is the value of rank correlation coefficient?

$N = 8$

$\Sigma d_i^2 = 50$

$$\gamma_R = \frac{1 - 6\Sigma(d_i^2)}{n(n^2-1)}$$

$$= \frac{1 - 6(50)}{8(63)} \Rightarrow 0.4047$$

Example: For a number of towns, the coefficient of rank correlation between the people living below the poverty line and increase of population is 0.50. if the sum of squares of the differences in ranks awarded to these factors is 82.50, find the number of towns.

(a) 10 (b) 12 (c) 8 (d) None

$r = 0.50$

$\Sigma d_i^2 = 82.50$

$n = ?$

$$r = 1 - \frac{6\Sigma d_i^2}{n[n^2-1]}$$

$$0.50 = 1 - \frac{6 \times 82.50}{n[n^2-1]}$$

$$n[n^2-1] = 990$$

Try option

Example: While computing rank correlation coefficient between profits and investment for 10 years of a firm, the differences in rank for a year was taken as 7 instead of 5 by mistake and the value of rank correlation coefficient was computed as 0.80. what would be the correct value of rank correlation after rectifying the mistake?

$r = 0.80$

$$r = 1 - \frac{6\Sigma d_i^2}{n[n^2-1]}$$

$$0.80 = 1 - \frac{6\Sigma d_i^2}{10 \times (10^2-1)}$$

$$0.80 = 1 - \frac{6x}{990}$$

$$1 - 0.80 = -\frac{6x}{990}$$

$$+0.20 = +\frac{6x}{990}$$

$$\frac{0.20 \times 990}{6} = x$$

$x = 33$ → Σd_i^2 → 9 correct

$= 33 - (7)^2 + (5)^2$

$\Sigma d_i^2 = 33 - 49 + 25$

$\Sigma d_i^2 = 9$ → Correct

$$r_R = 1 - \frac{6\Sigma d_i^2}{n(n^2-1)}$$

$$= \frac{1 - 6 \times 9}{990}$$

$r_R = 0.945$

Coefficient of Concurrent Deviation

- This Method is Suitable when we are **not serious** about **Magnitude** of the **Observation.**

Formula

$$\gamma_{CCD} = \pm\sqrt{\pm\frac{(2C-M)}{M}}$$

C = No. of +ve Deviations
M = No. of Pairs of Product [n-1]

If (2C-M) → +ve → +,+ | If (2C-M) → -ve → -,-

Example: Find the coefficient of concurrent deviation from the following data:

Price:	25	28	30	23	35	38	39	42
Demand:	35	34	35	30	29	28	26	23

P	D	D_P	PP	
25	35	Nil	Nill	Nill
28 ↓+ve	34	+ve	−	−
30 ↓+ve	35	+ve	+	+
23 ↓+ve	30	-ve	−	+
35	29	+ve	−	+
38	28	+ve	−	−
39	26	+ve	−	−
42	23	+ve	−	−

C = 2
M = 7

$$\pm\sqrt{\pm\frac{(2C-M)}{M}}$$

$$-\sqrt{\frac{4-7}{7}} = \sqrt{+\frac{3}{7}}$$

$$\Rightarrow -\sqrt{+\frac{3}{7}}$$

$$\Rightarrow -\sqrt{\frac{3}{7}}$$

$$\simeq -0.659$$

Common Property

Coefficient of Correlation remains **Unchanged** due to change of **Origin** and **Scale**, but it is affected due to sign of "b" and "d"

Step-1: 1st of all write equation in **Standard form**

$u = a + bx$ $\quad$ $v = c + dy$

Step-2: Now **Consider** the sign of "b" and "d"

$$r_{uv} = r_{xy}\,[\text{sign of } x][\text{sign of } y]$$

Example: Given that the correlation coefficient between x and y is 0.8, write down the correlation coefficient between u and v were

(i) $2u + 3x + 4 = 0$ and $4v + 16y + 11 = 0$
(ii) $2u - 3x + 4 = 0$ and $4v + 16y + 11 = 0$
(iii) $2u - 3x + 4 = 0$ and $4v - 16y + 11 = 0$
(iv) $2u + 3x + 4 = 0$ and $4v - 16y + 11 = 0$

i) $r_{xy} = 0.80$, r_{uv}

$2u + 3x + y = 0$

$2u = -3x - y$

$u = \frac{-3}{2}x - \frac{y}{2}$

$4v + 16y + 11 = 0$

$4v = -16y - 11$

$v = \frac{-16}{4}y - \frac{11}{4}$

$r_{uv} = r_{xy}\,(\text{sign of } x)(\text{sign of } y)$

$= 0.80\,(-)(+)$

$r_{uv} = +0.80$

ii) $2u - 3x + 4 = 0$ | $4v + 16y + 11 = 0$

$u = \frac{3}{2}x - \frac{4}{2}$ | $v = \frac{-16}{4}y - \frac{11}{4}$

$r_{uv} = 0.80 \times (+) \times (+)$

$= -0.80$

iii) $r_{xy} = 0.80$

iv) $r_{xy} = -0.80$

Errors

Probable Error

Standard Error

Properties of Probable Error

Correlation of Population

Example: If r = -0.8 and N = 36, calculate (a) standard error, (b) probable error and (c) limits of population correlation. Also state whether the value of r is significant.

Other Important Concepts

Coefficient of Determination

- It is **Square** of Coefficient of Correlation
- Coefficient of determination [ ] is **preferred** as compare to Coefficient of Correlation [ ] Because it explains the **Process of Variation** in the dependent variable

Coefficient of Non-Determination

- It is
- It is Ratio of

Note

Both **Coefficient of Determination** [ ] & **Coefficient of Non-Determination** [ ] Can **Never be -ve** and Can **Never Exceed 1**

Coefficient of Alienation

- It is

Example: If r = 0.8, what is the proportion of variation in the dependent variable (a) which is explained by the independent variable (b) which bis not explained by the independent variable?

Summary Revision Question

Example Calculate **(i)** standard deviation of X and y **(ii)** Co-variance of X & Y **(iii)** Coefficient of correlation using Co-variance in each of the following alternative cases:

	Given Information
A	$N = 5, \sum X = 15, \sum Y = 150, \sum X^2 = 55, \sum Y^2 = 5500, \sum XY = 540$
B	$N = 5, \sum X = 15, \sum Y = 150, \sum (X-3)^2 = 10\ \sum (Y-30)^2 = 1000, \sum (X-3)(Y-30) = 90$
C	$N = 5, \sum x^2 = 10, \sum y^2 = 1000, \sum xy = 90, \sum x = 0, \sum y = 0$

A

B

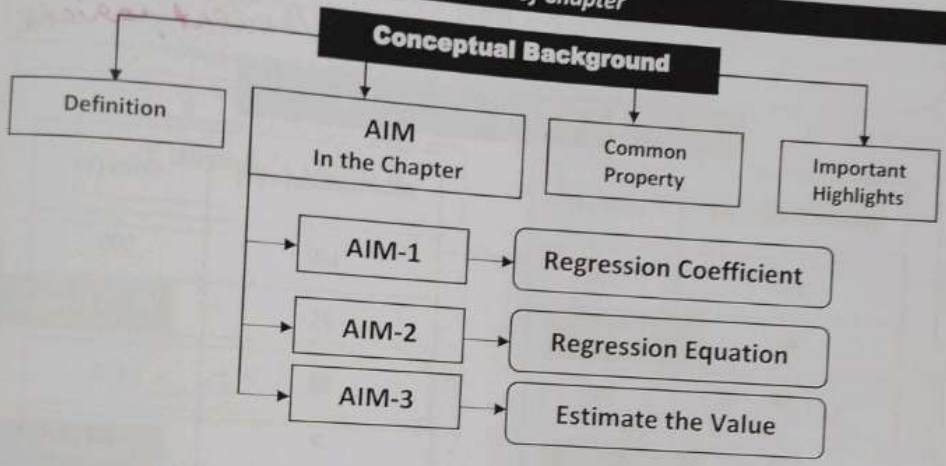
Ch. 13: Correlation Analysis
Aman Khedia

C

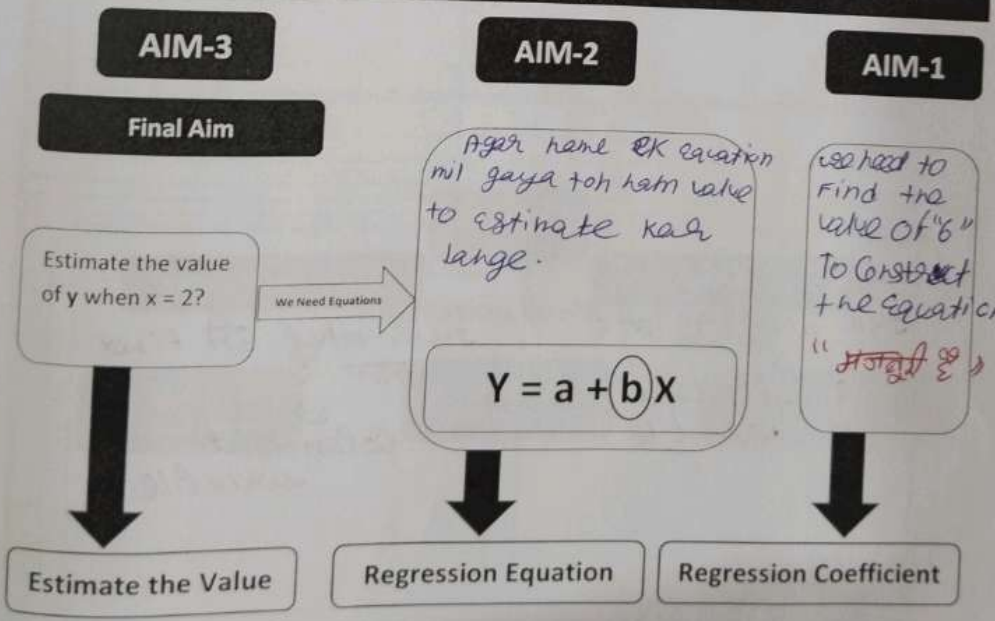
Regression Analysis

Chapter **14**

Background of Chapter



Flow of the Chapter



Definition

AIM-3 (Final Aim)

Regression means estimating the value of one variable {Dependent variable} for the corresponding value of another value {Independent variable}

Eg/ Find $x = ?$ when $y = 5$] $x \rightarrow$ Dependent variable, $y \Rightarrow$ Independent variable

Dependent & Independent Variable

Advertisement (X)	Sales (Y)
100	500
90	450
80	350
125	?

x = Independent variable
y = Dependent

Advertisement (X)	Sales (Y)
100	500
95	550
80	350
?	600

y = Independent
x = Dependent

CRUX

Independent Variable

Jiski value ~~hi~~ पता है
↓
Independent variable

Dependent Variable

Jiski value ~~hi~~ find करना है
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Dependent variable

lue of one variable
corresponding value
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ident variable
dependent variable

e

ement (X)	Sales (Y)
0	500
5	550
	350
	600

dependent
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Regression Equation

To Estimate the Value of ONE VARIABLE (Dependent Variable) we need to put value of ANOTHER VARIABLE (Independent Variable) in an **EQUATION**

So, we need to Have an Equation and that is **REGRESSION EQUATION**

AIM-2

Y depends Upon X

General Form

$y = a + bx$

Point Form

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

X depends Upon Y

General Form

$x = a + by$

Point Form

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

Here: **a** and **b** are Regression Parameters and **b**/b_{yx}/b_{xy} are Regression Coefficient

From above we can Clearly see that we cannot construct Equation Unless we have Value of Regression Coefficient (**b**/b_{yx}/b_{xy}) and Here Comes the Need of **New AIM** i.e.,

Regression Coefficient **AIM-1**

AIM-1 (Regression Coefficient)

AIM-2 (Regression Equation)

AIM-3 (Estimating the Value of Variable)

Example

Question: Find the value of "y" given that x = 25?

Solution: (Aim-3)

Nahi nikal Sakte → use need Equation → without value of "b" use cannot form Equation

$y = a + bx$

Regression Coefficient

Below Given all the formula Gives us same things, so apply formula as per **DATA** Given in Question

Formula

	b_{yx}	b_{xy}
If all OBS given	$\frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma(x^2) - (\Sigma x)^2}$	$\frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma(y)^2 - (\Sigma y)^2}$
If S.D & "r" is given	$r \cdot \frac{\sigma_y}{\sigma_x}$	$r \cdot \frac{\sigma_x}{\sigma_y}$
If Cov(xy) is given	$\frac{Cov(x,y)}{\sigma_x^2}$	$\frac{Cov(xy)}{\sigma_y^2}$

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1. Calculate regression co-efficient b_{yx} *Example*

Cases	Data Given	b_{yx}
Case (a)	$N = 5, \sum X = 15, \sum Y = 150,$ $\sum X^2 = 55, \sum Y^2 = 5500, \sum XY = 540$	$\frac{N\sum xy - \sum x \cdot \sum y}{N\sum x^2 - (\sum x)^2}$ $\Rightarrow \frac{5 \cdot 540 - 15 \cdot 150}{5 \cdot 55 - 225}$ $b_{yx} = 9$
Case (b)	$N = 5, \sum X = 15, \sum Y = 150,$ $\sum (X-3)^2 = 10$ $\sum (Y-30)^2 = 1000,$ $\sum (X-3)(Y-30) = 90$	$b_{yx} = \frac{Cov(x,y)}{\sigma x^2} = \frac{18}{2} = 9$ $Cov(x,y) = \frac{\sum (x-\bar{x})(y-\bar{y})}{n} = \frac{90}{5} = 18$ $\sigma x^2 = \sqrt{\frac{\sum (x-\bar{x})^2}{N}} = \sqrt{\frac{10}{5}} = (\sqrt{2})^2$ $\sigma y^2 = \sqrt{\frac{\sum (y-\bar{y})^2}{N}} = \sqrt{\frac{1000}{5}} = 200$
Case (c)	$N = 5, \sum (X-4) = 15,$ $\sum (Y-40) = -50,$ $\sum (X-4)^2 = 15,$ $\sum (Y-40)^2 = 15{,}000,$ $\sum (X-4)(Y-40) = 140$	σx^2 60

Case (d)	$N = 5, \bar{X} = 3, \bar{Y} = 30,$ $\sum x^2 = 10, \sum y^2 = 1000,$ $\sum xy = 90, \sum x = 0, \sum y = 0$	$b_{yx} = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma(x)^2 - (\Sigma x)^2}$ $= \frac{5 \times 90 - 0 \cdot 0}{5 \times 10 - 0}$ $= 9$ $b_{yx} = 9$
Case (e)	Covariance (X, Y) = 18, N = 5, $\Sigma X = 15, \Sigma Y = 150,$ variance of X = 2, $\Rightarrow (\sigma x)^2$ variance of Y = 200	$b_{yx} = \frac{Cov(x, y)}{\sigma x^2}$ $= \frac{18}{2} = 9$ $b_{yx} = 9$

Find the value of 'y' when $x = 2$.

Example: Find the the regression equation Y on X from the following data:

X:	2	4	5	5	8	10
Y:	6	7	9	10	12	12

x	y	xy	x^2	
2	6	12	4	
4	7	28	16	
5	9	45	25	
5	10	50	25	
8	12	96	64	
10	12	120	100	
34	56	351	234	

$\bar{x} = 5.66$

$\bar{y} = 9.33$

AIM-1 (1)

$$b_{yx} = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$

$$\frac{6 \times 351 - 34 \times 56}{6 \times 234 - (34)^2}$$

$$= \frac{202}{248}$$

$b_{yx} = 0.8145$ — Aim 1

AIM-2 y on x (2)

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$(y - 9.33) = 0.81(x - 5.66)$$

$$y - 9.33 = 0.81x - 4.5846$$

$$y = 0.81x - 4.5846 + 9.33$$

~~$y = 4.7454$~~

$y = 4.7456 + 0.81x$

Aim 2

AIM-3 (3)

$x = 1$ given

$y = 4.7456 + 0.81x$

$y = 6.3656$

Aim 3

Example: The following data relate to the mean and SD of the prices of two shares in a stock Exchange:

Share		Mean (in Rs)	SD (in Rs)
x	Company A	44	5.60
y	Company B	58	6.30

Coefficient of correlation between the share prices = 0.48. Find the most likely price of share A corresponding to a price of Rs 60 of share B and also the most likely price of share B for price of Rs 50 of share A.

AIM-1

$y = 60$ $x = ?$

x on y

$r = 0.48$ $\bar{x} = 44$ $\bar{y} = 58$

$\sigma_x = 5.60$

$\sigma_y = 6.30$

$$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} = 0.48 \times \frac{5.60}{6.30}$$

$$b_{xy} = 0.42$$

AIM-2

x on y

$$(x - \bar{x}) = b_{xy}(y - \bar{y}) \Rightarrow (x - 44) = 0.42(y - 58)$$

$$x = 0.42y - 24.36 + 44$$

$$x = 0.42y + 19.64$$

AIM-3

$y = 60$

$$x = 0.42y + 19.64$$

$$= 44.84$$

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Example: Cov (x, y) =16 — γ

Variance of x = 25, Variance of y = 16 and $\bar{X} = 20, \bar{Y} = 30$

Estimate y if x = 30

y on x

Cov(x, y) = 16

$\sigma_x = 25$ $\sigma_y = 16$

$\bar{x} = 20$, $\bar{y} = 30$

$x = 30$

AIM-1

$b_{yx} = \gamma \cdot \frac{\sigma_y}{\sigma_x}$

$= \frac{16 \times 16}{25}$

$\gamma = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y}$

$= \frac{16}{5 \times 4}$

$= 0.8$

$G_{yx} = \frac{Cov(x,y)}{\sigma_x^2} = \frac{16}{25}$

$= 0.64$

AIM-2

$(y - \bar{y}) = b_{xy}(x - \bar{x})$

$(y - 30) = 0.64(x - 20)$

$y - 30 = 0.64x - 12.8$

$y = 0.64x + 17.2$

AIM-3

$y = 0.64x + 17.2 \Rightarrow$ $y = 36.4$

Common Property

Co-efficient of Regression remains **Unchanged** due to **Change of Origin** but **Changes** due to **Change of Scale**

Shortcut

$$b_{uv} = b_{xy} \frac{M_x}{M_y}$$

$$b_{vu} = b_{yx} \frac{M_y}{M_x}$$

Note

Make Sure to Convert the Equations into ***Standard Form 1st***

$u = a + bx$

$v = c + dy$

Example

If the relationship between two variable x and u is **u +3x =10** and between y and v is **2y +5v =25** and the regression Coefficient of **y and x** is known as **0.80** then regression coefficient of v on u?

$b_{yx} = 0.80$

$u = 10 - 3x$

$5v = 25 - 2y$

$v = \frac{25}{5} - \frac{2}{5}y$

$b_{vu} = b_{yx} \frac{M_y}{M_x} \Rightarrow 0.80 \times \frac{-\frac{2}{5}}{-3}$

$= 0.80 \times \frac{2}{5} \times \frac{1}{3} = 0.1$

two shares in a

SD (in Rs)
5.60
6.30

f share B and also

$\bar{x} = 44$

$\bar{y} = 58$

60

30

2

.42x(y - 58)

Relationship Between Correlation & Regression

1. The correlation coefficient is the geometric mean of regression coefficient $r = \pm\sqrt{b_{yx} \times b_{xy}}$
2. $b_{yx} = r\frac{\sigma_y}{\sigma_x}$ and $b_{xy} = r\frac{\sigma_x}{\sigma_y}$
3. If $r = 0$, then the regression coefficient is zero
4. If $r = \pm 1$. Then lines are identical and if $r = 0$ then lines are perpendicular to each other

Example

Cases	Given Data	Computation of r
Case (a)	$b_{xy} = +.09$, $b_{yx} = +9$	$r = \pm\sqrt{.09 \times 9} = 0.9$
Case (b)	$b_{xy} = -.06$, $b_{yx} = -6$	$r = \pm\sqrt{-0.06 \times -0.6} = -0.18$
Case (c)	$b_{xy} = 0.6$, $\sigma_x = 3$, $\sigma_y = 4$	$b_{xy} = r\frac{\sigma_x}{\sigma_y} \Rightarrow 0.6 = r \times \frac{3}{4}$ $= 0.8$
Case (d)	$b_{yx} = 1.067$, $\sigma_x = 3$, $\sigma_y = 4$	$b_{yx} = r \times \frac{\sigma_y}{\sigma_x} = 1.067 = r \times \frac{4}{3}$ $= 0.8$
Case (e)	$b_{xy} = 0.6$, variance of X is $9/16^{th}$ of the variance of Y	$(\sigma x)^2 = \frac{9}{12} \times \sigma y^2$; $\sigma x = \frac{3}{4} \times \sigma y$; $\frac{3}{4} = 0.75$ $b_{xy} = r.\frac{\sigma x}{\sigma y}$; $0.6 = r.\frac{3}{4} \times \frac{\sigma y}{\sigma y}$; $= 0.8$
Case (f)	$b_{xy} = 0.4$, variance of Y is 4 times the variance of X	$(\sigma y)^2 = (2)^2 \times \sigma x^2$; $\sigma y = 2 \times \sigma x$ $b_{xy} = r.\frac{\sigma x}{\sigma y}$; $0.4 = r.\frac{\sigma x}{2 \times \sigma x}$; $0.4 \times 2 = r$; $r = 0.8$

Case (g)	**Regression equation of Y on X:** $45X - 5Y + 15 = 0$ **Regression equation of X on Y:** $9Y - 100X + 30 = 0$	y on x: $45x - 5y + 15 = 0$, $y = \frac{15}{5} + \frac{45}{5}x$, $b_{yx} = 9$ x on y: $9y - 100x + 30 = 0$, $x = \frac{30}{100} + \frac{9}{100}y$, $b_{xy} = 0.09$

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$\sqrt{9 \times 0.09} = \sqrt{0.81} \Rightarrow 0.9$$

$$r = 0.9$$

Important Highlights

1. If one of the regression coefficients is **greater than unity**, then the other is **less than unity.**
2. Arithmetic mean of the regression coefficient is **greater** than the correlation coefficient
3. Both the regression will have the **same sign** i.e., either both are positive or both are negative.
4. Sign of correlation coefficient (r) is same as that of regression coefficient i.e., $r > 0$ if $b_{yx} > 0$ *and* $b_{xy} > 0$ and $r < 0$ if $b_{yx} < 0$ *and* $b_{xy} < 0$
5. Two Regression lines Intersect each other at Arithmetic Mean $(\bar{X}, \bar{Y})$
6. Correct Regression Coefficient

$$Correct_{b_{xy}} = \frac{1}{Incorrect_{b_{yx}}}$$

$$Correct_{b_{yx}} = \frac{1}{Incorrect_{b_{xy}}}$$

How to identify the Regression Lines?

Step-1: 1st of all assume any one line as X on Y and another as Y on X (Hit & Trail) and convert them into Standard form

Step-2: Pick the Coefficient of X and Y Which will be b_{xy} & b_{yx}

Step-3: Now apply the formula $r = \sqrt{b_{yx} \times b_{xy}}$ if r Comes in Between **-1 to 1** then our assumption in Right else Reverse assumption would be correct

Example

For the variables x and y, the regression equation is given as $7x - 3y - 18 = 0$ and $4x - y - 11 = 0$

because to regression line are intersect at point it is A.M

iv. Find the arithmetic means of x and y.

$7x - 3y = 18$ → $28x - 12y = 72$

$4x - y = 11$ → $-28x + 7y = -77$

$-5y = 5$

$y = -1$

Putting (1)

$7x - 3 = 18$

$7x = 3$

$\bar{x} = 3, \bar{y} = -1$

ii. Identify the regression equation of y on x.

$7x - 3y - 18 = 0$ → y on x

$y = \frac{18}{-3} + \frac{7x}{+3}$ ⇒ $b_{yx} = \frac{7}{3}$

$4x - y - 11 = 0$ → x on y

$x = \frac{11}{4} + \frac{y}{4}$

$b_{xy} = \frac{1}{4}$ 0.25

$r = \sqrt{b_{yx} \times b_{xy}}$ ⇒

$\sqrt{2.33 \times 0.25}$

0.763

i. Compute the correlation coefficient between x and y.

$r_{xy} = 0.76$

iii. Given the variance of x is 9, find the SD of y.

$\sigma_x^2 = 9$

$\sigma_x = 3$

$b_{yx} = \frac{7}{3}$ $b = \frac{1}{4}$ $r = 0.763$

$b_{yx} = r = \frac{\sigma_y}{\sigma_y} = 2.33 = 0.76 \times \frac{\sigma_y}{3}$

$\sigma_y = 9.21$

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Example: A student of statistics treated the regression equation of X on Y as of Y on X and of Y on X as of X on Y and calculated $b_{xy} = -2/3$ and $b_{yx} = -6$

Required: Calculate **(a)** wrong correlation coefficient, **(b)** Correct regression coefficients, **(c)** correct correlation coefficient and **(d)** regression equations, if $\bar{X} = 4$ *and* $\bar{Y} = 7$.

Solution:

(a) *Wrong correlation coefficient:*

$$r = -\sqrt{\frac{-2}{3}(-6)} = -\sqrt{4} = -2$$

(b) *Correct regression coefficients*

$b_{yx} = -1.50$ $\quad b_{xy} = -0.166$

$$r = -\sqrt{-1.50 \times -0.166}$$

$$= \sqrt{.249} \quad = 0.498 \quad = -0.5$$

(c) *Correct correlation coefficient*

$$= -0.5$$

(d) *Regression equations of X on Y*

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$(x - 4) = -0.166(y - 7)$$

$$x = \frac{-0.166}{1/4}\left(\frac{y-7}{4}\right)$$

$$(x - 4) = \frac{-1}{6}(y - 7)$$

$$6x = -y + 7 + 24$$

$$x = \frac{31}{6} - \frac{y}{6}$$

(d) *Regression equations of Y on X*

Summary Notes

Important Note

Regression Equation

$Y = a + bX$

- y - intercept
- Regression Coefficient → slope

July-2021

Que. If the slope of regression line is calculated to be 5.5 and the intercept 15, then find the value of y when x is 6 ?

Solu. $b = 5.5$, $a = 15$

$y = a + bx$

$= 15 + 5.5 \times 6$

$x = 6$

$y = ?$

July-2021

Que. If $Y = 9X$ and $X = 0.01Y$ then r is equals to?

Solu. $y = 9x$ $\quad x = 0.01y$

$b_{yx} = 9$ $\quad b_{xy} = 0.01$

$r = +\sqrt{9 \times 0.01}$

$= 0.3$

What You Should Think If You See Following Inputs in Question?

1. $\Sigma(x - \bar{x})(y - \bar{y})$ → Cov(x, y)
2. $\Sigma(x - \bar{x})^2$ or $\Sigma(y - \bar{y})^2$ → σx or $\sigma y / \sigma x^2$ or σy^2
3. Regression Lines are Given $(2X + 3Y + 5 = 0)$ → Regression equation ka hona mtlb Regression Coefficient ka hona → Standard form

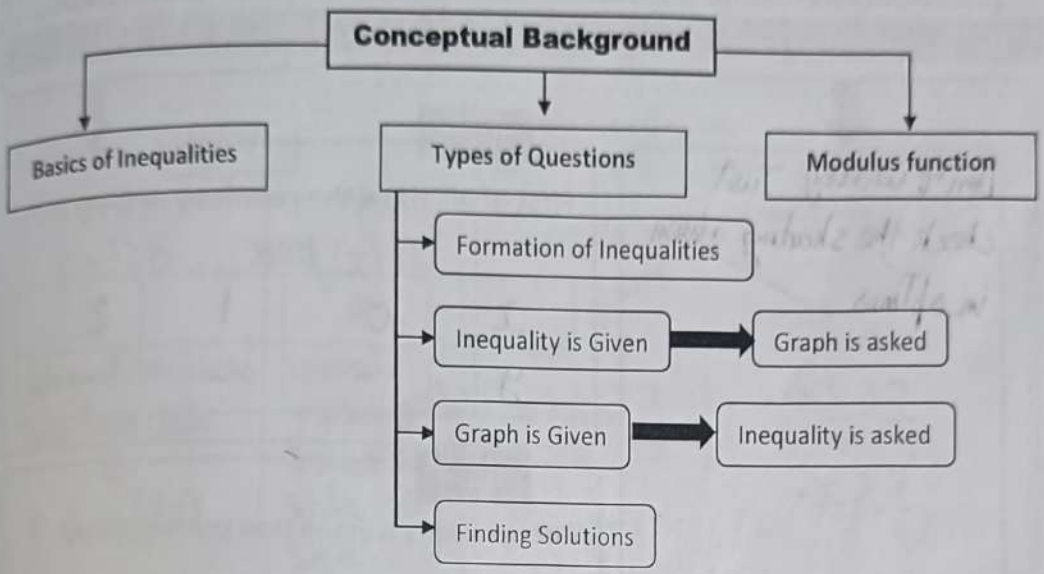
Regression Coefficient ka hona mtlb Correlation Coefficient ka hona

$r = \sqrt{b_{xy} \times b_{yx}}$

Inequalities

Chapter 4

Background of Chapter

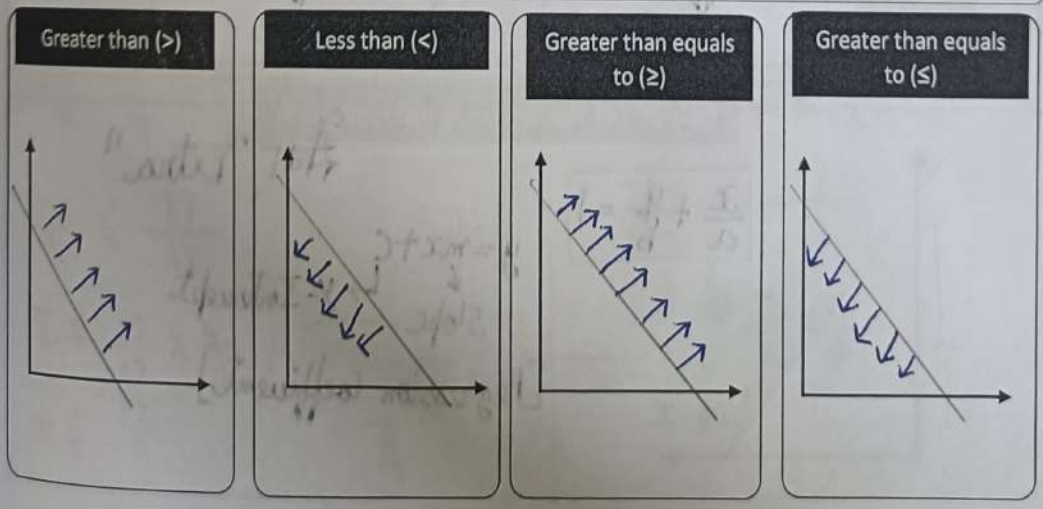


Basics of Inequalities

Any Linear equations formed using sign of any inequalities is termed as Linear Inequalities.

E.g., $2x + 3y > 5$, E.g., $2x + 3y < 5$, E.g., $2x + 3y \geq 5$

Signs of Inequalities: $>, <, \geq, \leq$



Inequalities is Given and Graph is Asked

Before starting the question look at the Options, whether all the options have Same Diagram (Geometry) or Different Diagram (Geometry)

Options Have Same Diagram

Don't worry Just check the shading given in options

Options Have Different Diagram

Step-1

First of all, draw the geometry of Diagram

How

x	0	1	2
y	-	-	-

If after putting x=0 y also becomes 0 then line passes from origin.

Step-2

After geometry is drawn Now look for Shading

Graph is Given and Inequalities is Asked

In this category question consist of graphs and in options we are given with inequalities.

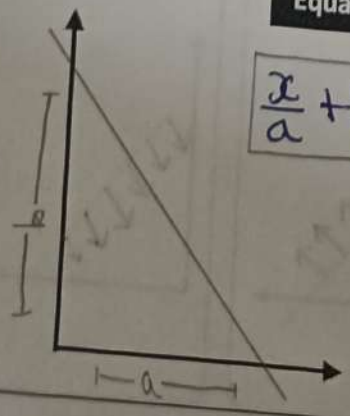
What we need to do is, we need to **figure Out the Sign** of **Inequalities** for given Equations

NOTE

Sloping Upward ⇓ ≥

Sloping Downward ⇓ ≤

Equation of Straight Line - Intercept form



$$\frac{x}{a} + \frac{y}{b} = 1$$

Hat "Extra"

$y = mx + C$

Slope ↳ y-Intercept

[Regression Coefficient]

Formation of Inequalities

Under this Category we are given with certain **Statements** [Conditions], what we need to do is, we need to convert them **into** equations with **Inequalities.**

Example: An employer recruits experienced (x) and fresh workmen (y) for his firm under the condition that he cannot employ more than 9 people. X and y can be related by the inequality

Modulus Functions

$|f(x)|$

$f(x) \geq 0$ ⇓ Convert modulus into Bracket ⇓ As it is $f(x)$

$f(x) < 0$ ⇓ Convert modulus into Bracket with -ve sign ⇓ $-f(x)$

Example: If $\left|x+\frac{1}{4}\right| > \frac{7}{4}$, then:

(a) $x < \frac{-3}{2}$ or $x > 2$ (b) $x < -2$ or $x > \frac{3}{2}$

(c) $-2 < x < \frac{3}{2}$ (d) none of these

+ve

$x + \frac{1}{4} > \frac{7}{4}$

$x > \frac{6}{4}$

$x > 3/2$

-ve

$-x - \frac{1}{4} > \frac{7}{4}$

$-x > \frac{8}{4}$

$-x > 2$

$x < -2$

Example: If $\left|\frac{3x-4}{4}\right| \leq \frac{5}{12}$, then solution set is:

+ve

$\frac{3x}{4} - \frac{4}{4} \leq \frac{5}{12}$

$\frac{3x}{4} \leq \frac{17}{12}$

$x \leq \frac{17}{12} \times \frac{4}{3}$

$x \leq \frac{17}{9}$

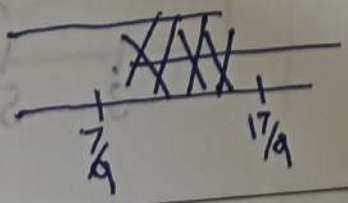
-ve

$-\frac{3x}{4} + \frac{4}{4} \leq \frac{5}{12}$

$-\frac{3x}{4} \leq \frac{-7}{12}$

$x \geq \frac{7}{12} \times \frac{4}{3}$

$x \geq \frac{7}{9}$



Ans $\left[\frac{7}{9}, \frac{17}{9}\right]$ OR $\frac{7}{9} \leq x \leq \frac{17}{9}$

Finding Solutions

Method to Draw Graph

Step-1: 1st of all convert the equations in intercept form. $\frac{x}{a} + \frac{y}{b} = 1$

Step-2: Now **Plot the points**, draw the diagram and then get the corner points of shaded region.

Exam Approach – Shortcut

Go from **Option to Question** and just check 2 things

- **All Points** of Options Should **Justify the inequality**
- Each Point should give exact **LHS = RHS** with 2 equations

Example: On solving the inequalities $6x + y \geq 18$, $x + 4y \geq 12$, $2x + y \geq 10$, we get the following

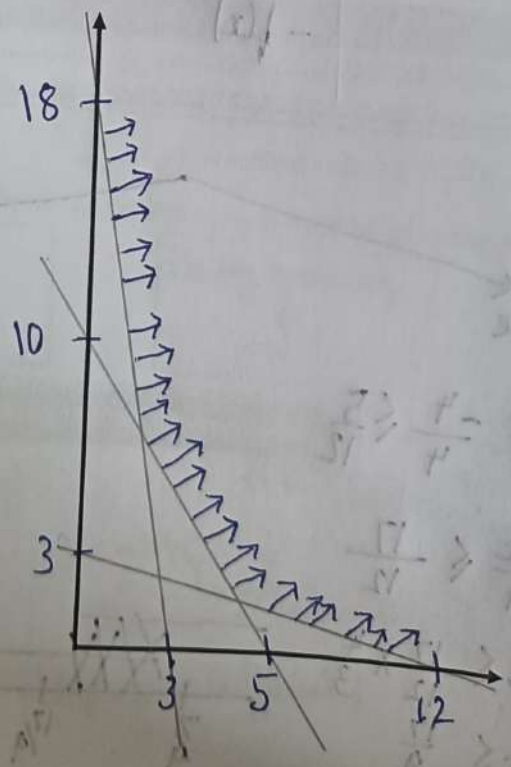
(a) (0, 18), (12, 0), (4, 2) and (2, 6)

$x \geq 0, y \geq 0$

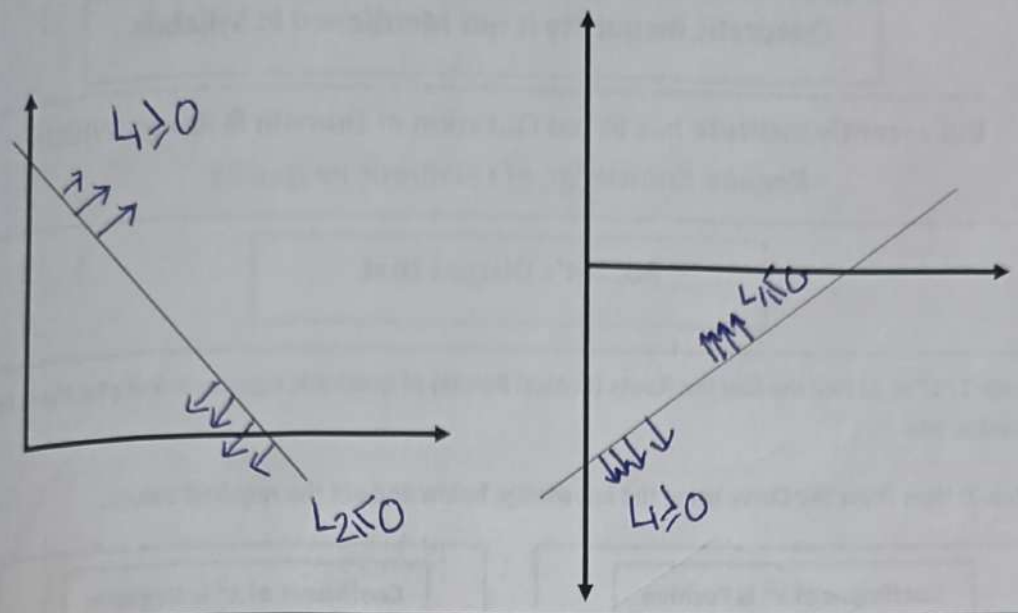
$6x+y \geq 18 \rightarrow \frac{x}{3} + \frac{y}{18} \geq 1$

$x+4y \geq 12 \rightarrow \frac{x}{12} + \frac{y}{3} \geq 1$

$2x+y \geq 10 \rightarrow \frac{x}{5} + \frac{y}{10} \geq 1$



Clarification Note



January-2021

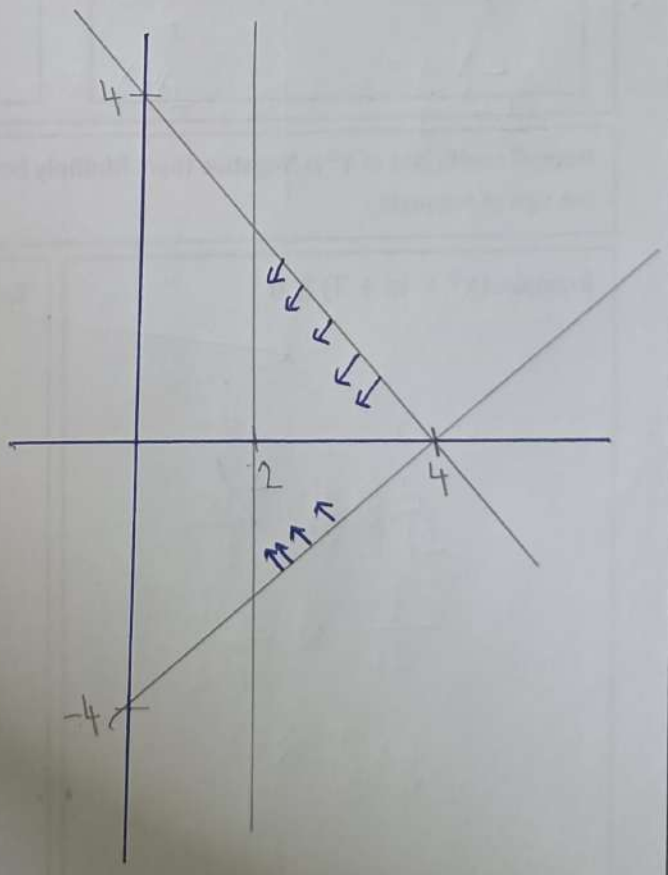
Example: The common region in the graph of the inequalities $x + y \leq 4, x - y \leq 4, x \geq 2$ is

(a) Equilateral triangle (b) Isosceles triangle (c) Quadrilateral (d) Square

$x+y \leq 4 \rightarrow \frac{x}{4} + \frac{y}{4} = 1$

$x-y \leq 4 \rightarrow \frac{x}{4} + \frac{y}{(-4)} = 1$

$x \geq 2 \rightarrow x = 2$



Equations

Chapter 3

Background of Chapter

Conceptual Background

- Linear Equation
- Quadratic Equation
- Cubic Equation

Linear Equation

- Definition
- Methods of Solving Simultaneous In **Two Variables**
  - Elimination Method
  - Cross Multiplication
- Methods of Solving Simultaneous In **Three Variables**
  - Elimination Method
  - Cross Multiplication
- Test of Consistency

Quadratic Equation

- Definition
- Methods of Solving **Quadratic Equation**
  - Factorization
  - Formula
- Relation Between Roots & Co-efficient
- Nature of Roots
- Useful Factors
- Formation of Equations

Cubic Equation

- Definition
- Methods of Solving **Cubic Equation**
  - Hit & Trail
  - Synthetic Division
- Relation Between Roots & Co-efficient
- Useful Factors
- Formation of Equations

Our AIM in Chapter

To find the value of "x" = Roots of Equation = Solution of the equation = Aise "x" ki value which will give us LHS=RHS

Linear Equation

Definition

Equation in which the highest power is **1** is called Linear Equation

General Form

$ax+by+c=0$ | $ax+by+cx=0$

a, b, C ⇓ Constant

Methods of Solving Simultaneous equation in two variables

Elimination Method

Any one Variable is eliminated by making its **Coefficient Equal** then we get equation containing only the other variable

Cross Multiplication Method

The coefficient of x & y is arranged & multiplied across where down-arrow indicates positive products & up-arrow indicates negative products

1st Approach → Reverse Approach

2nd Approach → Elimination Approach

Elimination Method

Example: Consider the system of equation 2x – y = 3 and 5x - 2y = 8 find value of x and y?

$2x-y=3 \longrightarrow 10x-5y=15$

$5x-2y=8 \longrightarrow 10x-4y=16$

$-y=-1$

$2x-1=3$

$2x=4$

$x=2$

$x=2$, $y=1$

Cross Multiplication Method

Example: Consider the system of equation 2x – y = 3 and 5x - 2y = 8 find value of x and y?

$x=2$ $y=1$

$4-1=3$ $10-2=8$

LHS = RHS

Methods of Solving Simultaneous equation in three variables

Elimination Method

Any one Variable is eliminated by making its **Coefficient Equal** then we get equation containing only the other variable

Cross Multiplication Method

The coefficient of x & y is arranged & multiplied across where down-arrow indicates positive products & up-arrow indicates negative products

We will always solve eq. in 3 variables with reverse approach. Option that will satisfy all the 3 equation, will be correct.

Example: Consider the system of equation find value of x and y and z?

$2x - y + z = 3$, $x + 3y - 2z = 11$, and $3x - 2y + 4z = 1$

a) 3, 2, −1

$3 + 6 + 2 = 11$

$11 = 11$

$6 - 2 - 1 = 3$

$3 = 3$

$9 - 4 - 4 = 1$

$1 = 1$

LHS = RHS

Test Of Consistency

NO. OF SOLUTIONS	CONDITION	SYSTEM OF EQUATIONS	LINES INTERSECT AT
Unique solution	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Consistent	One Point
No solution	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Inconsistent	Parallel
Infinite solutions	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Consistent	Coincident

Quadratic Equation

Definition

Equation in which the highest power is **2** is Known as Quadratic Equation

General Form:

$$ax^2 + bx + c = 0$$

Methods of Solving Quadratic Equation

Factorization Method

Take LCM of AXC & use the factors in such a way that [+,-] will give you "B" & multiplication will give you "AXC".

Formula Method

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Factorization Method

Example: Find the Solution of the equation Solve: $x^2 - 5x + 6 = 0$

$x^2 - 2x - 3x + 6 = 0$

$x(x-2) - 3(x-2) = 0$

$(x-2)(x-3) = 0$

$x - 2 = 0$, $x = 2$

$x - 3 = 0$, $x = 3$

Formula Method

Example: Find the Solution of the equation Solve: $x^2 - 5x + 6 = 0$

+ve

$$\frac{5 + \sqrt{25-24}}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$$

-ve

$$\frac{5 - \sqrt{25-24}}{2} = \frac{5-1}{2} = \frac{4}{2} = 2$$

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$$(x-2)(x-3) = 0$$

$x - 2 = 0$, $x = 2$

$x - 3 = 0$, $x = 3$

Formula Method

Example: Find the Solution of the equation Solve: $x^2 - 5x + 6 = 0$

+be

$$\frac{5 + \sqrt{25-24}}{2}$$

$$= \frac{5+1}{2} = \frac{6}{2} = 3$$

-be

$$\frac{5 - \sqrt{25-24}}{2}$$

$$= \frac{5-1}{2} = \frac{4}{2} = 2$$

TIP: $(ab)^n = a^n b^n$, $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Nature of Roots

$D = b^2 - 4ac$ is called discriminant of the equation $ax^2 + bx + c = 0$

The nature of roots of a quadratic equation can be known from its discriminant as follows:

DISCRIMINANT	NATURE OF ROOTS
D = 0	Real & Equal
Perfect Square	Real, unequal & rational
D>0 Not a Perfect Square	Real, Unequal, Irrational **(Conjugate Surds)**
D < 0	Imaginary (Not real) (Conjugate complex nos.)

What if the questions say **ROOTS ARE REAL** nothing else is mentioned?

$D \geq 0$

Example

Equation	D = b^2 - 4ac	Nature of Roots	Value of Roots
$x^2 - 6x + 9 = 0$	$D = b^2 - 4ac$ $= 36 - 36$ $= 0$	Real & Equal	$x^2 - 3x - 3x + 9 = 0$ $x(x-3) - 3(x-3) = 0$ $(x-3)(x-3) = 0$ $x = 3, 3$
$x^2 - 6x - 16 = 0$	$D = b^2 - 4ac$ $= 36 + 64$ D>0 $= 100$	Real, Unequal & Rational	$x^2 - 8x + 2x - 16 = 0$ $x(x-8) + 2(x-8) = 0$ $(x-8)(x+2) = 0$ $x = 8, -2$
$x^2 - 6x + 7 = 0$	$D = b^2 - 4ac$ $= 36 - 28$ D>0 $= +8$	Real, unequal & Irrational	$\frac{6+\sqrt{8}}{2} = 4.4142$ $\frac{6-\sqrt{8}}{2} = 1.5857$
$x^2 - 6x + 13 = 0$	$D = b^2 - 4ac$ $= 36 - 52$ D<0 $= 16$	Imaginary	$\frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm \sqrt{+1 \times 16}}{2}$ $= \frac{6 \pm 4i}{2} = 3 \pm 2i$

TIP: $\sqrt{-1} = i$

Points to Remember

- If $p + \sqrt{q}$ is a root then $p - \sqrt{q}$ is also a root — If nature of roots is irrational
- If one root is reciprocal of other then $a = c$
- If roots are equal in magnitude but opposite in sign then "b" will be 0
- If α, β are roots of $ax^2 + bx + c = 0$ then $\frac{1}{\alpha}, \frac{1}{\beta}$ will be roots of $cx^2 + bx + a = 0$

Relation Between Roots & Coefficient

Equation - $ax^2 + bx + c = 0$ → α, β

$\alpha + \beta = \frac{-b}{a}$ $\quad \alpha\beta = \frac{c}{a}$

Formation of equation With Roots

$x^2 - [\text{Sum of the Roots}]\,x + \text{Product of Roots} = 0$

Example: Find the equation whose one root are 2 & 3.

$x^2 - 5x + 6 = 0$

Example: Find the equation whose one root is $3 + \sqrt{2}$.

$2^{nd} = 3 - \sqrt{2}$

$x^2 - 6x + 7 = 0$

Useful Factors

1	$\alpha^2 + \beta^2$	$(\alpha+\beta)^2 - \alpha\beta$
2	$\alpha^3 + \beta^3$	$(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta)$
3	$\alpha^3 - \beta^3$	$(\alpha-\beta)[(\alpha+\beta)^2 - \alpha\beta]$
4	$\alpha - \beta$	$\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

Example: Roots of equation $2x^2 + 3x + 7 = 0$ are α and β. The value of $\alpha\beta^{-1} + \beta\alpha^{-1}$ is

$\alpha+\beta = -3/2$ $\quad \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ $\quad \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$\alpha\beta = 7/2$ $\quad \frac{9 - 7}{7/2}$ $\quad \frac{\alpha^2+\beta^2}{\alpha\beta}$

$=$

Manipulation in formation of Equations with Useful Factors

When Roots are **Given** in Questions **Directly.**	When Roots are **Not Given** Directly but depends upon **Roots** of **Another equation.**	When Roots are **Not Given** Directly but depends upon **Roots** of **Another equation.**
	And it is possible to factorise the given eq. & get value of $\alpha\beta$ By factorization	Not possible by factorization. Then get required value by useful factor

Example: Find the equation whose one root are 7 & 5.

$x^2 - 12x + 35 = 0$

Example: If α, β are the roots of equation $x^2 - 5x + 6 = 0$ & $\alpha, > \beta$ then the equation with roots $(\alpha^2 + \beta)$ and $(\alpha + \beta^2)$ is

$x^2 - 5x + 6 = 0 \quad \alpha, \beta$

$x^2 - 2x - 3x + 6 = 0$

$x(x-2) - 3(x-2) = 0$

$(x-2)(x-3) = 0 \quad 2, 3$

$\alpha = 3, \beta = 2$

$\alpha^2 + \beta = 11 \quad \alpha + \beta^2 = 7$

$x^2 - 18x + 77 = 0$

Example: If α and β be the roots of the equation $x^2 + 3x + 4 = 0$, then find the equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$

$\alpha + \beta = -3 \quad (\alpha+\beta)^2 = 9$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= 9 - 16 = -7$

$(\alpha+\beta)^2 = 9 \quad (\alpha-\beta)^2 = -7$

$x^2 - 2x - 63 = 0$

Approach to Solve Complex Quadratic Equations

Whenever question & option both are given in form of variables.

In that case question will always give you condition in roots of equation.

In that case, assume the roots that fulfills the condition given in questions.

⇓

Now form eq. using that roots

⇓

Now Use a,b,c to get

Example: The condition that one of $ax^2+bx+c=0$ the roots of is thrice the other is

(a) $3b^2=16ca$ (b) $b^2=9ca$ (c) $3b^2=-16ca$ (d) $b^2=-9ca$

Roots = 1, 3

$x^2-4x+3=0$

$a=1, b=-4, c=3$

Example: If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals then $\frac{b^2}{ac}+\frac{bc}{a^2}$ is equal to

(a) 2 (b) -2 (c) 1 (d) -1

Roots = 1, 1

$x^2-2x+1=0$

$a=1, b=-2, c=1$

$\frac{4}{1}+\left(\frac{-2}{1}\right)=4-2=2$

Example: If the roots of $ax^2+bx+c=0$ are in the ratio $\frac{p}{q}$ then the value of $\frac{b^2}{(ca)}$ is

(a) $\frac{(p+q)^2}{(pq)}$ (b) $\frac{(p+q)}{(pq)}$ (c) $\frac{(p-q)^2}{(pq)}$ (d) $\frac{(p-q)}{(pq)}$

Roots = 1, 2

$x^2-3x+2=0$

$a=1, b=-3, c=2$

$\frac{9}{(2\times1)}=\frac{9}{2}$

Cubic Equation

Definition

Equation in which the highest power is 3 is called Cubic Equation

General Form: $ax^3 + bx^2 + cx + d = 0$

↓

Has 3 Roots = α, β and γ

Relation Between Roots and Coefficients → Extra "~~Rat~~"

- $\alpha + \beta + \gamma = \frac{-b}{a}$
- $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
- $\alpha\beta\gamma = \frac{-d}{a}$

$$x^3 - [\alpha+\beta+\gamma]x^2 + [\alpha\beta+\beta\gamma+\gamma\alpha]x - \alpha\beta\gamma = 0$$

Methods of Solving Quadratic Equation

Hit & Trial Method

Go from Option to Question
wo option jiske 3 roots
LHS = RHS kardenge wo option right.

Synthetic Division

Nahi Padega Thuje ⇒ Will Use Trick
↓
Very Lengthy

Example: Solve $x^3 + x^2 - x - 1 = 0$ Root $x=1$, Factor $x-1=0$

	1	1	−1	−1
		↓		
1		1	2	1
	1	2	+1	X

$x^2 + 2x + 1 = 0$

$x^2 + x + x + 1 = 0$

$x(x+1) + 1(x+1) = 0$

$(x+1)(x+1) = 0$

$x = -1, -1$

Word Problems

Reverse Approach

Go from option to Question. The option then satisfy all the conditions given in ~~that~~ question that option will be correct.

Equation Approach

Step 1: Allot the variables

Step 2: While satisfying the cond make sure that you justify the 'equal to sig

Example: Find the fraction which is equal to ½ when both its numerator and denominator are increased by 2. It is equal to ¾ when both are increased by 12.

~~(a)~~ 3/8 (c) 2/8

(b) 5/8 (d) 2/3

Example: A number consisting of two digits is four times the sum of its digits and if 27 be added to it the digits are reserved. The number is :

(a) 63 ~~(c)~~ 36

(b) 35 (d) 60

Example: Two numbers are such that twice the greater number exceeds twice the smaller one by 18 and 1/3rd of the smaller and 1/5th of the greater number are together 21. The numbers are:

~~(a)~~ (36, 45)

~~(b)~~ (45, 36) (c) (50, 41)

(d) (55, 46)

Example: The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age.

(a) 60 years (c) 51 years

(b) 52 years ~~(d)~~ 50 years

$x = 2y$

$x - 5 = 3(y - 10)$

$x - 5 = 3y - 30$

$y = 25$

$x = 2 \times 25$

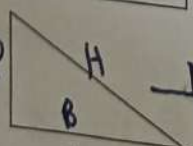
$\boxed{x = 50}$

Some Basic Knowledge

1. 

Area = L × B

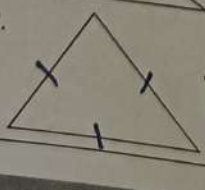
Perimeter = 2(L + B)

2. 

$H^2 = B^2 + P^2$

Area of Δ = $\frac{1}{2} \times B \times P$

In Right Angle Δ Hypotenuse is the largest si

3. 

Equilateral Δ

Number Series

Chapter 19A

Patterns in Number Series

- Addition Pattern
- Subtraction Pattern
- Multiplication Pattern
- Division Pattern
- Square /Cube Pattern
- Combination of Above Patterns
- Even / Odd / Prime Pattern

How to Think About Correct Patterns

There is **No Fixed No of Patterns** in this Chapter There Can be **Infinite Patterns** and **Any Question** Can be Solved from **Various Patterns** So Will be Cover Questions of Large Patterns **Relevant** for **ICAI Exam** and Below Discussed Logic Will Help you to Think about **Correct Pattern**.

- Numbers are **Increasing Slowly** → Additional Logic
- Numbers are **Decreasing Slowly** → Subtraction Logic
- Numbers are **Increasing Moderate** → Multiplication Logic
- Numbers are **decreasing Moderate** → Division Logic
- Numbers are **Increasing Very Fast** → Square/Cube Logic

Example: 20, 30, 42, 56, 72, ? → 90 (+10, +12, +14, +16, +18)

Logic-1	4×5	5×6	6×7	7×8	8×9	9×10
Logic-2	4^2+4	5^2+5	6^2+6	7^2+7	8^2+8	9^2+9
Logic-3	5^2-5	6^2-6	7^2-7	8^2-8	9^2-9	10^2-9

Example-1: 12, 36, 80, 150, 252, ? 392

D: 24 44 70 102 140

D: 20 26 32 38 (+6 +6 +6)

Example-2: 29, 40, 53, 68, 85, ? 104

D: 11 13 15 17 19

Example-3: 13, 25, 37, 49, 61, ? 73

12 12 12 12 12

Example-4: 4, 11, 30, 67, 128, ? 219

7 19 37 61 91

12 18 24 30

Example-5: 15, 17, 22, 32, 49, ? 75

2 5 10 17 26

3 5 7 9

Example-6: 108, 128, 150, 174, 200, ? 228

20 22 24 26 28

Example-7: 17, 19, 33, 67, 129, ? 227

2 14 34 62 98

12 20 28 36

Example-8: 124, 160, 224, 324, 468, ? 664

36 64 100 144 196

6^2 8^2 10^2 12^2 14^2

Example-9: 4, 4, 6, 12, 30, ? 90

Multi: 1 1.5 2 2.5 3.5

Example-10: 17, 52, 158, 477, 1435, ? 4310

×3+1 ×3+2 ×3+3 ×3+4 ×3+5

Example-11: 287, 250, ?, 190, 167, 148, 131 219

37 31 29 23 19 17 [Prime No.s]

Example-12: 7, 6, 11, 13, 15, 20, 19, 27, 23, ? 34

+4 −1 +4 −1 +4 −1 +4

Example-13: 12, 6, 6, 9, 18, ? 45

0.5 1 1.5 2 2.5

Some Common Observations

- From Increasing to **Decreasing** [Check pattern from reverse]
- When Given Series is **Lengthy** than Usual [Alternate there can be 2 series]
- When Starts with Same Number → Multiplication with $1, 1^2, 1^3$
- When Increase-Decrease-Increase then [×1, ×1.5, ×2, ×2.5, ×3]

Questions From ICAI Module

Question-1: 6, 11, 21, 36, 56, ? 81

5 10 15 20 25

Question-2: 10, 100, 200, 310, ? 430

90 100 110 120

10 10 10

Question-3: 11, 13, 17, 19, 23, 25, 29, ? 31

2 4 2 4 2 4 2

Question-4: 6, 12, 21, 33, ? 48

6 9 12 15

Question-5: 2, 5, 9, 14, ? 20, 27

3 4 5 6

Question-6: 6, 11, 21, ? 35, 56, 81

5 10 15 20 25

Question-7: 10, 18, 28, 40, 54, ? 70, 88

8 10 12 14 16 18

Question-8: 120, 99, ? 80, 63, 48, 35

21 19 17 15 13

2 2 2

Question-9: 22, 24, 28, 36, ? 52, 84

2 4 8 16 32

Question-10: 4832, 5840, 6848, 7856, ? 8864

1008 1008 1008 1008

Question-11: 10, 100, 200, 310, 430, ? 560

90 100 110 120 130

Question-12: 28, 33, 31, 36, 34, ? 39

+5 -2 5 -2 5 -2

Ratio

Question-13: 120, 80, 40, 45, ? 25, 5

$45 - x = x - 5$

$2x = 50$

$x = 25$

2AP's

1st 3 Term-AP$_1$

Next 3 Term-AP$_2$

Question-14: 2, 15, 41, 80, 132, ? 197

13 26 39 52 65

Question-15: 1, 4, 10, 22, ?, 94

46

3 6 12 24 48

×2 ×2 ×2 ×2

Question-16: 48, 24, 96, ?, 192

48

0.5 4 0.5 4

Question-17: 6, 17, 39, ?, 116

72

11 22 33 44

Question-18: 7, 26, 63, 124, 215, ?, 511

342

19 37 61 91 133

18 24 36 42

Question-19: 3, 7, 15, 31, ? 127

63

4 8 16

Question-20: 2, 7, 27, 107, 427, ?

1707

5 20 80 320 1280

Question-21: 5760, 960, 192, ? 16, 8

48

6 5 4 3 2

Question-22: 48, 24, 96, ?, 192

Same as Q-16

Question-23: 165, 195, 255, 285, ?, 375

345

30 60 30 60 30

Question-24: 6, 13, 28, 59, ?

122

7 15 31 63

8 16 32

Question-25: 4, 12, 36, ?, 324

108

3 3 3 3

Question-26: 6, 13, 28, 59, ?

122

×2+1 ×2+2 ×2+3 ×2+4

Auctual Logic of Question

Aman Khedia

Some Advance Pattern Questions

Till Now You Must Have Seen that it was **Possible to Solve** Question from **Addition Logic** Even through **Actual Logic was different** (may be lengthy and Difficult to Decode) But Now we Will Practice Some Question Based on Some **Advance Logic** & Have **No Alternative Logic**, Students are Advised to **Practice** Below Question **Multiple Times.**

Question-27: 1, 1, 4, 8, 9, ?, 16, 64

27

1^2 1^3 $(2)^2$ $(3)^3$ 3^3 4^2 4^3

3^2

Question-28: 4, 9, 25, 49, ?, 169, 289, 361

121

2^2 3^2 5^2 7^2 13^2 17^2 19^2

$\times 9^2$ 11^2

Question-29: 8, 28, 116, 584, ? 3508

$\times 3+4$ $\times 4+4$ $\times 5+4$ $\times 6+4$

Question-30: 6, 13, 28, 59, ?

Same

Ratta

Question-31: 5, 2, 7, 9, 16, 25, 41, ? 66

$+$ $+$ $+$ $+$ $+$ $+$ $+$

Ratta

Question-32: 1, 2, 6, 7, 21, 22, 66, ?, 201

67

$+1$ $+1$ $+1$ $\times 3$ $+1$ $\times 3$

$\times 3$ $\times 3$

Question-33: 2, 3, 3, 5, 10, 13, 39, ?, 172, 177

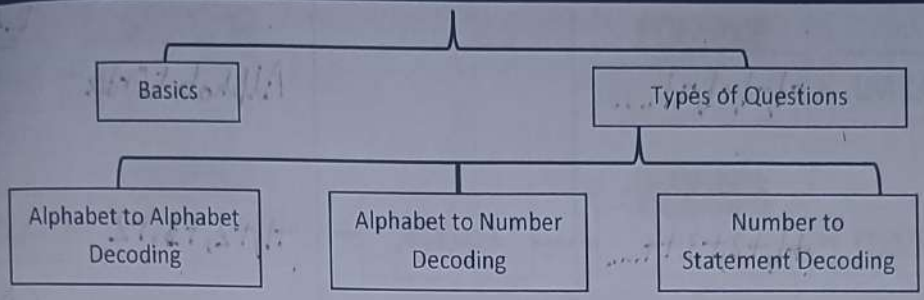
43

$+1$ $\times 1$ $+2$ $\times 2$ $+3$ $\times 3$ $+4$ $\times 4+5$

Coding Decoding

Chapter **19B**

Background



Basics

Order of Alphabets

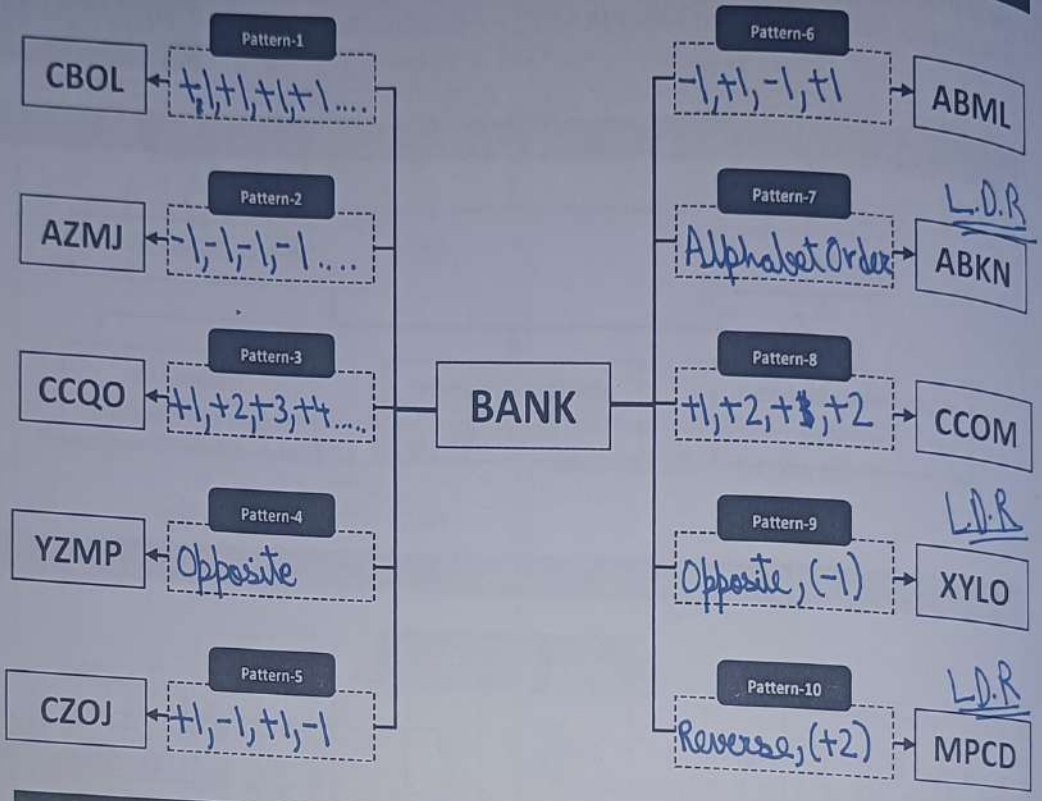
A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
Z	Y	X	W	V	U	T	S	R	Q	P	O	N
26	25	24	23	22	21	20	19	18	17	16	15	14

Trick to Remember Reverse Order

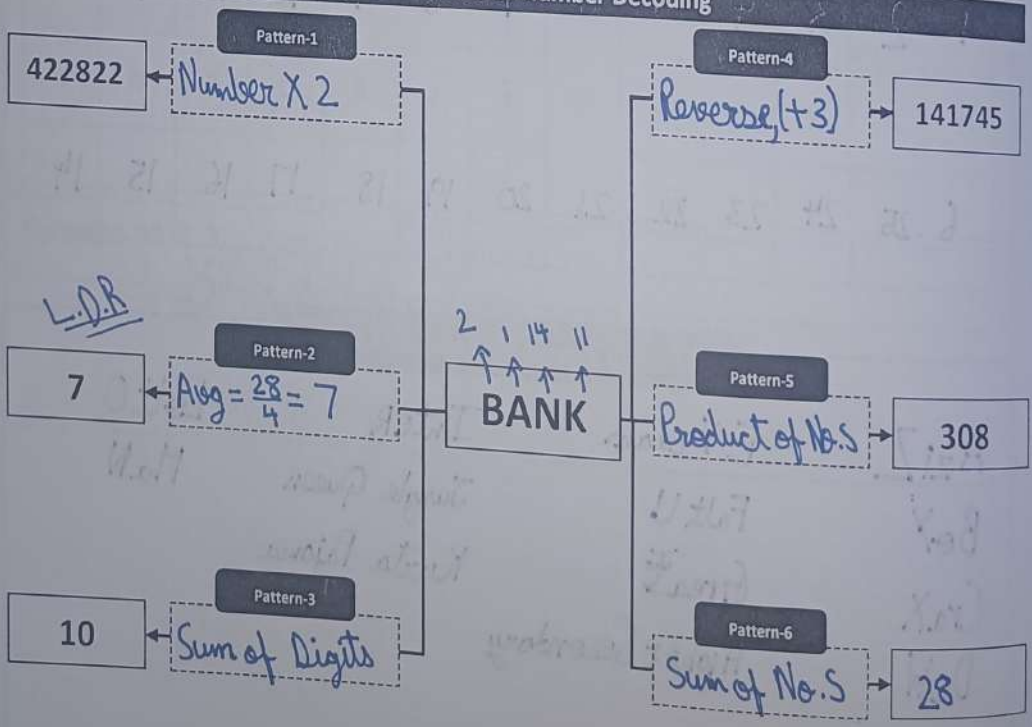
AziZ
BeY
CraX
DeW
EVeaan
FatU
GreaT
Higher Secondary
InteR
Jungle Queen
Kurta Pajama
LottO
MaN

Different Patterns in Coding & Decoding

Type-1-Alphabet to Alphabet Decoding



Type-2-Number to Number Decoding



Example: In a certain language, MADRAS is coded NBESBT, how DELHI is coded in that code?
(a) EMMJI (b) EFMIJ
(c) EMFIJ (d) JIFEM

MADRAS
↓ ↓ ↓ ↓ ↓ ↓
NBESBT

DELHI
↓ ↓ ↓ ↓ ↓
EFMIJ

Example: If GOLD is written as IQNF, how WIND can be written as code?
(a) YKPF (b) VHCM
(c) XJOE (d) DNIW

GOLD
↓ ↓ ↓ ↓ +2
IQNF

WIND
↓
YKPF

Example: If ROSE is written as TQUG, how BISCUIT can be written in that code?
(a) DKUEWKV (b) CJTDVJU
(c) DKVEWKV (d) DKUEWKY

ROSE
↓ +2
TQUG

BISCUIT
↓
DKVEWKW

Example: If DELHI is coded as CCIDD, how would you encode BOMBAY?
(a) AJMTVT (b) AMJXVS
(c) MJXVSU (d) WXYZAX

DELHI
-1 -2 -3 -4 -5
CCIDD

BOMBAY
↓
AMJXVS

Example: If MEKLF is coded as 91782 and LLLJK as 88867, how can IHGED is coded as?
(a) 97854 (b) 64512
(c) 54310 (d) 75632

13 5 11 12 6 | 12 12 12 10 11
MEKLF | LLLJK
91782 | 88867

9 8 7 5 4
IHGED
5 4 3 1 0

Example: If in a certain code language NAME is written as 4258 then what is coded as MEAN?
(a) 2458 (b) 5842
(c) 8524 (d) 5824

NAME | MEAN
4258 | 5824

Pattern-7 Jo hona chaiye aur jo allot kiye hai usme difference hai!

Example: If RAMAN is written as 12325 and DINESH as 675489 how HAMAM is written?

(a) 92323 (b) 92233
(c) 93233 (d) 93292

RAMAN 12325
DINESH 675489

HAMAM
92323

Example: If BROTHER is coded 2456784, SISTER coded as 919684, what is coded for BORBERS?

(a) 2542849 (b) 2542898
(c) 2454889 (d) 2524889

BROTHER 2456784
SISTER 919684

BORBERS
2542849

Example: If DELHI is coded 73541 and CALCUTTA as 82589662, How can CALICUT be coded?

(a) 5279431 (b) 5978213
(c) 8251896 (d) 8543962

DELHI 73541
CALCUTTA 82589662

CALICUT
8251896

Example: If CLOCK is coded 34235 and TIME is 8679, what will be code of MOTEL?

(a) 72894 (b) 77684
(c) 72964 (d) 27894

CLOCK 34235
TIME 8679

MOTEL
72894

Example: If PALE is coded as 2134 and EARTH is coded as 41590, how is PEARL is code?

(a) 29530 (b) 24153
(c) 25430 (d) 254313

PALE 2134
EARTH 41590

PEARL
24153

Example: If LOSE is coded as 1357 and GAIN is coded as 2468, what do figure 82146 stands for?

(a) NGLAI (b) NGLIA
(c) GNLIA (d) GNLIA

LOSE 1357
GAIN 2468

82146
NGLAI

Example: In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code?

(a) 318826 (b) 318286
(c) 618826 (d) 338816

RIPPLE 613382

LIFE 8192

PILLER
3 1 8 8 2 6

Example: If PALAM could be given the code number 43, what code number can be given to SANTACRUZ?

(a) 123 (b) 85
(c) 120 (d) 125

16 1 12 1 13
PALAM
43

19 1 14 20 1 3 18 21 26
SANTACRUX

Example: If A = 1, FAT = 27, FAITH =?

(a) 44 (b) 45
(c) 46 (d) 36

6 1 20
A FAT
1 27

6 1 9 20 8
FAITH

OR

Example: If RED is coded as 6720 then GREEN would be coded as

(a) 9207716 (b) 167129
(c) 1677209 (d) 1672091

18 5 4
RED
6720

18+2 = 20
5+2 = 7
4+2 = 6

7 18 5 5 14
GREEN

7+2 = 9
18+2 = 20
5+2 = 7

5+2 = 7
14+2 = 16

1677209

Type-3

Example: In a certain code '256' means 'you are good', '637' means 'we are bad' and '358' means 'good and bad'. Which of the following represents 'and' in that code?

(a) 2 (b) 5 (c) 8 (d) 3

256 = You are good - (1)
637 = We are bad - (2)
358 = Good and bad - (3)
xX .

6 → Are
3 → Bad
5 → Good

And = 8

Example: If 'Sky is star', 'Star is Cloud, 'Cloud is earth', ''earth is tree', and 'tree is book'. Then where do birds fly?

(a) Cloud **(b)** Sky **(c)** Star **(d)** Fire

Odd Man Out

Chapter 19C

Patterns in Odd Man

Divisible by Specific No/ Number Comes on Table of Any Number

Pattern-1

You will find all the numbers are divisible by common number, only one element will not be divisible by Such Specific No ➡ **ODD MAN**

E.g., Find the **ODD MAN** 10,25,45,54,60,75,80

All are divisible by 5 → Odd = 54

E.g., Find the **ODD MAN** 3,6,9,12,15,18,21,28,30

All are divisible by 3 except 28 = Odd

Series of Prime Number

→ Divisible by 1
→ Divisible by itself

Pattern-2

There will be series of Prime Number and among them there will be a **Non-Prime No-ODD MAN**

E.g., Find the **ODD MAN** 41,43,47,53,61,71,81

All are prime no.s except 81 = Odd

E.g., Find the **ODD MAN** 3,5,7,12,17,19

All are prime except 12 = Odd

Even and Odd Series

Pattern-3

Even no. ke beech mai Odd No.

Odd No. ke beech mai Even No.

E.g., Find the **ODD MAN** 10,14,16,18,21,24,26

All are even no.s except 21 = Odd

E.g., Find the **ODD MAN** 3,5,7,8,9,11

All are odd no.s except 8 = Odd

Perfect Square Series

Pattern-4

E.g., Find the **ODD MAN** 16,25,36,72,144,196,225

All are perfect square except 72 = Odd

Perfect Cube Series

Pattern-5

E.g., Find the **ODD MAN** 8,27,64,100,125,216

All are perfect cube except 100 = Odd

Perfect Square/Perfect Cube Series - Alternatively

Pattern-6

E.g., Find the **ODD MAN** 1,4,27,64,125,~~256~~ 512,343

↓

Odd Man

Sum of Digits

Pattern-7

E.g., Find the **ODD MAN** 78,96,69,52,87

All are having sum 15 except 52 = Odd

Three Number Series

LDR → DMAS

Pattern-8

Generally: We will perform "ALGEBRA" on the **EXTREME digits** to get **MID-digits**

E.g., Find the **ODD MAN** 835,734,642,751,853,981,532

8-5 7-3 6-2 7-1 8-3 9-1 5-2

= 3 = 3 = 4 = 6 = 5 = 8 = 3

All the extreme when subtracted give us mid-point except = 751 → Odd

E.g., Find the **ODD MAN** 331,482,551,263,383,362,284

3×3 4×2 5×1

= 3 = 8 = 5

All the extreme when multiplied give us mid-term except = 383 → Odd

E.g., Find the **ODD MAN** 396,462,572,427,671,264

3+6 4+2 5+2 4+7

= 9 = 6 = 7 = 11

All the extreme when added gives us mid term except = 427 → Odd

Pair Pattern

(Not very high changes)

Pattern-9

Whenever elements are given in Pairs, we will see that 1st **element** of the pair leads us tom 2nd **element**

E.g., Find the **ODD MAN** 12-21, 23-32, 34-43, 55-56 → Odd Man

R R R XR

E.g., Find the **ODD MAN** 15-46, 12-37, 9-28, 8-33 → Odd Man

×3+1 ×3+1 ×3+1 ×3+1

×

E.g., Find the **ODD MAN** 15-45, 12-36, 9-27, 8-32 → Odd Man

×3 ×3 ×3 ×3

×

Other Patterns

Till Now we have Covered Patterns **Based on Numbers** in ODD MAN OUT in Module Question are Based on Number Patterns Only, still there Can **Various Questions** on ODD MAN Which Has to Solved from **Common Session** So Let's Practice Some Questions on that.

Example: Choose out the odd one of the following:
(a) December = 31
(b) February = 28 → Odd
(c) March = 31
(d) July = 31

Example: Choose out the odd one of the following:
(a) June = 30 → Odd
(b) July = 31
(c) Aug = 31
(d) Oct = 31

Example: Choose out the odd one of the following:
(a) Curd (Milk Prod.)
(b) Butter (Milk Prod.)
~~(c)~~ Oil
(d) Cheese (Milk Prod.)

↓
Odd

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Example: Choose out the odd one of the following:
(a) Zebra [H]
(b) Giraffe [H]
(c) Horse [H]
~~(d)~~ Tiger [C]

↓
Odd

Basic Mathematical Tools

Chapter **Basics**

Basic of Maths

*Hello students, Before Starting Our **CA Foundation** Syllabus it is necessary to make our Basics of Math's Strong So Let's Discuss Concepts inspired from **Class 8th to 10th** and Use of **Calculator Tricks.***

Product of Algebra

$(a + b)c =$ $ac+bc$

$(a + b)(c + d) =$ $a[c+d] + b[c+d] = ac+ad+bc+bd$

Example: Find the Product of Following

1 $(5 - 2x)(3 + x)$

$5[3+x] - 2x[3+x]$

$= 15+5x-6x-2x^2$

$= -2x^2-x+15$

2 $(x^2 - 5)(x + 5) + 25$

$x^2(x+5) - 5(x+5) + 25$

$= x^3+5x^2-5x-25+25$

$= x^3+5x^2-5x$

3 $(p^2 - q^2)(2p + q)$

$p^2(2p+q) - q^2(2p+q)$

$2p^3+p^2q-2pq^2-q^3$

Crux

In Case of Multi/Division ➡ Power Changes — eg:- $x[x] = x^2$

In Case of Addition/Sub ➡ Coefficient changes — eg:- $x+x = 2x$

Negative Number Having Power

$(-ve)^{Even\ Number}$ ⬇ +ve

$(-ve)^{Odd\ Number}$ ⬇ -ve

Example: $(-2)^2$ ⇓ 4

Example: $(-2)^3$ ⇓ -8

Example: $(-5)^6$

Example: $(-3)^4$

Inverse of Power

$a^{-n} = \frac{1}{a^n}$ | $\frac{1}{a^{-n}} = a^n$ | $\frac{1}{a^n} = (a)^{-n}$

Whenever we **reciprocal** the **Sign** of Power **Changes**

Example: 2^{-4}

$\frac{1}{(2)^4}$

Example: $\frac{1}{7^{-2}}$

$(7)^2$

Example: $(-2)^3 \times (-2)^4$

$\frac{1}{(-2)^3} \times (-2)^4$

$= \frac{-1}{8} \times 16 = -2$

Example: $3^2 \times 3^{-5} \times 3^6$

Basics of Indices

We Will Study this **Chapter in detail** but let's discuss basics **used** in various chapters as **Raw Work**.

Laws		
$a^m \times a^n$	=	a^{m+n}
$\frac{a^m}{a^n}$	=	a^{m-n}
$(a^m)^n$		$a^{m \times n}$

Example: $\frac{(25)^{7.5} \times (5)^{2.5}}{(125)^{1.5}} = 5^x$ find value of x?

$\frac{(5)^{2\times7.5} \times (5)^{2.5}}{(5)^{3\times1.5}} = \frac{(5)^{15} \times (5)^{2.5}}{(5)^{4.5}}$

$(5)^{15+2.5-4.5}$

$=(5)^{13}=(5)^x$

$x=13$

Example: The value of $[(10)^{150} \div (10)^{146}]$ is:

$\frac{10^{150}}{10^{146}} = 10^4 = 10{,}000$

Simplification of Division

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}$ | $\frac{\frac{a}{b}}{\frac{c}{1}} = \frac{a}{b} \times \frac{1}{c}$ | $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Example: $\frac{\frac{4x}{7}}{\frac{6xy}{5}}$

$\frac{4xy}{7} \times \frac{5}{6xy}$

Example: $\frac{m}{16} \div \frac{5m}{12}$

$\frac{m}{16} \times \frac{12}{5m}$

Example: $\frac{6x+3}{8} \div \frac{2x+1}{12}$

$\frac{6x+3}{82} \times \frac{12}{2x+1}$

$= \frac{27x+9}{4x+2}$

LCM

LCM means Least Common Multiple

Example-1
4,8,16 → LCM: 16

Example-2
3,4,6,18 → LCM: 36

Example-3
5,10,15,20 → LCM: 60

Example-4
2,3,6,7,12 → LCM: 84

Example-5
3,5,7 → LCM: 105

Example-6
12,24,36,48 → LCM: 144

Example-7
5,15,25,45 → LCM: 225

Note

In Case of **2-digit** number just take LCM as multiple of Both Numbers and Cross Multiply.

Example: $\frac{5}{3}+\frac{7}{2}$

$\frac{10+21}{6}=\frac{31}{6}$

Example: $\frac{1}{72}+\frac{2}{114}$

$\frac{114+144}{8208}=\frac{258}{8208}$

Example: $\frac{5}{7}+\frac{3}{21}$

$\frac{15+3}{21}=\frac{18}{21}$

Example: $1+\frac{1}{2}+\frac{1}{8}-\frac{1}{16}$

$\frac{16+8+2-1}{16}=\frac{25}{16}$

Example: $\frac{3}{x+1}+\frac{x+7}{(x+1)(x+2)}$

$\frac{3(x+2)+x+7}{(x+1)(x+2)}=\frac{3x+6+x+7}{(x+1)(x+2)}$

Square Root & Nth Root

Square Root: $\sqrt{a}=a^{\frac{1}{2}}$

Cube Root: $\sqrt[3]{a}=a^{\frac{1}{3}}$

Nth Root: $\sqrt[n]{a}=a^{\frac{1}{n}}$

Example: $5\sqrt{4}$

$5\times\sqrt{4}$

Example: $\sqrt[5]{32}$

$(32)^{1/5}$

Calculator Shortcuts

M+	Memory Plus	MRC	Memory Recall
M-	Memory Minus	GT	Grand Total

Example:

X	F	
15	3	45
12	2	24
8	5	40
6	7	42
18	4	72
Total		223

Example:

$$\frac{23+75-54}{2\times 11} \Rightarrow 2$$

Example:

$$\frac{4+55-9}{(15\times 2)-5} \Rightarrow 2$$

Example: $(2)^3$

8

Example: $(25)^3$

15,625

Example: $(0.07)^5$

0.0000016807

Example: $(3.5)^2$

12.25

Example:

$$\frac{110.16}{(0.06)^2(3.06)} = 10000$$

Example:

$$\frac{1.50}{(0.05)^2} = 600$$

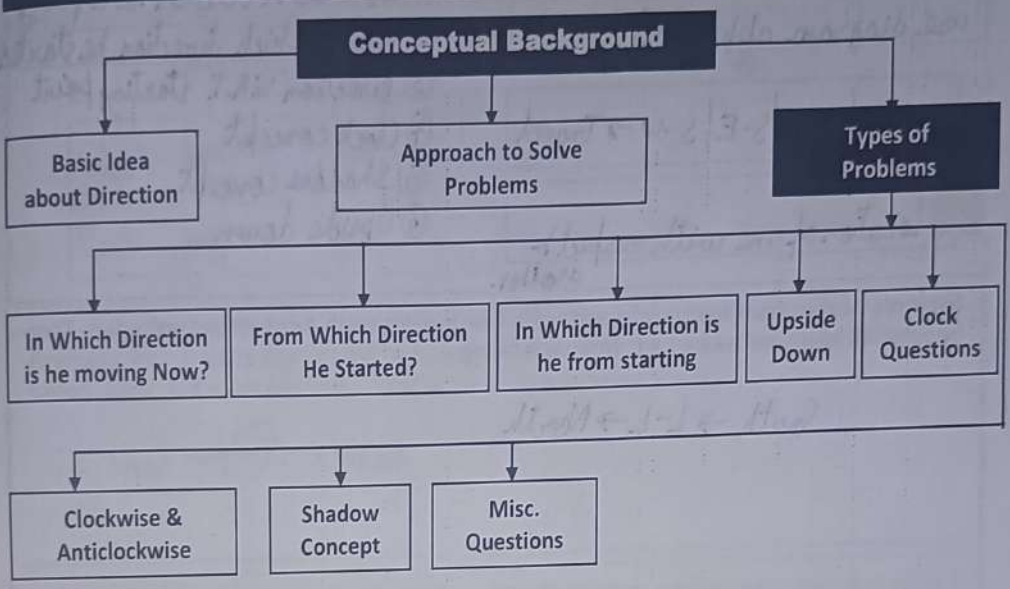
Example:

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = 7.87298$$

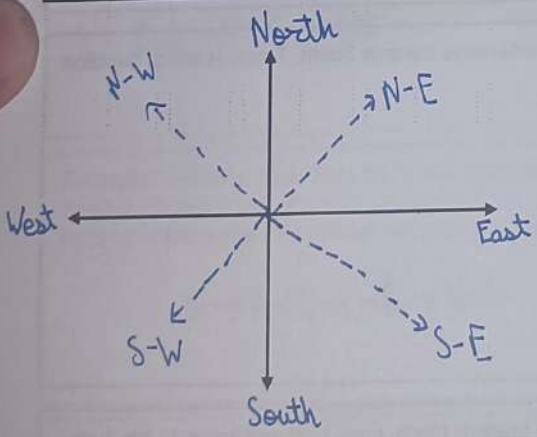
Direction Sense Test

Chapter 19

Background of Chapter



Basic Idea about Direction



You face in any direction

Right Movement	Clockwise
Left Movement	Anti-Clockwise

Examples

North	R	East	South	R	West	North	R-R	South
North	L	West	South	L	East	East	R-L	East
East	R	South	West	R	North	West	L-L-L	North
East	L	North	West	L	South	South	R-R-R-R-L	East

Approach to Solve Problems

Diagram Approach

If you are confused you can always use diagram approach.
[In below two cases, we have to use diagram approach]

1. N-E/N-W/S-E/S-W → Travel
2. Location of one with respect to another.

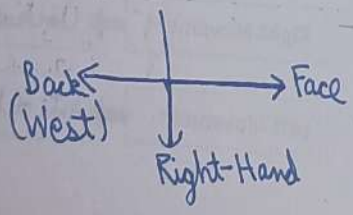
Shortcut Approach

You use shourcuts in all the cases as we have discussed)
↳ ① In which direction he is facing now?
② From which direction he started?
③ Direction W.R.T. starting point
④ Clock concept
⑤ Shadow concept
⑥ Upside down

Example: Mohan starts from point A and walks 1 km towards south, turns left and walks 1km. Then he turns left again and walks 1 km. Now he is facing.

South → L-L → North

Example: I stand with my right-hand extended side-ways towards South. Towards which direction will my back be?



Example: A man starts from a point, walk 8 km towards North, turns right and walks 12 km, turns left and walks 7 km turns and walks 20 km towards South, turns right and walks 12 km. In which direction is he from the starting point?

$N_8 E_{12} N_7 S_{20} W_{12} = S_5$

~~North → R E~~

In which direction is he facing now?

Same Movement	Add	R,R
Opposite Movement	Subtract	R, L

Example

North	R	L	R	R	South
West	R	R	L	L	West
East	L	L	L	R	West

Example: Roopa starts from a point and walks 15 meters towards west, turns left and walks 12 meters, turns right again and walks. What is the direction she is now facing?

West → L - R → West

Example: A man starts his journey facing the sun early morning. He then turns right and walks 2 km. He then walks 3 km after turning right again. Which is the direction he is facing now?

East → R - R → West

Example: Sanmitra starts from his house and walks 3 km towards north. Then he turns right and walks 2 km and then turns right and walks 5 km, then turns right and walks 2 km and then again turns right and walks 2 km. Which direction is he facing now?

N → R - R - R - R → North

Example: A man started to walk East. After moving a distance, he turned to his right. After moving a distance, he turned to his right again. After moving a little he turned in the end to his left. In which direction was he going now.?

East → R - R - L → South

From which direction he Started?

Same Movement →	Add	R, R
Opposite Movement →	Subtract	R, L

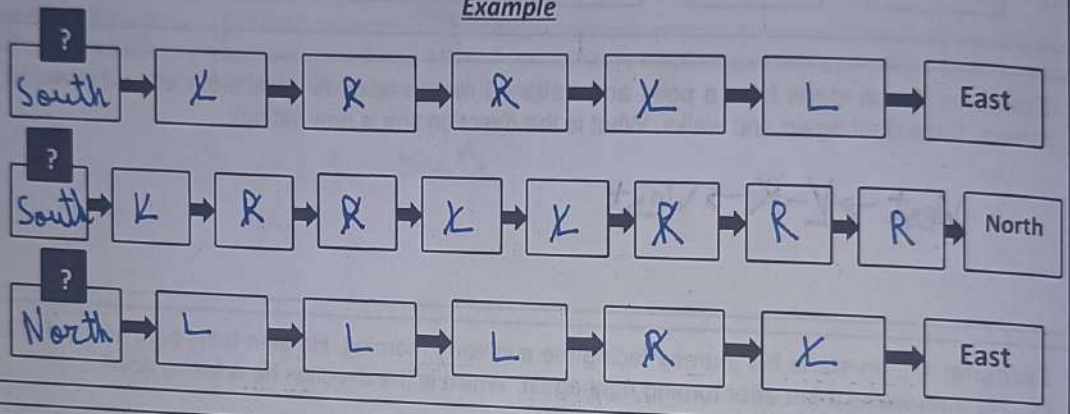
Shortcut

Reverse the Outcome → R → Left, L → Right

3R → 3L

2L → 2R

Example



Example: Ramu after walking 10m, turns right, turns left, turns right, turns left, finally turns left. In the end, he was moving towards the East. From which direction did Ramu Started his journey?

South → R - L - R - L - L → East

Example: Salman After walking 25m, turns left, turns left, turns right, turns left, turns right, turns left, finally turns left. In the end, he was moving towards the North. From Which direction did Salman starts his journey?

West → L - L - R - L - R - L - L → North

In which direction is he from starting point?

Magnitude of Movement is Given

- Same Direction → Add (NN, SS, EE, WW)
- Opposite Direction → Subtract (NS, EW)

Magnitude of Movement is Not Given

Use diagram approach

Following Questions Can be asked

- In Which direction is he from **Starting point**?
- **Distance** between starting point & Starting point?
- In Which direction is starting point from ~~starting~~ End point?

Example

$N_{40}E_{60}S_{40}W_{35}$ → E_{25}

1	2	3
East	25	West

$E_{30}S_{50}W_{65}N_{70}E_{35}$ → N_{20}

1	2	3
North	20	South

What if 2 Direction are left at End?

How to find the distance if 2 directions are left at the end?

↓

Apply Phythagoras Theorem

$H^2 = B^2 + P^2$

$H = \sqrt{(B^2)+(H^2)}$

Example

$S_{90}E_{85}N_{50}W_{45}$

$S_{40}E_{40}$ = South-East

$H = \sqrt{(40)^2+(40)^2} = \sqrt{3200} = 56.56$

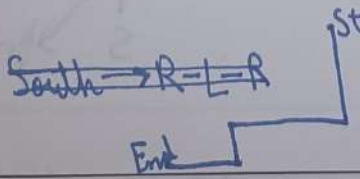
Example: Rahul walked 40m towards north. Then he turned right and walked 55m.Then he turns right and walks 60m.Then he turns right and walks 45m.In which direction and how many meters is he from the starting position?

$N_{40}E_{55}S_{60}W_{45} = S_{20}E_{10}$ = South-East

$H = \sqrt{20^2+10^2} = \sqrt{500} = 22.36$

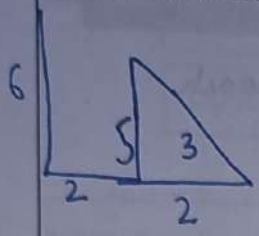
Example: A walk southwards, then turns right, then left and then right. In which direction is he from the starting point?

~~South → R – L – R~~



South-West

Example: Roy walks 2 km to East, then turns North-West and walks 3 km. Then he turns South and walks 5 km. Then again, he turns West and walks 2 km. Finally, he turns North and walks 6 km. In which direction, is he from the starting point?



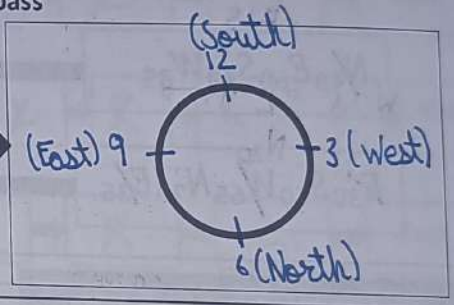
North-West

Clock Based Questions

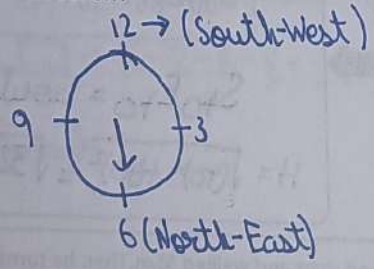
While solving questions on Clock always keep the **Clock Diagram Constant** and Frame the **Directions** as per the **Question Demand** & **Compass**

Example: Aman Sir put the clock on the table in such a way that at 6 P.M hour hand point to north. In which direction the minute hand will point 9.15 P.M?

West



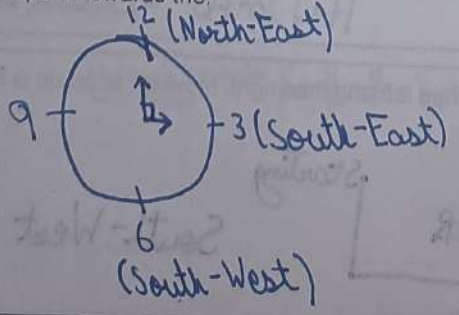
Example: At 2.30 A.M the minute hand of clock face North-East. At 7.00 AM the minute hand will be in which direction?

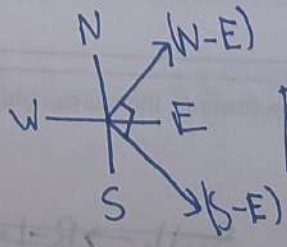


South-West

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Example: It is 3'o clock in watch. If the minute hand points towards the North-East then the hour hand will point towards the:





South-East

Clockwise & Anticlockwise Questions

When person turns clockwise and anticlockwise it means he **Rotating** on its **Own axis**. It doesn't mean that he travels and move unless mentioned in question.

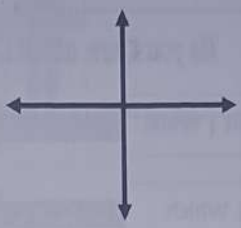
Shortcut

Clockwise	Add
Anticlockwise	Subtract

Example: A person is facing north direction now he turns 90° Clockwise then 45° Anticlockwise then 90° Clockwise then 180° Anticlockwise, In which diagram is he facing now?

North-West

Method



Shortcut

90
(45)
90
(180)
-45

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Example: A man is facing west he turns 45 degrees in the clockwise direction and then another 180 degrees in the same direction and then 270 degrees in the anticlockwise direction. Which direction is he facing now?

(a) South-west (b) North-west (c) West (d) South

45
180
(270)
(45)

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Example: A Person facing in North moves 70° in clockwise direction. He again moved 300° in anti-clockwise direction. In which direction is he facing now?

(a) North-West (b) South-East (c) North-East (d) South-West

70
(300)
(230)

Location of One with Respect to Another

Whenever question demands Position of One Person with respect to another person always draw the diagram and form opinion.

All of the following statement means the same

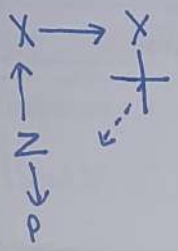
In which direction is "B" w.r.t "A"?	In which direction is "B" from "A"?	In which direction of "A" is "B"?

Conclusion You have to draw a compass at "A" and look towards B.

Do you Know about Connectors?

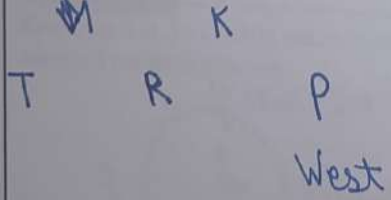
And | But | While	1st Person
Who | Which	2nd Person

Example: Y is East of X Which is to the North of Z. If P is to the south of Z in which direction of Y is P?



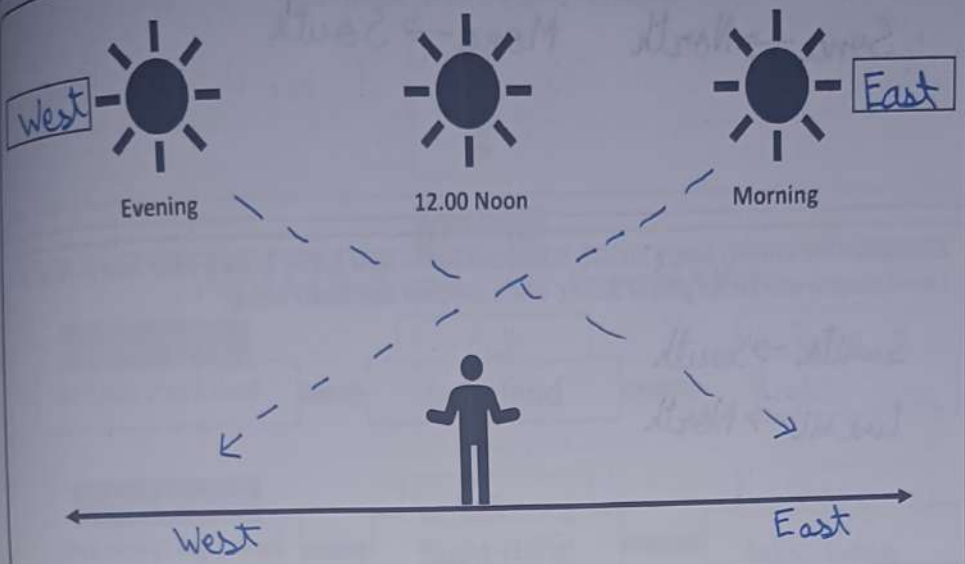
South-West

Example: K is a place which is located 2 km away in the north-west direction from the capital P. R is another place that is located 2 km away in the south-west direction from K. M is another place and that is located 2 km away in the north-west direction from R. T is yet another place that is located 2 km away in the south-west direction from M. In which direction is T located in relation to P?



West

Shadow Concept



Conclusions from Above

Facing ↓	Morning Shadow	Evening Shadow
North	Left	Right
South	Right	Left

NOTE

If your Shadow is **Front** or **Back** it means you are travelling **East** or **West**

NOTE

If your Shadow is **Left** or **Right** it means you are travelling **North** or **South**

NOTE: *Shadows Will be On the Same Side*

Example: *One Morning Aman and Pawan were talking to each other while walking on ground. If Aman's shadow was exactly to the right of Pawan, which direction was Aman Facing?*

Pawan → South, Aman → South

Example: *One Morning Raju and kaju were talking each other face to face at a crossing. If kaju's shadow was exactly to the left of Raju, which direction was Raju facing?*

Right → North Kaju → South

Example: One evening before sunset, Mona and Sonam were talking to each other face to face If Mona shadow was exactly to the right of Sona, Which direction was Mona facing?

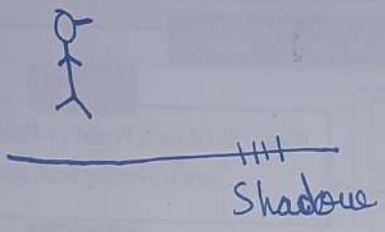
Sona → North Mona → South

Example: One evening before sunset, Sunita and Laxmi were talking to each other face to face If Laxmi shadow was on left side of Sunita, which direction was Sunita facing?

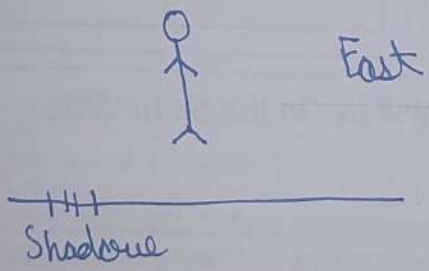
Sunita → South

Laxmi → North

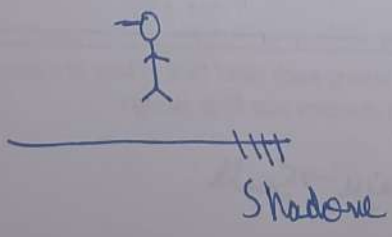
Example: If at 5 PM the shadow of a person is on his front then the person is facing which direction?



Example: If at 9 AM the shadow of a person is on his back then the person is facing which direction?



Example: If at 5PM the shadow of a person is on his back then the person is facing which direction?



Upside Down Concept

Upside Means Person is Rotated **180 degree**



Shortcut

1st of all find the **direction** in which the **person is facing**

Question Demanding	Cut	Turn
Direction of **Left Hand** →	~~Left Hand~~ →	Right Turn

Question Demanding	Cut	Turn
Direction of **Right Hand** →	~~Right Hand~~ →	Left Turn

Example: *A person is standing upside down facing North than in which direction is his left hand would be?*

North R → East

Example: *A person is standing upside down facing East than in which direction is his Right hand would be?*

East L → North

Example: *A man is standing upside down facing you than in which direction his left hand would be?*

South R → West

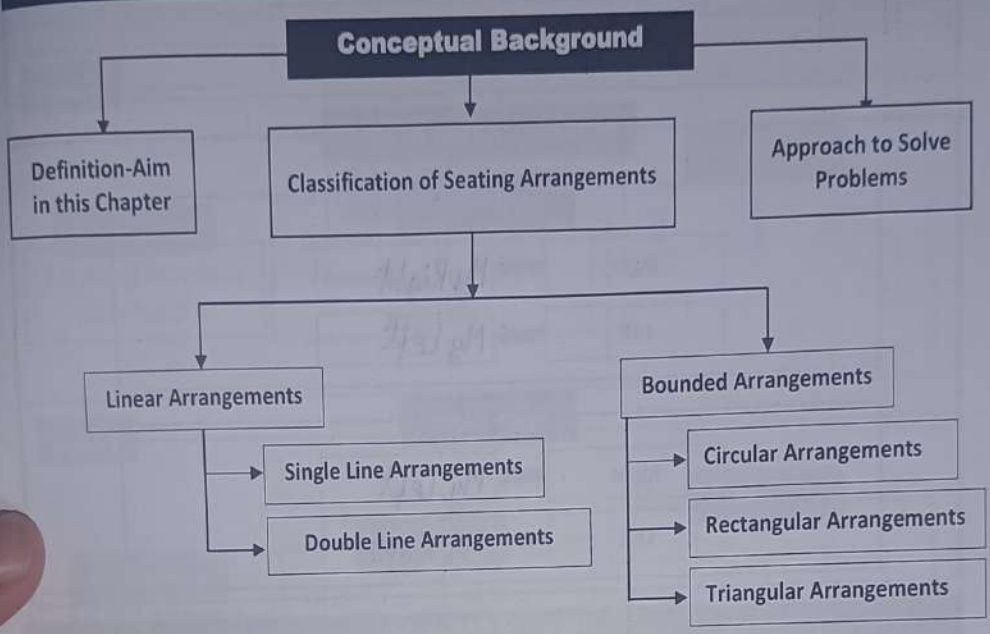
Example: *In the morning A man was performing Yoga with Head down and legs up if at this time face towards West than in which direction his left hand would be?*

West R → North

Seating Arrangements

Chapter 20

Background of Chapter



Definition/Aim in this Chapter

In this chapter we need to arrange certain Objects/persons after complying with certain Given conditions.

⬇

We will achieve this target by Converting given Conditions into Picture form.

Linear Arrangement

In this type of arrangement, we arrange objects/people in a line or row, in this type of arrangement, we take directions according to our left and right.

Note: If the question is silent, we will solve questions assuming Object/People are facing **NORTH** Direction

Movement of Right & Left

North Facing

Right	My Right
Left	My Left

South Facing

Right	My Left
Left	My Right

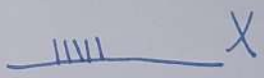
Points to Remember

Point-1

Given Condition
There are 5 Persons
A, B, C, D, E

Conclusion
Never Draw Marks in the Beginning

Diagram

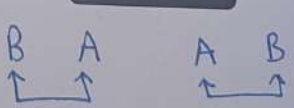


Point-2

Given Condition
A is Sitting next to B

Conclusion
A & B are Sitting together
"It doesn't mean A is forward of "B" he can be backward also

Diagram



Point-3

Given Condition
B is Sitting to the right/left of A

Conclusion
It Does not mean that B is sitting immediate Right or Left of A

Diagram
A B ✓
A C B ✓

Point-4

Given Condition
B is Sitting immediate right/left of A

Conclusion
It means there is no person Between them they are neighbors

Diagram
A B ✓

Point-5

Given Condition
E is Sitting between A & B

Conclusion
There will be 2 Possibilities

Diagram
A E B ✓
B E A ✓

Point-6

Given Condition

Conclusion

Diagram

Point-7

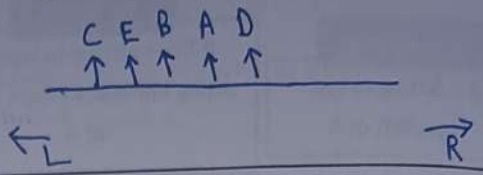
Given Condition

Conclusion

Diagram

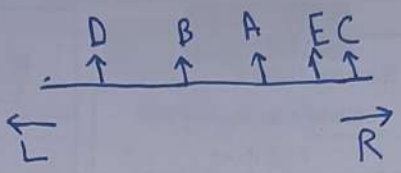
Example: Five boys A, B, C, D and E are sitting in a row A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end? (U.P.B. Ed 2013)

(a) D (b) A (c) E (d) B



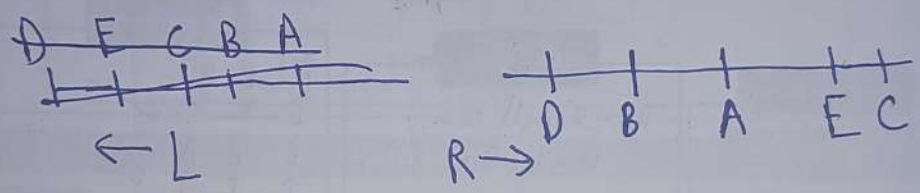
Example: There are five different houses, A to E, in a row. A is to the right of B and E is to the left of C and right of A, B is to the right of D. Which of the houses is in the middle? IB CA (IO) 2013)

(a) A (b) B (c) C (d) D



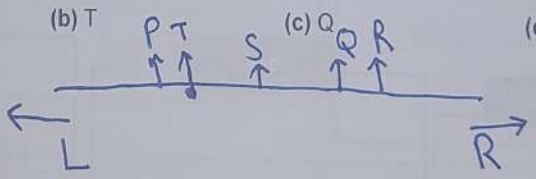
Example: There are five different houses, A to E, in a row. A is to the right of B and E is to the left of C and right of A, B is to the right of D. Which of the houses is in the middle? IB CA (IO) 2013)

(a) A (b) B (c) C (d) D



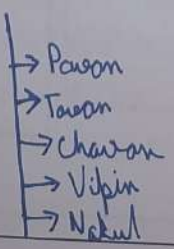
Example: Five friends P, Q, R, S and T are sitting in a row facing North. Here, S is between T and Q and Q is to the immediate left of R. P is to the immediate left of T. Who is in the middle? (SSC (Multi Task)2014)

(a) S (b) T (c) Q (d) R



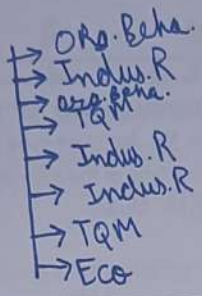
Example: Five boys are standing in a row facing East. Pavan is left of Tavan, Vipin and Chavan to the left of Nakul. Chavan is between Tavan and Vipin. Vipin is fourth from the left, then how far is Tavan from the right? (CLAT 2014)

(a) First (b) Second (c) Third (d) Fourth



Example: There are eight books kept one over the other. Two books are on Organization Behavior, two books on TQM, three books on Industrial Relations and one book is on Economics. Counting from the top, the second, fifth and sixth books are on Industrial Relations. Two books on Industrial Relations are between two books on TQM. One book of Industrial Relations is between two books on Organizational Behavior while the book above the book of Economics is a book of TQM. Which book is the last book from the top? (MAT 2011)

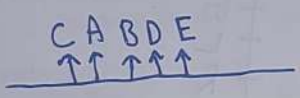
(a) Economics (b) TQM
(c) Industrial Relations (d) Organizational Behavior



2 → Org. Beha.
2 → TQM
3 → Indus. Relation
1 → Eco.

Example: Five friends are sitting on a bench. A is to the immediate left of B but on the immediate right of C, D is to the immediate right of B but on the immediate left of E. Who are at the extremes?

(a) A,B (b) A,D (c) C,E (d) B,D

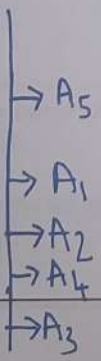


Example: Five boys A_1, A_2, A_3, A_4 and A_5 are sitting in a stair in the following way. (RRB (TC/CC)

I. A_5 is above A_1.
II. A_4 is under A_2
III. A_2 is under A_1
IV. A_4 is between A_2 and A_3.

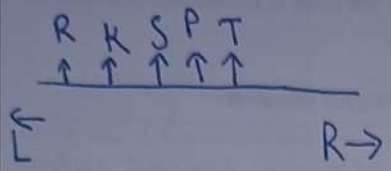
Who is at the lowest position of the stair?

(a) A_1 (b) A_3 (c) A_5 (d) A_2



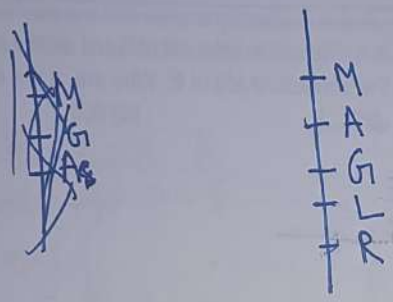
Example: Five children are sitting in a row. S is sitting next to P but not T. K is sitting next to R, who is sitting on the extreme left and T is not sitting next to K. Who is/are adjacent to S? (NIFT (UG) 2014)

(a) K and P **(b)** R and P **(c)** Only P **(d)** P and T



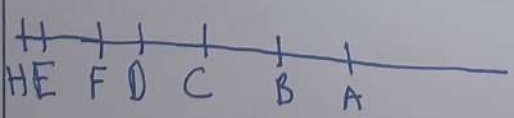
Example: Five senior citizens are living in a multistoried building. Mr. Muan lives in a flat above Mr. Ashokan, Mr. Lokesh in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat above Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives in the topmost flat? (MAT 2013).

(a) Mr. Lokesh **(b)** Mr. Gaurav **(c)** Mr. Muan **(d)** Mr. Rakesh



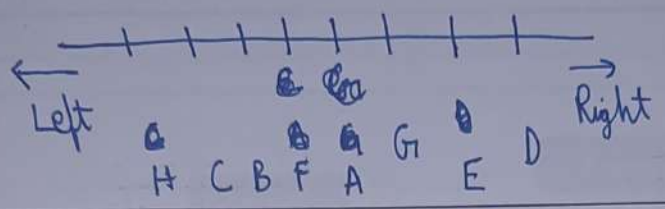
Example: In a gathering seven members are sitting in a row. 'C' is sitting left to 'B' but on the right to 'D'. 'A' is sitting right to 'B', 'F; is sitting right to 'E' but left to 'D'. 'H' is sitting left to 'E'. Find the person sitting in the middle (SSC (10+2) 2013)

(a) C **(b)** D **(c)** E **(d)** F



Example: Study the following information carefully to answer the given questions.
A to H is seated in straight line facing North. C sits fourth left of G. D sits second to right of G. Only two people sit between D and A. B and F are immediate neighbors of each other. B is not an immediate neighbor of A. H is not neighbor of D. (GIC 2012)

Diagram



1. Who amongst the following sits exactly in the middle of the persons who sit fifth from the left and the person who sit sixth from the right?
   (a) C **(b)** H **(c)** E **(d)** F

2. Who amongst the following sits third to the right of C?
   (a) B **(b)** F **(c)** A **(d)** E

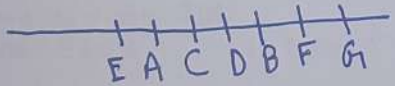
3. Which of the following represents persons seated at the two extreme ends of the line?
   (a) C, D **(b)** A, B **(c)** B, G **(d)** D, H

4. What is the position of H with respect to F?
   (a) Third to the left **(b)** Immediate right **(c)** Second to right **(d)** Fourth to left

5. How many persons are seated between A and E?
   (a) One **(b)** Two **(c)** Three **(d)** Four

Example: A group of singers, facing the audience, are standing in line on the stage as follows.
I. D is to the immediate right to C
II. F is standing beside G.
III. B is to the immediate left of F
IV. E is to the immediate left of A
V. C and B have one person between them
VI. A and D have one person between them

Diagram



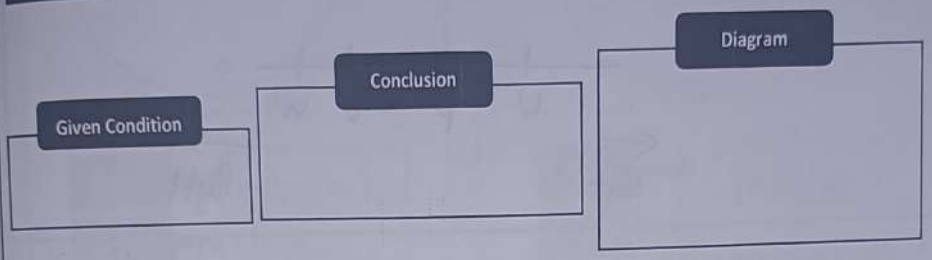
1. Who is on the Second extreme right?
   (a) D (b) F (c) G (d) E

2. If we start counting from the left, on which number is B?
   (a) 1st **(b)** 2nd **(c)** 3rd **(d)** 5th

Double Line Arrangements

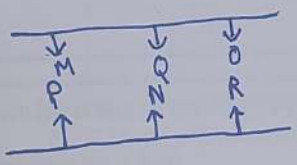
In this type of arrangement, we arrange objects/people in 2 rows facing each other (one is facing north & another is facing south)

Point-8



Example: Six persons M, N, O, P, Q and R are sitting in two rows with three persons in each row. Both the row are in front of each other. Q is not at the end of any row. P is second the left of R. O is the neighbor of Q and diagonally opposite to P. N is the neighbor of R. Who is in front N? (UPSC)

(a) R (b) Q (c) P (d) M



Example: Six persons A, B, C, D, E and F are sitting in two rows, three in each row. (MAT 2011)

(I) E is not at the end of any row

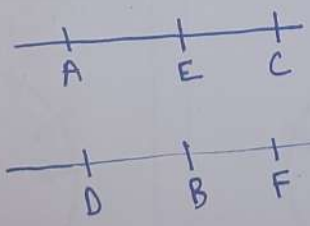
(II) D is second to the left of F

(III) C, the neighbor of E, is sitting diagonally opposite to D

(IV) B is the neighbor of F.

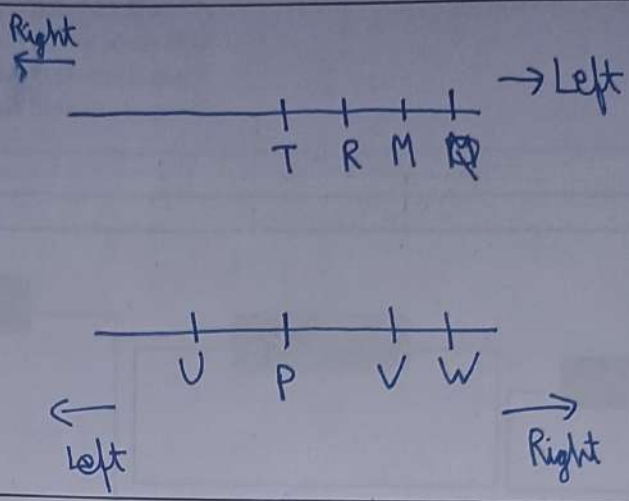
Which of the following are in one of the two rows?

(a) D, B and F (b) C, E and B (c) A, E and F (d) F, B



Example: Eight persons P to W are sitting in front of one another in two rows. Each row has four persons. P is between U and V and facing North. Q, who is to the immediate left of M is facing W. R is between T and M and W is to the immediate right of V. **(UCO Bank 2011)**

Diagram



1. Who is sitting in front of R?
   (a) U **(b)** Q **(c)** V **(d)** P

2. Who is to the immediate right of R?
   (a) M **(b)** U **(c)** M or W **(d)** None of these

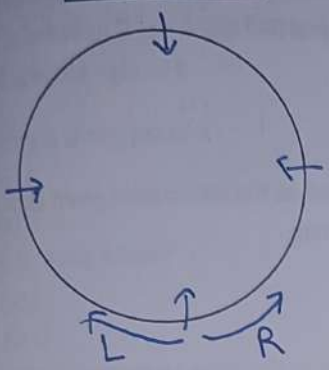
3. In which of the following pairs, persons are sitting in front of each other?
   (a) MV **(b)** RV **(c)** TV **(d)** UR

Example: Four girls A, B, C, D are sitting around a circle facing the center. B and C Infront of each other, which of the following is definitely true? (MAT 2009)
(a) A and D Infront of each other **(b)** A is not between B and C
(c) D is left of C **(d)** A is left of C

Circular Arrangement

Movement of Right & Left

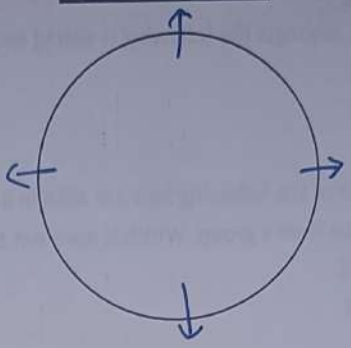
Facing Inwards



Right → Anti-Clockwise

Left → Clockwise

Facing Outwards



Right → Clockwise

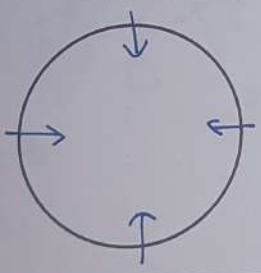
Left → Anti-Clockwise

Notes

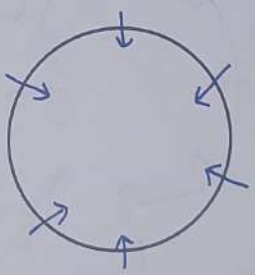
- In Circular always draw the places in beginning.
- If there is multiple possibility then draw multiple diagram wrong will eliminate automatically.

No of Persons and their Presentation

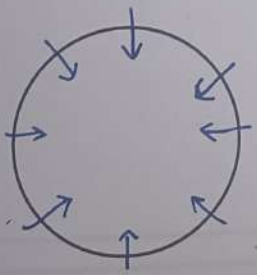
4 persons



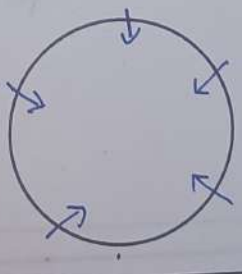
6 persons



8 persons



5 persons



Example: Eight friends A, B, C, D, E, F, G and H are sitting in a circle facing the Centre, not necessarily in the same order. D sits third to the left of A. E sits to the immediate right of A. B is third to left of D. G is second to the right of B. C is neighbors of B. C is third to left of H.

1. Who amongst the following is sitting exactly between F and D?
   (a) C (b) E
   (c) H (d) A

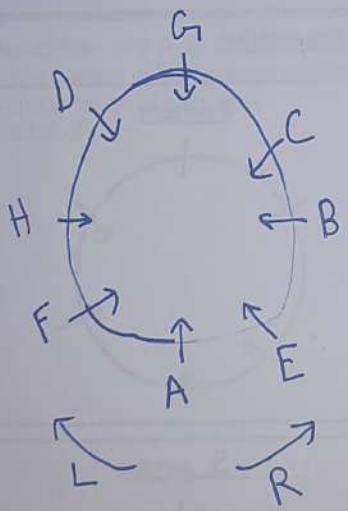
2. Three of the following four are alike in a certain way based on the information given above and so from a group. Which is does not belong to that group.
   (a) DC (b) AH
   (c) EF (d) CB

3. Who amongst the following second to the left of H?
   (a) E (b) B
   (c) A (d) none of these (G)

4. Who amongst the following are immediate neighbors of G?
   (a) CA (b) AF
   (c) DC (d) DF

5. Who amongst the following is sitting third to the right of A?
   (a) F (b) B
   (c) H (d) C

Diagram



Example: Four ladies & A, B, C and D and four gentlemen E, F, G and H are sitting in a circle around a table facing each other.

I. No two ladies or gentlemen are sitting side by side.

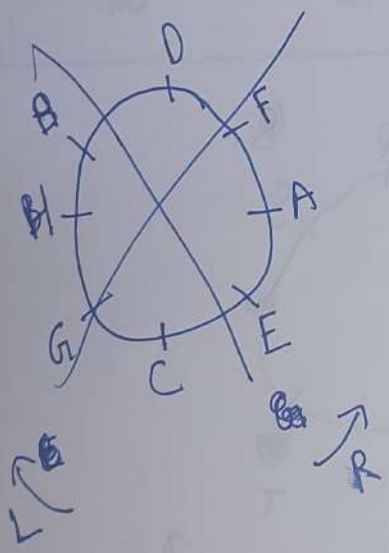
II. C, who is sitting between G and E, is facing D.

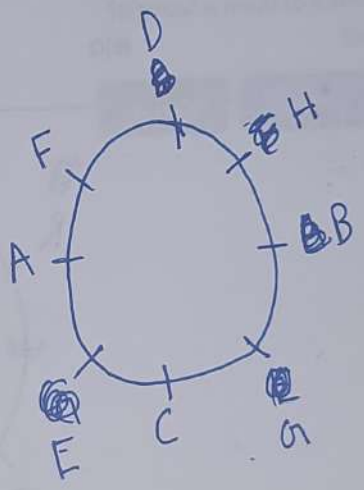
III. F is between D and A and facing G.

IV. H is to the right of B.

1. Who is sitting left of A?
   (a) E (b) F
   (c) G (d) H
2. E is facing whom?
   (a) F (b) B
   (c) G (d) H
3. Who is immediate neighbors of B?
   (a) G and H (b) E and F
   (c) E and H (d) F and H

Diagram

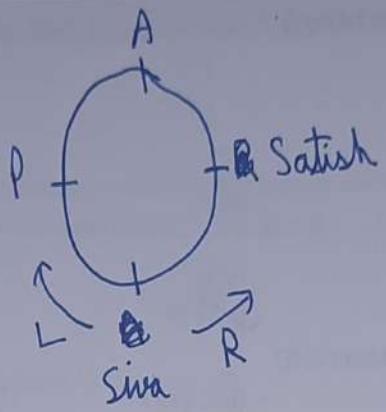




Example: Siva, Satish, Amar and Praveen are playing cards. Amar is to the right of Satish who is to the right of Siva. Who is to the right of Amar?

(a) Satish **(b)**Amar **(c)** Praveen **(d)** Shiva

Diagram

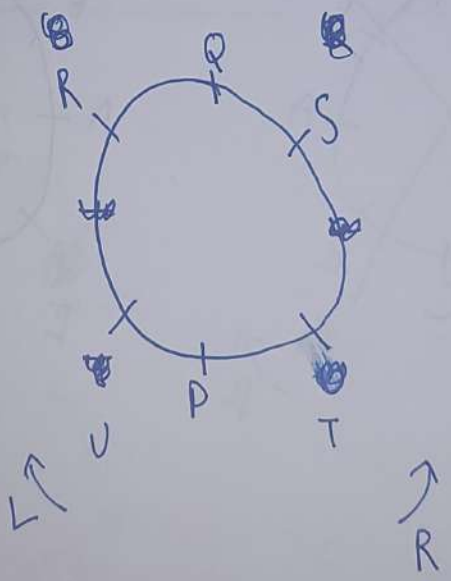


Example: If six person are sitting in a hexagonal table are P,Q,R,S,T,U each facing the center. P is seated to opposite to Q who is b/w R & S. P is b/w T & U. T is the left of S.

Which of them is facing R?

(a)P **(b)**Q **(c)**U **(d)**T

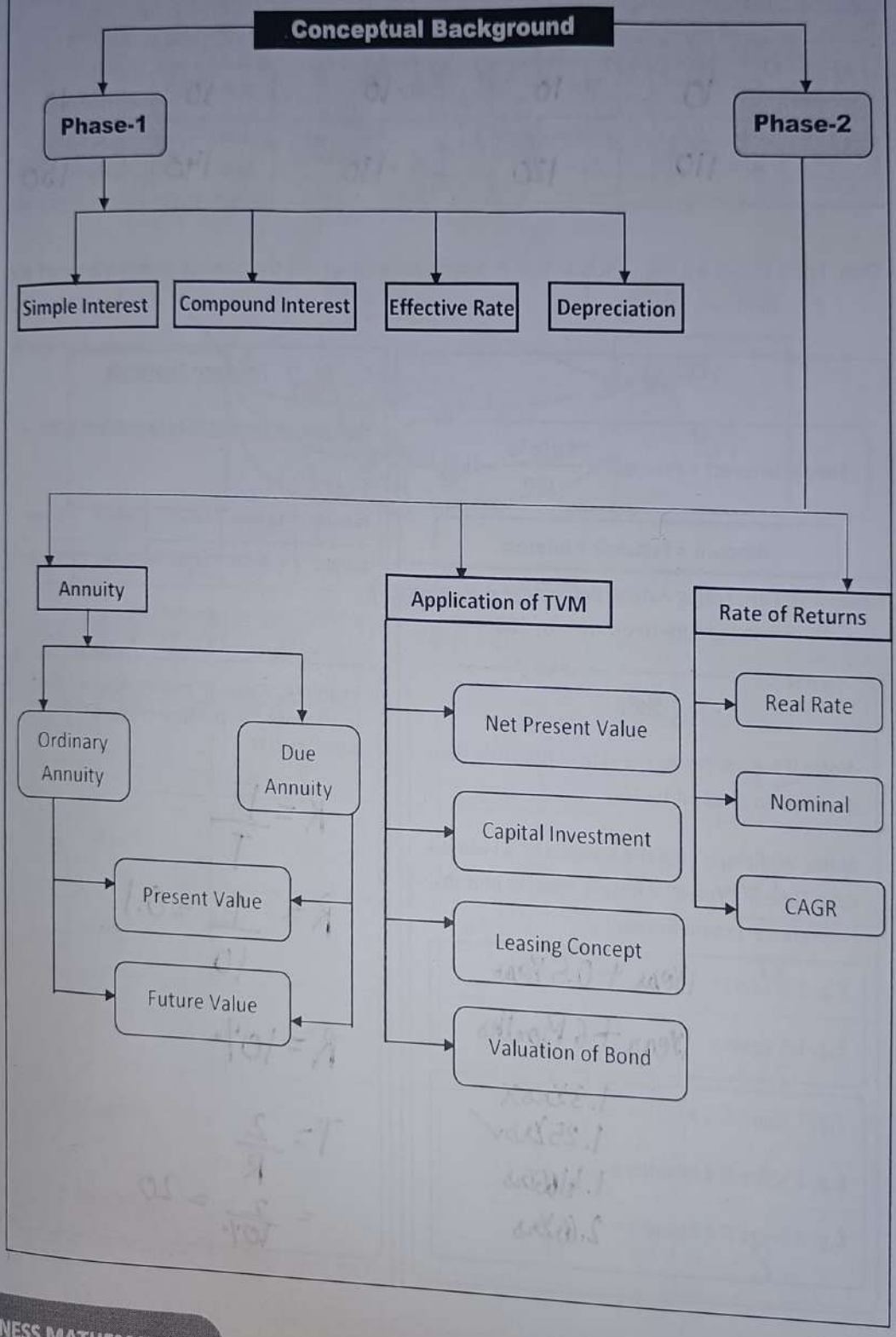
June-22 **Diagram**



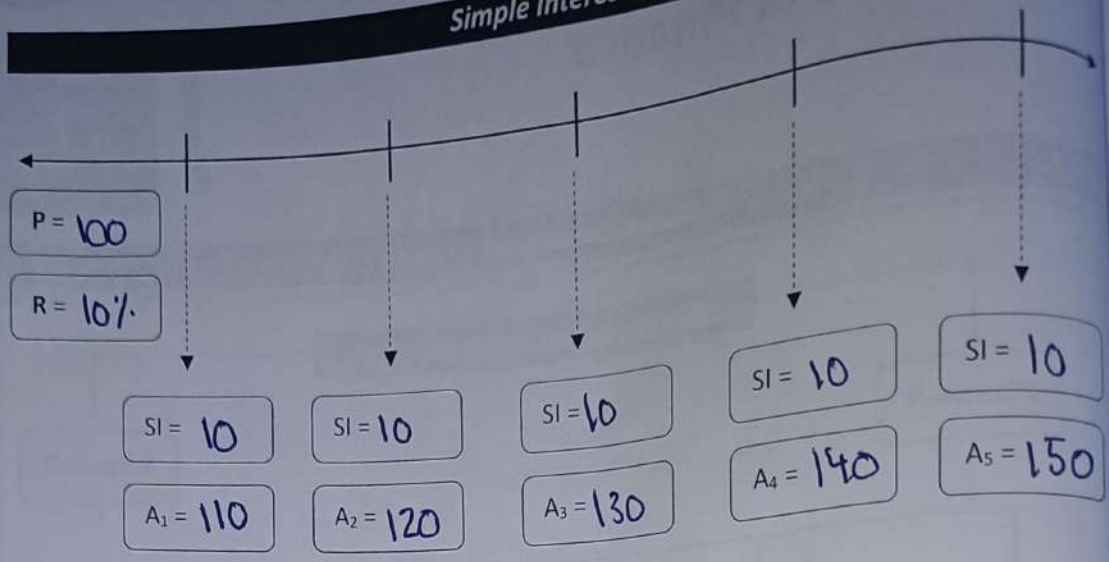
Mathematics of Finance

Chapter 5

Background of Chapter



Simple Interest



Crux: From above we can conclude that in Simple interest we get the interest on same amount (i.e. **Principle)** every year

Formula

$$\text{Simple Interest} = \text{Principle} \times \frac{Rate\ \%}{100} \times \text{Time}$$

Amount = Principle + Interest

Note: if you are taking Rate in Percentage then make sure to divide it by 100

Note

Note: If we are taking the rate in **decimals** then **no need** to divide it by 100

Note: While applying the formula of S.I always take Time in **"Years"** if **time** is given in months, then convert them in Years

E.g. 1.5 Years = 1 Year + 0.5 Year

E.g. 1.5 Years = 1 Year + 6 Months

E.g. 1 Years & 3 Months = 1.3 Yrs X

E.g. 1 Years & 5 Months = 1.25 Yrs ✓

E.g. 2 Years & 8 Months = 1.416 Yrs

E.g. 2 Years & 8 Months = 2.66 Yrs

Advance Shortcuts

How to find Time or Rate to multiple a sum at S.I.

Particular	1.5 Times	Double	Trebled	4 Times
Time Req.	$T = \frac{0.5}{R}$ yrs.	$T = \frac{1}{R}$ yrs.	$T = \frac{2}{R}$ yrs.	$T = \frac{3}{R}$ yrs.

Example: A sum of money doubles itself in 10 years. The number of years it would triple itself is

$$R = \frac{1}{T}$$

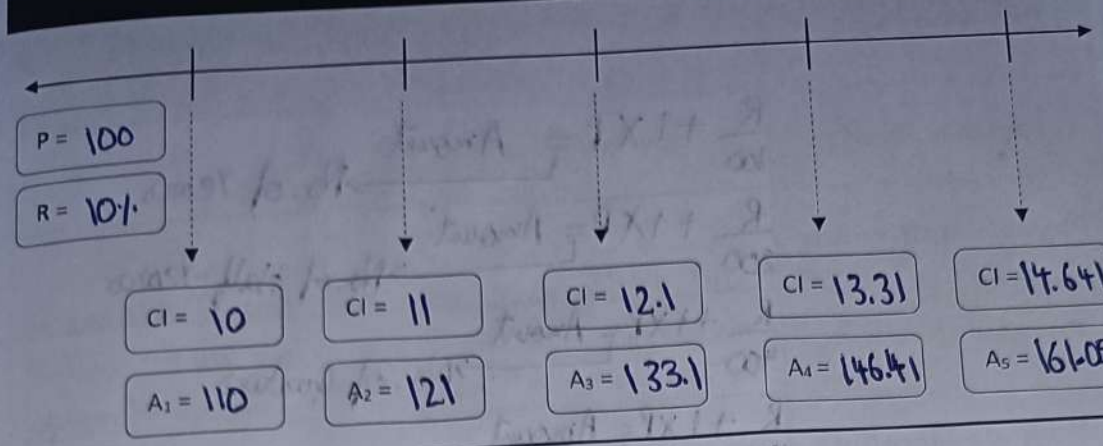
$$R = \frac{1}{10} = 0.1$$

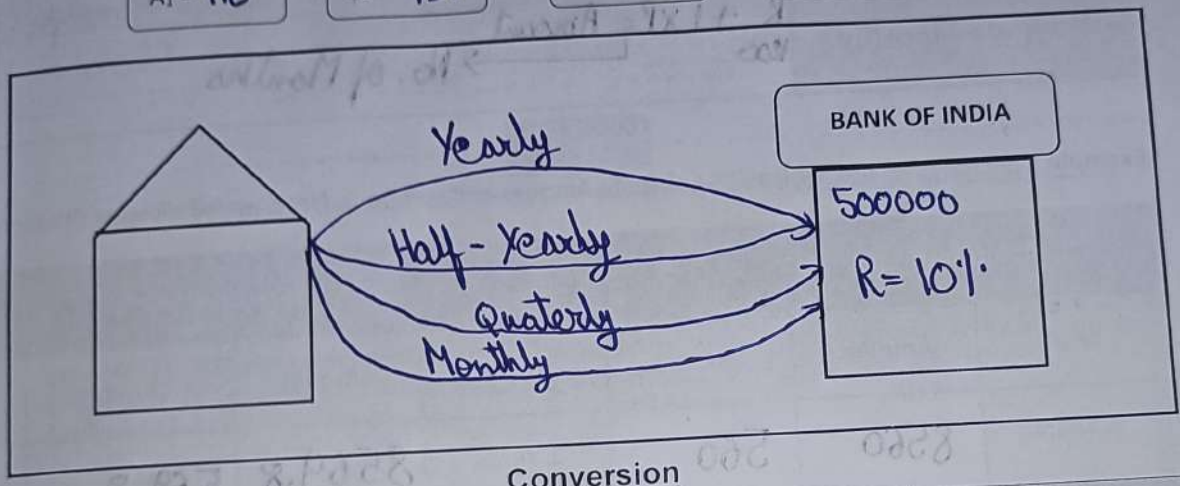
$$R = 10\%$$

$$T = \frac{2}{R}$$

$$= \frac{2}{10\%} = 20$$

Compound Interest





Conversion

Year	Half Year	Quatre	Months
1	2	4	12
2	4	8	24
1.5	3	6	18
2.5	5	10	30
6 Months	1	2	6
3 Months		1	3

Calculator Trick to Compute Compound Interest

Payment Period	Calculator Trick
Payable Yearly	$\frac{R}{100} + 1 \times P =$ Amount → No. of Years
Payable Half Yearly	$\frac{R}{200} + 1 \times P =$ Amount → No. of Half-Years
Payable Quarterly	$\frac{R}{400} + 1 \times P =$ Amount → No. of Quaters
Payable Monthly	$\frac{R}{1200} + 1 \times P =$ Amount → No. of Months

Example A person Invested 8000 at 7% Calculate Amount and compound interest in following situatio

A. Interest is payable Yearly

Years	Amount	Interest
1 Years	8560	560
1.5 Years	8859.6	859.6
2 Years	9159.2	1159.2
2.5 Years	9479.772	1479.772

B. Interest is payable half yearly

Years	Amount	Interest
1 Years	8569.8	569.8
2 Years	9180.1840	1180.1840
1.5 Years	8869.743	869.743
6 Months	8280	280

C. Interest is payable Quarterly

Years	Amount	Interest
1 Years	8574.87	574.87
1.5 Years	8877.61	877.61
6 Months	8282.45	282.45

D. Interest is payable Monthly

Years	Amount	Interest
1 Years	8578.32	578.32
1.5 Years	8882.97	882.97
6 Months	8284.11	284.11

Reverse Approach

Whenever question Demands Anything Backwards of equals to Button (i.e., P, R, T)

⬇

We Will Use Reverse Approach

$$\frac{R}{100} + 1 \times P = \text{Amount}$$

⇓

Whenever Question demands anything Backward of "=" Button → we will use reverse approach i.e. Go from option to question. The option that gives us LHS = RHS that option is correct.

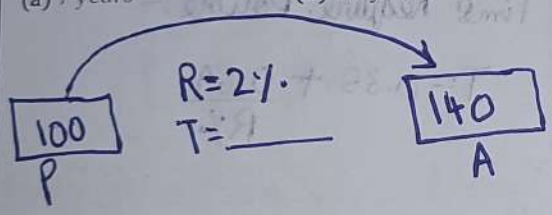
Note

When the Question Does not give you the exact Data what to do?

In that case assume the **Principle = 100** and construct the question according to the information given in the question.

Example: The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is

(a) 7 years (b) 10 years (c) 17 years (d) none



Note

When Years are in Decimal?

Example: The Amount on Rs. 16000 for 11.6 years at 10% p.a. Compounded annually?

Advance Shortcuts of Compound Interest

1 Amounts in C.I Forms GP

$P, A_1, A_2, A_3, A_4 \ldots = G.P$ (ratio r)

$r - 1 = i$

$A_2, A_4, A_6 \ldots = G.P$ (ratio r)

$(r)^{1/n} - 1 = i$

Forward Term of GP: $\times r$

Backward Term of GP: $\div r$

Take R in Decimals

2 Difference Between C.I & S.I

$CI_2 - SI_2 = PR_2$

$CI_3 - SI_3 = PR^2[R+3]$

$$R = \frac{2[CI_2 - SI_2]}{SI_2}$$

Time Require Double :-

$$T = 0.35 + \frac{0.69}{R\%}$$

3 If Rate Keeps on Changing every year?

Differential Rate of Interest: $P + R_1\% + R_2\% + R_3\% \ldots R_n = A_n$

4 L.D.R

If a sum of money amounts to "**m times**" in "**n years**" then "**q years**" it will amount to:

$$A_q = (m)^{q/n}$$

Effective Rate of Interest

Effective rate of interest can be defined as the equivalent annual rate of interest compounded annually if interest is compounded more than once in a year

Calculator Trick to Compute Effective Rate [Base = 1 Yr.]

Payment Period	Trick
Payable Yearly	$\frac{R}{100} + 1 \times 100 = -100 = E.Rate$ → 1 Times
Payable Half Yearly	$\frac{R}{200} + 1 \times 100 = -100 = E.Rate$ → 2 Times
Payable Quarterly	$\frac{R}{400} + 1 \times 100 = -100 = E.Rate$ → 4 Times
Payable Monthly	$\frac{R}{1200} + 1 \times 100 = -100 = E.Rate$ → 12 Times

Note:

Depreciation

Calculator Trick to Compute Depreciation

$[1 - R\%] \times M.V. = Scrap$ → Life of Machine

Reverse Approach - Go from option to Question. To bhi option Dega LHS=RHS, use option correct

When Years are in Decimal?

Example: Machine Value 10,000, Rate of Depreciation 10 % Calculate Scrape at the end of 5.8 years.

Sol:

Closing Value of Machine @ 5th Year = 5904.9 → M+

(-) Depreciation for 0.8 years = (472.392) → M−

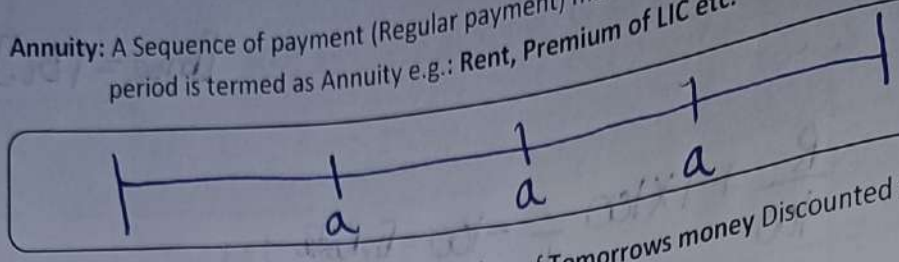
[5904.9 × 10% × 0.8]

Value of Machine after 5.8 Yrs 5432.508 → MRC

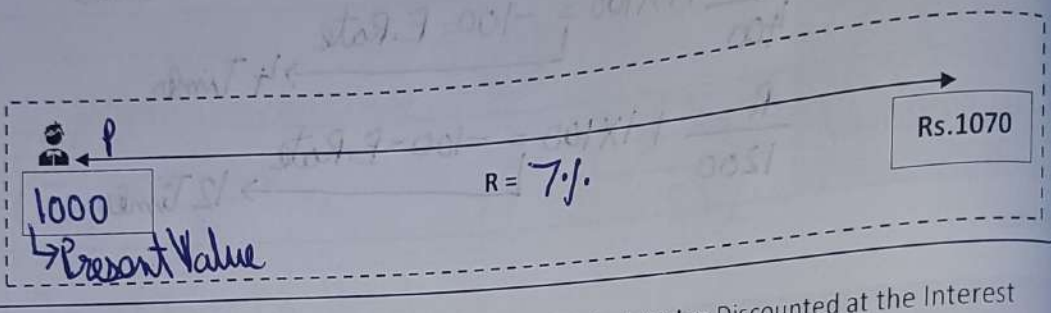
Annuity

Some Important Definition

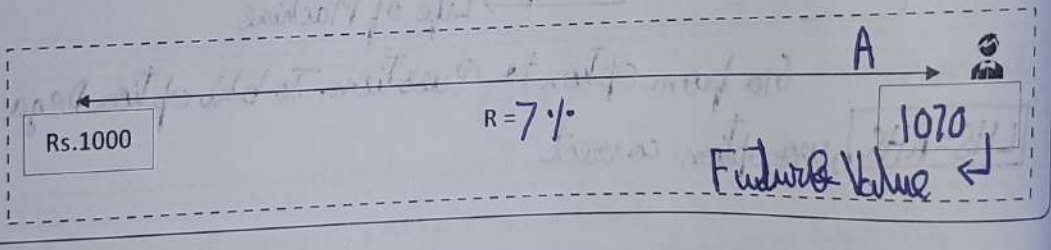
Annuity: A Sequence of payment (Regular payment) made at the end or beginning of periodical period is termed as Annuity e.g.: **Rent, Premium of LIC etc.**



Present Value: Present value is **Today's Value** of **Tomorrows money** Discounted at the Interest Rate.



Future Value: Future value is **Tomorrow's money** of **Today's Value** Discounted at the Interest Rate.



Types of Annuity

Ordinary Annuity

When the Annuity is Paid or Received at the **End** of Periodical term it is termed as Ordinary Annuity

Annuity Due (Starting from Today / Immediate)

When the Annuity is Paid or Received at the **Beginning** of Periodical term it is termed as Annuity Due

Note: If the question is **silent** then we will **Assume** it to be ordinary annuity

Present Value

Calculator Trick to Compute Present Value

Ordinary annuity

Annuity to Present Value

R/100 +1 ÷ = GT X a = PV

No. of Installments

Present Value to Annuity

R/100 +1 ÷ = GT [M+] → PV/MRC = a

No. of Installments

Annuity Due

Annuity to Present Value

1+ = R/100 +1 ÷ = GT X a = PV

N-1

Present Value to Annuity

1+ = R/100 +1 ÷ = GT [M+] → PV/MRC = a

N-1

Note: Use 100 for annual ,200 for Semiannual ,400 for Qtrly, & 1200 for Monthly Compounding

Future Value

Calculator Trick to Compute Future Value

Ordinary annuity

Annuity to Future Value

1+ = R/100 +1X1 = GT X A = FV

N-1

Future Value to Annuity

1+ = R/100 +1X1 = GT [M+] → FV/MRC = A

N-1

Annuity Due

Annuity to Future Value

R/100 +1X1 = GT X A = FV

N

Future Value to Annuity

R/100 +1X1 = GT [M+] = FV/MRC = A

N

Present Value

Example: Given R = 8 % & Time = 3 Years
Compute Following

Ordinary Annuity			
Given A = 500 Find PV?	Yearly 1288.54	Half Yrly 2621.66 ~~3875.54~~	Qtrly 5287.67
Given PV = 10,000 Find A?	3880.33	1907.61	945.59

Annuity Due			
	Yearly	Half Yrly	Qtrly
Given A = 500 Find PV?			
Given PV = 10,000 Find A?			

Future Value

Example: Given R = 8 % & Time = 3 Years
Compute Following

Ordinary Annuity			
Given A = 500 Find FV?	Yearly 1623.2	Half Yrly ~~3866.48~~	Qtrly ~~6706.04~~ ~~4488.56~~
Given FV = 10,000 Find A?	3080.33	~~1363.15~~ 1507.61	745.59

Annuity Due			
	Yearly	Half Yrly	Qtrly
Given A = 500 Find FV?			
Given FV = 10,000 Find A?			

Perpetual Annuity

Formula to Compute Perpetual Annuity

Particulars	Ordinary annuity	Due Annuity
Without Growth	$PV_\infty = \frac{a}{i\%}$	$PV_\infty = a + \frac{a}{i\%}$
With Growth	$PV_\infty = \frac{a}{(i-g)\%}$	$PV = a + \frac{a}{(i-g)\%}$

Note

Annually	Half Yearly	Quarterly	Monthly
r %	$\frac{r\%}{2}$	$\frac{r\%}{4}$	$\frac{r\%}{12}$

Note: As there is No Future date, we **Cannot** calculate **Future value** of perpetuity **only Present value** of Perpetuity can be Computed Known as **Perpetual Annuity**

Example: If money is worth 6 % per annum, find the present value of a perpetuity of Rs.3300 Payable annually.
Sol:

$$PV_\infty = \frac{a}{i\%} = \frac{3300}{6\%} = 55{,}000$$

Example: Assuming that the discount rate is 7% p.a how much would you pay to receive Rs 50 Growing at 5% annually forever?
Sol:

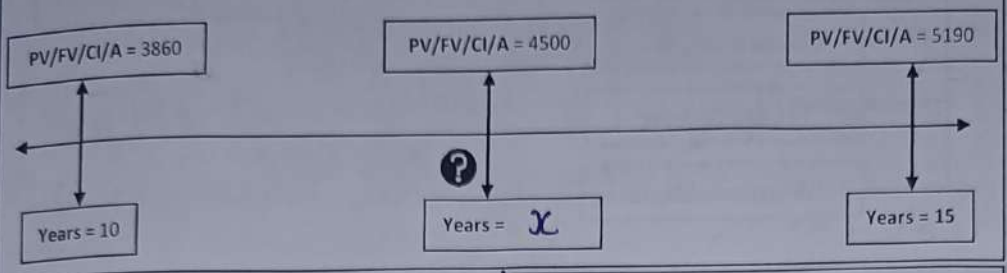
$$PV_\infty = \frac{50}{(7-5)\%} = 2500$$

What if Years are in Decimals (INTERPOLATION TECHNIQUE)

When the **Years are in decimal** then we will apply **Interpolation** techniques, basically interpolation method is used for finding any value between any two known values.

Let's understand it from situation below

Find the value of x?



$$\frac{4500-3860}{x-10} = \frac{5190-4500}{15-x}$$

$$\frac{640}{x-10} = \frac{690}{15-x}$$

$$640[15-x] = 690[x-10]$$

$$9600-640x = 690x-6900$$

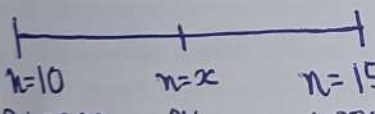
$$9600+6900 = 1330x$$

$$\frac{16500}{1330} = x$$

$$12.40 \text{ Yrs} = x$$

Example: A company borrows Rs. 10000 on condition to repay it with compound interest at 5% p.a. by annual installments of Rs. 1000 each. The number of years by which the debt will be clear is

(a) 14.2 yrs. (b) 10 yrs. (c) 12 yrs. (d) None of these



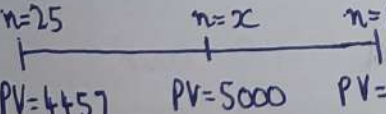
$$\frac{10000-7722}{x-10} = \frac{10380-10000}{15-x}$$

$$\frac{2278}{x-10} = \frac{380}{15-x}$$

$$x = 14.2 \text{ Yrs}$$

Example: Given annuity of Rs. 100 amounts to Rs. 5000 at 4.5% p.a C.I. The number of years will be

(a) 25 yrs. (appx.) (b) 20 yrs. (appx.) (c) 26.2 (d) none



$$\frac{5000-4457}{x-25} = \frac{6101-5000}{30-x}$$

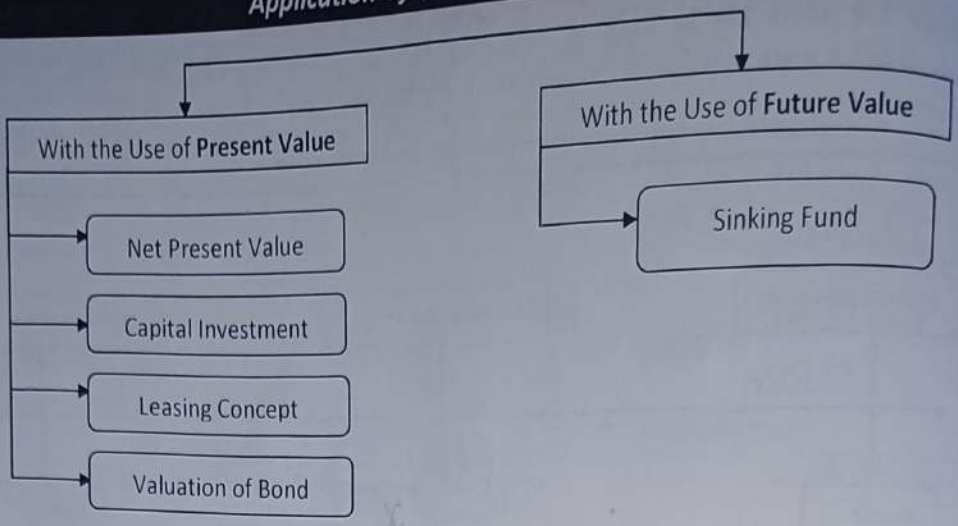
$$\frac{543}{x-25} = \frac{1101}{30-x}$$

$$30-x(543) = x-25(1101)$$

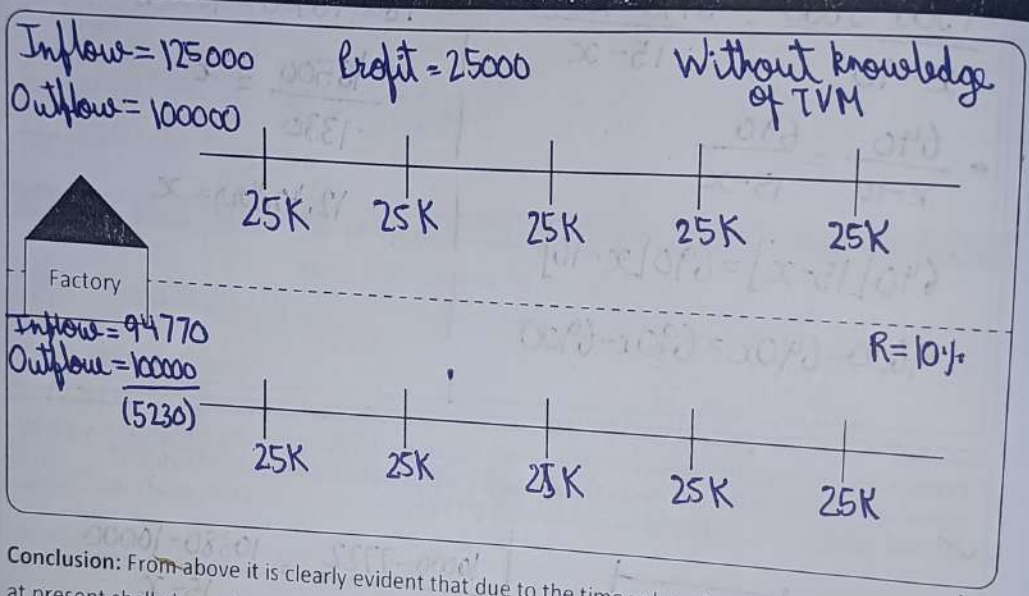
$$16290-543x = 1101x-27525$$

$$43815 = 1644x$$

Application Of Time Value of Money



Net Present Value (NPV)



Conclusion: From above it is clearly evident that due to the time value of money, the value of money at present shall always have more value than the same amount of money in future. Due to Time value of Money, a person would prefer to receive the money at present rather than in future and would like to earn interest on the money held.

Formula to Compute Net Present Value

$$NPV = PVCI - PVCO$$

Present Value of Cash Inflow

Present Value of Cash Outflow

Note

- **Same Cash Inflow** → Annuity to PV Trick
- **Different Cash Inflow** → Table Concept

Present Value of Future Money

PV of FM = Discounting factor of that year × Future Money

Example: Find the present value of Rs.10,000 to be required after 5 years if the interest rate by 9%.

PV of FM = PF @ 5th Year × FM
= 0.6499 × 10000 = 6499.31



Example: Assume cash outflow are Rs. 1,20,000 followed by cash inflows of Rs. 25,000 per year for 8 years and cost of capital is 11% what is the net present value?

Sol:

NPV = PVCI − PVCO
= 128653 − 120000
= 8,653

Example: Assume that ABC Inc is considering two projects namely Project X and wants to calculate the NPV for the project. Project X is four-year project and cash flows for four years are given below:

R = 10%

Year	0	1	2	3	4
Cash Flows	(Rs 10,000)	Rs. 1000	Rs. 3000	Rs. 4000	Rs. 6750

Sol:

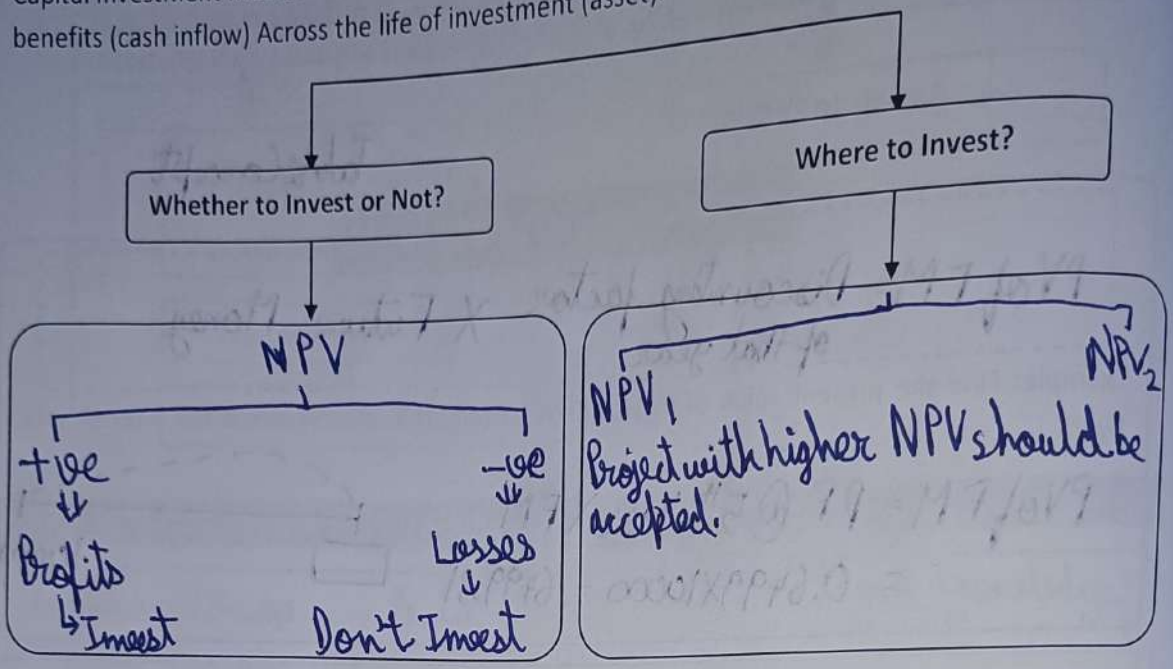
Year	Cash Flows	D. Fact.	PVCI
1	1000	0.9091	909.1
2	3000	0.8264	2479.2
3	4000	0.7513	3005.2
4	6750	0.6830	4010.25
P.V. of CI			11003.75 → MRC

→ MT

NPV = PVCI − PVCO
= 11003.75 − 10000
= 1003.75

Capital Investment

Capital Investment means Purchasing an asset (Which results in Outflow) today in anticipation of benefits (cash inflow) Across the life of investment (asset).



Example: A Project will cost Rs. 1,50,000 followed by cash inflows of Rs. 45,000 per year for 5 years and cost of capital is 12% Should you accept the project?

Sol:

$NPV = PVCI - PVCO$

$= 1,62,215 - 150000$

$= 12215$

Yes, Accept the Project

Example: A machine with useful life of 7 years costs Rs. 10,000 while another machine with useful life of 5 years costs Rs. 8,000. The first machine saves labor expenses of Rs. 1900 annually and the second one saves labor expenses by Rs. 2,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% per annum.

Sol:

$NPV_1 = PVCI - PVCO$

$= 9250 - 10000$

$= (750)$

$NPV_2 = PVCI - PVCO$

$= 8340 - 8000$

$= 340$

Preffered Course of Action is Machine 2.

Leasing Concept

Investor have two alternatives to Occupy particular asset one is **Purchasing option** and another is taking it on **lease**, He have to decide Which option is Preferable to him.

Lessee Point of View
(For Lessee LR are Expense)

If PVLR < Buying Cost

Leasing is Preffered

Lessor Point of View
(For Lessor LR are Income)

If PVLR > Buying Cost

Leasing is Preffered

Example: A company is considering proposal of purchasing a machine either by making full payment of Rs. 4,000 or by leasing it for 4 years at an annual rent of Rs. 1,250. Which course of action is preferable, if the company can borrow money at 14% per annum? [Given: $(1.14)^4 = 1.68896$]

Sol: 3642 < 4000

∴ Leasing is Preffered

Example: A person wants to lease out a machine costing Rs. 5,00,000 for a 10-year period. It has fixed a rental of Rs. 51,272 per annum payable annually starting from the end of first year. Suppose rate of interest is 10% per annum compounded annually on which money can be invested. To whom this agreement is favorable?

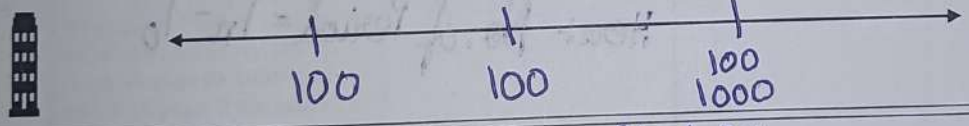
Sol: Cost → 500000

PVLR → 3,15,044

It is favourable for Lessee

Valuation of Bond

A bond is a debt security in which the issuer owes the holder a debt & is obliged to repay principle (at face value of bond) & **Interest.**



Value of Bond = PV of Invest + PV. of RV

a → PV + Disc F. × RV

Example: An investor intends purchasing a three-year Rs. 1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

VOB = PV. of I + PV. of RV

= 232.16 + 674.97

= 907.13

Sinking Fund

Sinking fund is a fund created to accumulate the specified amount of sum by the way of regular periodic payment for some specific purpose

Use trick of PV/FV as Required

Rate of Returns

Nominal Rate of Return

A nominal rate of return is nothing but the total amount of money that is earned from a particular investing activity before taking, inflation, into the due consideration.

NRR = RRR − [−IR]
NRR = RRR + IR

Real Rate of return

The real rate of return is the actual annual rate of return after taking into consideration the factors that affect the rate like inflation.

RRR = NRR + [−IR]
RRR = NRR − IR

CAGR Model

Compound annual growth rate, or CAGR, is the mean annual growth rate of an investment over a specified period of time longer than one year.

Formula Below

Formula

$$CAGR = \left[\left[\frac{\text{Current Value}}{\text{Initial Value}}\right]^{\frac{1}{\text{No. of Periods}}} - 1\right] \times 100$$

Here:- No. of Period $= T_n - T_0$

Calculator Trick to Compute any Power

$(a)^n$

a → 12 Times, −1 × Power + 1 = → 12 times

Let's Remove Confusion whether Question is of
PRESENT VALUE | FUTURE VALUE | AMOUNT | PRINCIPLE

*1st of all look read the question very carefully and check whether question is of **Phase-1** or **Phase-2***

Phase-1

If Question Consist of **Single** Payment / Receipt

Phase-2 Comes in Discussion Only

Phase-2

If Question Consist of **Multiple** Payment / Receipt **i.e. Annuity**

Principle
- Sum Invested
- Deposit

Amount
- It will become
- Amounts to

Present Value

There is a multiple payment and questions demanding initial deposit.

Future Value

There is multiple payment and question in demanding amount i.e. Bank Balance ↓ Future Value

S.no	Question	Phase	What is Given	What is asked	Answer
1.	The amount of Rs. 150 after 12 years at 3.5% p.a. C.I is	I	Principal	Amount	266.66
2.	The amount of an annuity certain of Rs. 150 for 12 years at 3.5% p.a. C.I is	II	Annuity	Future Value	2190.29
3.	You invest Rs 3000 in a two-year investment that pays you 12% per annum. Calculate the future value of investment	I	3000 → Single Deposit ↓ Principle	Amount	3763.2
4.	What is present value of Rs.1 to be received after two years compounded annually at 10% interest rate?	II	Future Money	Present Value	0.8264
5.	How much amount is required to be invested every year so as to accumulate Rs.300000 at the end of 10 years if the interest is compounded annually at 10%?	II	Future Value	Annuity	18823
6.	How much amount should I deposit today so as to accumulate Rs.300000 at the end of 10 years if the interest is compounded annually at 10%?	II	Future Money	Present Value	115663
7.	Raju buys a house for which he agrees to pay Rs. 5000 at the end of every year for 8 years. If money is worth 12, what is the capital value of the house?	II	Annuity	Present Value	24838

8.	Y bought a TV costing Rs. 13,000 by making a down payment of Rs. 3000 and agreeing to make equal annual payment for four years. How much would be each payment if the interest on unpaid amount be 14% compounded annually?	II	Present Value	Annuity	3432
9.	A certain sum invested at 4% per annum compounded semi-annually amounts to Rs78030 at the end of one year. Find the sum.				72250
10.	At the Beginning of each Period Consisting of 6-months, Rs 500 is deposited into saving account that pays 5% compounded half-yearly. Find the balance in the account at the end of 5 year.	II	Annuity	Future Value	5741.7
11.	Rs 8000 at 10% per annum interest compounded half yearly will become at the end of one year	I	Principle	Amount	8820
12.	Find the present value of Rs 1,00,000 to be required after 5 years if the interest rate be 9%.	II	Future Money	Present Value	64993
13.	Rs 2,500 is paid every year for 10 Years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually?	II	Annuity	Present Value	13040
14.	Assuming that discount rate is 7% p.a. how much would pay to receive Rs 200 growing at 5% annually forever?	II	Perpetual Annuity	Present Value	10000
15.	A company needs Rs 10,000 in five years to replace an equipment. How much should be invested now at an rate of 8% p.a. in order to provide for this equipment?	II	Future Money	Present Value	6805
16	R needs money to pay Rs 5,00,000 in 10 years. He invested a sum in a scheme at 9% rate of compounded half-yearly. How much amount he invested?	II	Future Money	Present Value	207321
17.	If the desired future value after 5 Years with 18% interest rate is Rs 1,50,000, then the present value is	II	Future Money	Present Value	65566

Formula Practice

Compound Interest

$A_n = P(1+i)^n$

$CI = P[(1+i)^n - 1]$

Effective Rate of Interest

$ER = (1+i)^n - 1$

Present Value

$$PV = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

Future Value

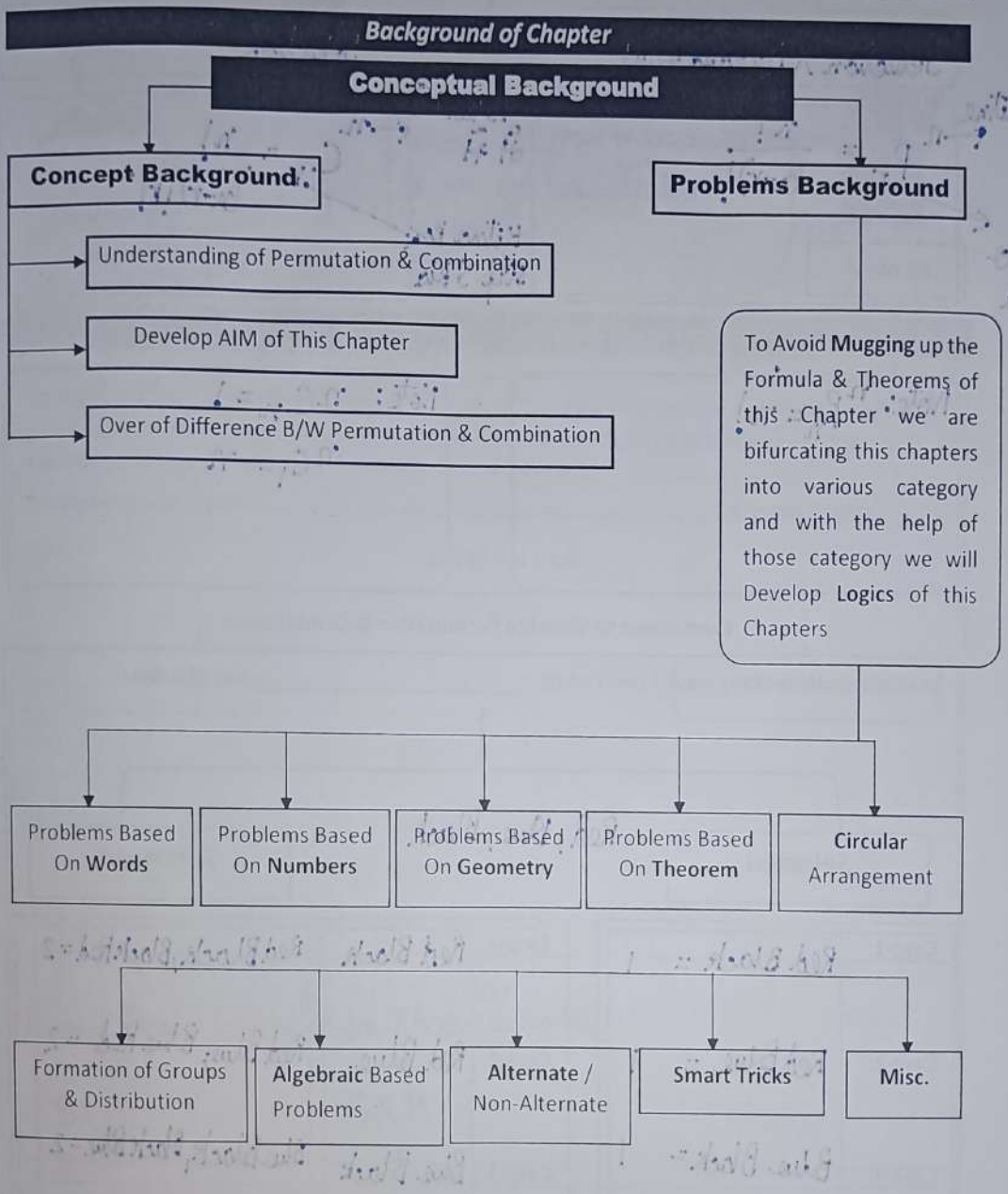
$$FV = \frac{a}{i}\left[(1+i)^n - 1\right]$$

Important Conclusions

1. Sinking Fund Factor is reciprocal of present value interest factor of an annuity.
2. Present Value of Annuity Due > Present Value of ~~Ord~~ Ordinary due
3. F.V. of Ordinary Annuity > FV of Annuity due

Permutation & Combination

Chapter 6



Aman Khedia

$0! = 1$, $1! = 1$

Understanding of Permutation & Combination

Permutation

Selection X Arrangement

Total Kitne ki ?? ← n

Kitno ko SXA kia ← $P_r = \frac{n!}{(n-r)!}$

Combination

Selection

Total Kitne ki ?? ← n

Kitno ko sirf S kia ← $C_r = \frac{n!}{(n-r)!r!}$

Point to Remember

Note: ${}^nP_n = n!$

Note: ${}^nC_n = 1$

${}^nC_1 = n$

Experiment to Visualize Permutation & Combination

Example: In How Many ways 2 pen Can be _____ out of 3 Pens?

Red, Blue, Black

Selected

Case-1: Red Black = 1

Case-2: Red Blue = 1

Case-3: Blue Black = 1

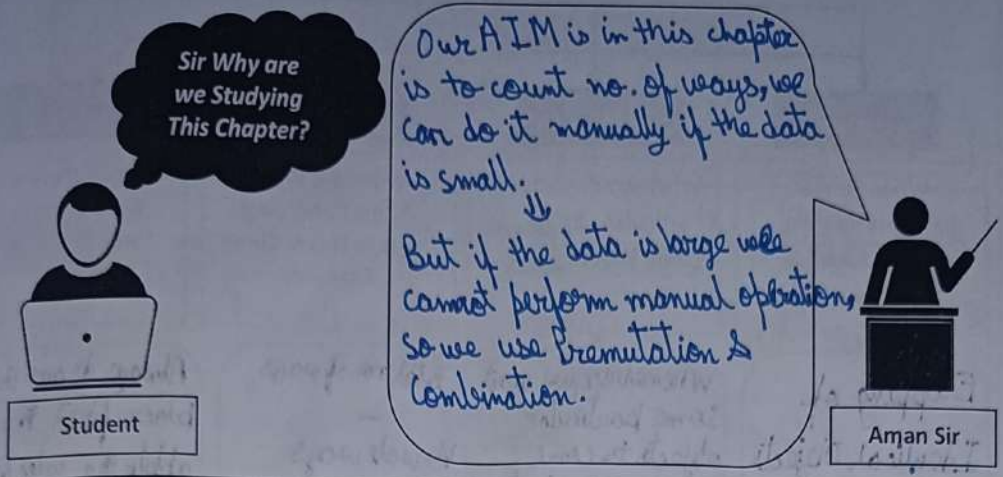
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Arranged

Case-1:	Red Black	Red Black, Black Red = 2
Case-2:	Red Blue	Red Blue, Blue Red = 2
Case-3:	Blue Black	Blue Black, Black Blue = 2

6

Clarification Note

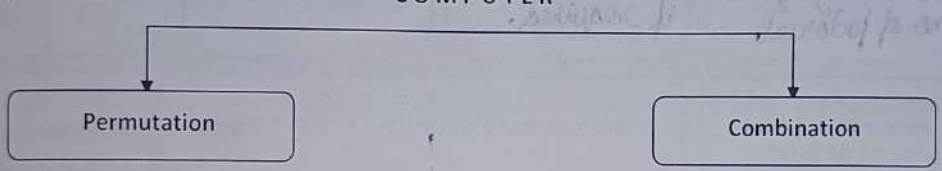


Over of difference Between Permutation & Combination

Students Usually get Confused Later on That **When To apply** Permutation and When to apply Combination When question **Does Not Clearly Use** the Word Like Selection and Arrangement Let's Remove this Confusion with the Help of One Example

Example: How many different Words Can be formed from the letters of the word "**COMPUTER**"?

Sol:



Crux

Permutation = Combination × Arrangement

↓ ↓

S × A S × A

Recommendation

If question is silent whether arrangement is required or NOT, or we are confused, that shall we use arrangement or NOT, we will prefer Combination & if answer doesn't match multiply it with Arrangement.

Problems Based on Words

Logic-1

When **Similar** (Identical) item Occurs **Multiple Times**

Clipping of Identical Objects will not give anything NEW.

↓

So, we will remove these arrangements by dividing by such no. of factorial.

E.g.
Arrangement of Word "A M A N"

$4P_4 = \frac{4!}{2!}$

$= 12$ Ways

Logic-2

When we Want Some Particular Items to **Come Together**

Whenever, we want some particular objects to come together, then put them in Box ☐ count that Box as unit

↓

Apply arrangement inside box **Only** if required.

E.g.
Arrangement of Word "M O B I L E" So that Vowel Come Together

DIE M, B, L

$4P_4 \times {}^3P_3$

$= 4! \times 3! = 24 \times 6$

$= 144$ ways

Logic-3

When we Want Some Particular Items to **Never Come Together**

Total no. of ways
−
No vowels ways
To hame nahi chaiye !

E.g.
Arrangement of Word "M O B I L E" So that Vowel Never Come Together

Total − When Vowels Come Together Ways

$6P_6 - 144$

$= 720 - 144$

$= 576$ Ways

Logic-4

When there is **Restriction** in Some **Particular Places**

Always draw the places first then apply the selection in place having restriction & then multiply them to get them arranged

E.g.
Arrangement of Word "S U N D A Y"
So that Word Start with S but Does not End with N

↓	↓	↓	↓	↓	↓
1	4	3	2	1	4

96 Ways

Logic-5

C-1 When there are **Many Alphabets (Items)** but the question Demands to Select & Arrange **Few Only**

C-2 There are identical objects [alike objects]

Case 1: 2 Alike, 2 Alike

Case 2: 2 Alike, 2 Different

Case 3: All 4 Different

E.g., The ways 4 letter can be _____ out of letters of the word **EXAMINATION**?

AA, II, NN E, X, M, TO

Selecting

Case-1	2 Alike, 2 Alike	${}^3C_2 = 3$
Case-2	2 Alike, 2 Diff	${}^3C_1 \times {}^7C_2 = 63$
Case-3	All 4 Different	${}^8C_4 = 70$
	Total	136

Arranging

Case-1	2 Alike, 2 Alike	${}^3C_2 \times \frac{4!}{2!\times 2!} = 18$
Case-2	2 Alike, 2 Diff	${}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$
Case-3	All 4 Different	${}^8C_4 \times 4! = 1680$
	Total	2454

Clarification Note

When Question Demands Some Particular Objects to Comes in **ODD/EVEN** Place you can apply **Logic no-4** but it will be time taking so prefer to Select & Arrange them in Particular Places Using ${}^nP_r$ & ${}^nC_r$

E.g., If the letters of word **DAUGHTER** are to be arranged so that vowels occupy the odd places then number of different words are

DAUGHTER

J — J — J — J —

${}^4P_3 \times {}^5P_5 = 2,880$

Clarification Note

When Question Demands **Rearrangements** of Objects not arrangement (It Means Question is Demanding all The Possible Arrangements Other Then the Way Which is already given, so in that Case Eliminate **(By Subtracting 1)** that Only if it is Included.

E.g., The number of ways the letters of the word COMPUTER can be rearranged is:

${}^8P_8 = 8! - 1 = 40320 - 1 = 40319$

Problems Based on Number

Logic-6

With/Without Restriction

While solving the problems based on numbers, always draw the places 1st. Then apply selection in each individual places, multiply them to get them arranged.

Example: Given Digits 2,3,4,0,5,7,8

1. How many 4 digit no can be formed?

$\downarrow\ \downarrow\ \downarrow\ \downarrow$

$6 \times 6 \times 5 \times 4 = 720$

2. How many 4 digit no can be formed greater than 5000 can be formed?

$\downarrow\ \downarrow\ \downarrow\ \downarrow$

$3 \times 6 \times 5 \times 4 = 360$

3. How many numbers can be formed between 10 & 1000?

$6 \times 6 = 36$; $6 \times 6 \times 5 = 180$

$36 + 180$

216

4. How many numbers greater than 2000 can be formed using digits 0,1,2,4,5?

$3 \times 4 \times 3 \times 2$; $4 \times 4 \times 3 \times 2 \times 1$

$72 + 96$

168

Logic-7

2 Place Restriction

Jab second place ki selection depend karti hai ki 1st Place par kaun aaya in that case further split the cases

Refer Example Below

5. How many numbers greater than 23,000 can be formed with 1, 2...5? 1,2,3,4,5

Starting with 2 ; Starting with 3,4,5

$1 \times 3 \times 3 \times 2 \times 1$; $3 \times 4 \times 3 \times 2 \times 1$

$18 + 72$

90 ways

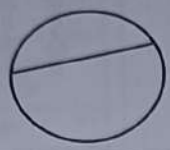
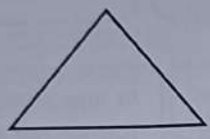
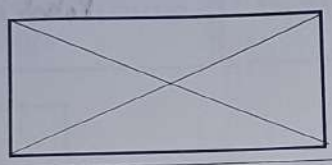
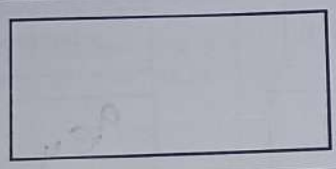
Conclusion

Note: If digits are not given in the question, then we will consider all are available, 0,1,2,3,4,5,6,7,8,9.

Note 2: If question is silent, then we will assume repetition not allowed.

Problems Based on Geometry

Basics

Geometry	Diagram	Requirement
Straight Line		2 Points
Chord		2 Points from circumference of circle.
Triangle		• 2C, 1NC • 2NC, 1C • 3NC We don't get the Δ when all 3 points are collinear
Diagonal		We need 2 Corner Points (→) No. of Sides
Parallelogram		• 2-11 lines on x-axis & • 2-11 lines on y-axis

Conclusion

While solving problems based on geometry, we will always prefer to apply "Combination" because flipping of some diagram will not give us anything NEW.

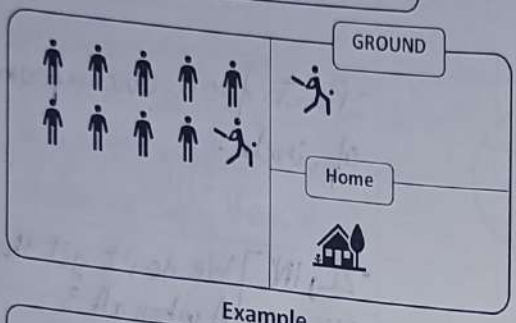
Logic-8

No of Ways in Geometry → Total C Requirement ↓ Requirement to form Geometry.

Problems Based on Theorem

Logic-9

When we want particular person/things to **always occur**



Example

In how many ways a team of 4 person can be formed out of 10 person so that one particular person (Crayonk) **Always occur** in a team?

- Selection: $^9C_3$
- Arrangement: $^9C_3 \times 4!$

Example

$^9C_3$

$\frac{9!}{6! \times 3!}$

$\Downarrow$

84

$84 \times 4!$

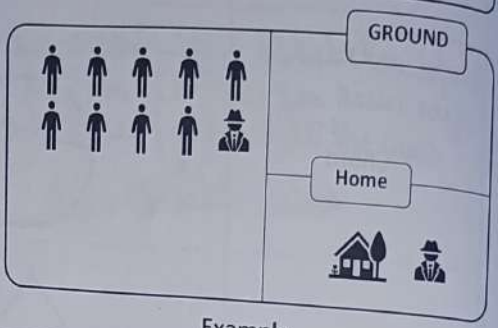
84×24

$\Downarrow$

2016

Logic-10

When we want particular person/things to **Never always occur**



Example

In how many ways a team of 4 person can be formed out of 10 persons so that one particular person (Rahul) **Never Always occur** in a team?

- Selection: $^9C_4$
- Arrangement: $^9C_4 \times 4!$

Example

$^9C_4$

$\frac{9!}{5! \times 4!}$

$\Downarrow$

126

$126 \times 4!$

126×24

$\Downarrow$

3024

Selection of Alike Objects

Logic-11

Alike Objects 

Total No of Ways of Selecting of **ZERO OR MORE** From "n" ***alike things*** [**NO Condition of (at least One)**]	No of Ways of Selecting of **ONE OR MORE THAN ONE** From "n" ***alike things*** [**Condition of (at least One)**]
"n+1" Ways	"n" ways

Example: In how many ways can zero or more letters be selected from the letters AAAAA

Solution: (5+1) = 6 Ways

Example: In how many ways can One or more letters be selected from the letters AAAAA

Solution: 5 ways

Multiple Alike Objects

The total number of ways in which a selection can be made of p+q+r things of which p are alike, q are alike, r are alike is

L.O.R

Total Ways (Zero or More Things)	No of Ways (One or More Things)
(P+1)(Q+1)(R+1)	[(P+1)(Q+1)(R+1) − 1]

Example: From 5 apples, 4 oranges and 3 mangoes how many selections of fruits can be made?

(a) 120 (b) 119 (c) 118 (d) None

Solution:

P = 5
Q = 4
R = 3

[(P+1)(Q+1)(R+1) − 1]
= 6 × 5 × 4 − 1
= 120 − 1 = 119 Ways

Selection of Different Objects

Logic-12

Different Objects



Total No of Ways of Selecting of **ZERO OR MORE** From "n" ***Different things*** [**NO Condition of (at least One)**]	No of Ways of Selecting of **ONE OR MORE THAN ONE** From "n" ***Different things*** [**Condition of (at least One)**]
${}^4C_0+{}^4C_1+{}^4C_2+{}^4C_3+{}^4C_4 \Rightarrow 2^n$	${}^4C_1+{}^4C_2+{}^4C_3+{}^4C_4 \Rightarrow 2n-1$

Example: In how many ways can zero or more mobiles can be selected from 4 different mobile phones?

Solution:

$2^4 = 16$

Example: In how many ways can One or more mobiles can be selected from 4 different mobile phones?

Solution:

$2^4 - 1 = 15$ Ways

Example: An examination paper with 10 questions consists of 6 questions in mathematics and 4 questions in statistic part. At least one question from each part is to be attempted in how many ways can this be done?

Solution:

10 → 6(M), 4(S)

$M \rightarrow 2^6 - 1 = 63$

$S \rightarrow 2^4 - 1 = 15$

$63 \times 15 = 945$ Ways

Combined Concept of Alike & Different Objects (Once in a bluemoon)

Number of ways of selecting things from **p identical things** of one type, **q identical things** of one type and **r identical things** of one type and **n different things** is given by L.D.R

Total Ways (Zero or More Things)	No of Ways (One or More Things)
$(P+1)(Q+1)(R+1)2^n$	$[(P+1)(Q+1)(R+1)2^n - 1]$

Example: There are 3 Math's Books, 2 Accounts Book & 1 Book of Law & Economics each in how many ways one or more books can be selected?

$(3+1)(2+1)2^2 - 1 = 48 - 1 = 47$ Ways

Problems Based on Grouping & Distribution

Logic-13
Grouping

Number of ways of dividing (p + q + r = n) **distinct things** into **Groups** of size p, q and r

$\frac{n!}{p!\,q!\,r!\,m!}$

Extra Factorial
When there are equal groups.
No. of eq. Groups

Logic-14
Distribution

Fixed Distribution
Number of ways of Distributing (p + q + r = n) **distinct things** among **m persons** (Say 3 Persons) So that they get "p things" "q things" and "r things"

$\frac{n!}{p!\,q!\,r!\,\boxed{m!}} \times {}^{n}P_{r}$

$\downarrow$

$G \times A = 0$

Not a Fixed Distribution
Number of ways in which **"n distinct"** Objects (Say Balls) can be distributed among **"r distinct"** Persons (Say Boxes).

Each Person Can Get "Zero" of "n things"
Box can be Empty

$(r)^n$

Each Person Can Get "at least" 1 of "n things"
Box cannot be Empty

$(r)^n - [{}^{r}C_1(r-1)^n - {}^{n}C_2(r-2)^n + \ldots$ Till ${}^{r}C_r$ Equal

Clarification Note

Till Now **ICAI** has Given **very less Questions** on this Category (in Module, PYQ & Other ICAI Sources Like MTP etc.) and Students apply one general logic on all the questions and they don't get the answer and latter on they get confused & Panic.

We Have Covered a Scenario When Objects & Persons are Different if they are Identical then above concept will not apply and lots of Cases can be there which has not been asked yet.

June-2022

SOLID CONFUSION

Sometimes Students Get **Solid Confusion** When to Thing About **(r^n)** think about it Only there is distribution of (**"n"** objects) among (**"r"** Persons) else don't apply it (Apply Normal Logics & **${}^{n}P_{r}$ or ${}^{n}C_{r}$**)

Example: There are 5 questions each have four options. Then in how many different ways can we answer the questions?

(a) 20 (b) 120 (c) 1024 (d) 60

$4 \times 4 \times 4 \times 4 \times 4 = 1024 \Rightarrow (4)^5$

S.no	Question	Group/Dist	Type	Answer
1.	In how many can a pack of 52 cards be divided equally into four group?	Grouping	Equal	$\frac{52!}{13!\times13!\times13!\times13!\times4!}$
2.	In how many ways can a pack of 52 cards be divided equally among four players in order?	Distribution	Fixed	$\frac{52!}{13!\times13!\times13!\times13!\times4!}\times4!$
3.	The number of ways in which 12 students can be equally divided into three groups is	Grouping	Equal	$\frac{12!}{4!\times4!\times4!\times3!}$
4.	The number of ways in which 15 mangoes can be equally divided among 3 students is	Distribution	Fixed	$\frac{15!}{5!\times5!\times5!\times3!}\times3!$
5.	In how many ways can 7 departments be distributed among three ministers if every minister gets at least one but not more than 3 departments?	Distribution	Not Fixed	2,2,3 $\frac{7!}{2!\times2!\times3\times2!}\times3!$ = 630 + 1,3,3 $\frac{7!}{1!\times3!\times3!\times2!}\times3!$ = 336 ⇓ 966
6.	**Five balls** of different colors are in be placed in **three boxes** of different sizes. In how many they can be arranged if each box can have zero or all the 5 balls? **ICAI Module** (a) 243 (b) 120 (c) 150	Distribution	Not Fixed	$(r)^n = (3)^5$ = 243 Ways
7.	**Example:** In how many ways can 9 letters be posted in 4 letter boxes? **ICAI (AQB)** (a) 4^9 (b) 4^5 (c) ${}^9P_4$	Distribution	Not Fixed	$(r)^n \rightarrow (4)^9$ = 262144
8.	**Example: Five balls** of different colors are in be placed in **three boxes** of different sizes. Each box can hold all the five balls. In how many different ways can we place the balls **so that no box remains empty?** **ICAI Module** (a) 100 (b) 120 (c) 150	Distribution	Fixed Not	$(3)^5 - [{}^3C_1(2)^5 - {}^3C_2(1)^5 + {}^3C_3(0^5]$ ⇓ 243 − [96 − 3 + 0] 243 − 93 = 150 Ways

Problems on Alternate & Non-Alternate

Logic-16

Non-Alternate

Arrangement of 2 kinds of things (say Boy & Girl) In such a way that question Demand any **One Particular Kind (Say Boys Never come together)** to never come together

Plot the quantity which is more in number i.e Boys = 6, Now count the places for girls, include both the corner places & apply arrangement for them.

Example

In how many ways 6 boys and 5 girls can be arranged in a row so that **no two girls** are together?

$B_1\ B_2\ B_3\ B_4\ B_5\ B_6$

$^6P_6 \times {}^7P_5$

Logic-17

Alternate

Arrangement of 2 kinds of things (say Boy & Girl) In such a way that question Demand **Both the Kinds** (No two boys & girls) to never come together

L.D.B

Diff. Qty	Same Qty.
6 → Boys 5 → Girls	5 → Boys 5 → Girls
Don't consider any corner places.	Consider anyone corner place [Ans] × 2

Example

In how many ways 6 boys and 5 girls can be arranged in a row so that **no two boys and no two girls** are together?

$B_1\ B_2\ B_3\ B_4\ B_5\ B_6$

$^6P_6 \times {}^5P_5$

Example

In how many ways 5 boys and 5 girls can be arranged in a row so that **they occur alternatively?**

$B_1\ B_2\ B_3\ B_4\ B_5$

$(^5P_5 \times {}^5P_5) \times 2$

Problems on Algebraic Equations

Combination

1. $^nC_r = {}^nC_{n-r}$
2. Pascal Law: $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
3. $^nC_r = {}^nC_s$
   - $\therefore r = s$
   - $\therefore r+s = n$

Permutation

1. XXX
2. XXX
3. $^nP_r = {}^nP_s$
   - $\therefore r+s = 2n-1$

Important Algebraic Ratio

4. $\frac{^nP_r}{^nC_r} = r!$

5. Not asked in exam usually
   - $\frac{^nC_r}{^{n+1}C_{r+1}} = \frac{r+1}{n+1}$
   - $\frac{^nC_r}{^{n-1}C_{r-1}} = \frac{n}{r}$

6.
   - $\frac{^nC_r}{^nC_{r+1}} = \frac{r+1}{n-r}$
   - $\frac{^nC_r}{^nC_{r-1}} = \frac{n-r-1}{r}$

NOTE
LCM: Factorial of Greatest Integer

NOTE
HCF: Factorial of Smallest Integer

Clarification Note

If question demand value of one variable, then we can also go from option to question i.e. LHS = RHS

Que: If $^nP_{13} : {}^{n+1}P_{12} = 3:4$ then the Value of 'n' will be:

(a) 13 (b) 15 (c) 18 (d) 31

$$\frac{^{15}P_{13}}{^{16}P_{12}} = \frac{\frac{15!}{2!}}{\frac{16!}{4!}} = \frac{15!}{2!} \times \frac{4!}{16!} = \frac{15! \times 24}{2 \times 16 \times 15!} = 0.75$$

6.13

Problems on Circular Permutation

Logic-18

Person

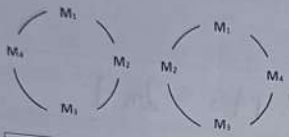
$(n-1)!$

Logic-19

Necklace

$\frac{(n-1)!}{2}$

Special Case



Clockwise | Anticlockwise

In Circular Permutation when we do **(n-1)!**

It Counts Everything Clockwise as well as anticlockwise, It Means Everything is Counted **2 Times**

And if the Question Demands **Different neighbours,** it means we want either only clockwise or only anticlockwise

$\frac{(n-1)!}{2}$

E.g., The number of ways in which 7 girls form a ring is

$(7-1)! = 6! = 720$

E.g., The number of ways in which 8 different beads be strung on a necklace is

$\frac{(8-1)!}{2} = \frac{7!}{2} = 2,520$

Clarification Note

Apply arrangement first in those, who are not having any kind of restriction.

E.g., In how many ways 6 men and 6 women sit at a round table so that no two men are together?

$(6-1)! \times 6!$

↓

We will not do $-(-1)$ because Circle is already formed

E.g., In how many ways 4 men and 3 are arranged at a round table if the women's never sit together

$(4-1)! \times {}^4P_3$

$3! \times 24$

144 Ways

E.g., In how many ways 6 men can sit at a round table so that all shall not have the same neighbors in any two occasions?

$\frac{(6-1)!}{2} = \frac{5!}{2} = 60$

Smart Tricks

Ranking of the word in Dictionary

Steps	C	H	A	L	K
Allot ascending order A-Z	2	3	1	5	4
Aage Dekho	1	1	0	1	0
Begin R-L start with 0!	4!	3!	2!	1!	0!
Multiply	24 + 6 + 0 + 1 + 0				
Add +1	=31+1=32				

Sum of the Numbers Formed from Digits

(Sum of Digits) × (n-1)! × [1111] =

If the Digits are **Repeated** then divide by such no of factorial to remove the arrangement of identical objects

Special Case

What if the Given Digits Consist of "0" in?	In that case we have to apply trick as explained below.	Let's understand from the example below

Example: Find the sum of four-digit numbers made by given digits 1,3,3,0?

Let's find out the sum of all the "4 Digit No" [Considering "0" comes in 1st Place]

$= (1+3+3) \times (4-1)! \times 1111$

$= \frac{46662}{2!} = 23331$

(-) Sum of 3 digit no.

$= (1+3+3) \times 2! \times 111$

$= \frac{1554}{2} = (777)$

22554

Number of Divisors

Step-1: Take LCM

Step-2: Write Factors in the form of $(a)^n (b)^q (c)^r \ldots$

Number of Divisors $= (P+1)(Q+1)(r+1) - 1$

ICAI Module

Example: The number of different factors the number 75,600 has is

(a) 100 (b) 121 (c) 119 (d) None of these

Solution: 756×100

$(2)^2 (3)^3 (7)^1 (10)^2$

$(2)^2 (3)^3 (7)^1 (2)^2 (5)^2$

$(2)^4 (3)^3 (5)^2 (7)^1$

$(4+1)(3+1)(2+1)(1+1) - 1$

$120 - 1$

119

2	756	3	21
2	378	7	7
3	189		1
3	63		
3	21		

Other Useful Concepts

Q] How many no.s can be formed between 10 & 1000 using the digits 0, 1, 2, 4, 5, 7 & 8

$6 \times 6 \rightarrow 36$

$6 \times 6 \times 5 \rightarrow 180$

$36 + 180$

$216 - 1 = 215$

Sequence & Series

Chapter 7

Background of Chapter

Conceptual Background

- Definition
- Mind-Map
- Assuming Value Technique
- Advance shortcuts
- Calculator Trick for AP & GP
- Alternate terms in infinite terms of GP
- Divisibility Test
- Special Series
- Relation B/W AM, GM

- Arithmetic Progression: Sequence = ✓, Formula = ✓ ⇓ Formula Based
- Geometric Progression: Sequence = ✓, Formula = ✓ ⇓ Formula Based
- ∞ [MP]: Seq = ✓ For = ?, For = ✓ Seq = ?, For = ✓ For = ?, Invention Based

Definition

Sequence

Anything associated with a common pattern is a Sequence.

H.AB, A.B, Abhi.B .... Sequence

F-S F-S

Series

Sum of all the terms of sequence is termed as series

2+4+6+8..... Series

Mind-Map

Arithmetic Progression (AP)

Nth Term Formula

$N^{th}\ Term = a + (n-1)d$

- $N^{th}\ Term$ → 37th Place par baithne wala person
- n → Put Place

Sum (S_n) Formula

$S_n = \frac{n}{2}[2a + (n-1)d]$

- n → Put Place
- 37th Place tak ka Sum

$S_n = \frac{n}{2}[a + l]$

Geometric Progression (GP)

Nth Term Formula

$N^{th}\ Term = ar^{n-1}$

- $N^{th}\ Term$ → 37th Place par baithne wala person
- $n-1$ → Put Place

Sum (S_n) Formula

$S_n = \frac{a[r^n - 1]}{r-1}$ $r > 1$

$S_n = \frac{a[1 - r^n]}{1-r}$ $r < 1$

$S_\infty = \frac{a}{1-r}$

Calculator Tricks for AP

n^{th} Term of AP in Calculator

$a \pm d = = = = \ldots$ → t_2, t_3, t_4, t_5, t_n

S_n of AP in Calculator

$a + =, \pm d, = = = = \ldots$ GT → $t_1, t_2, t_3, t_4, t_5, t_n$

Calculator Tricks for GP

n^{th} Term of GP in Calculator

$r \times a = = = = \ldots$ → t_2, t_3, t_4, t_5, t_n

S_n of GP in Calculator

$a + =, r \times a, = = = = \ldots$ → $t_1, t_2, t_3, t_4, t_5, t_n$

Shortcut to find Place of any n^{th} Term

Place of Last Term $= \frac{\text{Last Term}}{\text{It should be multiple of}}$

Note

Place of Last Term is No of Numbers

Mind-Map

Crux

N^{th} Term

We put the place in "n" & formula gives us 37th Place par baithne wala person

S_n Term

We put the place in "n" & formula gives us 37th Place tak ka sum.

MP

Sequence or Series is Given & Formula is Asked

N^{th} Term formula is Asked

We will put the place in given options [Say n = 1, 23] and Jo bhi option hame dega 37th place par baithne wala person ⇒ Right

SUM formula is Asked

We will put the place in the given options [Say n = 1, 2, 3] & Jo bhi option hame dega 37th place tak ka sum dega wo option ⇒ right

Formula is Given & Sequence or Series is Asked

N^{th} Term formula is Given

We will put the place in given formula ... say [n = 1, 2, 3 ...] & sequence is invented automatically

$n=1 \rightarrow T_1$ $\quad T_1, T_2, T_3 \ldots\ldots$

$n=2 \rightarrow T_2$

$n=3 \rightarrow T_3$

SUM formula is Given → L.D.R

We will put the place in given formula [Say n = 1, 2, 3 ...] & Sequence is invented as following... $T_1, T_2, T_3 \ldots\ldots$

$n=1 \rightarrow S_1 = T_1$

$n=2 \rightarrow S_2 - S_1 = T_2$

$n=3 \rightarrow S_3 - S_2 = T_3$

One Formula is Given & Another Formula is Asked

It is impossible to jump from One Formula to another Directly .... ⇒ Insert Sequence 1st.

Arithmetic & Geometric Mean

Arithmetic Mean

Numbers inserted between two terms which will construct AP

If a, b, c are in AP then b is AM Between a & b

$$b = \frac{a+c}{2}$$

Insertion of 'n' AM's between 'a' & 'b' $d = \frac{b-a}{n+1}$

Geometric Mean

Numbers inserted between two terms which will construct GP

If a, b, c are in AP then b is GM Between a & b

$$b = \sqrt{ac}$$

Insertion of 'n' GM's between 'a' & 'b' $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

* You can ignore this formula and check option for AP or GP.

Assuming Value Technique

Assuming Value Technique

Sir, we are having high fear of variables.....
Don't worry:- Convert the variables into numbers, in such a way that it satisfies a question's demand.

Note

Don't assume any numbers, which are undefined....
$\sqrt{-ve}$ X $\frac{n}{0}$ X

Example: If p, q and r are in A. P. and x, y, z are in G.P. then $x^{q-r} \cdot y^{r-p} \cdot z^{p-q}$ is equal to?

P=1 Q=2 R=3 x=1 y=2 z=4

$(1)^{-1} \times 2^2 \times 4^{-1}$

$1 \times 4 \times \frac{1}{4} = \frac{4}{4} = 1$

Example: if a, b, c are the p^{th}, q^{th}, r^{th} term of G.P. respectively the value of $a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$ is ____

a = 1, b = 2, c = 4

P=1 Q=2 R=3

$1^{-1} \times 2^2 \times 4^{-1}$

$1 \times 4 \times \frac{1}{4} = 1$

Example: If 'S' be the sum, 'P' the product and 'R' the sum of the reciprocals of n terms in a G.P. then 'P' is the _____ of S^n and R^{-n}.

(a) Arithmetic Mean ~~(b)~~ Geometric Mean (c) Harmonic Mean (d) None

1, 2, 4

S = 7, n = 3 $S^n = 7^3 = 343$

P = 8 $R^{-n} = \frac{1}{(1.75)^3} = 0.1865$

R = 1.75

343, 8, 0.1865

Advance Shortcuts

1

If $t_n = \frac{1}{m}$ & $t_m = \frac{1}{n}$

- $t_{m*n}^{th} = 1$
- $S_{m+n} = \frac{-[m+n+1]}{2}$

Example

$t_2 = \frac{1}{3}$ & $t_3 = \frac{1}{2}$ Then

- $t_6 = 1$
- $S_5 = \frac{-7}{2}$

2

If $S_n = S_m$

- X
- $S_{m+n} = 0$

Example

If $S_{10} = S_{19}$ *then*

- X
- $S_{29} = 0$

3

If $S_n = m$ & $S_m = n$

- X
- $S_{m+n} = -[m+n]$

Example

If $S_2 = 5$ & $S_5 = 2$ then S_7

4

L.D.R

If **ratio** of S_n of 2 A. P's $= \frac{(An+Bn)}{(Cn+Dn)}$

Then ratio of them $t_m = \frac{[A(2m-1)+B]}{[C(2m-1)+D]}$

Replace $n = 2m-1$

Example

Ratio of sum of 2 A.P is $\frac{(n+2)}{(n-1)}$ then ratio of 10^{th} **term**

$T_{10} = \frac{2m-1+2}{2m-1-1} = \frac{2M+1}{2M-2} = \frac{21}{18}$

5

In a **G.P** if $(p+q)^{th}$ term is **m** & $(p-q)^{th}$ term is **n** then

- $p^{th} = \sqrt{mn}$
- X

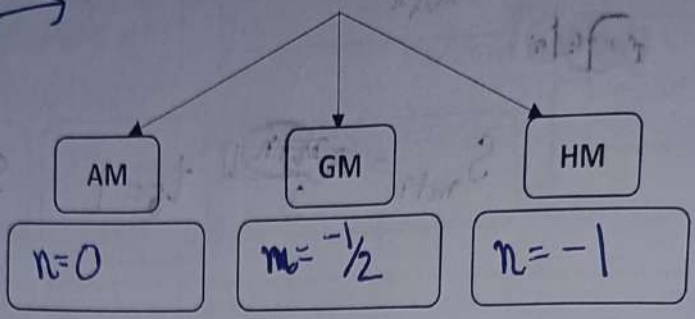
Example

- X
- X

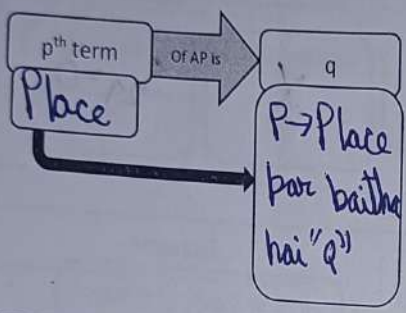
Last asked in year 2009

6 Find the value of "n" if $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is in

Learn →



7 **General Approach to Solve Advance Problems**



Step 1:- 1st of all identify "No. of places" & allot numbers to their places.
[Say:- p^{th} term = q, q^{th} term = p]

$\frac{2}{P=1}$ $\frac{1}{q=2}$

Step 2:- Now allot the nos & form the sequence

Example: If p^{th} term of an AP is q and its q^{th} term is p, then what will be the value of $(p + q)^{th}$ term?

$\frac{2}{P=1}$, $\frac{1}{Q=2}$, $\frac{0}{P+Q=3}$

Example: If p^{th} term of an AP is 1/q and its qth term is 1/p, then what will be the value of $(p+q)^{th}$ term is?

(a) $\frac{1}{2}(pq+1)$ (b) $\frac{1}{2}(pq-1)$ (c) $pq+1$ (d) $pq-1$

$\frac{1/2}{P=1}$, $\frac{1}{Q=2}$, $\frac{1.5}{P+Q=3}$

Example: If p^{th} term of an AP is q and its qth term is p, then value of r^{th} term is?

(a) $p-q-r$ (b) $p+q-r$ (c) $p+q+r$ (d) *None*

$\frac{2}{P=1}, \frac{1}{Q=2}, \frac{0}{R=3}$

L.D.R

Example: The sum of p term of an AP is q and the sum of q term p, The sum of p+q term is?

(a) $-(p+q)$ (b) $p+q$ (c) $(p-q)^2$ (d) p^2-q^2

$d=-3$

$\frac{2}{P=1}, \frac{-1}{Q=2}, \frac{-4}{P+Q=3}$

$S_2=1$

$S_2-S_1=T_2$

$1-2=T_2$

$T_2=-1$

L.D.R

Example: If the p^{th} term of a G.P is x and q^{th} term is y, then the n^{th} term is _____

(a) $\left(\frac{x^{n-p}}{y^{n-q}}\right)^{\frac{1}{p-q}}$ (b) $\left(\frac{x^{n+p}}{y^{n+q}}\right)^{\frac{1}{p-q}}$ (c) $\left(\frac{x^{n-q}}{y^{n-p}}\right)^{\frac{1}{p-q}}$ (d) $\left(\frac{x^{n-q}}{y^{n-p}}\right)^{\frac{1}{p+q}}$

$\frac{1=x}{P=1}, \frac{2=y}{Q=2}, \frac{4}{N=3}$

Example: If S_1, S_2, S_3 are S_n of 3 A. P., where 'a' of all is 1 & 'd' is 1, 2, 3 resp. then $\frac{(S_1+S_3)}{S_2}$ is.

(a) 1 (b) 2 (c) 3 (d) none

S_1 $d=1$ $d=3$ S_2 S_3

AP_1 AP_2 AP_3

1, 2, 3 1, 3, 5 1, 4, 7

$S_1=6$

$S_2=9$

$S_3=12$

$\frac{S_1+S_3}{S_2}=\frac{6+12}{9}$

$=2$

Some Questions of AP & GP

Sometimes Question demands Sum of the Numbers Between two Particular numbers and they form either AP or GP but solving them through formula is a takes time as we have to apply formula 2 times ones to get the **No of Numbers** and then **Formula of Sum** so to save time, we will Go from **CALCULATOR TRICK** as Discussed in AP & GP **MIND-MAP**

Example: The Sum of all the natural numbers between 500 & 1000, Which are divisible by, 13 is_

From	Between
507, 520.....988	
⇓	Same
28,405	

Example: The Sum of all the natural numbers between 100 & 300, Which are divisible by, 4 is___

From	Between
100, 104..... 300	104, 108..... 296
⇓	⇓
10,200	9,800

Example: The Sum of all the natural numbers from 100 & 300, excluding those Which are divisible by, 4 is____

Total Sum	−	Divisible by 4
⇓		⇓
100, 101.....300		100, 104.....300
↓		↓
40,200	−	10200

= 30,000

Example: The Sum of all the natural numbers between 100 & 300, Which are divisible by 5, is__

From	Between
100, 105.... 300	105, 110, ...295
⇓	⇓
8,200	7800

Example: The Sum of all the natural numbers from 100 & 300, Which are divisible by, 4 or 5 is_____

Divisible by 4	Divisible by 5	Divis by 4&5
100, 104...300	100, 105...300	100, 120, 300
⇓	⇓	⇓
10200 +	8,200 −	2200

16,200

Example: The Sum of all the natural numbers from 100 & 300, Which are divisible by, 4 & 5 is_____

2200

(Not will be in exam)

Alternate terms in Infinite terms of G.P & AGP

There is Only One Series whose **Sum upto Infinite** Terms can be computed & that is **G.P**

→ But Sometimes question don't give us **G.P directly** them form **alternate terms in G.P** or any other Patter

→ Seperate the two "G.P" & formula of sum of ∞ G.P. two times & then solve further.

Example: The sum up to infinity of the series
$(1 + 2^{-2}) + (2^{-1} + 2^{-4}) + (2^{-2} + 2^{-6}) + ...$
(a) 7/3 (b) 3/7 (c) 4/7 (d) none

Example: The sum up to infinity of the series
$4/7 - 5/7^2 + 4/7^3 - 5/7^4 + ...$ is
(a) 23/48 (b) 25/48 (c) ½ (d) none

Divisibility Test

Put n=1 → [ ] → Outcome is your answer.
If outcome is "0" then
Put n=2 → [ ] → Outcome is your answer

Example: $2^{4n} - 1$ is divisible by

$2^4 - 1 = 16 - 1 = 15$

Sum of Special Series

S.no	Particulars	Series	Formula
1.	1st n natural	$1 + 2 + 3 + \ldots\ldots + n$	$= \frac{n(n+1)}{2}$
2.	1st n odd number	$1 + 3 + 5 + \ldots + (2n - 1)$	$= n^2$
3.	Squares of the first, n natural nos.	$1^2 + 2^2 + 3^2 + \ldots + n^2$	$= \frac{n(n+1)(2n+1)}{6}$
4.	Cube of the first, n natural nos.	$1^3 + 2^3 + 3^3 + \ldots + n^3$	$= \left\{\frac{n(n+1)}{2}\right\}^2$

Relation Between AM GM & HM

If A.M. & G.M. of 2 nos. is known, the two nos. are: A.M. $\pm \sqrt{(A.M.)^2 - (G.M.)^2}$

If A.M. & G.M. of 2 nos. is in ratio m:n, then nos. are in ratio $(m+\sqrt{m^2-n^2}) : (m-\sqrt{m^2-n^2})$

For any two distinct positive numbers; A.M>G.M>H.M.& $(G.M.)^2 = A.M. \times H.M.$

"K" Concept

Whenever question says Something = Something = Something then assume them equals to "K".

MOTIVE:-

Our motive is to get the values of all the variables in the form of "K" [So that we can apply BODMAS & Indices]

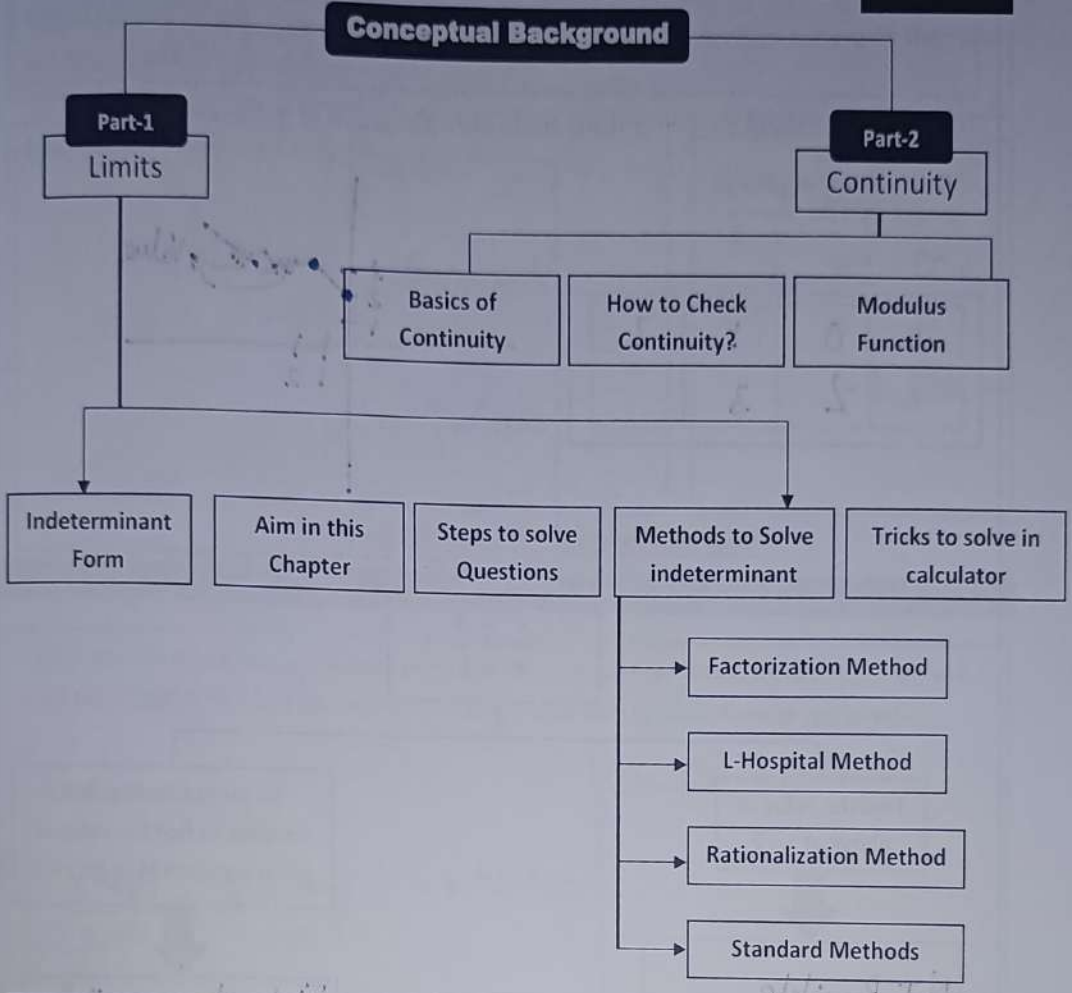
Note: If we want a sequence which is A.P. & G.P. Both assume all terms are same. [eg. 5,5,5,5]

Ref. Q-53

Note: If Question says a^2, b^2, c^2 are in AP then assume $a=1$, $b=5$, $c=7$

Limits and Continuity

Chapter 8C



Indeterminant Form

$\frac{4}{1} = 4$ $\quad\quad$ $\frac{0}{5} = 0$

$\frac{5}{2} = 2.5$ $\quad\quad$ $\frac{0}{0} =$ Undefined

$\frac{7}{3} = 2.\overline{33}$ $\quad\quad$ $\frac{\infty}{\infty} =$ Undefined

There are some forms whose values cannot be computed because and those forms are termed as indeterminant forms (There are total 7 Indeterminant forms)

$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0^0, \infty^0, 0^\infty, 1^\infty$

TIP: $\frac{0}{\text{No.}} = 0$, $\frac{\text{No.}}{\infty} = 0$

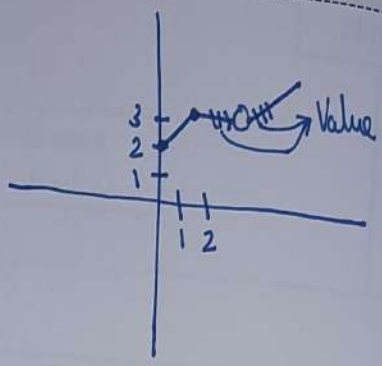
Aim in this Chapter

To find the limit, i.e. to find the value, near about that no. ($x \to 2$)

To better understand the concept lets try to draw the graph of below equation

$$y = \frac{x^2 - 4}{x - 2}$$

X	0	1	2
y	2	3	



$$\frac{x^2 - 4}{x - 2}$$

Find the value of above at x = 2

Not Possible

As we can see it is **Not Possible** to find the value of given equation at exact x = 2

Find value near "2"

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

Let's Conclude the AIM from above

Limit will exsist only if following condition is true

LHL = RHL

$f(a^-) = f(a^+)$

Steps to Solve Questions of Limit

Step-1: 1st of all check whether the question forms the indeterminable form of not, if not then no need to think about any method, value computed is only your answer.

Step-2: Give 1st preference to **L-HOSPITAL** Method and if that method not works then go for any other Method or Trick

Example: $\lim_{x\to 2}(3x+2)$

$3(2)+2$

$=8$

Example: $\lim_{x\to -2}\frac{x^2-4}{x+2}$

Undefined

Methods

Example: $\lim_{x\to\infty}\frac{3}{x^2}+2$

$\frac{3}{\infty}+2=0+2$

$=2$ Ans

Methods to Solve Indeterminant form

Factorization Method

Under this method, we will **Factorize** the numerator and denominator and then we will **CUT** the **COMMON FACTOR** and we will find that indeterminant form is removed

Example: $\lim_{x\to 2}\frac{x^2-4}{x-2}$

$\frac{x^2-(2)^2}{x-2}=\frac{(x+2)(x-2)}{(x-2)}=x+2=4$ Ans

Calculator Trick

Example: $\lim_{x\to 2}\frac{2x^2-7x+6}{5x^2-11x+2}$

~~$2x^2-4x$~~

Calculator

Example: $\lim_{x\to 4}\frac{x^2-16}{x-4}$

$\frac{x^2-(4)^2}{x-4}$

$\frac{(x+4)(x-4)}{(x-4)}$

$4+4=8$

Calculator Trick

L-Hospital Method

This method is **applicable** only when indetermination is of the form $\frac{0}{0}, \frac{\infty}{\infty}$

Under this Method we will **DERIVATE** numerator and denominator respectively and we will find that indeterminate form is removed

Example: $\lim_{x \to 2} \frac{x^2-4}{x-2}$

$\frac{2x}{1} = \frac{4}{1} = 4$ Ans

Example: $\lim_{x \to 1} \frac{x^3-5x^2+2x+2}{x^3+2x^2-6x+3}$ is =

$\frac{3x^2-10x+2}{3x^2+4x-6} = \frac{-5}{1} = -5$ Ans

Rationalization Method

If the question consists of **IRRATIONAL** part either in Numerator or Denominator in that case, we will **RATIONALIZE** it and we will that indeterminable form is removed.

How to rationalise?

Example: $\frac{\sqrt{a}+\sqrt{b}}{c}$

$\frac{\sqrt{a}+\sqrt{b}}{c} \times \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}} =$

TIP:- $(a-b)(a+b) = a^2-b^2$

$(\sqrt{a})(\sqrt{a}) = a$

Example: $\lim_{x\to 0} \frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2} =$

$$\frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2} \times \frac{\sqrt{1+2x^2}+\sqrt{1-2x^2}}{\sqrt{1+2x^2}+\sqrt{1-2x^2}}$$

$$= \frac{1+2x^2-[1-2x^2]}{x^2\left[\sqrt{1+2x^2}+\sqrt{1-2x^2}\right]}$$

$$\frac{1+2x^2-1+2x^2}{x^2\left[\sqrt{1+2x^2}+\sqrt{1-2x^2}\right]}$$

$$\frac{4x^2}{x^2\left[\sqrt{1+2x^2}+\sqrt{1-2x^2}\right]}$$

$$\frac{4}{1+1} = \frac{4}{2} = \underline{2 \text{ Ans}}$$

Trick to Solve Question in just 10 Sec

Put $x = 0.01$

$$\frac{MRC}{0.0001} = \underline{2 \text{ Ans}}$$

Standard Function

1.	$\lim_{x\to 0} \frac{e^{f(x)}-1}{f(x)}$	1
2.	$\lim_{x\to 0} \frac{a^{f(x)}-1}{f(x)}$	$\log a$
3.	$\lim_{x\to 0} \frac{\log[1+f(x)]}{f(x)}$	1
4.	1^{∞}	
5.	$\lim_{x\to a} \frac{f(x)^n - a^n}{f(x)-a} = \frac{x^n-a^n}{x-a}$	$n(a)^{n-1}$

How to solve Indeterminant of the form 1^{∞}

$$\lim_{x\to\infty}[f(x)]^{g(x)} = e^{[f(x)-1]\,g(x)}$$

Example: $\lim_{x\to\infty}\left(1+\frac{9}{x}\right)^{x}$

$$e^{\left(1+\frac{9}{x}-1\right)x} = e^{\frac{9}{x}\times x}$$

Examples on Standard Function

1.	$\lim_{x\to0}\frac{e^{x}-1}{x}$	1
2.	$\lim_{x\to0}\frac{e^{2x}-1}{x}$	$\frac{e^{2x}-1}{2x}\times 2 = 1\times 2 = 2$
3.	$\lim_{x\to0}\frac{2^{x}-1}{x}$	$\log 2$
4.	$\lim_{x\to0}\frac{5^{3x}-1}{x}$	$\frac{5^{3x}-1}{3x}\times 3 = \log 5\times 3 = 3\log 5$
LDR 5.	$\lim_{x\to0}\frac{5^{x}+3^{x}-2}{x}$	$\frac{5^{x}+3^{x}-1-1}{x} = \frac{5^{x}-1}{x}+\frac{3^{x}-1}{x} = \log 5+\log 3$ OR $\log 15$
6.	$\lim_{x\to0}\frac{\log(1+x)}{x}$	1
7.	$\lim_{x\to0}\frac{\log(1+2x)}{x}$	$\frac{\log(1+2x)}{2x}\times 2 = 1\times 2 = 2$
LDR 8.	$\lim_{x\to0}\frac{\log(1-2x)}{x}$	$\frac{\log(1+(-2x))}{-2x}\times -2 = 1\times(-2) = -2$
9.	$\lim_{x\to2}\frac{x^{5}-2^{5}}{x-2}$	$5(2)^{5-1} = 5(2)^{4} = 80$

Tricks to solve Lengthy Question

In case of: $\frac{\infty}{\infty}$

As we know it is possible to apply L-HOSPITAL Method, but if we feel it is going to be lengthy method. In (If there is **UNDER ROOT** in Numerator) in that case what to do?

Solution

1st Pref.: Calculator Trick
2nd Pref.: Take highest power common from N & D

Example: $\lim_{x\to\infty} \frac{\sqrt{3x^4+5x^2+7x+5}}{4x^2}$

$$\frac{\sqrt{x^4\left[3+\frac{5}{x^2}+\frac{7}{x^3}+\frac{5}{x^4}\right]}}{4x^2}$$

$$\frac{x^2\sqrt{\left[3+\frac{5}{x^2}+\frac{7}{x^3}+\frac{5}{x^4}\right]}}{4x^2}$$

$= \sqrt{3}/4$

Calculator Trick

$x = 100$

$$\frac{\sqrt{300040705}}{4\times100\times100} = 0.4330$$

$\frac{\sqrt{3}}{4} = 0.4330$

Calculator Tricks to solve Limits

$\lim_{x\to a}$

Put $x = a.01$

$\lim_{x\to 2} \Rightarrow 2.01$

$\lim_{x\to 0} \Rightarrow 0.01$

$\lim_{x\to\infty}$

Put $x = 100$

1^∞

Aman Sir Shortcut

Example:

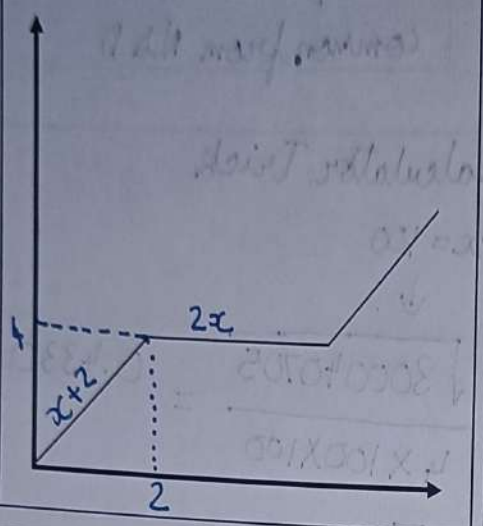
Example:

Continuity

AIM

Our AIM in this Part of Chapter is only to Check Whether the given **Graph** is **CONTINUOUS** or not at any Given Particular Point

Logic



Logic

Comment whether the given f(x) is continuous or not at x = 2?

$$f(x)=\begin{cases} x+2 & x<2 \\ 2x & x>2 \\ 4 & x=2 \end{cases}$$

$f(2) = f(2^+) = f(2^-)$

$4 = 4 = 4$

How to Check Continuity ?

$f(a) = f(a^+) = f(a^-)$

What Does this Notations Mean?

f(a) = Exact "a" par kya value hai → 2

$f(a)^+$ = a^+ means a se thodi badi value → 2.01

$f(a)^-$ = a^- means a se thodi choti value → 1.99

Example: A function f is defined as follows:

$$f(x)=\begin{cases} 2x \text{ if } x<2 \\ 2 \text{ if } x=2 \\ x^2 \text{ if } x>2 \end{cases}$$

Discuss its continuity at x = 2.

$f(2^-) = f(2) = f(2^+)$

$4 = 2 = 4$

↳ Discontinous

What if question has given modulus function?

In That Case Question will not give the Conditions and if conditions are not given, we will be Helpless to check Continuity So 1st of all we will create Conditions & Equations

$y = |x|$

$x \geq 0$: x

$x < 0$: $-x$

$f(0) = f(0^+) = f(0^-)$

$0 = 0 = 0$

$y = |2 - x|$

If $2 - x \geq 0$

$2 \geq x$

Eq. $2 - x$

$2 - x < 0$

$2 < x$

$-2 + x$

Example: The function f (x) = | x | + x is for every value of x not equal to zero is:

at $x = 0$

$x \geq 0$: $x + x = 2x$

$x < 0$: $-x + x = 0$

$f(0) = f(0^+) = f(0^-)$

$0 = 0 = 0$

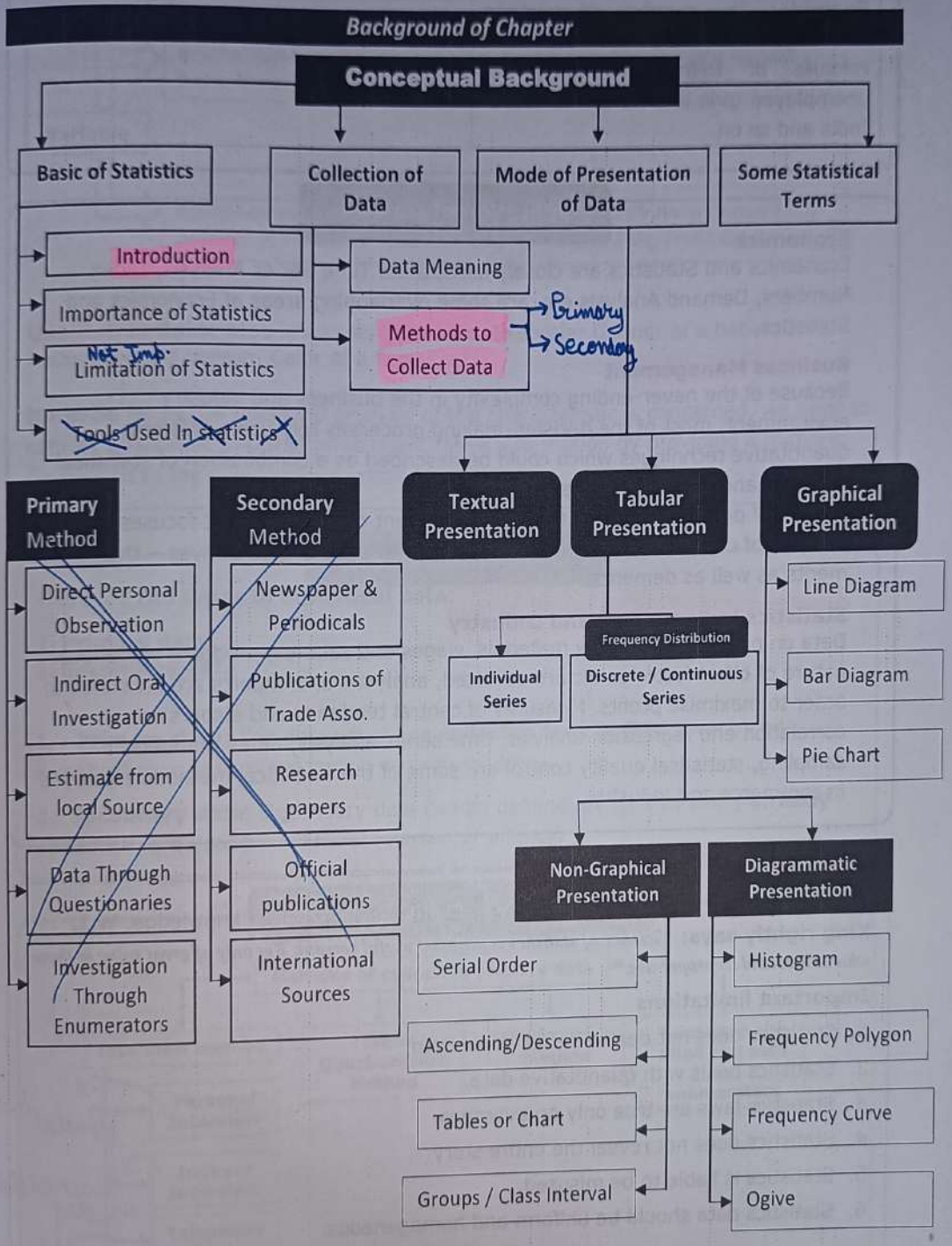
Continous

If Point of Discontinuity is asked go from option to question

Statistical Description of Data

Chapter **11**

↳ Derived from → Latin Word ⇒ Status
→ Italian Word ⇒ Statista
→ German Word ⇒ Statistik



Introduction

Plural Sense	Singular Sense
It refers to the **numerical data (qualitative as well as quantitative)** collected in a systematic manner with some definite aim or object in view. **Example**: The number of persons suffering from malaria in different colonies of Delhi or number of unemployed girls in different states of India and so on.	**Scientific method** that is employed for collecting, analyzing and presenting data, leading finally to drawing statistical inferences about some important characteristics.

Application of statistics

Economics

Economics and Statistics are closely associated. Time Series Analysis, Index Numbers, Demand Analysis etc. are some overlapping areas of Economics and Statistics.

Business Management

Because of the never-ending complexity in the business and industry environment, most of the decision-making processes rely upon different quantitative techniques which could be described as a combination of statistical methods and operations research techniques.

Statistical decision theory is another component of statistics that focuses on the analysis of complicated business strategies with a list of alternatives – their merits as well as demerits.

Statistics in Commerce and Industry

Data on previous sales, raw materials, wages and salaries, products of identical nature of other factories etc are collected, analysed and experts are consulted in order to maximise profits. Measures of central tendency and dispersion, correlation and regression analysis, time series analysis, index numbers, sampling, statistical quality control are some of the statistical methods employed in commerce and industry.

Statistics and its techniques are widely used in every branch of knowledge. **W.I. King rightly says:** *"Science of statistics is the most useful servant. But only of great value to those who understand its proper use."*

Important limitations

1. Statistics does not deal with individual item.
2. Statistics deals with quantitative data.
3. Statistics laws are true only on averages.
4. Statistics does not reveal the entire story.
5. Statistics is liable to be misused.
6. Statistics data should be uniform and homogeneous.

Methods of Collecting Data

Data: The word data means information (its literary meaning is given as facts). This can be defined as the information collected through censuses and surveys or in a routine manner or other sources is called data.

Quantitative Data/ Quantitative Characteristic: Also known as a **variable** or in other words, a **variable is a measurable quantity**.

Variable

- **Discrete Variable:** When a variable assumes a finite or a countably infinite number of isolated values. **Examples**: Number of misprints a book, Number of road accidents in a particular locality etc.
- **Continuous Variable:** When a variable assumes infinite Uncountably number of values. **Examples**: Height, Weight, Sale, Profit etc.

Qualitative Data: Also known as attributes. Example: Gender of a baby, Nationality of a person, Color of a flower etc.

Note: So far as the statistical analysis of the characteristic is concerned, we need to convert qualitative information to quantitative information by providing a numeric description to the given characteristic.

Types of statistical Data

There are two **types of statistical data**:

1. **Primary data**
2. **Secondary data**

1. **Primary data:** The data which are **collected for the first time** by an investigator or agency are known as primary data.

2. **Secondary data:** Secondary data can be defined as, data as being **already collected**, are used by a different person or agency.

Methods of Collecting Primary Data

Methods of collecting primary data

- **Interview method**
  - Personal Interview
  - Indirect Interview
  - Telephone Interview
- **Mailed Questionnaire Method**
- **Observation method**
- **Questionnaire filled and sent by enumerators**

Natural Calamity

Not able to recall person

Quickest

1. **Personal Interview:** the investigator meets the respondents directly and collects the required information then and there from them.

   Application: In case of a natural calamity like a super cyclone or an earthquake or an epidemic like plague, we may collect the necessary data much more quickly and accurately by applying this method.

2. **Indirect Interview:** Under this method, investigator collects the necessary information from the persons associated with the problems.

   Application: if there are some practical problems in reaching the respondents directly, as in case of rail accident.

Quickest

3. **Telephone Interview**: Under this method, relevant information can be gathered by the researcher himself by contacting the interviewee over the phone.

   Advantage: ~~Quick, Wide Coverage & Non-Expensive~~
   Disadvantages: ~~Non-Responses~~

4. **Mailed Questionnaire: Mailed** questionnaire method comprises of framing a well-drafted and soundly-sequenced questionnaire covering all the important aspects of the problem under consideration and sending them to the respondents with pre-paid stamp after providing all the necessary guidelines for filling up the questionnaire. To cover wide area Non-responsiveness

5. **Observation Method:** Under this method data are collected by direct observation or using instrument.
   Example: Data on the height and weight of a group of students

6. **Questionnaire filled and sent by enumerators:** Questionnaire form of data collection is used for larger enquiries from the persons who are surveyed. Enumerators collects information directly by interviewing the persons having information, Question are explained and hence data is collected

Most Imp. Method to Collect Secondary Data

Methods of Collecting Secondary Data

(a) International sources like WHO, ILO, IMF, World Bank etc.

(b) Government sources like Statistical Abstract by CSO, Indian Agricultural Statistics by the Ministry of Food and Agriculture and so on.

(c) Private and quasi-government sources like ISI, ICAR, NCERT etc.

(d) Unpublished sources of various research institutes, researchers etc.

Above list is not complete list of secondary data. It can be many → But above eg.s we have to remember for exams.

Mode of Presentation of Data

- Textual Presentation
- Tabular Presentation
- Graphical Presentation

This is the 1st Step of collecting Info.

Textual Presentation

This method comprises presenting data with the help of a paragraph or a number of paragraphs.

Example: The official report of an enquiry commission, Government Reports etc.

Example: 'In 2009, out of a total of five thousand workers of Roy Enamel Factory, four thousand and two hundred were members of a Trade Union. The number of female workers was twenty per cent of the total workers out of which thirty per cent were members of the Trade Union.

In 2010, the number of workers belonging to the trade union was increased by twenty per cent as compared to 2009 of which four thousand and two hundred were male. The number of workers not belonging to trade union was nine hundred and fifty of which four hundred and fifty were females.'

Merits

- **simplicity** even a layman can present and understand the data by this method.
- The observations with **exact magnitude** can be presented with the help of textual presentation.
- This type of presentation can be taken as the **first step** towards the other methods of presentation

Demerits

- Dull
- Monotonous
- Does not facilitate Comparison
- For manifold classification this method cannot be recommended.

Tabular Presentation

Tabulation may be defined as logical and systematic arrangement of statistical data in **rows** and **Columns.**

Objective

To **simplify** the complex data.	To **clarify** the objective of investigation.	Economize space.	To facilitate **comparison**.	To depict **trend** a pattern of data
To help reference for **future studies**.	To facilitate statistical **analysis**.	To **detect errors** and omissions		To clarify the **characteristics** of data.

Essential parts of a statistical Table

Format of a Table

Table No. XXX

Title: [Head note or prefatory Note]

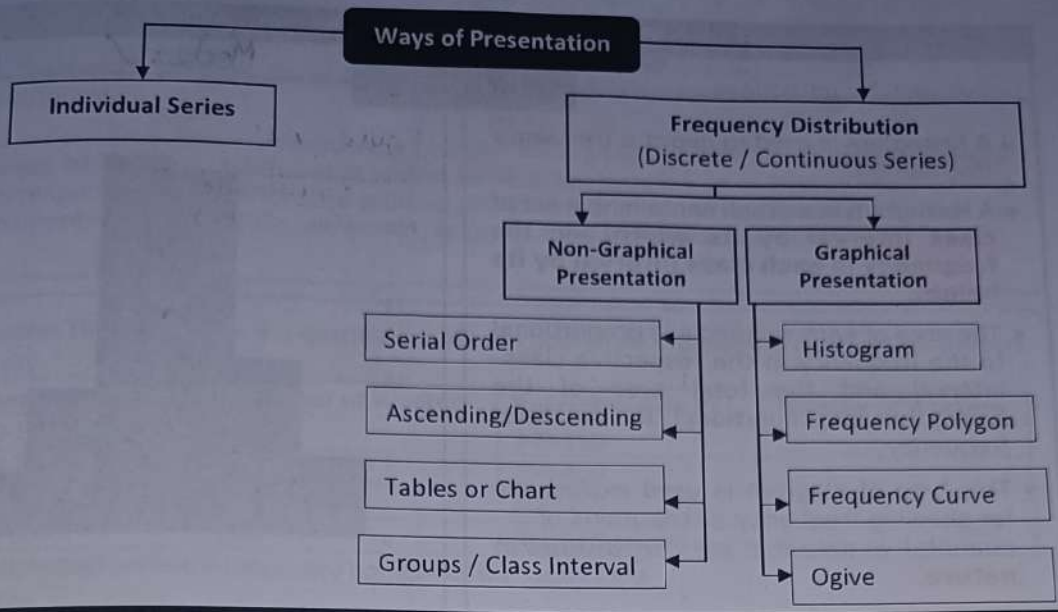
Stub Heading	Caption				Total (5)
	Sub-Heads		Sub-Heads		
	Column Head (1)	Column Head (2)	Column Head (3)	Column Head (4)	
Row 1					
Row2		Body	of	Table	
Row 3					
Total					

Foot Note

Source Note:

Number and Date: No. Dc 1172-25/2/2006

Table Number	A table should be numbered for **identification** especially when there are a large number of tables in a study. The number may be put at the **center**, **above the title** or at the **bottom of the table.**
Table Title	Every table should have a title It should be clear brief and **self-explanatory**. The title should be set in **bold** type so as to give it prominence.
Date	The date of preparation of table should always be written on the table. It enables to recollect the **chronological order** of the table prepared.
Stubs (Row Heading)	Each row of the table must have a heading. The designations of the rows are called stubs or stub items
Captions (Column Heading)	A table has many columns; sub-headings of the columns are called captions or headings.
Body	It is the most vital part of the table It contains the numerical information. It should be made as comprehensive as possible.
Unit of Measurement	The unit of measurement should always be stated along with the title if this is uniform throughout
Source Notes	A note at the bottom of the table should always be given to indicate the primary source as well as the secondary source from where the data has been taken particularly. When there is more than one source.
Foot notes & Refrennses	It is always placed at the bottom of the table. It is a statement which contains explanation of some specific items which cannot be understood by the reader from the title or captions and stubs. The unit of measurement should always be stated along with the title if this is uniform throughout



Non-Graphical Methods

Serial Order

When the raw data are arranged in systematic manner it is said to be in serial order

Ascending/Descending

A raw data can be arranged in Ascending or Descending order of Magnitude

Tables Or Chart

In order to maker the data easily understandable, we tabulate the data in the form of tables or charts. A Table has three columns.

1.Variable

Any character which can vary from one individual to another is called a Variable.
For eg. Age, Income, Height etc.
Variables with numbers as possible values are called Quantitative Variable whereas those with name of place, quality etc are called Qualitative Variable.

2.Tally Marks

It is a method of keeping count in blocks of Five.

3.Frequency

The number of times an observation occurs in the given data is called the frequency of the observation.

Groups Or Class Interval

0-10
10-20
20-30

Diagrammatic Presentation

Histogram

Mode = ✓

- A histogram is used to depict a frequency distribution.
- A Histogram is a graph containing a set of **class interval by its width** and the **frequency in each class interval by its height**.
- The area of each rectangle is proportional to the frequency in the respective class-interval and the total area of the histogram is proportional to the total frequency.
- This type of diagram is used exclusively for showing frequency distributions of quantitative data that are **continuous in nature**.

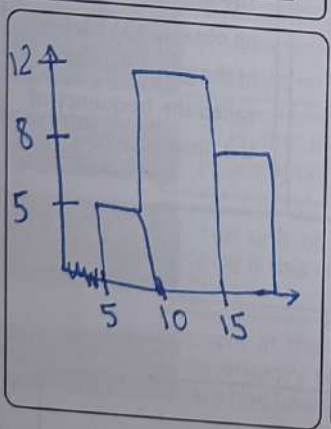


Histogram With Equal Class Interval

Yes Possible

Exclusive Series

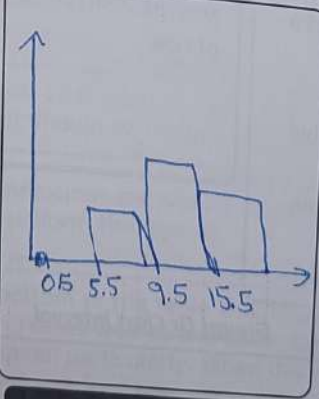
x	1-5	5-10	10-15
f	5	12	8



Histogram With Unequal Class Interval

1st of all convert into Exclusive series

x	1-5	6-9	10-15
f	5	12	8

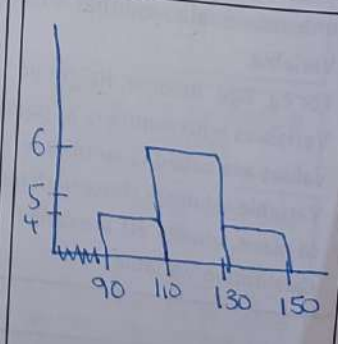


Histogram When Mid-Points are Given

Calculate $H = \frac{20}{2} = 10$

90-110 | 110-130 | 130-150

x	100	120	140
f	5	6	4



Key Points to Remember

- ✓ **Different Scales** May be taken on the 2 axes (i.e., x-axis & y-axis)
- ✓ **Kink mark ()** is used if class interval is **not** starting from **0**
- ✓ In **Histogram** both **height as well as width** of each rectangle matters Whereases in **bar** diagram **only height** of each bar matters

Limiting form of histograph

Frequency Polygon

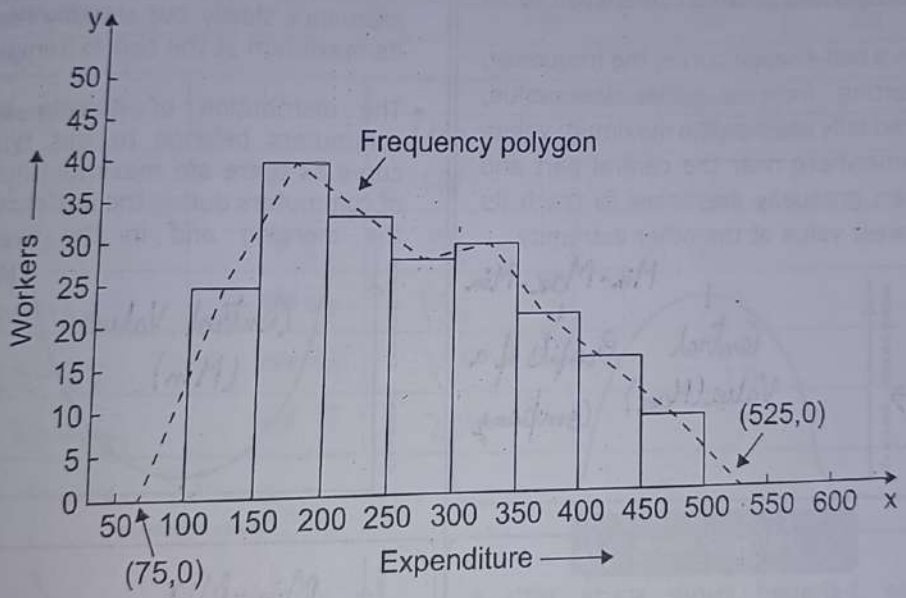
- A frequency polygon is a line graph for graphical representation of frequency distribution
- In order the to draw a frequency polygon **mid-point of the class interval (mid-point of the top horizontal sides of the rectangles in a histogram)** and its corresponding frequency The plotted points are the joined successively by the line segments and the figure, so drawn, is given the shape of a polygon

Example: The following is the distribution of total household expenditure (in ₹) of 202 workers in a city.

Expenditure (in ₹)	Number of Workers	Expenditure (in ₹)	Number of Workers
100-150	25	300-350	30
150-200	40	350-400	22
200-250	33	400-450	16
250-300	28	450-500	8

Draw a histogram and a frequency polygon of the above data

Solution



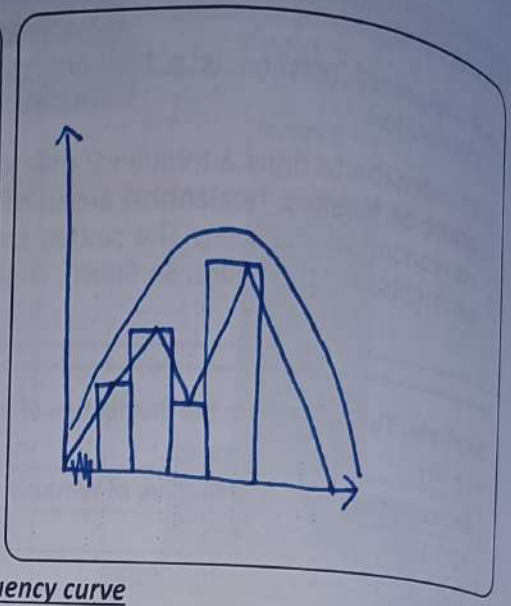
Note

To draw frequency polygon without constructing histogram compute **class mark** (Mid-Point of class interval) and then plot class mark and frequency in the graph paper.

Limiting form of frequency curve & Histogram

Frequency Curve

- A frequency curve is drawn by smoothing the frequency polygon. It is smoothed in such a way that the sharp turns are avoided
- The curve is drawn in such a way that area included under the curve is approximately the same as that of the frequency polygon
- To draw the frequency curve, firstly draw a **histogram**, then draw a **frequency polygon** and **lastly smooth** it to get frequency curve.

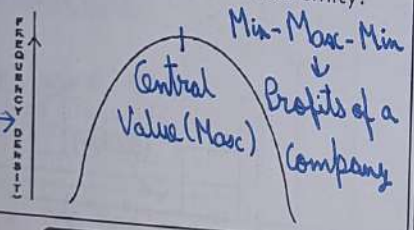


Types of frequency curve

Bell-Shaped Curve

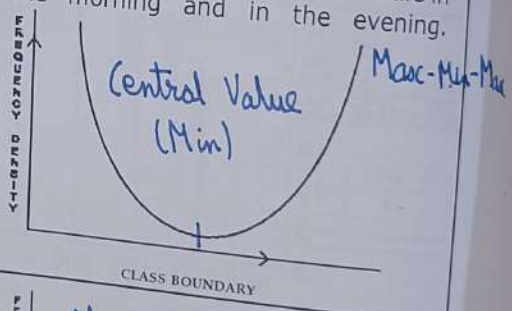
- Most of the commonly used distributions provide bell-shaped curve
- On a bell-shaped curve, the frequency, starting from a rather low value, gradually reaches the maximum value, somewhere near the central part and then gradually decreases to reach its lowest value at the other extremity.

Most commonly used curve →



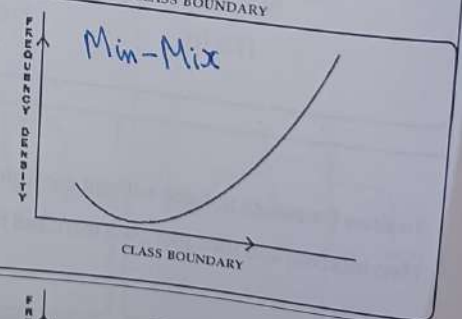
U-Shaped Curve

- For a U-shaped curve, the frequency is minimum near the central part and the frequency slowly but steadily reaches its maximum at the two extremities
- The distribution of Kolkata bound commuters belongs to this type of curve as there are maximum number of commuters during the peak hours in the morning and in the evening.



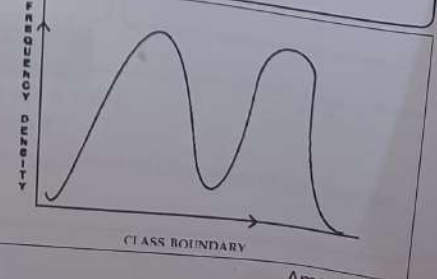
J-Shaped Curve

- The J-shaped curve starts with a minimum frequency and then gradually reaches its maximum frequency at the other extremity.
- The distribution of commuters coming to Kolkata from the early morning hour to peak morning hour follows such a distribution. Sometimes, we may also come across an inverted J-shaped



Mixed Curve

Lastly, we may have a combination of these frequency curves, known as mixed curve. These are exhibited in the following figures.



Ogive or Cumulative frequency

It is a graph which represent the data of cumulative frequency distribution. By plotting cumulative frequency against the respective class boundary, we get ogives.

Less than Ogive

If cumulative frequencies are plotted with upper limit of class intervals, then it is the less than Ogive

More than Ogive

If cumulative frequencies are plotted with lower limit of class intervals, then it is the less than Ogive

Assumption

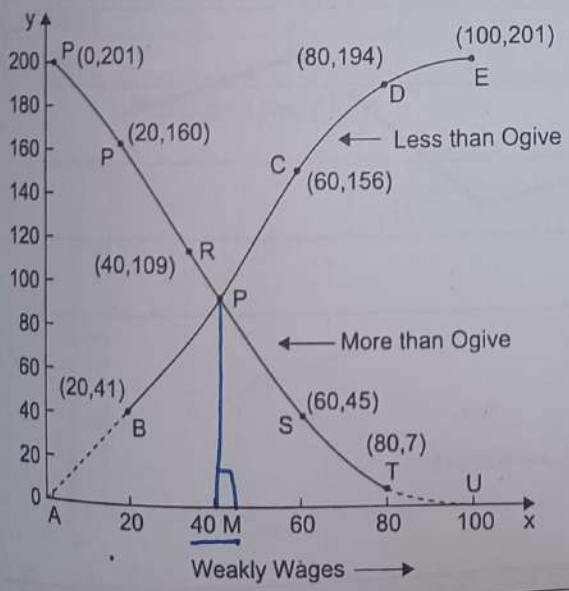
It is assumed that the class preceding the first class in the classification exists and its frequency is zero.

Example: Draw a less than Ogive and More than Ogive from the following distribution

Weekly Wages	20	40	60	80	100
No of Workers	41	92	156	194	201

Solution

Weekly Wages	F	Cumulative frequency	
		Less than	More than
0-20	41	41	201
20-40	~~92~~ 51	92	160
40-60	~~156~~ 64	156	109
60-80	~~194~~ 38	194	45
80-100	~~201~~ 7	201	7
	201		



Point to Remember

Ogives is used to find Median, quartiles, deciles and percentiles

Draw a Straight line from point of intersection of two Ogives on X-axis

That point on X-axis is Required Median OR Partition Value

Diagrammatic Presentation of Data

- Line Diagram
- Bar Diagram
- Pie Chart

Line Diagram

It is **simplest** of all diagram. Line diagram are simple mathematical graph drawn on the graph paper by plotting **one variable on x-axis** and **another on y-axis.**

This diagram felicitate **comparison** & no technical Skill is required to draw

When we observe the values of a variable at **different period of time**, the series so formed is known as **time series.** We take time on X-axis and Value of Variable on Y-axis. The **Time series** graph is also known as **Histogram**

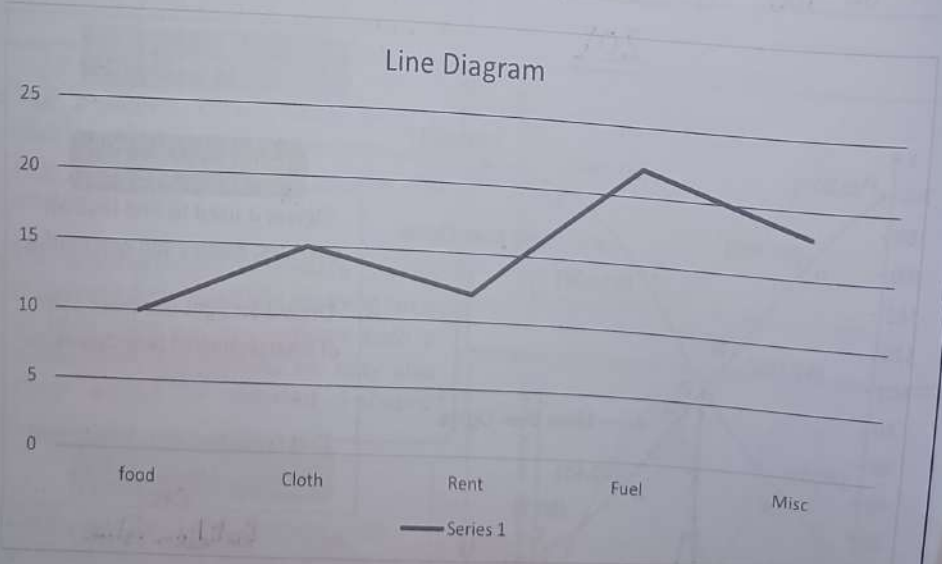
Time Series Graph Can be Constructed

- **Natural Scale** → Absolute Changes are Shown
- **Ratio Scale** → Relative Changes are Shown

Example: Draw a line diagram from the following distribution

Commodity	Food	Cloth	Rent	Fuel	Misc.
Price	10	15	12	22	18

Solution:



Bar Diagram

Bars i.e., rectangles of equal width and usually of varying lengths are drawn either horizontally or vertically.

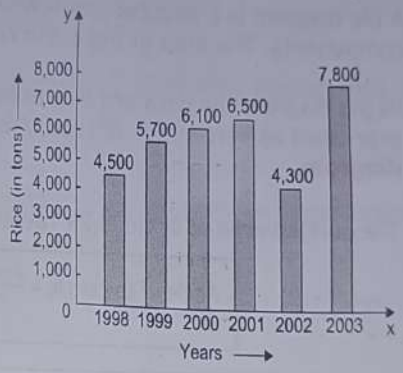
- It is especially useful in comparing qualitative data or quantitative data of discrete type.
- While horizontal bar diagram is used for qualitative data or data varying over space, the vertical bar diagram is associated with quantitative data or time series data.

Types of Bar diagram

1. Simple Bar Diagram:
2. Multiple or Grouped Bar Diagram:
3. Divide or component Bar Diagram.
4. Percentage subdivided Bar Diagram

Example: Draw a bar chart on procurement of rice in India

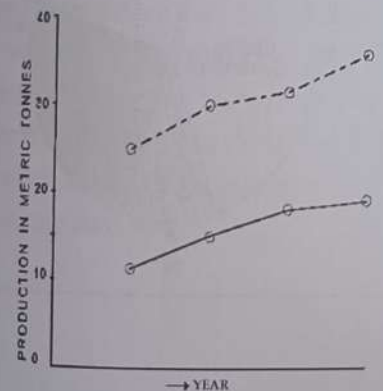
Year	1998	1999	2000	2001	2002	2003
Rice	4500	5700	6100	6500	4300	7800



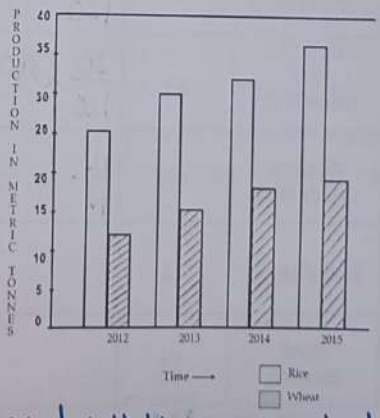
Example: The production of wheat and rice of a region are given below:

Years	2012	2013	2014	2015
Wheat	12	15	18	19
Rice	25	30	32	36

Multiple Line Chart



Multiple Bar Diagram



Note: Multiple line chart is used for 2 or more time series, variables having same units

Multiple axes line chart used for 2 or more time series variables have different units

Difference Between Histogram & Bar Chart

Histogram (2) Dimension	Bar Chart (1)
Refers to graphical representation, that displays data by way of bars to show frequency of numerical data	Refers to Pictorial representation of data that uses bars to compare different category of data
Distribution of Non-discrete Variable (Continuous Variable)	Distribution of Discrete Variable
Quantitative Data	Categorial Data
Bars Touches each other's	Bars Don't touch each other's
Bars Can't be reordered	Bars Can be reordered
Width of Bars can or cannot be same	Width of Bars is same

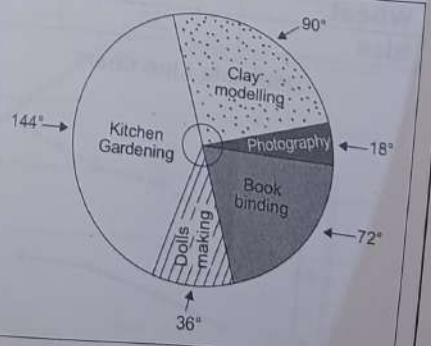
Pie Diagram (2 Dimensions)

- A pie diagram is a circular graph which represent the total value with its components. The area of the circle represents the different parts.
- In pie diagram the data are expressed as **percentage**. Each Component is expressed as percentage of total value. The pie diagram is also known as **angular diagram**
- The surface area of circle is 2π radians or 360 degrees

$$\text{Angle of the circle} = \frac{frequency\ of\ sector}{Total\ Frequency} \times 360^0$$

Example: 120 students of a college were asked to opt different work experience. The details of these options are as under. Represent in Pie diagram

Area of work	No of Students	
Photography	6	$\frac{6}{120} \times 360 = 18°$
Clay modelling	30	$\frac{30}{120} \times 360 = 90°$
Kitchen Gardening	48	$\frac{12}{12}$ 144°
Doll Making	12	
Book Binding	24	
	120	



Some Statistical Terms

1. **Raw Data** A raw data is a statistical data in original form before any statistical technique is applied to redefine, process or summarize it.

1. **Variable or Variant** Any character which can vary from one individual to another is called variable or variate. For example, age, income, height, intelligence, color, etc.,

2. **Discrete Series** When the discrete variables are arranged in the form of a series, it is called a discrete or discontinuous series.

3. **Continuous Series** When the continuous variables are arranged in the form of a series, it is called continuous series or exclusive series.

4. **Array** An array is an arrangement of data in order of magnitude either in descending or ascending order.

5. **Descending order** When data is arranged from the highest value to the lowest value, the array so formed is in descending order.

6. **Ascending Order** When the data is arranged from the lowest value to the highest value, the array so formed is in ascending order.

> **Example:** If the given data is 17, 7, 11, 5, 13, 9 then
>
> Array in Ascending order: 5, 7, 9, 11, 13, 17
>
> Array in Descending order: 17, 13, 11, 9, 7, 5

7. **Range** It is difference between the largest and the smallest number in the given data. The range of the data given on illustration 2 is $17 - 5 = 12$.

8. **Class /Class Limit** If the observations of a series are divided into groups and the groups are bounded by limits, then each group is called a class. The end values of a class are called the class limits. In other words, the two end limits of each class are called class limits. The smaller value of the two limits is called the lower limit and the higher value of the same is called the upper limit of the class. These two class limits are sometimes called the stated class limits.

9. **Class interval** The difference between the lower limit (L) and the upper limit (U) of the class is known as **Class interval (I). Thus: I = U – L.** In other words, the range of a class is called its class interval.

Example: The given data is

Marks obtained	Tally Marks	No. of Students
1 – 10	𝍸 I	6
11 – 20	III	3
21 – 30	𝍸 II	7
31 - 40	II	2
Total		**16**

In the above data the **classes are:** 1 – 10, 11 – 21, 21 – 30, 31 – 40.

10. **Actual Class Limit or Class Boundaries** In the Above example, there is a gap of 1 mark between the limits of any two adjacent classes. This gap may be filled up by extending the two limits of each class by half of the value of the gap.

Thus

Lower class boundary = Lower class limit - $\frac{1}{2}$ of the gap

Upper class boundary = Upper class limit + $\frac{1}{2}$ of the gap

The class boundary of the class 11 to 20 is

Lower class boundary = 11 - $\frac{1}{2}$ of 1 = 11 - 0.5 = 10.5

Upper class boundary = 20 + $\frac{1}{2}$ of 1 = 20 + 0.5 = 20.5

In other words, the class boundaries are the limits up to which the two limits, (actual) of each class may be extended to fill up the gap that exists between the classes. The class boundaries of each class, so obtained are called the Actual class limits or True class limits.

True lower-class limit = lower class limit - $\frac{1}{2}$ of the gap

True upper-class limit = upper class limit + $\frac{1}{2}$ of the gap

Note 1. In the case of exclusive series True class limits are the same as class limits.

Example

Class Interval	Class Boundaries
11 – 20	10.5 – 20.5
21 – 30	20.5 – 30.5
31 – 40	30.5 – 40.5

11. **Class- mark or Mid-point or Mid-value** The central value of the class interval is called the mid-point or mid-value or class mark. It is the arithmetic mean of the lower class and upper-class limit of the same class.

Mid – value of class = $\frac{\text{lower class limit+upper class limit}}{2}$

Or Class mark = $\frac{\text{True upper class limit+True lower class limit}}{2}$

The class mark of the class $11-20$ **is** $\frac{11+20}{2} = 15.5$

12. **Class Magnitude** It is the difference between the upper-class boundary and the lower-class boundary of the class.

In the above example the class magnitude of the class, $20.5 - 30.5$ is $(30.5 - 20.50) = \mathbf{10}$.

Best Method to Present Data (Most Accurate)
Most attractive (Hidden Trends)
Charts uses logarithm
To check consistency & Accuracy
Internal consistency can be checked

Tabular
Digramatic
Ratio Charts
~~Social~~ Security
From a No. of related series are given

Exclusive Classification
Excludes the upper class but includes the lower class

Mutually Inclusive ⇓ Discrete Variable

Mutually Exclusive ⇓ Continous Variable

Relation b/w Range & Class Interval L.D.R

Range = No. of class Interval × Class Length

Formula to compute No. of Class Interval L.D.R

No. of Class Interval = $1 + 3.22 \log N$ Here: N = Total frequency

Requisites of Good Classification

Unambiguous ← U S E → Exhaustive & Mutually Exclusive

U → Stable & Flexible

H R → Revealing Classification

H → Homogeneous

13. Frequency Density = $\frac{\text{Class frequency}}{\text{Class Length}}$

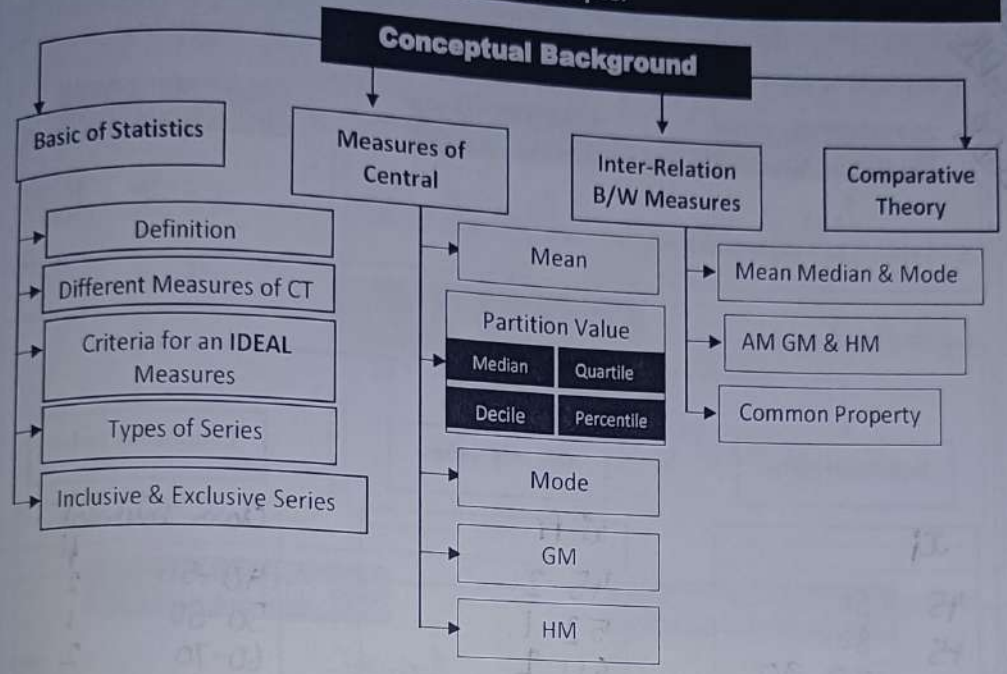
14. Relative frequency for particular class: Lies b/w 0 & 1

15 Frequency Distribution (Grouped Frequency)
- → Arrange obs. in increasing order
- → Arrange obs. in Items of No. of Group
- → Relates to measurable characteristics

Measures of Central Tendency

Chapter **12A**

Background of Chapter



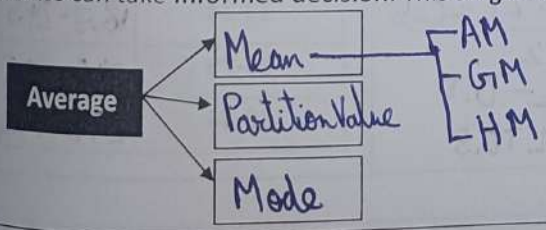
Basics of Statistics

Definition

Statistics is many concerned, with **Decision making**, but it is not possible to form decision by looking at huge amount of data, and here comes the need of Single Value
So that by looking at that single value we can form an **Informed decision**

How

We need a **Single value**, which can represent the characteristics of entire population, so that we can take **Informed decision**. This **Single value** in statistics is known as **Average**



Criteria for an IDEAL Measures

1. It Should be **Rigidity Defined.**
2. It should be **easy** to **understand** and **Compute**
3. It should be based on **all the observations**
4. It should be **least affected** by **sampling fluctuation**
5. It should be **unduly affected** by **extreme values**
6. It should be **easy** to **interpret**

Types of Data

Individual Series

x_i

45
45
52
60
60
80
80
85
85
90

Discrete Series

x_i	f_i
45	2
52	1
60	1
80	2
85	2
90	1

Continuous Series

Class Interval	f_i
40-50	2
50-60	1
60-70	2
70-80	0
80-90	4
90-100	1

Inclusive & Exclusive Series

Inclusive Series

Last Member is: Included

E.g. 2-7 → [2, 3, 4, 5, 6, 7]

Class Interval	Frequency
19-24	
25-29	
30-34	

How

$\frac{25-24}{2} = 0.5$

LL − 0.5

UL + 0.5

Exclusive Series

Last Member is: Not Included

E.g. 2-7 → [2, 3, 4, 5, 6]

Class Interval	Frequency
19-24	18.5 - 24.5
25-29	24.5 - 29.5
30-34	29.5 - 34.5

Mean

Definition

It is sum of all the observation, divided by no. of observation.
[X]

Formula

Individual Data

$$\bar{X} = \frac{\Sigma x_i}{N}$$

Discrete Data

$$\bar{X} = \frac{\Sigma f_i x_i}{N}$$

$$N = \Sigma f_i$$

Continuous Data

$$\bar{X} = \frac{\Sigma f_i x_i}{N}$$

x_i = Mid Point $\Rightarrow \frac{LL + UL}{2}$

$$N = \Sigma f_i$$

Don't mug up this for. see example below

Corrected Mean

$$\bar{X} = \frac{\left[\Sigma x_{(Incorrect)} - \text{Incorrect obs} + \text{Correct obs}\right]}{N}$$

Combined Mean

$$\bar{X}_{12} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2}$$

G_1: $\bar{X}_1$, n_1 — G_2: $\bar{X}_2$, n_2

Important Highlights

1. The algebraic sum of deviation from mean is **ZERO**
2. If all the Observations are Same "Say-a" then **AM = a**
3. COMMON PROPERTY

Aman Khedia

Examples

1. **Individual Series**

Following are the daily wages in rupees of a sample of 9 workers: 58, 62, 48, 53, 70, 52, 60, 84, 75, compute the mean wages.

$$\bar{X} = \frac{\sum x_i}{N} = \frac{562}{9} = 62.44$$

2. **Discrete Series**

Compute the mean weight of a group of BBA students of St. Xavier's College from the following data:

Weight in kgs	45	50	62	58	70	72
No. of students	3	4	5	7	9	8

x_i	f_i	$f_i x_i$	
45	3	135	
50	4	200	
62	5	310	
58	7	406	
70	9	630	
72	8	576	
		2257	

$$\bar{X} = \frac{\sum f_i x_i}{N} = \frac{2257}{36} = 62.69$$

3. Continuous Series

Find the AM for the following distribution:

Class Intervals	350-369	370-389	390-409	410-429	430-449	450-469	470-489
Frequency	23	38	58	82	65	31	11

CI	Exclusive	x_i	f_i
350-369	349.5-369.5	359.5	23
370-389	369.5-389.5	379.5	38
390-409		399.5	58
410-429		419.5	82
430-449		439.5	65
450-469		459.5	31
470-489		479.5	11
			308

$$\bar{X} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{128346}{308} = 416.70$$

4. Combined Mean – Calculations

The mean salary for a group of 40 female workers is Rs 5200 per month and that for a group of 60 male workers is Rs 6800 per month. What is the combined salary?

Female

$n_1 = 40$

$\bar{X}_1 = 5200$

Male

$n_2 = 60$

$\bar{X}_2 = 6800$

$$\bar{X}_{12} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2} = \frac{616000}{100} = 6160$$

$$\boxed{\bar{X}_{12} = 6160}$$

5. Corrected Mean

The mean of 10 Observations was found to be 28. Later, it was discovered that one observation 14 was misread as 24 find the correct mean?

$\bar{X} = 28$, $n = 10$

$\bar{X} = \frac{\Sigma x}{N}$

$28 = \frac{\Sigma x}{10}$

$\Sigma x = 280$

$\bar{X}_{(Ic)} = 280 - 24 + 14$

$\bar{X}(\text{correct}) = 270$

$\bar{X} = \frac{\Sigma x}{n} = \frac{270}{10} = 27$ Ans

Partition Value

Median

It Divides the series into

2 equal Parts

Quartile

It Divides the series into

4 Equal Parts

Decile

It Divides the series into

10 Equal Parts

Percentile

It Divides the series into

100 Equal Parts

Point to Remember While Applying Formula

In case of Individual Series

1. 1st of all arrange the data in **Ascending order**.
2. Now apply the formula, but you will get the position, So the Observation corresponding to that position will be Your Required Partition Value

In case of Discrete Series

1. Step-1 & 2 are same but, we will also Calculate **CUMMULATIVE FREQUENCY TABLE** and check the outcome of formula in CUMMULATIVE FREQUENCY TABLE, the **Observation** Corresponding to such **C.F** is Your **Answer**

Formula

Individual Series

Median	Quartile	Decile	Percentile
$M_d = \left(\frac{N+1}{2}\right)^{th}$ Obs.	$Q_K = K\left(\frac{N+1}{4}\right)^{th}$ Obs.	$D_K = K\left(\frac{N+1}{10}\right)^{th}$ Obs.	$P_K = K\left(\frac{N+1}{100}\right)^{th}$ Obs.
	$K = 1, 2, 3$	$K = 1, 2 \ldots\ldots 9$	$K = 1, 2 \ldots\ldots 99$

Discrete Series

Median	Quartile	Decile	Percentile
$M_d = \left(\frac{N+1}{2}\right)^{th}$ Obs.	$Q_K = K\left(\frac{N+1}{4}\right)^{th}$ Obs.	$D_K = K\left(\frac{N+1}{10}\right)$ Obs.	$P_K = K\left(\frac{N+1}{100}\right)^{th}$ Obs.
	$K = 1, 2, 3$	$K = 1, 2 \ldots\ldots 9$	$K = 1, 2 \ldots\ldots 99$

Continuous Series

Median	Quartile	Decile	Percentile
$L + \frac{(N/2) - CF_P}{f} \times i$	$L + \frac{\left(\frac{KN}{4}\right) - CF_P}{f} \times i$	$L + \frac{\left(\frac{KN}{10}\right) - CF_P}{f} \times i$	$L + \frac{\left(\frac{KN}{100}\right) - CF_P}{f} \times i$

In case of Continuous Series

1. If the series is not exclusive then Make it Exclusive 1st
2. Compute $\left[\frac{N}{2}, \frac{KN}{4}, \frac{KN}{10}, \frac{KN}{100}\right]$ and mark the area ⇓ Median class

What if Formula Outcome is in Decimal?

↓

Only for Individual Series

Example: 5.6th Observation

5^{th} Obs $+ 0.6\,[6^{th}$ Obs $- 5^{th}$ Obs$]$

Important Highlights

- Median $= Q_2 = D_5 = P_{50}$
- $\Sigma f_i |x_i - A|$ is lowest where A is median
- Common Property

Examples

MEDIAN

Individual Series

6. Find the Median for the following OBSERVATION 8, 7, 3, 2, 15, 17, 20?

S-1: 2, 3, 7, 8, 15, 17, 20

S-2: $M_d = \left(\frac{N+1}{2}\right)^{th}$ Obs $= 4^{th}$ Obs. → 8 Median

7. Find the Median for the following OBSERVATION 3, 9, 11, 8, 15, 7?

S1: 3, 7, 8, 9, 11, 15

S2: $M_d = \left(\frac{N+1}{2}\right)^{th}$ Obs $= 3.5^{th}$ Obs

⇓

3^{rd} Obs $+ 0.5\,[4^{th}$ Obs $- 3^{rd}$ Obs$]$

$8 + 0.5\,[9-8]$

$8 + 0.5 = 8.05$ Median

Discrete Series

8. Find the Median

Weight in kgs	3	8	7	9	2	4
No. of students	4	3	2	5	7	3

x_i	f_i	C.F	
2	7	7	
3	4	11	
4	3	14	
7	2	16	
8	3	19	
9	5	24	
	24		

$M_D = \left(\frac{N+1}{2}\right)^{th}$ Obs. Median = 4

$= \left(\frac{24+1}{2}\right)^{th}$ Obs

$= 12.5^{th}$ Obs.

9. Continuous Series

Find the Median for the following distribution: Also compute P_{82}

Class Intervals	350-369	370-389	390-409	410-429	430-449	450-469	470-489
Frequency	23	38	58	82	65	31	11

CI		Frequency	CF
350-369		23	23
370-389		38	61
390-409	L	58	119 → CF_p
410-429	409.5 - 429.5	82 → f	201
430-449	429.5 - 449.5	65	266
450-469		31	297
470-489		11	308
		308	

Median Class

$\frac{N}{2} = \frac{308}{2} = 154^{th}$ Obs.

$M_d = L + \frac{N/2 - CF_p}{f} \times i$

$= 409.5 + \frac{154 - 119}{82} \times 20$

$= 418.03$

$\frac{KN}{100} = \frac{82 \times 308}{100} = 252.56$

$M_d = L + \frac{\frac{KN}{100} - CF_p}{f} \times i$

$= 429.5 + \frac{252.56 - 201}{65} \times 20$

$= 445.36$

Quartiles, Deciles & Percentiles

Individual Series

10. Following are the wages of the laborers: 82, 56, 90, 50, 120, 75, 75, 80, 180, 65. Calculate Q_1, Q_3 D_6, P_{82}.

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ Obs.

$= \left(\frac{10+1}{4}\right)^{th}$ Obs.

$= \frac{11}{4}$ Obs.

$Q_1 = 2.75^{th}$ Obs.

$\Downarrow$

2^{nd} Obs. $+ 0.75[3-2^{nd}$ Obs.$]$

$= 56 + 0.75[65-56]$

$= 62.75 Q_1$

$D_6 = \frac{6(N+1)^{th}}{10}$ Obs.

$= 6\left(\frac{11}{10}\right)^{th}$ Obs.

$= \frac{66}{10}^{th}$ Obs.

$= 6.6^{th}$ Obs.

$\Downarrow$

6^{th} Obs. $+ 0.6[7-6^{th}$ Obs.$]$

$= 80 + 0.6[82-80]$

$= 81.2 \rightarrow D_6$

$P_{82} = 82\left(\frac{N+1}{100}\right)^{th}$ Obs.

$= 82\left(\frac{11}{100}\right)^{th}$ Obs.

$= \frac{902}{100}^{th}$ Obs.

$= 9.02^{th}$ Obs.

$\Downarrow$

9^{th} Obs. $+ .02[10-9^{th}$ Obs.$]$

$= 120 + .02[180-120]$

$= 121.2 \rightarrow P_{82}$

$Q_3 = 3\left[\frac{N+1}{4}\right]^{th}$ Obs.

$= 3\left[\frac{11}{4}^{th}\right]$ Obs.

$= 8.25^{th}$ Obs.

8.25^{th} Obs

$\Downarrow$

8^{th} Obs. $+ 0.25[9-8^{th}$ Obs.$]$

$90 + 0.25[120-90]$

$97.5 \Rightarrow Q_3$

Compute Quartile deviation & its co-efficient also? (**This Topic Will be Covered in Measure of Dispersion**)

Quartile Deviation

$\frac{Q_3 - Q_1}{2} = \frac{97.5 - 62.75}{2}$

$= 17.375$

Coefficient of QD

$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100 = \frac{34.875}{158.25} \times 100$

$= 21.68$

Discrete Series

11. Compute Q1, D6, P90?

X	3	8	7	9	4	11
F	2	3	1	4	5	6

x_1	F_i	CF
3	2	2
4	5	7
7	1	8
8	3	11
9	4	15
11	6	21
	21	

$Q_1 = K\left(\frac{N+1}{4}\right)^{th}$ Obs.

$= \left(\frac{22}{4}\right)^{th}$ Obs

$= 5.5^{th}$ Obs

$\Downarrow$

$4 = Q_1$

$D_6 = K\left(\frac{N+1}{10}\right)^{th}$ Obs.

$= 6\left(\frac{22}{10}\right)^{th}$ Obs.

$= 14.2^{th}$ Obs.

$\Downarrow$

$9 = D_6$

$P_{90} = K\left(\frac{N+1}{100}\right)^{th}$ Obs.

$= 90\left(\frac{22}{100}\right)^{th}$ Obs.

$= 19.8^{th}$ Obs

$\Downarrow$

$11 = P_{90}$

$Q_3 = K\left(\frac{N+1}{4}\right)^{th}$ Obs

$= 3\left(\frac{22}{4}\right)^{th}$ Obs $= 16.5^{th}$ Obs

$= 11$

Compute Quartile deviation & its co-efficient also? (This Topic Will be Covered in Measure of Dispersion)

Quartile Deviation

$\frac{Q_3 - Q_1}{2}$

$= \frac{11-4}{2}$

$= 3.5$

Co-efficient of QD

$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100 = \frac{7}{15} \times 100$

$= 46.\overline{66}$

Mode

Definition

Mode is the value which repeats maximum no. of time.

Formula

Individual Data

Value which repeats Max. No. of Times

Discrete Data

Value which is having highest frequency.

Continuous Data

→ L.C.R.

Modal C = Class having highest freq.

Mode $= l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times i$

- l = Lower Limit of Modal Class
- f_m = frequency of Modal Class
- f_1 = frequency preceding to modal Class
- f_2 = frequency succeeding to Modal Class
- i = Class Interval of Modal Class

Examples

Individual Series

12. Computation Mode for following data 2,4,5,5,6,7,5,8,8,9 ?

Mode = 5

13. Computation Mode for following data?

X	3	8	7	9	4	11
F	2	3	1	8	5	6

Mode = 9

14. Computation of Mode

Compute mode for the following distribution –

Class intervals	0-10	10-20	20-30	30-40	40-50
Frequency	4	7	9	2	1

CI	F	
0-10	4	
10-20	7 → f_1	
20-30	9 → f_m	
30-40	2 → f_2	
40-50	1	

Modal Class

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times i$$

$$= 20 + \frac{2}{9} \times 10$$

$$= 22.22$$

Geometric Mean

Definition

It is the n^{th} root of "n" number of observations.

Formula

Individual Data

$$GM = [x_1 \times x_2 \times \dots \times x_n]^{1/n}$$

Discrete Data

$$GM = \text{Anti log}\left[\frac{\Sigma f_i \log x_i}{N}\right]$$

Tricks

Logarithm

$\sqrt{\sqrt{a}}$ -1X3558=

13 Times

Anti-Logarithm → 10^a

Antilog (a)

$\sqrt{\sqrt{10}}$-1XAntilog+1X=

12 Times 12 Times

Important Highlights

1. If all the Observations are Same "Say-a" then GM = a
2. If 2 variables are associated like $z = xy$ then $GM. of Z = GM. of X \times GM. of Y$
3. If 2 variables are associated like $z = \frac{x}{y}$ then $GM. of Z = GM of X / GM of Y$
4. Common Property
5. Combined $GM = \text{Antilog}\left[\frac{\Sigma n_1 \log GM_1 + n_2 \log GM_2}{N_1 + N_2}\right]$ (Only for theory) No need to learn this formula
6. It is considered as best method to compute average of Percentage.

Calculator Trick to Compute any Power

$\sqrt{\sqrt{a}}$ -1 XPower+1X=

12 Times 12 Times

Examples

Individual series

15. Computation of geometric mean

(a) Find the GM of 3, 6 and 12.

$GM = (3 \times 6 \times 12)^{1/3}$

$= (6)^{3 \times 1/3}$

$GM = 6$

16. Discrete series

(b) Find the GM for the following distribution:

X:	2	4	8	16
f:	2	3	3	2

x_i	f_i	$\log x_i$	$f_i \log x_i$
2	2	$\log^2 = 0.3010$	0.602 → M+
4	3	$\log^4 = 0.6021$	1.8063 → M+
8	3	$\log^8 = 0.9032$	2.7069 → M+
16	2	$\log^{16} = 1.2044$	2.4088 → M+
	10		7.5267 → MRC

$GM = \text{Antilog}\left[\frac{\sum f_i \log x_i}{N}\right]$

$= \text{Antilog}\left[\frac{7.5267}{10}\right]$

$= \text{Antilog}\,[0.75267]$

$GM = 10^{0.75267}$

$GM = 5.6587$

Harmonic Mean

Definition

It is reciprocal of sum of all the observations divided by no. of observations.

Individual Data

$$HM = \frac{N}{\Sigma 1/x_i}$$

Discrete Data

$$HM = \frac{N}{\Sigma \frac{f_i}{x_i}}$$

Important Highlights

1. If any observation is **"0"** then HM is **NOT DEFINED**
2. If all the observations are same say- a then **HM is also a**
3. It is the **BEST METHOD**, when we need to Compute **Average** of TWO RATIO'S

$$\text{Speed} = \frac{\text{Dis (Km)}}{\text{Time (Hr)}}$$

4. The **H.M** Between two number **a and b** is → $\frac{2ab}{a+b}$
5. COMMON PROPERTY

Combined H.M

G_1 n_1 H_1 — G_2 n_2 H_2

$$H_{12} = \frac{n_1+n_2}{\frac{n_1}{H_1}+\frac{n_2}{H_2}}$$

Examples

17. Computation of Harmonic Mean

Find the HM for 4, 6 and 10.

$$HM = \frac{N}{\Sigma 1/x_i} = \frac{3}{\frac{1}{4}+\frac{1}{6}+\frac{1}{10}} = 5.80$$

18. Find the HM for the following data

X	2	4	8	16
f	2	3	3	2

$$HM = \frac{N}{\sum \frac{f_i}{x_i}} = \frac{10}{\frac{2}{2} + \frac{3}{4} + \frac{3}{8} + \frac{2}{16}} = 4.44$$

19. Compute the Average Speed for the following Data-

Speed (km/hr.)	25	40	80	60
Distance (kms)	20	30	30	20

$$HM = \frac{N}{\sum \frac{f_i}{x_i}} = \frac{100}{\frac{20}{25} + \frac{30}{40} + \frac{30}{80} + \frac{20}{60}} = 44.28 \text{ Km/hr}$$

Relationship Between Mean Median & Mode

Symmetrical Data

The data having mean; Median and mode equal is called symmetrical data

i.e., Mean = Median = Mode

Asymmetrical Data

The data having mean; median and mode not equal is called Asymmetrical data.

Positively Skewed → Skew towards Right

Mean > Median > Mode

Negatively Skewed → Skew towards Left

Mean < Median < Mode

Moderately Skewed

Mode = 3Median - 2Mean

Examples

20. For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean mark and median mark were found to be 55.60 and 52.40. what is the modal mark?

Mode = 3Median − 2Mean

= 3(52.40) − 2(55.60)

= 46

Relationship Between AM GM & HM

Symmetrical Data

In Symmetrical Data (**all the observations are Same**) AM, GM & HM are equal

i.e., AM = GM = HM

Asymmetrical Data

For the set of distinct positive observation following result holds good.

i.e., AM > GM > HM

Note: If question silent whether data is symmetrical or Asymmetrical then always use below Inequality

i.e., AM ≥ GM ≥ HM

Practical Relation Between AM, GM & HM $GM^2 = AM \times HM$

Examples

21. Find the values of geometric mean GM, if the arithmetic mean AM is 7, and Harmonic mean HM is 48/7?

$GM^2 = AM \times HM$

$= 7 \times \frac{48}{7}$

$= 48$

$= \sqrt{48}$

$\boxed{GM = 6.92}$

22. The AM and GM for two observations are 5 and 4 respectively. Find the two observations.

$AM = 5 \rightarrow \frac{a+b}{2} = 5$ — ①

$GM = 4 \rightarrow \sqrt{ab} = 4$ — ②

We will go from option to Question

Common Property

Measure of Central Tendency gets **affected** due to change of **origin** as well as **Scale**

Standard Equation

$y = a + bx$

Always Convert in Standard Equation 1st

Property

$\bar{Y}_{MOCT} = a + b\,\bar{X}_{MOCT}$

$Y_{MEAN} = a + b\bar{X}$	$Y_{MODE} = a + b\,Y_{Mode}$
$Y_{MEDIAN} = A + BX_{Median}$	$Y_{GM} = a + b\,X_{GM}$
$Y_{HM} = a + b\bar{X}_{HM}$	

23. Computation of mode, median of related series

(a) If $y = 2 + 1.50x$ and mode of x is 15, what is the mode of y?

(b) If $y = 25X - 10$, the mean of x is 20, what is the mean of y?

~~LDR~~ (c) If median of $x = 25$ and $2x + 3y - 5 = 0$ then what is the median of y?

a) $y = 2 + 1.5x$ $\quad X_{Mode} = 15$, $Y_{Mode} = ?$

$Y_{MODE} = a + bX_{MODE}$

$= 2 + 1.5(15)$

$= 24.5$

b) $y = 25X - 10$ $\quad X_{Mean} = 20$, $Y_{mean} = ?$

$Y_{MEAN} = a + bX_{MEAN}$

$= -10 + 25(20)$

$= 490$

c) $2x + 3y - 5 = 0$ $\quad X_{MEDIAN} = 25$, $X_{MEDIAN} = ?$

$3y = 5 - 2x$

$y = \frac{5 - 2x}{3}$

$y = \frac{5}{3} - \frac{2x}{3}$

$Y_{MEDIAN} = A + bX_{MEDIAN}$

$= 5/3 + (-2/3)(25)$

$= -15$

Measure of Dispersion

Chapter **12B**

Background of Chapter

Conceptual Background

- Basics
  - Defination
  - Criteria for IDEAL Measure
- Measures of Dispersion
- Inter-Relation B/W Measures
- Comparative Theory

Absolute Measure	Relative Measure
Rage	Coefficient of Range
Mean Deviation	Coefficient of Mean Deviation
Quartile Deviation	Coefficient of Quartile Deviation
Standard Deviation	Coefficient of Variance $\frac{\sigma}{\bar{x}} \times 100$

σ^2

Definition

Definition

It is the **Scatternness** of Data, the extend to which all the individual observations are deviated from the appropriate MOCT: {Mean, Patriation Value & Mode}

Criteria for an IDEAL Measures

- Properly Defined
- Easy to Compute
- Based on all the Observation
- Least affected by Sampling fluctuation

Difference Between Absolute & Relative Measure

Absolute Measure	Relative Measure
Absolute measures are dependent on the units of the variable	Relative measures of dispersion are unit free
	Relative measures are used only for the purpose of comparison between two or more series with varying size or number of items or varying central values or varying units of calculations
	Relative measures of dispersion are difficult to compute and comprehend

Range

Definition

It is the **difference** Between **Largest Value** and **Smallest Value**

Formula

Range

$$\text{Range} = L - S$$

Coefficient of Range

$$\frac{L-S}{L+S} \times 100$$

Important Highlights

Note: It is not possible to compute range in case of **Open Class Interval**

Note: *In Case of Continuous Series*: Make Series **Exclusive** 1st

$$UCI - LCI \qquad \frac{UCI-LCI}{UCI+LCI} \times 100$$

Individual Series

Example: Following are the wages of 8 workers expressed in rupees: 82, 96, 52, 75, 70, 65, 50, 70. Find the range and also its coefficient.

Range

L - S

= 96 - 50

= 46

Coefficient of Range

$\frac{L-S}{L+S} \times 100$

$\frac{46}{146} \times 100 = 31.50$

Continuous Series

Example: What is the range and its coefficient for the following distribution of weights?

49.5-54.5 ... 69.5-74.5

Weights in kgs	50-54	55-59	60-64	65-69	70-74
No. of students	12	18	23	10	3

Range

L - S

= 74.5 - 49.5

= 25

Coefficient of Range

$\frac{L-S}{L+S} \times 100$

$= \frac{25}{124} \times 100$

$= 20.16$

Property Based Question

Example: If the relationship between x and y is given by $2x + 3y = 10$ and the range of x is Rs 15, what would be the range of y?

$2x + 3y = 10$

$3y = 10 - 2x$

$y = \frac{10}{3} - \frac{2x}{3}$

↓ Origin ↓ Scale

$X_{Range} = 15$

$Y_{Range} = ?$

$Y_{Range} = |b| \times Range$

$= \frac{2}{3} \times 15$

$= 10$

Mean Deviation

Definition

Mean Deviation is **AM** of all the Observation, Deviated from an **Appropriate measure** of Central Tendency.

→ Mean
→ PV
→ Mode

Formula

Mean Deviation

Individual

$$MD_A = \frac{\Sigma|x_i - A|}{N}$$

Discrete

$$MD_A = \frac{\Sigma f_i|x_i - A|}{N}$$

Coefficient of Mean Deviation

$$\frac{MD_A}{A} \times 100$$

Important Highlights

1. Sum of all the observation from there mean is "0"
2. M.D of 1st n natural no $\frac{n^2-1}{4n}$

Individual Series

Example: What is the Mean Deviation about Mean for the numbers: 5, 8, 10, 10, 12, 9.

x_i	$x_i - A$
5	4
8	1
9	0
10	1
10	1
12	3

$\bar{x} = 9$

$$MD_A = \frac{\Sigma|x_i - A|}{N}$$

$$= \frac{10}{6} = 1.66$$

$$\text{Coefficient of MD} = \frac{MD_A}{A} \times 100$$

$$= \frac{1.66}{9} \times 100 = 18.44$$

Example: Find Mean Deviations about Median and also the corresponding coefficient for the following profits ('000 Rs) of a firm during a week: 82, 56, 75, 70, 52, 80, 68.

52, 56, 68, 70, 75, 80, 82

$A = \frac{(n+1)}{2} = \frac{8}{2} = 4^{th}$ Obs.

$A = 70$

$MD_A = \frac{\Sigma |x_i - A|}{N}$

$= \frac{61}{7}$

$= 8.71$

Coefficient of Mean Deviation

$\frac{MD_A}{A} \times 100$

$= \frac{8.71}{70} \times 100$

$= 12.44\ \%$

Property Based Question

Example: If x and y are related as $4x + 3y + 11 = 0$ and mean deviation of x is 5.40, what is the mean deviation of y?

$4x + 3y + 11 = 0$

$3y = -11 - 4x$

$y = \frac{-11}{3} - \frac{4}{3}x$

Origin — Scale

$MD_y = |b| \times MD_x$

$= \frac{4}{3} \times 5.40$

$= 7.2$

Quartile Deviation

Definition

It is **half of difference** between Upper Quartile (Q_3) and Lowe Quartile (Q_1) Based on 50% of Series, Also Known as **Semi-Inter Quartile Range**

Formula

Quartile Deviation

$$QD = \frac{Q_3 - Q_1}{2}$$

Coefficient of Quartile Deviation

$$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

Coefficient of Quartile Variation

Important Highlights

1. It is **Best MOD** for **Open Class Interval**
2. It is Based on Middle **50%** of Observation
3. Q_1 & Q_3 are **equidistance** from Median $Med - Q_1 = Q_3 - Med \rightarrow Med = \frac{Q_3 + Q_1}{2} = Q_2$

Individual Series

Example: Following are the marks of the 10 students: 56, 48, 65, 35, 42, 75, 82, 60, 55, 50. Find the Quartile Deviation and also its coefficient.

35, 42, 48, 50, 55, 56, 60, 65, 75, 82

$Q_1 = K\left(\frac{N+1}{4}\right) = \left(\frac{11}{4}\right)^{th}$ Obs. $= 46.5$

$Q_3 = K\left(\frac{N+1}{4}\right) = 3\left(\frac{11}{4}\right)^{th}$ Obs. $= 67.5$

Quartile D. $\frac{Q_3 - Q_1}{2} = 10.5$

Coefficient of QD $\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100 = \frac{21}{114} \times 100 = 18.42$

Continuous Series QD

Example: Find an appropriate measure of dispersion from the following data:

Daily wages (Rs)	Upto 20	20-40	40-60	60-80	80-100
No. of workers	5	11	14	7	3

Property Based Question

Example If the Quartile Deviation of x is 6 and $3x + 6y = 20$, what is the Quartile deviation of y?

$3x + 6y = 20$

$6y = 20 - 3x$

$y = \frac{20}{6} - \frac{3}{6}x$

Origin Scale

$Y_{QD} = |b| \times X_{QD}$

$= \frac{3}{6} \times 6$

$= 3$

Standard Deviation

Definition

It is **square root** Average of **Square of Deviation** from there **Mean** $[\sigma]$

Formula

Standard Deviation

Method-1 → Indirect Method

$$\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{N}}$$

Method-2 → Direct Method

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2}$$

Coefficient of Standard Deviation

Relative measure of SD is known as coefficient of variance.

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

Discrete Series

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$$

Continuous Series

Use same formula just calculate midpoint.

Important Highlights

1. S.D of 1st "n" natural no = $\sqrt{\frac{n^2 - 1}{12}}$
2. For any 2 numbers S.D is always **Half of Range**

   e.g. There are 2 numbers a, b then

Combined Standard Deviation

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_1^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$d_1 = \bar{X}_{12} - \bar{X}_1$

$d_2 = \bar{X}_{12} - \bar{X}_2$

Aman Khedia

Variance

Variance

$[SD]^2 = \sigma^2$

Formula

Coefficient of Variance

$CV = \frac{SD}{Mean} \times 100 = \frac{\sigma}{\bar{X}} \times 100$

Important Highlights

1. Coefficient of Variance help us to determine the "Consistency of Data"

Less C.V → More Consistent Data

Individual Series

Example: Find the standard deviation and the coefficient of variation for the following numbers: 5, 8, 9, 2, 6.

$\bar{X} = \frac{\Sigma X}{N} = 6$

x_i	x^2	$(x-\bar{x})^2$
5	25	1
8	64	4
9	81	9
2	4	16
6	36	0

$DM = \sqrt{\frac{\Sigma x^2}{N} - (\bar{X})^2} = \sqrt{\frac{210}{5} - 36}$

$= \sqrt{6} = 2.45$

$IM = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{N}} = \sqrt{\frac{30}{5}} = \sqrt{6} = 2.45$

$CV = \frac{\sigma}{\bar{X}} \times 100$

$= 40.83$

Continuous Series

Example: Find an appropriate measure of dispersion from the following data:

Daily wages (Rs)	10-20	20-30	30-40	40-50	50-60
No. of workers	5	12	15	20	4

$\bar{X} = 36.07$

x_i	$x_i - \bar{x}$	f_i	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
15	21	5	441	2205
25	11	12	121	1452
35	1	15	1	15
45	9	20	81	1620
55	19	4	361	1444
		56		6736

$\sigma = \sqrt{\frac{\sum f(x_i - \bar{x})^2}{N}} = \sqrt{\frac{6736}{56}}$

$= \sqrt{120.28} = 10.96$

$\sigma \simeq 11$

$MD_A = \frac{\sum f_i |x_i - \bar{x}|}{N}$

$= \frac{508}{56} = 9.07$

L.D.R

Correct Standard Deviation

Example: A students obtained the mean and standard deviation of 100 observations as 40 and 5.1 respectively. It was later discovered that he had wrongly copied down an observation 50 instead of 40. Calculate the correct mean and standard deviation

$n = 100$

$\bar{X}_{old} = 40$

$\sigma = 5.1$

$\bar{X}_{New} = 39.9$

$\sigma = \sqrt{\frac{\sum X^2}{n} - (\bar{X})^2}$

$5.1 = \sqrt{\frac{\sum X^2}{100} - 1600}$

$\sum X^2 = 162601$

$\sum X^2_{Correct} = 162601 - 2500 + 1600$

$= 161701$

$\sigma_{correct} = \sqrt{\frac{\sum X^2}{n} - (\bar{X})^2}$

$= \sqrt{\frac{161701}{100} - 1592.01}$

$= \sqrt{25} = 5$

Combined Standard Deviation

Example: For a group of 60 boy students, the mean and SD of statistics marks are 45 and 2 respectively. The same figures for a group of 40 Girl students are 55 and 3 respectively. What is the mean and SD of Marks if the two groups are pooled together?

$$\bar{X}_{12} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

$$= 49$$

$$d_1 = \bar{X}_{12} - \bar{X}_1 = 4$$

$$d_2 = \bar{X}_{12} - \bar{X}_2 = -6$$

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}}$$

$$= \sqrt{30} = 5.48$$

Property Based Question

Example: If AM and coefficient of variation of x are 10 and 40 respectively, what is the variance of (15-2x)?

Common Property

Measure of Dispersion Remains **Unaffected** due to change of **origin** but is **affected** due to change of **Scale**

Standard Equation

$y = a + bx$

Always Convert in Standard Equation 1st

Property

$Y_{MOD} = |b| X_{MOD}$

$Y_{RANGE} = \|b\| X_{Range}$	$Y_{QD} = \|b\| X_{QD}$
$Y_{MD} = \|b\| X_{MD}$	$Y_{SD} = \|b\| X_{SD}$

CRUX

No common property for variance

Clarification Note

It is impossible to calculate σy^2 if relation b/w x & y is given :- We will calculate σ_y from property ⇒ Then take square of it to get variance

Relationship Between MD, QD & SD

QD : MD : SD = 10 : 12 : 15

Note:

① Mean cannot be calculated ⇒ Graphically

② The sum of deviation is Minimum when taken from "Median"

$\Sigma (x_i - Md) =$ Minimum

③ The sum of square of deviation is minimum when taken from "AM"

$\Sigma (x - \bar{x})^2 \rightarrow$ Minimum